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Data-driven H_∞ predictive control for constrained systems: a Lagrange duality approach

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Abstract The current article proposes a novel data-driven H_∞ control scheme for time-domain constrained systems based on model predictive control formulation. The scheme integrates H_∞ control into a minimax model predictive control framework, enabling more effective handling of external disturbances and time-domain constraints. First, by leveraging input-output-disturbance data, the scheme ensures the H_∞ performance of the closed-loop system. Subsequently, the minimax optimization problem is converted into a more manageable minimization problem via Lagrange duality, which reduces conservatism typically associated with ellipsoidal evaluations of time-domain constraints. The study examines key closed-loop properties, including stability, disturbance attenuation, and constraint satisfaction, achieved by the proposed data-driven moving horizon predictive control algorithm. The effectiveness and advantages of the proposed method are demonstrated through numerical simulations of a batch reactor system, confirming its robustness and feasibility under noisy conditions.

Keywords data-driven control, linear matrix inequality, H_∞ performance, model predictive control, dissipativity

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1 Introduction

With the development of data science and artificial intelligence, the analysis and control of systems based on data-driven control frameworks have become increasingly popular in recent years. This trend is attributable to the abundance of easily collected system data, which enables bypassing the challenges associated with creating highly accurate models. Traditional control methods rely on predefined system models and rule-based algorithms, which may lead to poor control performance due to the inherent variability in certain complex scenarios. In contrast, data-driven control is derived directly from the data generated by the system [1–6]. Control strategies based on data-driven control are widely applied in many fields, including power systems, biomedical engineering, and robotic control [7–10]. Extensive research has been conducted on the analysis and control of systems using data-driven control frameworks, including robust control [11], nonlinear control [12], optimal control [13], and model predictive control (MPC) [14].

Owing to its robust ability to handle multivariable systems and constraints, MPC has emerged as a leading control strategy in the realm of advanced process control. MPC is an optimization-based control approach that utilizes a dynamic model of the system to predict and optimize future behavior over a finite time horizon [15]. Hence, the operation of MPC typically depends on a system model, which can be derived either from first principles or system identification using available data [16]. Linear time-invariant (LTI) models play a vital role in this context, serving as the cornerstone of numerous industrial MPC applications. Their prevalence stems from their mathematical tractability, well-established stability theory, and the ability to effectively linearize complex processes around operating points [17–20]. Recently, data-driven approaches for MPC design have gained increasing attention. These methods utilize available data to solve an optimization problem that determines the optimal control actions [21–28]. Typical examples of these methods include learning-based or adaptive MPC [29–31], Koopman operator-based MPC [32,33], behavioral systems theory-inspired MPC [34], methods handling noisy data [35], and stochastic data-driven MPC [36].

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However, existing data-driven MPC approaches still face several limitations. In learning-based MPC [28, 29], updating complex models often requires significant computational resources, which may hinder real-time implementation. Moreover, learning models—particularly neural networks—lack rigorous theoretical guarantees of stability, making the closed-loop system prone to instability or uncontrollable behavior. For Koopman operator-based MPC [32, 33], achieving sufficient approximation accuracy typically requires many basis functions, which substantially increases the dimension of the lifted state and the size of the MPC optimization problem, thereby reducing real-time implementation. Because Koopman models are typically obtained through data-driven regression, measurement noise or disturbances in the training data can introduce large modeling errors in the lifted linear system, thereby weakening robustness. Compared with traditional MPC, Koopman-based methods lack rigorous stability and robustness guarantees under model uncertainties and external disturbances. These drawbacks highlight an open research gap and motivate the development of new data-driven MPC methodologies.

Moreover, real-world systems often encounter uncertainties, disturbances, and model inaccuracies that can compromise the performance of standard MPC. To address these challenges, robust MPC (RMPC) has been developed. RMPC extends the traditional MPC framework by incorporating robustness into the optimization process, ensuring reliable performance even under uncertain conditions. By considering worst-case scenarios and employing robust optimization techniques, RMPC maintains system stability and performance despite external disturbances and parameter variations [37, 38]. To address situations where the system model cannot be accurately obtained, RMPC based on data-driven control offers a feasible approach. Currently, research on data-driven RMPC (DDRMPC) has been conducted from various perspectives, including stability-guaranteed frameworks [3], Koopman-based formulations [39], and other recent variants [26, 40, 41]. For example, in [42], an integrated MPC and H_∞ control scheme was designed to achieve online disturbance attenuation. In [43], an event-triggered DDRMPC method ensuring recursive feasibility was developed. In [44], a robust iterative learning predictive control approach for nonlinear systems with actuator constraints was proposed.

In this context, this study not only addresses stability but also leverages H_∞ control and MPC to tackle disturbance suppression and constraint satisfaction in data-driven control. Regarding data-driven H_∞ control, Ref. [4] introduced a method based on noise-corrupted data, effectively addressing the common issues of high dimensionality and robustness degradation caused by noise in data-driven control. However, the approach in [4] does not consider time-domain constraints nor achieve dynamic optimization performance. These limitations are addressed in [42], where a data-driven H_∞ MPC method is proposed. Nevertheless, when the system encounters more disturbances, such as output disturbances or a non-identity system disturbance matrix, the control method in [42] fails to perform effectively. To address this gap, this article investigates an H_∞ predictive control scheme based on a data-driven framework, providing a more comprehensive solution to disturbance suppression and constraint handling in data-driven control. The scheme employs a Lagrange duality approach to convert a minimax optimization into a minimization problem, which not only alleviates the challenges induced by infeasibility and time-domain constraints but also reduces the computational burden. Utilizing input-state-disturbance data, the proposed moving horizon predictive control algorithm ensures closed-loop guarantees, including time-domain constraint satisfaction, stability, and H_∞ control performance. The detailed contributions are summarized as follows.

(1) In this study, a novel H_∞ predictive control scheme for constrained systems based on input-state-disturbance data is proposed. Specifically, the study addresses more general system scenarios where existing methods [42] become inapplicable. Using the matrix Finsler's lemma [45] combined with Schur complement techniques and nontrivial algebraic transformations, a less conservative stability criterion and an H_∞ disturbance attenuation control scheme for output-constrained unknown systems are developed.

(2) The proposed scheme employs a Lagrange duality approach within a data-driven control framework to transform a minimax optimization problem into a more tractable minimization problem. This transformation not only reduces the conservatism introduced by ellipsoidal constraint analysis but also facilitates feasibility analysis, making the overall control design more practical and efficient.

(3) The study presents a moving horizon predictive control algorithm by introducing a dissipativity inequality constraint into the data-driven H_∞ minimization problem, ensuring sustained disturbance suppression over time. The closed-loop properties are then analyzed, including the recursive feasibility of the optimal control problem, stability of the closed-loop system, adaptive disturbance attenuation performance of H_∞ control, and satisfaction of output constraints.

The rest of the paper is organized as follows. Section 2 outlines the issue addressed in this paper, including a key assumption and a supporting lemma. Section 3 first introduces our approach for synthesizing an H_∞ controller of an unknown system utilizing offline data. Second, the relaxation of the minimax problem is addressed using a Lagrange duality method, and a tractable minimization problem based on a data-driven control framework is subsequently proposed. In the third part of Section 3, a data-driven moving horizon optimal control algorithm

is presented, and the closed-loop properties of the control algorithm are proposed. Section 4 demonstrates the effectiveness and superiority of the control algorithm through a practical example. Finally, Section 5 concludes the paper.

2 Notation

The symbol $\mathbb{R}^p$ represents the real number space of dimension p, where the superscript p denotes its dimension. For example, $\mathbb{R}^q$ and $\mathbb{R}^{p\times q}$ represent q-dimensional real vectors and $p\times q$ real matrices, respectively. The notation $||f(x)||$ denotes the pointwise Euclidean norm of the function f, I denotes the identity matrix of appropriate dimensions, and 0 denotes a zero matrix of appropriate dimensions. For a matrix N, N^{-1}, N^{+}, and $N^{\top}$ indicate its inverse, pseudo-inverse and transpose, respectively. Additionally, $N>0$ ($N\geqslant 0$) implies that N is positive (semi-)definite. The symbol $*$ denotes the symmetric term corresponding to a specific term in a symmetric matrix.

3 Problem formulation

Consider a discrete-time LTI system as follows:

$$x(t+1)=A_\nu x(t)+B_\nu u(t)+E_\nu w(t), \tag{1a}$$
$$y_1(t)=C_1x(t)+D_1u(t)+E_1w(t), \tag{1b}$$
$$y_2(t)=C_2x(t)+D_2u(t), \tag{1c}$$

where $x\in\mathbb{R}^p$ is the state, $u\in\mathbb{R}^q$ represents the input, and $w\in\mathbb{R}^l$ denotes an external disturbance. The matrices $A_\nu\in\mathbb{R}^{p\times p}$, $B_\nu\in\mathbb{R}^{p\times q}$, and $E_\nu\in\mathbb{R}^{p\times l}$ denote the state, input and disturbance matrices, respectively, which are unknown; C_1, C_2, D_1, D_2, and E_1 are assumed to be known matrices. The vector $y_1\in\mathbb{R}^{q_1}$ is the performance output vector, and $y_2\in\mathbb{R}^{q_2}$ represents the output vector constrained by time-domain constraints as follows:

$$|y_{2m}(t)|\leqslant y_{2m,\ \max},\quad \forall t\geqslant 0,\quad m=1,\,2,\,\ldots,\,q_2. \tag{2}$$

The basic assumptions for the system are that (A_ν,B_ν) is stabilizable and (C_1,A_ν) is observable.

In the article, the fundamental goal is to design a controller that ensures closed-loop internal stability for the LTI system (1a) using a certain number of sampling sequences of states and inputs of the system. To achieve this goal, we assume that

$$\begin{aligned}
\hat{X}&:=[x(0)\quad x(1)\quad\cdots\quad x(T)],\\
\hat{U}&:=[u(0)\quad u(1)\quad\cdots\quad u(T-1)],\\
\hat{W}&:=[w(0)\quad w(1)\quad\cdots\quad w(T-1)],\\
\hat{X}_-&:=[x(0)\quad x(1)\quad\cdots\quad x(T-1)],\\
\hat{X}_+&:=[x(1)\quad x(2)\quad\cdots\quad x(T)],
\end{aligned} \tag{3}$$

where $\hat{U}$, $\hat{W}$, and $\hat{X}_-$ denote the sampled data sequences of previous inputs, disturbances, and states over the time interval $t\in[0,\,T-1]$; $\hat{X}_+$ stands for the sampled data sequence of next states over the time interval $t\in[1,\,T]$. Hence, the following equation can be derived:

$$\hat{X}_+=A_\nu\hat{X}_-+B_\nu\hat{U}+E_\nu\hat{W}. \tag{4}$$

Note that the matrices A_ν, B_ν are unknown, while $\hat{X}$, $\hat{U}$, and $\hat{W}$ are sampled. The following assumption on the disturbance $\hat{W}$ is introduced.

Assumption 1. In this article, the disturbance samples $w(0)$, $w(1)$, $\ldots$, $w(T-1)$, which are collected in the matrix $\hat{W}$, are assumed to satisfy

$$\sum_{t=0}^{T-1}||w(t)||^2\leqslant\alpha, \tag{5}$$

where $||w(t)||^2\leqslant\alpha_t$ for a given scalar $\alpha_t\geqslant 0$. This implies that the disturbances are energy-bounded (with an overall bound $\alpha\geqslant 0$). As $T\to\infty$, we obtain $w\in L_2[0,\ \infty)$.

For a more general situation, we introduce a set Π to denote all systems $(A,\ B,\ E)$ compatible with the measurement sequence $\hat{X}_-$, $\hat{X}_+$, $\hat{U}$ and $\hat{W}$; that is to say, we obtain a general system

$$\hat{X}_+ = A\hat{X}_- + B\hat{U} + E\hat{W}, \tag{6}$$

where $\hat{W}$ satisfies Assumption 1. The set Π is formulated as

$$\Pi = \{(A,\ B,\ E)|\ (6)\ \text{holds}\}. \tag{7}$$

For the set Π, if we can design a controller to stabilize all systems within it, then the true system $(A_\nu,\ B_\nu,\ E_\nu)$ can also be stabilized.

Based on (4) and (5), we can not only achieve the fundamental goal below (2) but also extend the result to disturbance attenuation controller design with the help of the following lemmas.

Lemma 1 (see [45]). Consider symmetric matrices $G, H \in \mathbb{R}^{(p+k)\times(p+k)}$ partitioned as follows:

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{12}^\top & G_{22} \end{bmatrix}, \quad H = \begin{bmatrix} H_{11} & H_{12} \\ H_{12}^\top & H_{22} \end{bmatrix},$$

where $G_{11},\ H_{11} \in \mathbb{R}^{p\times p}$. Assume that

(1) $G_{12} = 0$, $G_{22} \leqslant 0$;

(2) $H_{22} \leqslant 0$ and $H_{11} - H_{12}H_{22}^{+}H_{12}^\top = 0$;

(3) $\exists F$ such that $G_{11} + F^\top G_{22} F > 0$ and $H_{22}F = H_{12}^\top$.

Then,

$$\begin{bmatrix} I \\ F \end{bmatrix}^\top G \begin{bmatrix} I \\ F \end{bmatrix} \geqslant 0, \quad \forall F \in \mathbb{R}^{p\times k},$$

such that

$$\begin{bmatrix} I \\ F \end{bmatrix}^\top H \begin{bmatrix} I \\ F \end{bmatrix} = 0,$$

if and only if there exists $\lambda \in \mathbb{R}$ such that $G - \lambda H \geqslant 0$.

Lemma 2 (see [46]). Consider a symmetric matrix M partitioned as

$$M = \begin{bmatrix} A & B \\ B^\top & C \end{bmatrix},$$

where A and C are square matrices. If A is invertible, then the following statements are equivalent:

(1) M is positive definite, i.e., $M > 0$;

(2) $A > 0$ and the Schur complement of A in M, given by $S = C - B^\top A^{-1}B$, satisfies $S > 0$;

(3) $C > 0$ and the Schur complement of C in M, given by $T = A - BC^{-1}B^\top$, satisfies $T > 0$.

Similarly, for negative definiteness ($M < 0$), the conditions hold with the inequalities reversed.

Remark 1. In data-driven control systems, traditional model-based controllability and observability analysis, which relies on explicit system matrices, is replaced by methods that directly utilize input-output data. Instead of constructing controllability and observability matrices using system dynamics, information can be extracted from collected trajectories to infer these properties.

The purpose of introducing the above lemmas is to link the Lyapunov stability condition presented in the next section with measurement data in (3), thereby enabling the stability analysis of the system (1) and its extension to controller design. In the following, we focus on the optimization problem with the performance output $y_1(t)$, and design an optimal controller that ensures closed-loop stability and dissipation by combining the results discussed above.

4 Main results

In this section, we first address the stabilization and H_∞ control of unknown systems based on measurement data. We then derive the Lagrange dual formulation of the minimax optimization problem with respect to performance output. Subsequently, we formulate the optimization problem with performance and output constraints. Finally, we present a moving horizon predictive control algorithm and analyze the robust performance of the controlled system.

4.1 Data-driven stabilization and H_∞ control

Through the above discussion, it becomes clear that the primary task is to find the appropriate controller gain K that can stabilize any $(A, B, E) \in \Pi$. To ensure stabilization and disturbance attenuation, the H_∞ performance level γ from the external input $w(t)$ to the control output $y_1(t)$ should be minimized. According to the Lyapunov stability theorem, define the Lyapunov function $V(t) = x^\top(t)Px(t)$. Then, the inequality $x^\top(t)Px(t) - (Ax(t) + Bu(t))^\top P(Ax(t) + Bu(t)) > 0$ implies that the stabilization of a closed-loop system can be achieved if there exists a matrix $P = P^\top > 0$ and a feedback gain K such that

$$P - A_U^\top P A_U > 0, \tag{8}$$

where $A_U = A + BK$. According to Proposition 3.12 in [46], for zero initial conditions, the system (1) has H_∞ performance level γ from $w(t)$ to $y_1(t)$ if and only if $\sum_{t=0}^{\infty} \gamma^2 w^\top(t)w(t) \geqslant \sum_{t=0}^{\infty} y_1^\top(t)y_1(t)$, which can be presented in linear matrix inequality (LMI) form as follows:

$$\begin{bmatrix} P - A_U^\top P A_U - C_U^\top C_U & -A_U^\top PE - C_U^\top E_1 \\ -E^\top P A_U - E_1^\top C_U & -E^\top PE - E_1^\top E_1 + \gamma^2 I \end{bmatrix} > 0, \tag{9}$$

where $C_U = C_1 + D_1 K$. From (9), it follows that

$$\begin{bmatrix} P - C_U^\top C_U & -C_U^\top E_1 \\ -E_1^\top C_U & \gamma^2 I - E_1^\top E_1 \end{bmatrix} - \begin{bmatrix} A_U^\top P A_U & A_U^\top PE \\ E^\top P A_U & E^\top PE \end{bmatrix} > 0. \tag{10}$$

Furthermore, Eq. (10) is equivalent to

$$\begin{bmatrix} P - C_U^\top C_U & -C_U^\top E_1 \\ -E_1^\top C_U & \gamma^2 I - E_1^\top E_1 \end{bmatrix} - \begin{bmatrix} A_U^\top \\ E^\top \end{bmatrix} P \begin{bmatrix} A_U^\top \\ E^\top \end{bmatrix}^\top > 0. \tag{11}$$

Then, using the Schur complement lemma twice and algebraic computation, Eq. (11) is equivalent to the following LMI:

$$\begin{bmatrix} I \\ A^\top \\ B^\top \\ E^\top \end{bmatrix}^\top \underbrace{\begin{bmatrix} P^{-1} & 0 \\ 0 & -G_* \end{bmatrix}}_{:=G} \begin{bmatrix} I \\ A^\top \\ B^\top \\ E^\top \end{bmatrix} > 0, \tag{12}$$

where

$$G_* = \begin{bmatrix} I & 0 \\ K & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} P - C_U^\top C_U & -C_U^\top E_1 \\ -E_1^\top C_U & \gamma^2 I - E_1^\top E_1 \end{bmatrix}^{-1} \begin{bmatrix} I & 0 \\ K & 0 \\ 0 & I \end{bmatrix}^\top,$$

which is the same condition as (9). Then, we can summarize the conclusion.

Lemma 3. Suppose that there exist scalars λ and β, matrices $Q = Q^\top > 0$ and L, such that Eq. (13) holds. Then, the data $(\hat{U}, \hat{X}, \hat{W})$ can be utilized for H_∞ control of a closed-loop system with a performance index γ.

$$\begin{bmatrix} Q - \beta I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & Q & 0 & 0 \\ * & * & 0 & 0 & L & 0 & 0 \\ * & * & * & 0 & 0 & I & 0 \\ * & * & * & * & Q & -C_L^\top E_1 & C_L^\top \\ * & * & * & * & * & \gamma^2 I - E_1^\top E_1 & 0 \\ * & * & * & * & * & * & I \end{bmatrix} + \lambda \begin{bmatrix} \hat{X}_+ \\ -\hat{X}_- \\ -\hat{U} \\ -\hat{W} \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \hat{X}_+ \\ -\hat{X}_- \\ -\hat{U} \\ -\hat{W} \\ 0 \\ 0 \\ 0 \end{bmatrix}^\top \geqslant 0, \tag{13a}$$

$$\begin{bmatrix} Q & -C_L^\top E_1 & C_L^\top \\ * & \gamma^2 I - E_1^\top E_1 & 0 \\ * & * & I \end{bmatrix} > 0, \tag{13b}$$

where $C_L = C_1 Q + BL$.

Proof. We first prove the conditions that satisfy the demands of Lemma 1. Set

$$G = \left[\begin{array}{c:c} G_{11} & G_{12} \\ \hdashline G_{12}^\top & G_{22} \end{array}\right] := \left[\begin{array}{c:c} P^{-1} & 0 \\ \hdashline 0 & -G_* \end{array}\right],$$
$$H = \left[\begin{array}{c:c} H_{11} & H_{12} \\ \hdashline H_{12}^\top & H_{22} \end{array}\right] := \xi \begin{bmatrix} 0 & 0 \\ 0 & -I \end{bmatrix} \xi^\top,$$

where

$$\xi = \left[\begin{array}{c:ccc} I & 0 & 0 & 0 \\ \hdashline \hat{X}_+^\top & -\hat{X}_-^\top & -\hat{U}^\top & -\hat{W}^\top \end{array}\right]^\top.$$

From (11) and (12), it is clear that $-G_* \leqslant 0$, then the condition $G_{22} \leqslant 0$ of Lemma 1 can be verified. According to the construction of H, the second condition of Lemma 1 is satisfied. As for the third condition, it can be achieved using (12) and the construction of H. Define

$$F = \begin{bmatrix} A_\nu^\top \\ B_\nu^\top \\ E_\nu^\top \end{bmatrix}.$$

By (12), $G_{11} + F^\top G_{22} F$ is equivalent to

$$P^{-1} - \begin{bmatrix} A_\nu^\top \\ B_\nu^\top \\ E_\nu^\top \end{bmatrix}^\top G_* \begin{bmatrix} A_\nu^\top \\ B_\nu^\top \\ E_\nu^\top \end{bmatrix} > 0.$$

Then,

$$H_{22} F = \begin{bmatrix} -\hat{X}_- \\ -\hat{U}_- \\ -\hat{W}_- \end{bmatrix} \begin{bmatrix} -\hat{X}_- \\ -\hat{U}_- \\ -\hat{W}_- \end{bmatrix}^\top \begin{bmatrix} A_\nu^\top \\ B_\nu^\top \\ E_\nu^\top \end{bmatrix}, \; H_{12} = \begin{bmatrix} -\hat{X}_- \\ -\hat{U}_- \\ -\hat{W}_- \end{bmatrix} - \hat{X}_+^\top,$$

where

$$-\hat{X}_+^\top = \begin{bmatrix} -\hat{X}_- \\ -\hat{U}_- \\ -\hat{W}_- \end{bmatrix}^\top \begin{bmatrix} A_\nu^\top \\ B_\nu^\top \\ E_\nu^\top \end{bmatrix}.$$

We can verify $H_{22} F = H_{12}^\top$.

Hence, by Lemma 1, it follows that

$$G - \lambda H \geqslant \begin{bmatrix} \beta I & 0 \\ 0 & 0 \end{bmatrix} \tag{14}$$

for some $\lambda \in \mathbb{R}$ and $\beta \in \mathbb{R}$. By the Schur complement lemma, Eq. (14) is equivalent to

$$G' - \lambda H' \geqslant 0, \tag{15}$$

where

$$G' = \begin{bmatrix} P^{-1}-\beta I & & 0 & \\ & 0 & \begin{bmatrix} I & 0 \\ K & 0 \\ 0 & I \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} I & K^\top & 0 \\ 0 & 0 & I \end{bmatrix} & \tilde{G} & \\ & 0 & & \end{bmatrix},$$

$$H' = \xi' \begin{bmatrix} 0 & 0 \\ 0 & -I \end{bmatrix} \xi'^\top,$$

$$\tilde{G} = \begin{bmatrix} P & -C_U^\top E_1 & C_U^\top \\ -E_1^\top C_U & \gamma^2 I - E_1^\top E_1 & 0 \\ C_U & 0 & I \end{bmatrix},$$

$$\xi' = \left[\begin{array}{c|cccccccc} I & 0 & 0 & 0 & 0 & 0 & 0 \\ \hat{X}_+^\top & -\hat{X}_-^\top & -\hat{U}^\top & -\hat{W}^\top & 0 & 0 & 0 \end{array}\right]^\top.$$

By setting $P^{-1} = Q$ and $K = LQ^{-1}$ and multiplying (15) from the left and right sides by a diagonal matrix $\mathrm{diag}\{I, I, I, I, Q, I, I\}$, the matrix G' can be transformed as follows:

$$\begin{bmatrix} Q-\beta I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & Q & 0 & 0 \\ * & * & 0 & 0 & L & 0 & 0 \\ * & * & * & 0 & 0 & I & 0 \\ * & * & * & * & Q & -C_L^\top E_1 & C_L^\top \\ * & * & * & * & * & \gamma^2 I - E_1^\top E_1 & 0 \\ * & * & * & * & * & * & I \end{bmatrix}.$$

This verifies that Eq. (13) implies (9). Consequently, the closed-loop system is internally stable and has an H_∞ performance level γ from $w(t)$ to $y_1(t)$.

In the next subsection, we present the Lagrange dual formulation of the minimax optimization and incorporate the above conclusion into the formulation.

4.2 Lagrange dual formulation of minimax optimization

The foundation of MPC involves the real-time solution of a constrained optimization problem, which is updated at each sampling interval based on the current state. Before the next sampling instant, the resulting control input is applied to the actual system. In the context of robust MPC, using the current state $x(t)$ within a moving horizon framework, our objective is to solve the minimax optimal control problem of the system (1),

$$\min_{u\in\mathbb{U}} \max_{w\in\mathbb{W}} \sum_{i=t}^{\infty} \|y_1(i)\|^2, \tag{16}$$

where $\mathbb{U}$ and $\mathbb{W}$ denote the set of all considered controls and permissible disturbances, respectively. In the subsequent analysis, a Lagrange duality is employed to obtain an approximation of the minimax problem (16) that is easier to solve.

According to Assumption 1, the allowable disturbances in system (1) can be presented as

$$\mathbb{W} = \left\{ w \in \mathbb{R}^{p_1\times[0,\infty)} \,\middle|\, \sum_{i=0}^{\infty} \|w(i)\|^2 \leqslant \alpha \right\}. \tag{17}$$

By (17), we can construct a Lagrangian by combining the original objective function with constraints for any $u \in \mathbb{U}$ and $w \in \mathbb{W}$ as follows:

$$\begin{aligned}\sum_{i=t}^{\infty} \|y_1(i)\|^2 &\leqslant \sum_{i=t}^{\infty} (\|y_1(i)\|^2 - \gamma^2 \|w(i)\|^2) + \gamma^2 \alpha \\ &\leqslant \max_{w \in L_2} \left(\sum_{i=t}^{\infty} \|y_1(i)\|^2 - \gamma^2 \|w(i)\|^2 + \gamma^2 \alpha \right) \end{aligned} \tag{18}$$

for a constant $\gamma > 0$. Then, we define a function

$$V(x) = \max_{w \in L_2} \left(\sum_{i=0}^{\infty} (\|y_1(i)\|^2 - \gamma^2 \|w(i)\|^2) \right) \tag{19}$$

for the system (1), where $x = x(0)$. Using dynamic programming, it can be derived that

$$V(x(t)) = \max_{w(t) \in \mathbb{R}^{p_1}} \left(\|y_1(t)\|^2 - \gamma^2 \|w(t)\|^2 + V(x(t+1)) \right). \tag{20}$$

Moreover, we can conclude that system (1) satisfies the dissipativity inequality

$$V(x(t+1)) - V(x(t)) \leqslant \gamma^2 \|w(t)\|^2 - \|y_1(t)\|^2, \tag{21}$$

for $w(t) \in L_2[0,\ \infty)$. According to (19), one has from (18) that

$$\sum_{i=t}^{\infty} \|y_1(i)\|^2 \leqslant V(x(t)) + \gamma^2 \alpha \tag{22}$$

for any $u \in \mathbb{U}$ and $w \in \mathbb{W}$. Hence, we can arrive at

$$V(x(t)) + \gamma^2 \alpha \geqslant \max_{w \in \mathbb{W}} \sum_{i=t}^{\infty} \|y_1(i)\|^2 \geqslant \min_{u \in \mathbb{U}} \max_{w \in \mathbb{W}} \sum_{i=t}^{\infty} \|y_1(i)\|^2. \tag{23}$$

Then, we can determine the optimal upper bound for (23) by the following minimization problem:

$$\min_{\gamma^2} V(x(t)) + \gamma^2 \alpha \quad \text{s.t. (21) for the system (1)}, \tag{24}$$

which serves as a Lagrange dual formulation for the minimax optimization problem (16).

Let us focus on the Lagrange dual formulation (24). The condition (9) precisely implies (21) in the scenario of $u(t) = Kx(t)$ by defining $V(t) = x^\top(t) P x(t)$ with a positive definite matrix P. Hence, the optimization problem (24) is equivalent to the following:

$$\min_{\lambda,\, \beta,\, \gamma^2} V(x(t)) + \gamma^2 \alpha \quad \text{s.t. (13) for the system (1)}. \tag{25}$$

Furthermore, we can derive the following result for H_∞ control using (25) by temporarily disregarding the time-domain constraints.

Lemma 4. For given offline data $(\hat{U},\ \hat{X},\ \hat{W})$ generated by system (1) and external disturbance satisfying Assumption 1, suppose that the LMI-based optimization problem (25) has an optimal solution $(\lambda_{\text{opt}},\, \beta_{\text{opt}},\, \gamma_{\text{opt}},\, Q_{\text{opt}},\, L_{\text{opt}})$. Then, the closed-loop system is internally asymptotically stable under the state feedback law $K_{\text{opt}} = L_{\text{opt}} Q_{\text{opt}}^{-1}$ and achieves an H_∞ performance level of at most γ_{opt} from w to y_1.

Proof. The proof follows directly from the discussion above.

To investigate the scenario with the time-domain constraint (2), we define an ellipsoid for the state $x(t)$ as

$$\Psi(P, \sigma_s) := \left\{ x \in \mathbb{R}^n | x^\top P x \leqslant \sigma_s \right\} \tag{26}$$

using a matrix P and a scalar $\sigma_s > 0$. Then, the formulation for the output constraints, subject to $x(t) \in \Psi(P, \sigma_s)$, can be presented as

$$\max_{t \geqslant 0} |y_{2m}(t)|^2 = \max_{t \geqslant 0} x^\top(t) C_{2,K}^\top C_{2,K} x(t)$$

$$\begin{aligned}&\leqslant \max_{x\in\Psi} x^\top C_{2,K}^\top C_{2,K} x\\&\leqslant y_{2m,\max}^2, \quad m=1,2,\ldots,q_2,\end{aligned} \tag{27}$$

where $C_{2,K}=C_2+D_2K$. In fact, Eq. (27) implies all x satisfying $V(x)=x^\top Px\leqslant\sigma_s$ such that $x^\top C_{2,K}^\top C_{2,K}x\leqslant y_{2m,\max}^2$. Using S-Lemma, we obtain

$$\begin{cases} y_{2i,\max}^2 - x^\top C_{2,K,i}^\top C_{2,K,i}x - \sigma_s\varphi + \varphi x^\top Px \geqslant 0,\\ i=1,2,\ldots,p_2\end{cases} \tag{28}$$

for a scalar $\varphi>0$, then (27) holds. For simplicity, suppose $\varphi=\frac{y_{2i,\max}^2}{\sigma_s}$, then the condition that makes (28) hold for all non-zero x is $\frac{y_{2i,\max}^2}{\sigma_s}P - C_{2,K,i}^\top C_{2,K,i}\geqslant 0$ or

$$\frac{y_{2i,\max}^2}{\sigma_s}Q - QC_{2,K,i}^\top C_{2,K,i}Q \geqslant 0. \tag{29}$$

With the help of the Schur complement lemma, a sufficient condition for (29) to hold is the existence of a symmetric matrix Λ, such that

$$\begin{cases}\begin{pmatrix}\frac{1}{\sigma_s}\Lambda & C_2Q+D_2L\\ (C_2Q+D_2L)^\top & Q\end{pmatrix}\geqslant 0,\\ \Lambda_{ii}\leqslant y_{2i,\max}^2, i=1,2,\ldots,q_2\end{cases} \tag{30}$$

holds for Q and L. Similarly, by the Schur complement lemma, finding the minimum lower bound for $x(t)^\top Px(t)\leqslant\sigma_s$ is equivalent to minimizing σ under the constraint

$$\begin{pmatrix}\sigma & x(t)^\top\\ x(t) & Q\end{pmatrix}\geqslant 0. \tag{31}$$

Therefore, the LMI-based optimization problem (25) with the time-domain constraint can be formulated as

$$\begin{aligned}&\min_{\sigma,\lambda,\beta,\gamma^2,L,Q}\ \sigma+\alpha\gamma^2\\&\text{s.t. (13), (30), (31) and } \sigma\leqslant\sigma_s.\end{aligned} \tag{32}$$

For generality, we introduce two weight parameters r_1 and r_2 and make the following result based on (32).

Theorem 1. Let $\sigma_s>0$ be a given scalar and Λ a matrix. Consider offline data ($\hat{U}$, $\hat{X}_-$, $\hat{W}$) generated by system (1) and an external disturbance satisfying Assumption 1. If the LMI-based optimization problem

$$\begin{aligned}&\min_{\sigma,\lambda,\beta,\gamma^2,L,Q}\ r_1\sigma+r_2\gamma^2\\&\text{s.t. (13), (30), (31) and } \sigma\leqslant\sigma_s\end{aligned} \tag{33}$$

has an optimal solution (σ_t, λ_t, β_t, γ_t, Q_t, L_t), then the closed-loop system satisfies the following properties:

(i) The closed-loop system is internally asymptotically stable under the state feedback law $K_{\text{opt}}=L_{\text{opt}}Q_{\text{opt}}^{-1}$;

(ii) The closed-loop system achieves the optimal H_∞ performance level γ_{opt} from w to y_1;

(iii) The constraint (2) is satisfied for all $t\geqslant 0$.

Proof. Based on Lemma 4, we can easily obtain conclusions (i) and (ii). From conclusion (iii), it can be derived that the closed-loop system satisfies dissipation inequality (21), and that solution (σ_t, λ_t, β_t, γ_t, Q_t, L_t) satisfies LMI (13b) and $V(x)\geqslant 0$.

Remark 2. The two weighting parameters r_1, r_2 introduced in (33) allow more flexibility in adapting this optimization problem to different scenarios. If a larger weight ratio is desired for σ—i.e., if the optimization of σ is prioritized—the parameter r_1 can be increased, and conversely, it can be decreased if less emphasis is desired. In addition, the optimization problem (33) can be transformed into different optimization problems depending on the values of r_1 and r_2. For example, setting $r_1=1$ and $r_2=\alpha$ reduces (33) to problem (32); setting $r_1=1$ and $r_2=0$ recovers the cost function in [47]; and setting $r_1=0$ and $r_2=1$ corresponds to the cost function in [48].

Remark 3. The inequality (10) implies the dissipation inequality (21), indicating that inequality (21) can be derived not only as shown in this paper but also from (10) (i.e., (8) and (9)). This method is detailed in [48]. Therefore, this paper leverages their correlation to propose a data-driven H_∞ optimal control method.

Remark 4. In minimax MPC, feedback predictive control is typically employed to mitigate the effects of disturbances and uncertainties while also reducing computational complexity [49]. However, implementing feedback minimax MPC may encounter feasibility and practical issues [50]. To address this, this study leverages Lagrange duality to relax the minimax problem into a more tractable minimization problem. This approach not only alleviates infeasibility but also reduces the conservatism introduced by the ellipsoidal evaluation in time-domain constraints.

4.3 Moving horizon predictive control

For moving horizon predictive control, the optimization problem (33) is solved in real time at each time instant $t \geqslant 0$ using the current state $x(t)$. This implementation enables the use of the current state variable $x(t)$ for feedback control. However, the dissipation property of the closed-loop systems may not be guaranteed under standard moving horizon optimization. To address this problem, according to [51], a dissipative constraint condition can be incorporated into the optimization problem (33) as

$$\begin{pmatrix} x(t)^\top P_{t-1}x(t) + p_0 - p_{t-1} & x(t)^\top \\ x(t) & Q \end{pmatrix} \geqslant 0 \tag{34}$$

for each time instant $t \geqslant 0$, where $p_0 = x(0)^\top P_0 x(0)$ and p_t is calculated according to the following equation:

$$p_t := p_{t-1} - \left[x(t)^\top P_{t-1}x(t) - x(t)^\top P_t x(t)\right]. \tag{35}$$

Then, the optimization problem (33) is formulated as

$$\begin{aligned} &\min_{\sigma,\lambda,\beta,\gamma^2,L,Q} r_1\sigma + r_2\gamma^2 \\ &\text{s.t. (13), (30), (31), (34) and } \sigma \leqslant \sigma_s \end{aligned} \tag{36}$$

for each time $t > 0$.

According to [48], to ensure that the closed-loop system satisfies constraint (2), the initial state of the system belongs to the ellipsoidal domain. Therefore, in case of unsolvable situations, we can avoid this problem by continuously increasing the value of σ_s. By incorporating a scalar $\eta \geqslant 0$, the optimization problem (36) can be reformulated into a more feasible version.

$$\begin{aligned} &\min_{\sigma,\lambda,\beta,\gamma^2,L,Q} r_1\sigma + r_2\gamma^2 \\ &\text{s.t. (13), (30), (31), (34), and } \sigma \leqslant \sigma_s(1+\eta). \end{aligned} \tag{37}$$

Furthermore, the following feasibility result can be established.

Lemma 5. For given $\sigma_s > 0$ and Λ, suppose that Assumption 1 holds and that the LMIs (13a), (13b), (31), and (30) are feasible with a solution (σ, λ, β, γ, Q, L) at time $t_0 \geqslant 0$. Then, the feasibility of the optimization problem (37) is guaranteed at every time $t_0 + n$ for some $n \geqslant 0$ and $\eta \geqslant 0$.

Proof. For $t = 0$, the feasibility of the optimization problem (33) implies the feasibility of the optimization problem (36). Let there exist a bounded initial state $x(0)$ such that Eq. (31) holds by defining $\sigma_0 = x^\top(0)P_0x(0)$. Because Eqs. (13) and (30) do not rely on the system state variable $x(k)$, it can be concluded that the problem is initially feasible and remains feasible at future time steps. Therefore, Eq. (33) is feasible for $\sigma_0 \leqslant \sigma_s$. In addition, the feasibility of (13) implies that Eq. (21) is feasible with $\gamma = \gamma_0$ and $V(x(t)) = x(t)^\top P_0 x(t)$, ensuring that $x(t)$ is bounded at $t = 1$.

When $t > 0$, assume that a bounded system state $x(t)$ exists such that the optimization problem (36) has a feasible solution (σ_t, λ_t, β_t, γ_t, Q_t, L_t). Then using (21), it follows that $x(t+1)$ is also bounded.

For $t = t+1$, let there exist a bounded $\sigma_{t+1} = x(t+1)P_t x(t+1)$ such that Eq. (31) holds. Then, the feasibility of (34) at time instant t implies that $p_0 - p_t \geqslant 0$ according to (35). In other words, at $t = t+1$, Eq. (34) remains feasible with $Q = Q_t$. Therefore, the optimization problem (36) has a feasible solution (σ_{t+1}, λ_t, β_t, γ_t, Q_t, L_t) corresponding to the system state variable $x(t+1)$ if $\sigma_{t+1} \leqslant \sigma_s$.

Based on this, the corresponding moving horizon predictive control algorithm is presented as Algorithm 1.

We now present the following conclusion regarding the properties of the closed-loop system based on the above discussion. This result integrates the findings of Theorem 1 into the MPC framework, enabling continuous optimization of the H_∞ performance.

Algorithm 1 Moving horizon control algorithm.

(1) Initialization. Set $t=0$ and given σ_s, Λ, initial state $x(0)$ and offline data $(\hat{U},\hat{X},\hat{W})$;
(2) For $t=0$, find a set of solutions $(\sigma_0,\lambda_0,\beta_0,\gamma_0,Q_0,L_0)$ to the optimization problem (33); if no feasible solution is found, substitute σ_s by $\sigma_s(1+\eta)$ and increase $\eta\geqslant 0$ slightly each time. Let $t_0=L_0Q_0$, $P_0=Q_0^{-1}$, $p_0=x^\top(0)P_0x(0)$ and go to **(4)**;
(3) For $t>0$, given $x(t)$, determine a set of solutions $(\sigma_t,\lambda_t,\beta_t,\gamma_t,Q_t,L_t)$ to the optimization problem (36). If the problem is not feasible, substitute (36) with (37). Then, set $K_t=L_tQ_t$, $P_t=Q_t^{-1}$, and $p_t:=p_{t-1}-x^\top(t)P_{t-1}x(t)+x^\top(t)P_tx(t)$;
(4) Compute the control input

$$u(t)=K_tx(t),\ \forall t\geqslant 0, \tag{38}$$

and apply it to the system. Let $t=t+1$, and then return to **(3)** to continue the procedure.

Theorem 2. For a given $\sigma_s>0$, suppose that

- Assumption 1 holds;
- The offline data set $(\hat{U},\ \hat{X},\ \hat{W})$ generated by system (1) is admissible;
- The optimization problem (37) is feasible with the solution $(\lambda_t,\ \beta_t,\ \sigma_t,\ \gamma_t,\ Q_t,\ L_t)$ at time $t\geqslant 0$;
- The set of optimal solutions $(\lambda_t,\ \beta_t,\ \sigma_t,\ \gamma_t,\ Q_t,\ L_t)$ is bounded.

Then, the closed-loop system under the controller (38) satisfies the following properties:
(i) The time-domain constraint is satisfied for all $t\geqslant 0$;
(ii) The system can be stabilized under finite energy disturbances;
(iii) For the discrete-time LTI system (1), the H_∞ norm is bounded above by $\gamma_\infty:=\lim_{t\to\infty}\max\{\gamma_t\}<\infty$;
(iv) The closed-loop system exhibits disturbance attenuation, i.e.,

$$\sum_{i=t_0}^{t}\|y_1(i)\|^2-\gamma^2\|w(i)\|^2\leqslant x(t_0)^\top P_{t_0}x(t_0)+p_0-p_{t_0} \tag{39}$$

holds with $\gamma\leqslant\infty$ and $p_0-p_{t_0}\geqslant 0$ for any $t\geqslant t_0\geqslant 0$.

Proof. For a given system state $x(t)$, property (i) can be clearly derived from the solvability of (30), i.e., the system state $x(t)$ and the calculated feedback gain K satisfy the given constraint condition.

Suppose there exists an optimal solution $(\sigma_t,\lambda_t,\beta_t,\gamma_t,Q_t,L_t)$, for any $t\geqslant t_0\geqslant 0$. Let $\gamma=\gamma_t$, $V(x)=x^\top(t)P_tx(t)$, and $P_t=Q_t^{-1}$. Then (13) implies (21), and it follows that

$$\begin{aligned}
&\|y_1(t_0)\|^2-\gamma_{t_0}^2\|w(t_0)\|^2\leqslant x(t_0)^\top P_{t_0}x(t_0)-x(t_0+1)^\top P_{t_0}x(t_0+1),\\
&\|y_1(t_0+1)\|^2-\gamma_{t_0+1}^2\|w(t_0+1)\|^2\leqslant x(t_0+1)^\top P_{t_0+1}x(t_0+1)-x(t_0+2)^\top P_{t_0+1}x(t_0+2),\\
&\quad\cdots\\
&\|y_1(t)\|^2-\gamma_t^2\|w(t)\|^2\leqslant x(t)^\top P_tx(t)-x(t+1)^\top P_tx(t+1),
\end{aligned}$$

and then

$$\begin{aligned}
\sum_{i=t_0}^{t}\Big(\|y_1(i)\|^2-\gamma_i^2\|w(i)\|^2\Big)\leqslant\ & x(t_0)^\top P_{t_0}x(t_0)-x(t+1)^\top P_tx(t+1)\\
&-\sum_{i=t_0+1}^{t}\Big(x(i)^\top P_{i-1}x(i)-x(i)^\top P_ix(i)\Big).
\end{aligned} \tag{40}$$

Furthermore, by the Schur complement lemma, the feasibility of (34) implies

$$p_0-p_{t-1}+x(t)^\top P_{t-1}x(t)-x(t)^\top P_tx(t)\geqslant 0. \tag{41}$$

Substituting (35) recursively into (41) from $t=0$ to $t=t_0$, the dissipation constraint yields

$$p_0-p_{t_0}+\sum_{i=t_0+1}^{t}\left[x(i)^\top P_{i-1}x(i)-x(i)^\top P_ix(i)\right]\geqslant 0. \tag{42}$$

Consequently, the inequality (40) can be reformed as

$$\sum_{i=t_0}^{t}\|y_1(i)\|^2-\gamma^2\|w(i)\|^2\leqslant x(t_0)^\top P_{t_0}x(t_0)+p_0-p_{t_0}-x(t+1)^\top P_tx(t+1) \tag{43}$$

for $\gamma_{\max} < \infty$. Because of the positive definiteness of P_t, Eq. (43) leads directly to conclusion (iv). Furthermore, at $t = t_0$, the solvability of (34) implies $p_0 - p_{t_0} \geqslant 0$ based on (35). For $t \to \infty$, we can conclude that the disturbances satisfying Assumption 1 are bounded, yielding

$$\sum_{i=0}^{\infty} \|y_1(i)\|^2 \leqslant x(0)^\top P_0 x(0) + \gamma_\infty^2 \sum_{i=0}^{\infty} \|w(i)\|^2. \tag{44}$$

This establishes conclusion (ii). Regarding conclusion (iii), when $x(0) = 0$, for finite-energy disturbances $w(t)$, we have

$$\gamma^2 \sum_{i=0}^{\infty} \|w(i)\|^2 \geqslant \sum_{i=0}^{\infty} \|y_1(i)\|^2. \tag{45}$$

Remark 5. Lemma 5 guarantees that the feasible set of the convex optimization problem (37) is non-empty at all time steps. Furthermore, the constraint $\sigma \leqslant \sigma_s$ (or other similar constraints) ensures that the feasible set is bounded. Because the objective function is continuous and minimized over a non-empty, closed, and bounded convex set, the existence of an optimal solution is guaranteed by the fundamental theorems of convex optimization [52].

5 Numerical example

In this subsection, we consider a batch reactor system [1, 53, 54] as a simulation example to demonstrate the effectiveness of the proposed scheme.

The open-loop unstable system represented by (1) is characterized by the matrices

$$A_\nu = \begin{bmatrix} 1.178 & 0.001 & 0.511 & -0.403 \\ -0.051 & 0.661 & -0.011 & 0.061 \\ 0.076 & 0.335 & 0.560 & 0.382 \\ 0 & 0.335 & 0.089 & 0.849 \end{bmatrix}, \; B_\nu = \begin{bmatrix} 0.004 & -0.087 \\ 0.467 & 0.001 \\ 0.213 & -0.235 \\ 0.213 & -0.016 \end{bmatrix}. \tag{46}$$

The other system parameters are considered as

$$C_1 = \begin{bmatrix} 1 & 0 & 1 & -1 \end{bmatrix}, \; C_2 = \begin{bmatrix} 0.5 & 0.5 & 1 & 1 \end{bmatrix}, \; D_1 = 0, \; D_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \tag{47}$$

and $E_\nu = E_1 = I$. These specific values indicate that the system is inherently unstable under open-loop conditions, necessitating further analysis and the design of a suitable control strategy to achieve desired performance and stability.

In this example, our objective is to determine a control gain that achieves optimal H_∞ performance and defines the state ellipsoid of an unknown system using the input-state-disturbance data. The parameters utilized in the simulations were carefully selected to satisfy the stability and convergence conditions derived in Section 4. The external disturbance $w(t)$ is assumed to satisfy $||w(t)||^2 \leqslant \alpha_t$ for all t. Input data were collected over a time horizon of $T = 20$, and initial state data were drawn from a standard normal distribution. The external disturbance data were randomly sampled to adhere to the above assumption and follow a standard normal distribution, while the corresponding state data were generated accordingly. The proposed data-driven control scheme is implemented for system (1) based on the offline data set $(\hat{U}, \; \hat{X}, \; \hat{W})$ with parameters $\alpha_t = 10^{-4}$, $\sigma_s = 10$, $\Lambda = 1.2I$, $r_1 = 1$, $r_2 = 1$, initial condition $x(0) = [1; \; -0.65; \; 0.4; \; -0.1]$ and random external disturbance $w(t)$. The optimization problem is solved by the YALMIP interface with the MOSEK solver in MATLAB. A stabilizing controller for system (1) is obtained using (38), as displayed in Figure 1(a). The figure clearly shows that the state responses of the closed-loop system converge to equilibrium points, demonstrating the effectiveness of the control input.

Figure 1(b) shows that the control output curve without the LMI constraint (30) ($Uncon\ y(t)$) exceeds the allowable limit, whereas the control output curve using the proposed method ($Con\ y(t)$) remains within the constraint. This demonstrates that the implementation of the moving horizon predictive control (37) successfully enforces the LMI constraints. Figures 2(a) and (b) further illustrate the advantages of the optimization problem (37) with moving horizon control, compared to the problem (33) without moving horizon control. Specifically, both methods achieved similar performance levels initially; however, as time progresses, the performance metrics (σ_t by (37)) and (γ_t by (37)) remain consistently optimal. Therefore, the moving horizon optimization problem (37) effectively improves performance levels while ensuring compliance with constraints. In addition, a smaller γ_t indicates better disturbance attenuation.

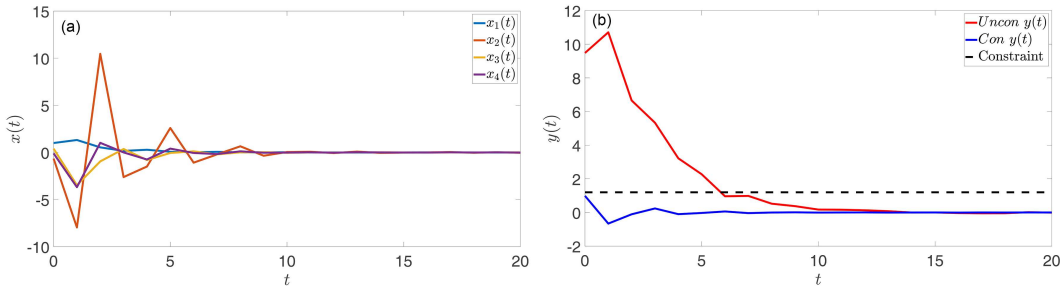

Figure 1 (Color online) (a) State trajectories under the applied control input and (b) the comparison of the control output between the unconstrained and constrained methods for parameter set 1 (see (46)).

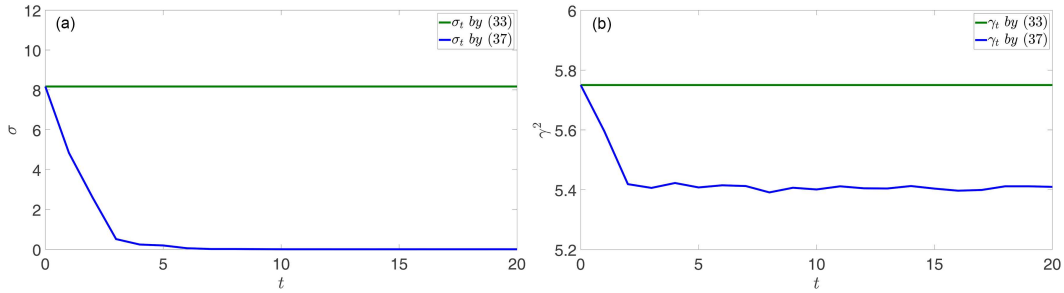

Figure 2 (Color online) Comparison of (a) σ_t and (b) γ_t between the moving horizon method and the static method for parameter set 1 (see (46)).

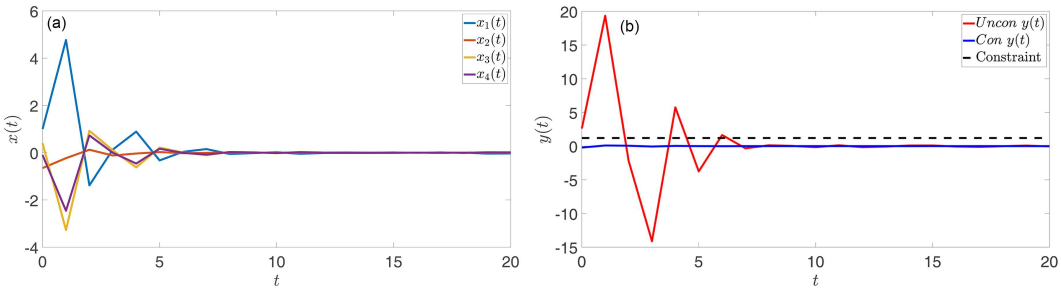

Figure 3 (Color online) (a) State trajectories under the applied control input and (b) the comparison of the control output between the unconstrained and constrained methods for parameter set 2 (see (48)).

Moreover, to further validate the robustness and optimality of the proposed controller (38), additional simulations were conducted under different working conditions of the batch reactor. The system parameters are referenced from [54] as follows:

$$A_\nu = \begin{bmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix}, \; B_\nu = \begin{bmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix}, \; C_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}, \tag{48}$$

and $\sigma_s = 10$, $\Lambda = 1.2I$. The other system and other predefined parameters are similar to the preceding system matrices and parameters. As shown in Figures 3 and 4, similar results to those obtained in the simulations can be achieved under different system parameters. Figure 3(b) shows that the control output curve without the LMI constraint (30) (*Uncon* $y(t)$) exceeds the allowable limit, while the control output curve using the proposed method (*Con* $y(t)$) remains within the constraint. Figures 4(a) and (b) demonstrate the superiority of the optimization problem (37) with moving horizon control, compared to the problem (33) without moving horizon control.

For comparison, the data-driven control schemes from [4,42] are substituted into the LMI (13) of the optimization problem (37). However, when applying the H_∞ design methods from [4,42] to the optimization problem (37) using the first system matrices and parameters (46) and (47), no feasible solution (σ, λ, β, γ, Q, L) was found. This indicates that the proposed method is less conservative than the methods in [4, 42].

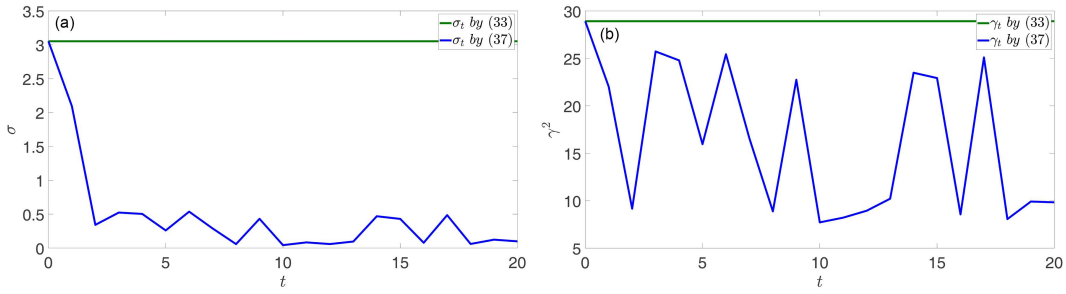

Figure 4 (Color online) Comparison of (a) σ_t and (b) γ_t between the moving horizon method and the static method for parameter set 2 (see (48)).

The low conservatism exists for two reasons. First, the output $y(t)$ in this paper, which comes from the state equation with $w(t)$, differs from the $y(t)$ in [4, 42]. In [4, 42], $y(t) = Cx(t) + Du(t)$, whereas in this paper, $y(t) = Cx(t) + Du(t) + Ew(t)$. The second reason is that the external disturbance data can be directly (or implicitly) utilized, whereas [4, 42] only requires $\|w(t)\| \leqslant \alpha$. This enables the proposed method to incorporate disturbance information more precisely. However, we note that the assumption of being able to use external disturbance data is strong, although it is practically achievable. Specifically, a disturbance estimator can be designed to recover the disturbance values from trajectory data—for instance, using the approach presented in Section IV of [36]. Additionally, the system models considered in this paper differ from those in [4, 42]. Specifically, the state-space model used in this paper includes an additional E matrix multiplying the $w(t)$ term, allowing the external disturbance to enter the state $x(t)$ from any direction. For example, if E is set to the identity matrix I, the external disturbance $w(t)$ will affect the state $x(t)$ from all directions. Conversely, when E is $[0; 0; 1]$, the external disturbance $w(t)$ will only influence the state component $x_3(t)$ (the choice of the dimensions of the matrix E here is for explanatory purposes only). Therefore, restricting the direction from which the disturbance can enter the system reduces the conservatism of the control method.

6 Conclusion

This article has presented a data-driven H_∞ predictive control scheme designed for unknown systems subject to time-domain constraints. By leveraging Lagrange duality, the approach transformed the minimax problem into a more tractable minimization problem, thus reducing the conservatism associated with ellipsoidal evaluations of time-domain constraints. Utilizing both input-output data and noisy data, our scheme achieved H_∞ performance in the closed-loop system. The comprehensive analysis demonstrates that the proposed control method guarantees closed-loop stability, effective disturbance attenuation, and satisfaction of constraints. The validity and advantages of the approach have been further confirmed through numerical simulations involving a batch reactor system, highlighting its robustness and feasibility in noisy environments.

This study contributes to the field of data-driven control by offering a robust and practical control strategy that can be applied to various real-world systems.

Furthermore, the deterministic robust control framework established in this study also provides a theoretical and methodological basis for future expansion toward fully stochastic, distributionally robust, or chance-constrained frameworks. Future research may seek to extend this framework through applications to more complex systems and to improve its capability in handling random uncertainties and noise. Such advances would further enhance the practicality and performance of data-driven control in complex and uncertain environments.

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