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• Supplementary File •

# A multiregional lagrangian-dual-based algorithm for dynamic constrained multiobjective optimization

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## Appendix A About "Related work"

### Appendix A.1 Problem formulation

DCMOPs are a kind of dynamic multi-objective optimization problems subject to constraints. In general, a DCMOP can be formulated as the following mathematical model [1]:

$$\begin{aligned} &\min\ F(\mathbf{x},t)=\left(\mathbf{f_1}\left(\mathbf{x},\mathbf{t}\right),\mathbf{f_2}\left(\mathbf{x},\mathbf{t}\right),\ldots\ldots,\mathbf{f_M}(\mathbf{x},\mathbf{t})\right)^{\top}(M\leqslant 2),\\ &s.t.\begin{cases} h_j\left(\mathbf{x},t\right)=0, & j=1,2,\ldots,n_h,\\ g_k\left(\mathbf{x},t\right)\leqslant 0, & k=1,2,\ldots,n_g,\\ x_i^{min}\leqslant x_i\leqslant x_i^{max}, & i=1,2,\ldots,N_{\mathrm{dim}}. \end{cases} \end{aligned} \tag{A1}$$

where $M$ is the number of objective functions that need to be optimized. $f_m(\mathbf{x},t)(m=1,2,...,M)$ represents the $m$th objective function at time $t$, where $t$ is a discrete time instance. $F(\mathbf{x},t)$ is $M$-dimensional minimization objective function. $h_j\left(\mathbf{x},t\right)=0$ represents the $j$th equation constraint at time $t$, where $j=1,2...,n_h$. $g_k\left(\mathbf{x},t\right)\leqslant 0$ represents the $k$th $(k=1,2,...,n_g)$ inequation constraint at time $t$. $\mathbf{x}$ is a $N_{dim}$-dimensional decision variable. The upper bound of the decision variable in the $i$th dimension is $x_i^{max}$ with the lower bound being $x_i^{min}$.

In DCMOP, if an individual, $\mathbf{x}_i$, satisfies all constraints at $t$, $\mathbf{x}_i$ is feasible; otherwise, it is infeasible. For two feasible individuals, $\mathbf{x}_1$ and $\mathbf{x}_2$, at $t$, $\mathbf{x}_1$ dominates $\mathbf{x}_2$, represented as $\mathbf{x}_1\prec\mathbf{x}_2$, if the following conditions are met:

$$\begin{cases} \forall j\in\{1,2,\ldots,M\},\ f_j\left(\mathbf{x}_1,t\right)\leqslant f_j\left(\mathbf{x}_2,t\right),\\ \exists k\in\{1,2,\ldots,M\},\ f_k\left(\mathbf{x}_1,t\right)<f_k\left(\mathbf{x}_2,t\right). \end{cases} \tag{A2}$$

If there are no other individuals that can dominate $\mathbf{x}_1$, then $\mathbf{x}_1$ is a nondominated solution (Pareto optimal solution). All Pareto optimal solutions compose the PS, denoted as $PS^t$ at $t$. The definition is as follows:

$$\mathrm{PS}^t=\left\{\mathbf{x}_1\middle|\ \nexists\mathbf{x}\in\mathbb{R}^{N_{dim}},\mathbf{x}_1\prec\mathbf{x}\right\}, \tag{A3}$$

If mapping $\mathrm{PS}^t$ into the objective space, we can obtain the PF at $t$, $\mathrm{PF}^t$, with its definition as follows:

$$\mathrm{PF}^t=\left\{F(\mathbf{x}_1,t)|\mathbf{x}_1\in PS^t\right\}, \tag{A4}$$

## Appendix B About "Multi-regional based prediction strategy"

### Appendix B.1 Population correction

## Appendix C About "Improved optimizer"

### Appendix C.1 Mating selection

The pseudocode of mating selection is exhibited in Alg. C1.

**Algorithm B1** Population Correction

**Input:** $P'$ and $N$;
**Output:** initial population, $P$;

Count the quantity of infeasible individuals, $n_{fea}$, in $P'$;
**if** $N_{fea} < n_{cor}$ **then**
  Discard infeasible individuals in $P'$;
  New individuals are randomly generated around feasible individuals and stored in $P'$;
  Randomly initialize $N - |P'|$ individuals and deposit them into $P'$;
  The infeasible in $P'$ are deposited in $A$, and the feasible are saved in $P$;
  **for** $i = 1 : |A|$ **do**
    Transfer the $i$th individual in $A$ to any one of $P$ using Eq. (18);
    The new individual are stored in $P$;
  **end for**
**else if** $N_{fea} \geqslant n_{cor}$ **then**
  The feasible individuals in $P'$ are moved into $P$;
  The individuals in $P$ are nondominated sorting and that individuals with ranks of 1 and 2 are deposited in $P_g$;
  Randomly initialize $N - |P'|$ individuals into $P'$;
  **for** $i = 1 : |P'|$ **do**
    Transfer the $i$th individual in $P'$ to any one of $P_g$ using Eq. (18);
    The new individual are stored in $P$;
  **end for**
**else if** $N_{fea} = 0$ **then**
  Randomly initialize $N - |P'|$ individuals, where infeasible and feasible ones are saved into $P'$ and $P$, respectively;
  **for** $i = 1 : |P'|$ **do**
    Transfer the $i$th individual in $P'$ to any one of $P$ using Eq. (18);
    The new individual are stored in $P$;
  **end for**
**end if**
**return** $P$.

**Algorithm C1** Mating Selection

**Input:** population, $P$;
**Output:** parents, $p_1$ and $p_2$;

Modify the objective values of the individuals in P using Eq. (18) and Eq. (19);
Perform nondominated sorting and crowding distance calculation on the modified $P$;
**for** $i = 1 : 2$ **do**
  Choose two different individuals, $\mathbf{x}_1$, $\mathbf{x}_2$, from $P$;
  **if** $\mathbf{x}_1 \prec \mathbf{x}_2$ **then**
    $p_i = \mathbf{x}_1$;
  **else if** $\mathbf{x}_2 \prec \mathbf{x}_1$ **then**
    $p_i = \mathbf{x}_2$;
  **else if** $d(\mathbf{x}_1) > d(\mathbf{x}_2)$ **then**
    $p_i = \mathbf{x}_1$;
  **else if** $d(\mathbf{x}_1) < d(\mathbf{x}_2)$ **then**
    $p_i = \mathbf{x}_2$;
  **else**
    Randomly select one individual from $\mathbf{x}_1$, and $\mathbf{x}_2$;
  **end if**
**end for**

**Algorithm C2** Offspring Generation

**Input:** population, $P$;
**Output:** offspring population, $Q$;

**for** $k = 1 : G_{max}$ **do**
  Use SBX and PM to generate offspring and deposit them into $Q$;
  **if** $G_{\max}\%5 = 0$ **then**
    **for** $i = 1 : n_{p1}$ **do**
      A parent $X$ is randomly selected from $P$;
      Generate a new individual $X'$ using Eq.(17) in the manuscript;
      Calculate the objective values $f_X^M$ and $f_{X'}^M$ of $X$ and $X'$, respectively;
      **if** $f_{X'}^M < f_X^M$ **then**
        Store $X'$ into $Q$;
      **end if**
    **end for**
  **end if**
  **for** $i = 1 : n_{p2}$ **do**
    $x' = P(i)$;
    **for** $j = 1 : N_{dim}$ **do**
      $index = 0$;
      $y = x'$, $\mathbf{x} = x'$;
      **while** $index < 2$ **do**
        **if** $index = 0$ **then**
          $y_j = \mathbf{x}_j + \delta$;
        **else**
          $y_j = \mathbf{x}_j - \delta$;
        **end if**
        Calculate the objective values of $y$;
        **if** $y_j \leqslant x_j^{\max}$ and $y_j \geqslant x_j^{\min}$ **then**
          **if** $y \prec \mathbf{x}$ **then**
            $\mathbf{x} = y$
          **else**
            $index++$;
          **end if**
        **else**
          $index++$;
        **end if**
        **if** $y \neq x'$ **then**
          Store $y$ into $Q$;
        **end if**
      **end while**
    **end for**
  **end for**
**end for**

## Appendix C.2 Offspring generation

The pseudocode for the offspring generation is presented in Alg. C2. In this method, parent $\mathbf{x}$ is first chosen and its offspring, $x'$, can be generated by adding a positive value, $\delta$, to one dimension of $\mathbf{x}$. If $x'$ is still within the boundary and dominates $\mathbf{x}$, $\delta$ is continuously added in the same dimension until the new individual is no longer improved. Otherwise, $\delta$ is reduced. The above procedure is repeated until all dimensions of $\mathbf{x}$ has been executed.

## Appendix C.3 Nondominated solution selection strategy

In this part, only $N_{non}$ nondominated individuals are reserved by nondominated sorting [2], rather than $N$, in order to release storage burden with an acceptable convergence and diversity. If the number of nondominated individuals is less than $N_{non}$, all of them will be stored in the approximated PS, $AS$. Otherwise, $N_{non}$ nondominated individuals lying on the first frontier are saved.

The pseudocode for the nondominated solution selection strategy is presented in Alg. C3.

**Algorithm C3** Nondominated Solution Selection Strategy

**Input:** population, $P$;
**Output:** approximated $PS$, $AS$;
1: Perform nondominated sorting and crowding distance calculation on feasible individuals in $P$;
2: **if** the number of nondominated solutions less than $N_{non}$ **then**
3: All nondominated solutions are stored in $AS$;
4: **else**
5: Select the first $N_{non}$ nondominated individuals to be stored in $AS$;
6: **end if**

# Appendix D About "Experimental setting"

## Appendix D.1 Test suites

There are eight benchmark test functions in [3] and we will select five benchmark test functions in [4]: DCTP2, DCTP3, DCTP5, DCTP6, and DCTP8. They are bi-objective dynamic constrained multi-objective problems (DCMOPs) designed with dynamic changes in the feasible region and the Pareto front (PF). The arithmetical formulation of these DCMOPs are listed in Tab. D1.

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**Table D1** Test suites in the experimental part.

| Case | Description |
|---|---|
| Ins1 | $\mathrm{s}(t) = \max(3.5 - 0.14 \times t, 0.7 + 0.14 \times t)$;<br>$m(t) = \max(1.43 - 0.05 \times t, 0.43 + 0.05 \times t)$;<br>$\mathrm{W}_t = 2; g(X,t) = \sum_{j=2}^{d} (x_j - (1 - 0.9 \times \sin(0.2\pi \times \mathrm{t})))^2$;<br>$Min(f_1(x,t), f_2(x,t))$<br>$f_1(x,t) = (1 + g(X,t))(x_1 + 0.05 \times \sin(\mathrm{w}_t \pi x_1))$;<br>$f_2(x,t) = (1 + g(X,t))(s(t) - x_1 + 0.05 \times \sin(W_t \pi x_1))$;<br>$s.j. \cos\frac{\pi}{4} \times f_2 - \sin\frac{\pi}{4} \times f_1 - m(t) \geqslant a \times \lvert \sin(\pi \times (\sin\frac{-\pi}{16} \times (f_2 - 1) + \cos\frac{-\pi}{16} \times f_1))\rvert^{0.5}$;<br>$f_1 + f_2 - 6 < 0; m(t) - 6 < 0$;<br>$\mathrm{x}_1 \in [0,1]; x_j \in [0,2], j = 2, ..., d$ |
| Ins2 | $\mathrm{s}(t) = \max(2.5 - 0.05 \times t, 1.5 + 0.05 \times t)$;<br>$m(t) = \max(1.16 - 0.075 \times t, -0.34 + 0.075 \times t)$;<br>$\mathrm{W}_t = 2; g(X,t) = \sum_{j=2}^{d} (x_j - (1 - 0.9 \times \sin(0.2\pi \times \mathrm{t})))^2$;<br>$Min(f_1(x,t), f_2(x,t))$<br>$f_1(x,t) = (1 + g(X,t))(x_1 + 0.05 \times \sin(\mathrm{w}_t \pi x_1))$;<br>$f_2(x,t) = (1 + g(X,t))(s(t) - x_1 + 0.05 \times \sin(W_t \pi x_1))$;<br>$s.j. \cos\frac{\pi}{4} \times f_2 - \sin\frac{\pi}{4} \times f_1 - m(t) \geqslant a \times \lvert \sin(\pi \times (\sin\frac{-\pi}{16} \times (f_2 - 1) + \cos\frac{-\pi}{16} \times f_1))\rvert^{0.5}$;<br>$f_1 + f_2 - 6 < 0; m(t) - 6 < 0$;<br>$\mathrm{x}_1 \in [0,1]; x_j \in [0,2], j = 2, ..., d$ |
| Ins3 | $\mathrm{s}(t) = \max(2.1 - 0.14 \times t, 4.9 + 0.14 \times t)$;<br>$m(t) = \max(0.93 + 0.05 \times t, 1.93 - 0.05 \times t)$;<br>$\mathrm{W}_t = 2; g(X,t) = \sum_{j=2}^{d} (x_j - (1 - 0.9 \times \sin(0.2\pi \times \mathrm{t})))^2$;<br>$Min(f_1(x,t), f_2(x,t))$<br>$f_1(x,t) = (1 + g(X,t))(x_1 + 0.05 \times \sin(\mathrm{w}_t \pi x_1))$;<br>$f_2(x,t) = (1 + g(X,t))(s(t) - x_1 + 0.05 \times \sin(W_t \pi x_1))$;<br>$s.j. \cos\frac{\pi}{4} \times f_2 - \sin\frac{\pi}{4} \times f_1 - m(t) \geqslant a \times \lvert \sin(\pi \times (\sin\frac{-\pi}{16} \times (f_2 - 1) + \cos\frac{-\pi}{16} \times f_1))\rvert^{0.5}$;<br>$f_1 + f_2 - 6 < 0; m(t) - 6 < 0$;<br>$\mathrm{x}_1 \in [0,1]; x_j \in [0,2], j = 2, ..., d$ |
| Ins4 | $\mathrm{s}(t) = \max(2 + 0.05 \times t, 3 - 0.05 \times t)$;<br>$m(t) = \max(0.41 + 0.075 \times t, 1.91 - 0.075 \times t)$;<br>$\mathrm{W}_t = 2; g(X,t) = \sum_{j=2}^{d} (x_j - (1 - 0.9 \times \sin(0.2\pi \times \mathrm{t})))^2$;<br>$Min(f_1(x,t), f_2(x,t))$<br>$f_1(x,t) = (1 + g(X,t))(x_1 + 0.05 \times \sin(\mathrm{w}_t \pi x_1))$;<br>$f_2(x,t) = (1 + g(X,t))(s(t) - x_1 + 0.05 \times \sin(W_t \pi x_1))$;<br>$s.j. \cos\frac{\pi}{4} \times f_2 - \sin\frac{\pi}{4} \times f_1 - m(t) \geqslant a \times \lvert \sin(\pi \times (\sin\frac{-\pi}{16} \times (f_2 - 1) + \cos\frac{-\pi}{16} \times f_1))\rvert^{0.5}$;<br>$f_1 + f_2 - 6 < 0; m(t) - 6 < 0$;<br>$\mathrm{x}_1 \in [0,1]; x_j \in [0,2], j = 2, ..., d$ |
| Ins5 | $\mathrm{s}(t) = \max(3.5 - 0.14 \times t, 0.7 + 0.14 \times t)$;<br>$m(t) = \max(1.43 - 0.05 \times t, 0.43 + 0.05 \times t)$;<br>$\mathrm{W}_t = 6 \times \sin(0.2 \times \pi(t+1)); g(X,t) = \sum_{j=2}^{d} (x_j - (1 - 0.9 \times \sin(0.2\pi \times \mathrm{t})))^2$;<br>$Min(f_1(x,t), f_2(x,t))$<br>$f_1(x,t) = (1 + g(X,t))(x_1 + 0.05 \times \sin(\mathrm{w}_t \pi x_1))$;<br>$f_2(x,t) = (1 + g(X,t))(s(t) - x_1 + 0.05 \times \sin(W_t \pi x_1))$;<br>$s.j. \cos\frac{\pi}{4} \times f_2 - \sin\frac{\pi}{4} \times f_1 - m(t) \geqslant a \times \lvert \sin(\pi \times (\sin\frac{-\pi}{16} \times (f_2 - 1) + \cos\frac{-\pi}{16} \times f_1))\rvert^{0.5}$;<br>$f_1 + f_2 - 6 < 0; m(t) - 6 < 0$; |

| | |
|---|---|
| | $\mathrm{x}_1 \in [0,1]; x_j \in [0,2], j = 2, ..., d$ |
| Ins6 | $\mathrm{s}(t) = \max(2.5 - 0.05 \times t, 1.5 + 0.05 \times t)$;<br>$m(t) = \max(1.16 - 0.075 \times t, -0.34 + 0.075 \times t)$;<br>$\mathrm{W}_t = 6 \times \sin(0.2 \times \pi(t+1)); g(X,t) = \sum_{j=2}^{d} (x_j - (1 - 0.9 \times \sin(0.2\pi \times \mathrm{t})))^2$;<br>$Min(f_1(x,t), f_2(x,t))$<br>$f_1(x,t) = (1 + g(X,t))(x_1 + 0.05 \times \sin(\mathrm{w}_t \pi x_1))$;<br>$f_2(x,t) = (1 + g(X,t))(s(t) - x_1 + 0.05 \times \sin(W_t \pi x_1))$;<br>$s.j. \cos\frac{\pi}{4} \times f_2 - \sin\frac{\pi}{4} \times f_1 - m(t) \geqslant a \times \lvert \sin(\pi \times (\sin\frac{-\pi}{16} \times (f_2 - 1) + \cos\frac{-\pi}{16} \times f_1))\rvert^{0.5}$;<br>$f_1 + f_2 - 6 < 0; m(t) - 6 < 0$;<br>$\mathrm{x}_1 \in [0,1]; x_j \in [0,2], j = 2, ..., d$ |
| Ins7 | $\mathrm{s}(t) = \max(2.1 - 0.14 \times t, 4.9 + 0.14 \times t)$;<br>$m(t) = \max(0.93 + 0.05 \times t, 1.93 - 0.05 \times t)$;<br>$\mathrm{W}_t = 6 \times \sin(0.2 \times \pi(t+1)); g(X,t) = \sum_{j=2}^{d} (x_j - (1 - 0.9 \times \sin(0.2\pi \times \mathrm{t})))^2$;<br>$Min(f_1(x,t), f_2(x,t))$<br>$f_1(x,t) = (1 + g(X,t))(x_1 + 0.05 \times \sin(\mathrm{w}_t \pi x_1))$;<br>$f_2(x,t) = (1 + g(X,t))(s(t) - x_1 + 0.05 \times \sin(W_t \pi x_1))$;<br>$s.j. \cos\frac{\pi}{4} \times f_2 - \sin\frac{\pi}{4} \times f_1 - m(t) \geqslant a \times \lvert \sin(\pi \times (\sin\frac{-\pi}{16} \times (f_2 - 1) + \cos\frac{-\pi}{16} \times f_1))\rvert^{0.5}$;<br>$f_1 + f_2 - 6 < 0; m(t) - 6 < 0$;<br>$\mathrm{x}_1 \in [0,1]; x_j \in [0,2], j = 2, ..., d$ |
| Ins8 | $\mathrm{s}(t) = \max(2 + 0.05 \times t, 3 - 0.05 \times t)$;<br>$m(t) = \max(0.41 + 0.075 \times t, 1.91 - 0.075 \times t)$;<br>$\mathrm{W}_t = 6 \times \sin(0.2 \times \pi(t+1)); g(X,t) = \sum_{j=2}^{d} (x_j - (1 - 0.9 \times \sin(0.2\pi \times \mathrm{t})))^2$;<br>$Min(f_1(x,t), f_2(x,t))$<br>$f_1(x,t) = (1 + g(X,t))(x_1 + 0.05 \times \sin(\mathrm{w}_t \pi x_1))$;<br>$f_2(x,t) = (1 + g(X,t))(s(t) - x_1 + 0.05 \times \sin(W_t \pi x_1))$;<br>$s.j. \cos\frac{\pi}{4} \times f_2 - \sin\frac{\pi}{4} \times f_1 - m(t) \geqslant a \times \lvert \sin(\pi \times (\sin\frac{-\pi}{16} \times (f_2 - 1) + \cos\frac{-\pi}{16} \times f_1))\rvert^{0.5}$;<br>$f_1 + f_2 - 6 < 0; m(t) - 6 < 0$;<br>$\mathrm{x}_1 \in [0,1]; x_j \in [0,2], j = 2, ..., d$ |
| DCTP2 | $Min(f_1(x,t), f_2(x,t))$<br>$f_1(x,t) = x_1$;<br>$f_2(x,t) = t \times g(x,t) \times (1 - \sqrt{\frac{f_1(x,t)}{g(x,t)}})$;<br>$g(x,t) = 1 + \sum_{i=2}^{D} (x_i - \sin(0.01\pi t))$;<br>$s.t., ((f_2(x,t)/t - 1) \times \cos(-0.2\pi)) - (f_1(x,t) \times \sin(-0.2\pi)) \geqslant$<br>$0.2 \times \lvert \sin(10\pi(((f_2(x,t)/t - 1) \times \sin(-0.2\pi)) + f_1(x,t) \times \cos(-0.2\pi)))\rvert^6$,<br>$x_i \in [0,1], i = 1, 2, ..., d$ |
| DCTP3 | $Min(f_1(x,t), f_2(x,t))$<br>$f_1(x,t) = x_1$;<br>$f_2(x,t) = t \times g(x,t) \times (1 - \sqrt{\frac{f_1(x,t)}{g(x,t)}})$;<br>$g(x,t) = 1 + \sum_{i=2}^{D} (x_i - \sin(0.01\pi t))$;<br>$s.t., ((f_2(x,t)/t - 1) \times \cos(-0.2\pi)) - (f_1(x,t) \times \sin(-0.2\pi)) \geqslant$<br>$0.1 \times \lvert \sin(10\pi(((f_2(x,t)/t - 1) \times \sin(-0.2\pi)) + f_1(x,t) \times \cos(-0.2\pi)))\rvert^{0.5}$,<br>$x_i \in [0,1], i = 1, 2, ..., d$ |
| DCTP5 | $Min(f_1(x,t), f_2(x,t))$<br>$f_1(x,t) = x_1$;<br>$f_2(x,t) = t \times g(x,t) \times (1 - \sqrt{\frac{f_1(x,t)}{g(x,t)}})$;<br>$g(x,t) = 1 + \sum_{i=2}^{D} (x_i - \sin(0.01\pi t))$; |

| | |
|---|---|
| | $s.t., ((f_2(x,t)/t-1)\times\cos(-0.2\pi))-(f_1(x,t)\times\sin(-0.2\pi))\geqslant$ |
| | $0.1\times\lvert\sin(10\pi(((f_2(x,t)/t-1)\times\sin(-0.2\pi))+f_1(x,t)\times\cos(-0.2\pi))^2)\rvert^2,$ |
| | $x_i\in[0,1], i=1,2,...,d$ |
| DCTP6 | $Min(f_1(x,t), f_2(x,t))$ |
| | $f_1(x,t)=x_1;$ |
| | $f_2(x,t)=t\times g(x,t)\times(1-\sqrt{\frac{f_1(x,t)}{g(x,t)}});$ |
| | $g(x,t)=1+\sum_{i=2}^{D}(x_i-\sin(0.01\pi t));$ |
| | $s.t., ((f_2(x,t)/t+2)\times\cos(0.1\pi))-(f_1(x,t)\times\sin(0.1\pi))\geqslant$ |
| | $40\times\lvert\sin(0.5\pi(((f_2(x,t)/t+2)\times\sin(0.1\pi))+f_1(x,t)\times\cos(0.1\pi)))\rvert^2,$ |
| | $x_i\in[0,1], i=1,2,...,d$ |
| DCTP8 | $Min(f_1(x,t), f_2(x,t))$ |
| | $f_1(x,t)=x_1;$ |
| | $f_2(x,t)=t\times g(x,t)\times(1-\sqrt{\frac{f_1(x,t)}{g(x,t)}});$ |
| | $g(x,t)=1+\sum_{i=2}^{D}(x_i-\sin(0.01\pi t));$ |
| | $s.t., ((f_2(x,t)/t+2)\times\cos(0.1\pi))-(f_1(x,t)\times\sin(0.1\pi))\geqslant$ |
| | $40\times\lvert\sin(0.5\pi(((f_2(x,t)/t+2)\times\sin(0.1\pi))+f_1(x,t)\times\cos(0.1\pi)))\rvert^2;$ |
| | $((f_2(x,t)/t)\times\cos(-0.05\pi))-(f_1(x,t)\times\sin(-0.05\pi))\geqslant$ |
| | $40\times\lvert\sin(2\pi(((f_2(x,t)/t)\times\sin(-0.05\pi))+f_1(x,t)\times\cos(-0.05\pi)))\rvert^6,$ |
| | $x_i\in[0,1], i=1,2,...,d$ |

## Appendix D.2 Performance indicators

Metrics of the Mean Inverted Generational Distance (MIGD) [5] and the Mean Hypervolume (MHV) [6] act as measurements to evaluate the performance of algorithms. For more details, please refer to Supplementary Materials. We hope to consider the effectiveness of the algorithm from convergence and distribution. So, we use the inverted generational distance (IGD) [7] and hypervolume (HV) [6]

### Appendix D.2.1 *Mean inverted generational distance (MIGD)*

IGD is a comprehensive index that measures the convergence and distribution of the Pareto optimal set obtained by the algorithm. The IGD indicator is calculated as Eq. (D1):

$$IGD(PF_t^*,\, PF_t)=\frac{\sum_{v\in PF_t^*} d(v,\ PF_t)}{|PF_t^*|}, \tag{D1}$$

Where $PF_t^*$ represents the set of uniformly distributed discrete points on the true PF and $PF_t$ is obtained by the algorithm at time $t$. $d(v,\ PF_t)=min_{u\in PF_t}||F(v)-F(u)||$ is the Euclidean distance between individual $u$ on $PF_t$ and the nearest individual $v$ on $PF_t^*$. $|PF_t^*|$ is the number of $PF_t^*$ that can be flexibly varied. It can be known from Eq.(D1) that the smaller the IGD value, the better the convergence and distribution of an algorithm.

Considering that each test problem includes $n_t$ dynamic changes. So, the mean value of all environmental IGD metrics (MIGD) [5] are used to reflect the comprehensive performance of the Pareto optimal set obtained by the algorithm. The MIGD is calculated as follows:

$$MIGD(PF_t^*,\, PF_t)=\frac{1}{|T|}\sum_{t\in T} IGD(PF_t^*,\, PF_t)\,, \tag{D2}$$

where $T$ is the set of discrete time point and $|T|$ is the quantity of $T$.

### Appendix D.2.2 *Mean hypervolume (MHV)*

HV is used to measure the convergence degree of the Pareto optimal set obtained by the algorithm, and quantify the difference between it and the true Pareto optimal front. The HV is calculated as Eq. (D3):

$$HV(PF_t,\, z^*)=\vartheta(\bigcup_{u=1}^{|PF_t|} V(u,z^*)), \tag{D3}$$

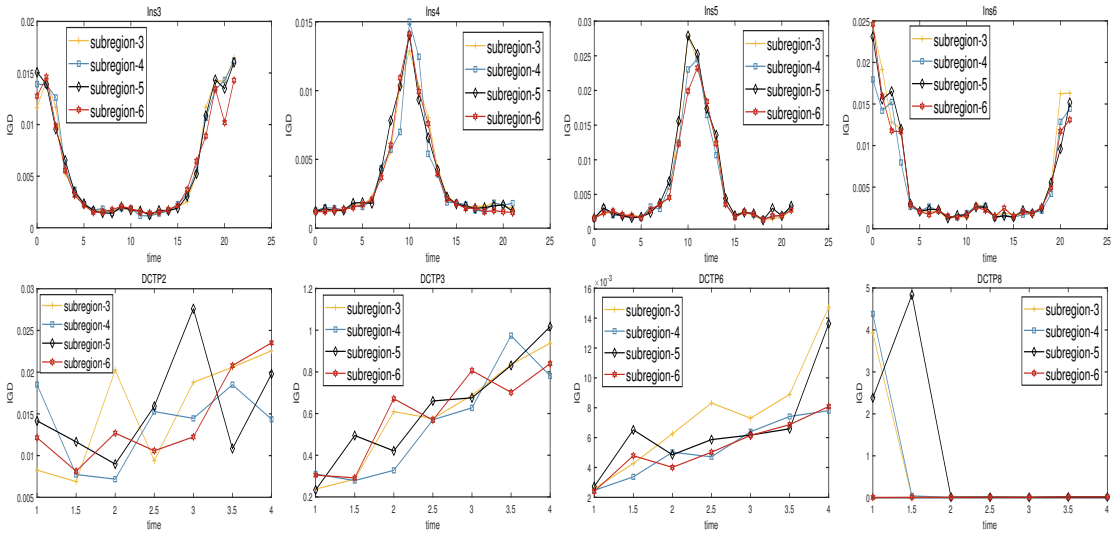



**Figure E1** The Curves of IGD over Time for MLDCMOA with Different Number of Sub-Regions on Ins3 to Ins6 with $n_t = 21$ and $\tau_t = 20$; DCTP2, DCTP3, DCTP6 and DCTP8 with $n_t = 21$ and $\tau_t = 10$, where the blue, yellow, black, and red curves correspond to results with the number of subregions as 3, 4, 5, and 6, respectively.

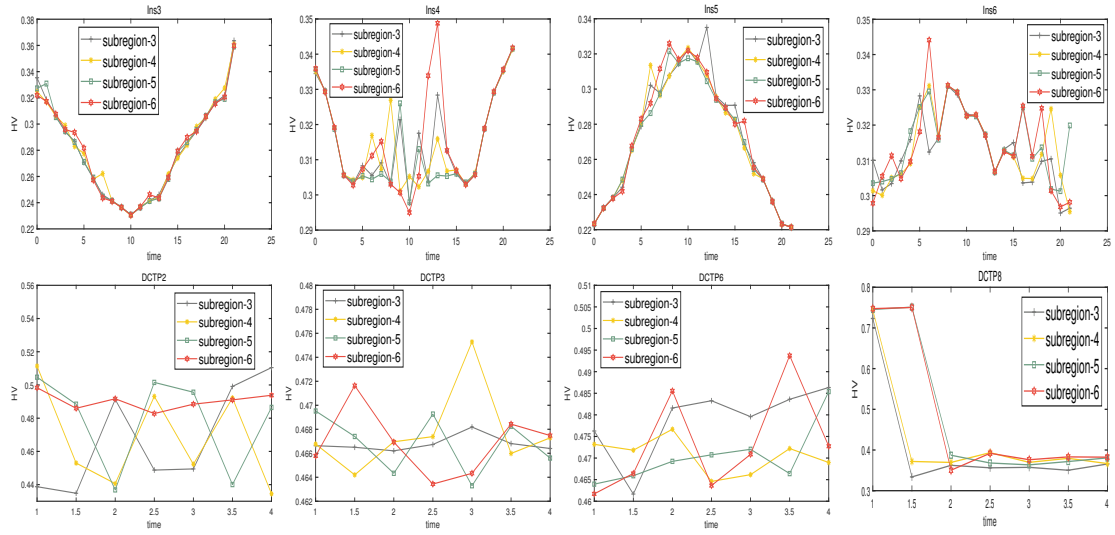



**Figure E2** The Curves of HV over Time for MLDCMOA with Different Number of Sub-Regions on Ins3 to Ins6 with $n_t = 21$ and $\tau_t = 20$; DCTP2, DCTP3, DCTP6 and DCTP8 with $n_t = 21$ and $\tau_t = 10$, where the gray, yellow, green, and red curves correspond to results with the number of subregions as 3, 4, 5, and 6, respectively.

where $\vartheta(\bullet)$ represents Lebesgue measure, $u$ represents an individual in the $PF_t$ at time $t$, $|PF_t|$ is the number of $PF_t$, and $V(u, z^*)$ represents the hypervolume of $u$ with the reference point $z^*(1, 1)$. It can be known from Eq.(D3) that larger HV values indicate better algorithm performance. Similarly, MHV is calculated as follows:

$$MHV(PF_t,\, z^*) = \frac{1}{|T|} \sum_{t \in T} HV(PF_t,\, z^*)\,, \tag{D4}$$

## Appendix E About "Experimental results and analysis"

### Appendix E.1 Comparison of different sub-regions

To investigate the impact caused by the number of sub-regions in MLDCMOA, in this section, we compare various settings, i.e., 3, 4, 5, and 6. Numbers over 6 are out of consideration since too many categories might in turn give rise to misleading, subsequently breaking the leverage between prediction and computational cost.

Fig. E1 and E2 illustrate the curves of IGD and HV over time on several test cases, i.e., Ins3-6, DCTP2, DCTP3, DTP6 and DCTP8. In Fig. E1, the changes from Ins3-6 seem not much difference. However, on the DCTP suite, when 6 sub-regions exist, the IGD results exhibit smoother and comparatively lower, especially on DCTP8. It is in consistence with observations in Fig. E2 of HV results, especially on Ins5, Ins6, DCTP2 and DCTP8. Finally, the number of sub-regions is as 6.

## Appendix E.2 Comparison with DCMOEAs

In this section, the mean and standard deviation (SD) results of MIGD and MHV obtained by MLDCMOA and its counterparts on the DCTP suite are investigated with different pairs of $(\tau_t, n_t)$ being listed in Table E1 to E2, described as mean(standard deviation), among which the best results are marked in bold. Without loss of generality, the Wilcoxon rank-sum test at the 0.05 confidence level [8] is exploited to determine whether MLDCMOA significantly differs from its comparisons. Herein, "+", "−", and "=" indicate MLDCMOA can achieve significantly better, worse, or equivalent results, respectively.

**Table E1** Mean and standard deviation of MIGD obtained by five algorithms on the DCTP suite [4].

| Prob. | $(\tau_t,n_t)$ | mEDCMOA | dCMOEA | DC-NSGA-II-A | DC-NSGA-II-B | MLDCMOA |
|---|---|---|---|---|---|---|
| DCTP2 | (5,21) | 0.1965(5.7172e-02)+ | 0.3137(7.6502e-02)+ | 0.1891(4.7957e-02)+ | 0.1792(5.8738e-02)+ | **0.0143(1.0242e-03)** |
| | (10,21) | 0.4180(1.0283e-01)+ | 0.4687(1.3225e-01)+ | 0.3382(1.2972e-01)+ | 0.3422(9.8878e-02)+ | **0.0139(1.9140e-03)** |
| | (20,21) | 0.6247(1.4853e-01)+ | 0.6140(1.6388e-01)+ | 1.0708(6.5493e-01)+ | 0.6077(2.0670e-01)+ | **0.0160(2.9608e-03)** |
| DCTP3 | (5,21) | 1.2288(2.5149e-01)+ | 1.7083(2.4153e-01)+ | 0.8216(1.9369e-01)+ | 0.7787(2.0150e-01)+ | **0.6307(4.1689e-02)** |
| | (10,21) | 1.8640(5.2034e-01)+ | 2.2012(3.9043e-01)+ | 1.5111(4.6721e-01)+ | 1.2960(3.7554e-01)+ | **0.6267(4.4396e-02)** |
| | (20,21) | 2.9644(6.9852e-01)+ | 2.8167(6.3665e-01)+ | 4.4057(2.7762e-00)+ | 2.6990(8.6214e-01)+ | **0.5952(7.6601e-02)** |
| DCTP5 | (5,21) | 0.3789(9.6474e-02)+ | 0.5016(9.5482e-02)+ | 0.3372(8.7015e-02)+ | 0.3185(8.8492e-02)+ | **0.0248(1.9580e-03)** |
| | (10,21) | 0.6471(1.8349e-01)+ | 0.7513(1.6486e-01)+ | 0.6059(2.6058e-01)+ | 0.6128(1.1194e-01)+ | **0.0281(3.0967e-03)** |
| | (20,21) | 1.0190(3.1167e-01)+ | 1.0775(2.6845e-01)+ | 1.8667(1.1909e-00)+ | 0.9737(4.1331e-01)+ | **0.0303(4.0605e-03)** |
| DCTP6 | (5,21) | 5.5445(9.8933e-01)+ | 5.8304(2.4524e-02)+ | 5.8093(2.7800e-02)+ | 1.1183(8.8518e-01)+ | **0.0065(6.8885e-04)** |
| | (10,21) | 5.8318(3.1157e-02)+ | 5.8224(4.2062e-02)+ | 5.8003(2.7748e-02)+ | 1.6011(1.1913e-00)+ | **0.0068(1.3361e-03)** |
| | (20,21) | 5.8328(4.8186e-02)+ | 5.8095(3.9784e-02)+ | 5.8049(3.9099e-02)+ | 2.1280(1.8623e-00)+ | **0.0066(1.9555e-03)** |
| DCTP8 | (5,21) | 12.5286(1.4645e-01)+ | 12.5579(3.5204e-02)+ | 12.5303(4.1259e-02)+ | 5.2956(3.0513e-00)+ | **0.2773(1.9487e-01)** |
| | (10,21) | 12.5826(7.0538e-02)+ | 12.5850(6.6446e-02)+ | 12.5944(8.2980e-02)+ | 6.4675(3.3730e-00)+ | **0.5225(6.3334e-01)** |
| | (20,21) | 12.4727(9.3954e-02)+ | 12.4957(9.4451e-02)+ | 12.5649(1.4284e-01)+ | 9.3554(3.9279e-00)+ | **0.7361(7.6435e-01)** |
| +/=/- | | 15/0/0 | 15/0/0 | 15/0/0 | 15/0/0 | |

**Table E2** Mean and standard deviation of MHV obtained by five algorithms on the DCTP suite [4].

| Prob. | $(\tau_t, n_t)$ | mEDCMOA | dCMOEA | DC-NSGA-II-A | DC-NSGA-II-B | MLDCMOA |
|---|---|---|---|---|---|---|
| DCTP2 | (5,21) | 0.4304(2.7672e-03)+ | **0.4770(8.3522e-03)=** | 0.4322(1.2328e-02)+ | 0.4283(1.6943e-02)+ | 0.4708(7.2878e-03) |
| | (10,21) | 0.4280(4.2912e-03)+ | **0.4688(1.5917e-02)=** | 0.4250(3.3461e-02)+ | 0.4268(2.6422e-02)+ | 0.4643(1.2556e-02) |
| | (20,21) | 0.4291(1.0117e-02)+ | 0.4442(1.4989e-02)+ | 0.4015(4.1157e-02)+ | 0.4266(5.1948e-02)+ | **0.4694(1.3733e-02)** |
| DCTP3 | (5,21) | 0.4534(2.9196e-03)+ | **0.4723(1.1347e-02)=** | 0.4535(1.0248e-02)+ | 0.4464(1.5543e-02)+ | 0.4674(7.6562e-04) |
| | (10,21) | 0.4515(8.6564e-03)+ | **0.4675(1.4077e-02)=** | 0.4341(5.8018e-02)+ | 0.4397(3.4105e-02)+ | 0.4674(1.3416e-03) |
| | (20,21) | 0.4462(7.0559e-03)+ | 0.4482(1.2151e-02)+ | 0.4114(4.2092e-02)+ | 0.4317(3.7904e-02)+ | **0.4669(1.8539e-03)** |
| DCTP5 | (5,21) | 0.4237(4.8956e-03)+ | **0.4604(1.1701e-02)=** | 0.4238(1.0415e-02)+ | 0.4216(1.1315e-02)+ | 0.4398(3.0164e-03) |
| | (10,21) | 0.4218(6.7794e-03)+ | **0.4556(1.9111e-02)=** | 0.3905(7.3322e-02)+ | 0.4194(2.5662e-02)+ | 0.4429(3.9810e-03) |
| | (20,21) | 0.4205(6.8384e-03)+ | 0.4290(1.5329e-02)+ | 0.3894(6.6236e-02)+ | 0.4070(5.0238e-02)+ | **0.4435(5.6813e-03)** |
| DCTP6 | (5,21) | 0.2438(2.8965e-02)+ | 0.2353(1.5591e-04)+ | 0.2355(1.3976e-04)+ | 0.3749(4.6655e-02)+ | **0.4768(3.1421e-03)** |
| | (10,21) | 0.2353(2.1222e-04)+ | 0.2354(1.9690e-04)+ | 0.2355(1.4649e-04)+ | 0.3534(4.7690e-02)+ | **0.4763(4.3157e-03)** |
| | (20,21) | 0.2355(3.0974e-04)+ | 0.2353(2.4538e-04)+ | 0.2356(2.4074e-04)+ | 0.3268(5.5738e-02)+ | **0.4705(4.1319e-03)** |
| DCTP8 | (5,21) | 0.2290(7.0797e-03)+ | 0.2275(1.7905e-04)+ | 0.2266(1.5086e-03)+ | 0.3023(4.4907e-02)+ | **0.4018(2.0310e-02)** |
| | (10,21) | 0.2274(2.0924e-04)+ | 0.2273(7.9348e-04)+ | 0.2235(6.1482e-03)+ | 0.2957(6.2104e-02)+ | **0.4115(4.9257e-02)** |
| | (20,21) | 0.2276(2.2101e-04)+ | 0.2274(2.1624e-04)+ | 0.2177(8.6158e-03)+ | 0.2617(6.2171e-02)+ | **0.4342(6.2212e-02)** |
| +/=/- | | 15/0/0 | 9/6/0 | 15/0/0 | 15/0/0 | |

## Appendix F About "Practical case"

In this section, a real-world case, Dynamic Environmental Economic Dispatch problem (DEED), is testified to evaluate and compare the efficiency of MLDCMOA with its counterparts. Herein, DEED experiences changing load demands in a variety of scheduling cycles with strong and dynamic constraints, whose mathematic model is formulated as Eq. (F1). We investigate the tracking capability of all algorithms and depict generator output changes during all cycles in Fig. F1. Following that, the approximated PFs with optimal results at one certain cycle are illustrated in Fig. F2 and Tab. F3.

## Appendix F.1 Model formulation

In this part, we consider the following DEED problem. The aim is to simultaneously minimize the fuel cost ($FC$) and the pollutant gas emission ($EM$). Herein, $n_u$ generating units with $n_t$ scheduling cycles are taken into account.

$$
\begin{aligned}
&M\text{in}(FC, EM)\\
&FC = \sum_{i=1}^{n} [a_i + b_i P_{i,t} + c_i P_{i,t}^2 + |d_i \sin(e_i(P_i^{\min} - P_{i,t}))|],\\
&EM = \sum_{i=1}^{n} [\alpha_i + \beta_i P_{i,t} + \gamma_i P_{i,t}^2 + \eta_i \exp(\delta_i P_{i,t})],\\
&s.j. P_{d,t} + P_{l,t} - \sum_{i=1}^{n} P_{i,t} \leqslant 0; P_{i,t} \leqslant \min(P_i^{\max}, P_{i,(t-1)} + UR_i),\\
&P_{i,t} \geqslant \max(P_i^{\min}, P_{i,(t-1)} - DR_i); (i = 1, 2, ..., n).
\end{aligned}
\tag{F1}
$$

where $P_i$,$_t$ represents the output power of $i$th generator at $t$th period. $P_d$,$_t$ and $P_l$,$_t$ denote the electricity load and the network loss of power transportation at $t$. Herein, $P_d$,$_t$ is a given parameter while $P_l$,$_t$ is calculated as Eq. (F2).

$$
P_{l,t} = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{i,t} B_{i,j} P_{j,t} + \sum_{i=1}^{n} B_{i0} P_{i,t} + B_{00}, \tag{F2}
$$

$P_i^{min}$ denotes the minimum output power of $i$. $a_i$, $b_i$, $c_i$, $d_i$, and $e_i$ are fuel cost factors of $i$. $\alpha_i$, $\beta_i$, $\gamma_i$, $\eta_i$, and $\delta_i$ are the pollutant emission factors of $i$. $UR_i$ and $DR_i$ represent the maximum power values for uphill and downhill of $i$, respectively. For more details with respect to values of related parameters, please refer to Supplementary Materials.

## Appendix F.2 Mathematical model

### Appendix F.2.1 *Objectives*

The fuel cost of per generating unit can be modeled as a quadratic and sinusoidal function in the condition of considering the valve point effect [9]. The total fuel cost (FC) in $H$ scheduling cycles can be expressed by Eq. (F3).

$$
FC = \sum_{t=1}^{H} \sum_{k=1}^{n} \left[ a_k + b_k P_{k,t} + c_k P_{k,t}^2 + \left| d_k \sin\left( e_k (P_k^{min} - P_{k,t}) \right) \right| \right], \tag{F3}
$$

where $P_k$,$_t$ represents the output power of the $k$th generator in the $t$th period. $P_k^{min}$ denotes the minimum output power of the $k$th unit. $a_k$, $b_k$, $c_k$, $d_k$, and $e_k$ are fuel cost factors of the $k$th generator.

The pollutant gas emission (EM) of generators $n$ in $H$ scheduling period can be obtained by Eq. (F4) [10].

$$
EM = \sum_{t=1}^{H} \sum_{k=1}^{n} \left[ \alpha_k + \beta_k P_{k,t} + \gamma_k P_{k,t}^2 + |\eta_k \exp(\delta_k P_{k,t})| \right], \tag{F4}
$$

where $P_k$,$_t$ represents the output power of the $k$th generator at $t$. $\alpha_k$, $\beta_k$, $\gamma_k$, $\eta_k$, $\delta_k$ are the pollutant emission factors of the $k$th generator.

### Appendix F.2.2 *Constraint conditions*

All generators in the DEED need to satisfy the following constraints:

(a) The power output of the $k$th generator at $t$, $P_k$,$_t$, should meet the upper and lower boundary limits:

$$
P_k^{min} \leqslant P_{k,t} \leqslant P_k^{max}, \tag{F5}
$$

(b) The power balance constraint of the power system at $t$:

$$
\sum_{k=1}^{n} P_{k,t} - P_{loss,t} - P_{d,t} = 0, \tag{F6}
$$

where $P_{d,t}$ and $P_{loss,t}$ denote the electricity load and the network loss of power transportation at $t$. Herein, the coefficient, $B$, is used to approximate the solution, which is shown in Eq. (F7).

$$
P_{loss,t} = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{i,t} B_{i,j} P_{j,t} + \sum_{i=1}^{n} B_{i,0} P_{i,t} + B_{0,0}, \tag{F7}
$$

(c) In the dynamic change from $(t-1)$ to $t$, due to the physical characteristics of the generator sets, the output power cannot be changed in a stepwise manner. Therefore, it is necessary to consider the adjustment ability of each generator in unit time, that is, the climbing constraints:

$$
\begin{cases}
P_{k,t} - P_{k,(t-1)} \leqslant UR_k,\\
P_{k,(t-1)} - P_{k,t} \leqslant DR_k.
\end{cases}
\tag{F8}
$$

where $UR_k$ and $DR_k$ represent the maximum power values of the $k$th unit for uphill and downhill, respectively.

$$B = 10^{-5} \times \begin{bmatrix} 4.9 & 1.4 & 1.5 & 1.5 & 1.6 & 1.7 & 1.7 & 1.8 & 1.9 & 2.0 \\ 1.4 & 4.5 & 1.6 & 1.6 & 1.7 & 1.5 & 1.5 & 1.6 & 1.8 & 1.8 \\ 1.5 & 1.6 & 3.9 & 1.0 & 1.2 & 1.2 & 1.4 & 1.4 & 1.6 & 1.6 \\ 1.5 & 1.6 & 1.0 & 4.0 & 1.4 & 1.0 & 1.1 & 1.2 & 1.4 & 1.5 \\ 1.6 & 1.7 & 1.2 & 1.4 & 3.5 & 1.1 & 1.3 & 1.3 & 1.5 & 1.6 \\ 1.7 & 1.5 & 1.2 & 1.0 & 1.1 & 3.6 & 1.2 & 1.2 & 1.4 & 1.5 \\ 1.7 & 1.5 & 1.4 & 1.1 & 1.3 & 1.2 & 3.8 & 1.6 & 1.6 & 1.8 \\ 1.8 & 1.6 & 1.4 & 1.2 & 1.3 & 1.2 & 1.6 & 4.0 & 1.5 & 1.6 \\ 1.9 & 1.8 & 1.6 & 1.4 & 1.5 & 1.4 & 1.6 & 1.5 & 4.2 & 1.9 \\ 2.0 & 1.8 & 1.6 & 1.5 & 1.6 & 1.5 & 1.8 & 1.6 & 1.9 & 4.4 \end{bmatrix} \tag{F11}$$

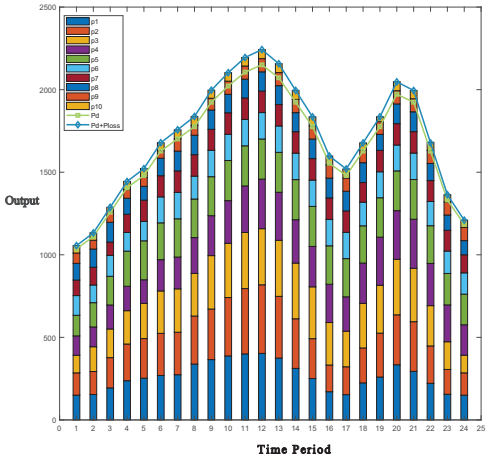



**Figure F1** Output allocation plans during all cycles.

### Appendix F.2.3 *Changing factors*

The power constraint of the power system is transformed into inequality constraints, which is shown in Eq. (F9).

$$P_{e,t} = P_{d,t} + P_{loss,t} - \sum_{k=1}^{n} P_{k,t} \leqslant 0, \tag{F9}$$

where $P_{e,t}$ is the micro-error of supply and demand at $t$. $P_{e,t} \leqslant 0$ refers to that the sum of the load on the demand side and the network loss of the system is slightly smaller than the total output of all generator sets. The purpose is to ensure that the power load on the demand side can be met in the case of random emergencies.

In the adjacent scheduling period, the range of the output power of unit $k$ at $t$ is expressed, which can be adjusted to the inequality shown in the Eq. (F10) by combining the ramping constraint of the generator sets.

$$\begin{cases} P_{k,t} \leqslant \min\left(P_k^{max}, P_{k,(t-1)} + UR_k\right), \\ P_{k,t} \geqslant \max\left(P_k^{min}, P_{k,(t-1)} - DR_k\right). \end{cases} \tag{F10}$$

In summary, constraints of DEED considered in this part consist of Eq. (F9) and Eq. (F10).

## Appendix F.3 Parameters settings

Coefficients are given in Tab. F1, and the correlation coefficients matrix required for its network loss is shown in Eq. (13). The load demand of the system is divided into 24 dispatch intervals, i.e. $H = 24$, with specific values and variations are presented in Tab. F2.

## Appendix F.4 Results analysis

The approximated PF obtained by MLDCMOA and the comparison algorithm at $T = 18$ is shown in Fig. F1. The best compromise solution is that lays around the middle part on the approximated PF, regarded as paying balanced attention on both $FC$ and $EM$ objectives. It can be seen that MLDCMOA provides more competitive allocation plans with smaller

**Table F1** Model coefficients for generator sets in the IEEE39 nodal system.

| k | $P_k^{max}$ | $P_k^{min}$ | $a_k$ | $b_k$ | $c_k$ | $d_k$ | $e_k$ |
|---|---|---|---|---|---|---|---|
| 1 | 470 | 150 | 786.7988 | 38.5397 | 0.1524 | 450 | 0.041 |
| 2 | 470 | 135 | 451.3251 | 46.1591 | 0.1058 | 600 | 0.036 |
| 3 | 340 | 73 | 1049.9977 | 40.3965 | 0.028 | 320 | 0.028 |
| 4 | 300 | 60 | 1243.5311 | 38.3055 | 0.0354 | 260 | 0.052 |
| 5 | 243 | 73 | 1658.5696 | 36.3278 | 0.0211 | 280 | 0.063 |
| 6 | 160 | 57 | 1356.6592 | 38.2704 | 0.0179 | 310 | 0.048 |
| 7 | 130 | 20 | 1450.7045 | 36.5104 | 0.0121 | 300 | 0.086 |
| 8 | 120 | 47 | 1450.7045 | 36.5104 | 0.0121 | 340 | 0.082 |
| 9 | 80 | 20 | 1455.6056 | 39.5804 | 0.109 | 270 | 0.098 |
| 10 | 55 | 10 | 1469.4026 | 40.5407 | 0.1295 | 380 | 0.094 |
| k | $\alpha_k$ | $\beta_k$ | $\gamma_k$ | $\eta_k$ | $\delta_k$ | $UR_k$ | $DR_k$ |
| 1 | 103.3908 | -2.4444 | 0.0312 | 0.5035 | 0.0207 | 80 | 80 |
| 2 | 103.3908 | -2.4444 | 0.0312 | 0.5035 | 0.0207 | 80 | 80 |
| 3 | 300.391 | -4.0695 | 0.0509 | 0.4968 | 0.0202 | 80 | 80 |
| 4 | 300.391 | -4.0695 | 0.0509 | 0.4968 | 0.0202 | 50 | 50 |
| 5 | 320.0006 | -3.8132 | 0.0344 | 0.4972 | 0.02 | 50 | 50 |
| 6 | 320.0006 | -3.8132 | 0.0344 | 0.4972 | 0.02 | 50 | 50 |
| 7 | 330.0056 | -3.9032 | 0.0465 | 0.5163 | 0.0214 | 30 | 30 |
| 8 | 330.0056 | -3.9032 | 0.0465 | 0.5163 | 0.0214 | 30 | 30 |
| 9 | 350.0056 | -3.9524 | 0.0465 | 0.5475 | 0.0234 | 30 | 30 |
| 10 | 360.0012 | -3.9864 | 0.047 | 0.5475 | 0.0234 | 30 | 30 |

**Table F2** Settings of load distribution in 24 dispatches.

| H | Pd (MW) | H | Pd (MW) | H | Pd (MW) |
|---|---|---|---|---|---|
| 1 | 1036 | 9 | 1924 | 17 | 1480 |
| 2 | 1110 | 10 | 2022 | 18 | 1628 |
| 3 | 1258 | 11 | 2106 | 19 | 1776 |
| 4 | 1406 | 12 | 2150 | 20 | 1972 |
| 5 | 1480 | 13 | 2072 | 21 | 1924 |
| 6 | 1628 | 14 | 1924 | 22 | 1628 |
| 7 | 1702 | 15 | 1776 | 23 | 1332 |
| 8 | 1776 | 16 | 1554 | 24 | 1184 |

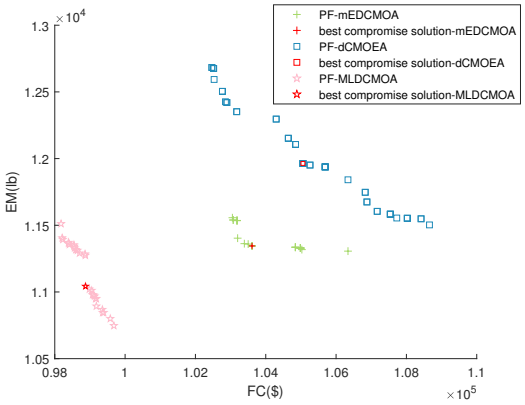



**Figure F2** Approximated PFs obtained by MLDCMOA and its counterparts at $T = 18$.

FC and economic EM tradeoff results and more sufficient alternatives. mEDCMOA takes second place in convergence, but the approximated PF obtained by dCMOEA have better diversity. The other two algorithms, NSGA-II-A and NSGA-II-B, can hardly generate tradeoff alternatives under this circumstance, probably due to lack of efficient prediction and diversity maintenance schemes. Wrong prediction misleads the search for real optimal regions, especially encountered with diversity loss resulted from dynamic environment and constraints.

**Table F3** Experimental results obtained by MLDCMOA with other algorithms at $T = 18$.

| Cyclicality | Algorithm | Optimal FC | | Optimal EM | | Best compromise solution | |
|---|---|---|---|---|---|---|---|
| | | FC($) | EM(lb) | FC($) | EM(lb) | FC($) | EM(lb) |
| T=1 | mEDCMOA | 62772 | 4478.2 | 64051 | 3929.2 | 63251 | 3998.7 |
| | dMOEA | 61907 | 4540.5 | 64606 | 3835.8 | 62632 | 4044.2 |
| | DC-NSGA-II-A | 63995 | 4318.9 | 64727 | 4238.2 | 64216 | 4270.3 |
| | DC-NSGA-II-B | 63080 | 5006.4 | 64252 | 3989.8 | 63161 | 6871.5 |
| | MLDCMOA | **62349** | **3899.2** | **62838** | **3830.1** | **62401** | **3881.4** |
| T=14 | mEDCMOA | 135868 | 19818.5 | 136092 | 19695.4 | 135868 | 19818.5 |
| | dMOEA | 131279 | 18122.3 | 133527 | 17841.8 | 132803 | 17896.9 |
| | DC-NSGA-II-A | - | - | - | - | - | - |
| | DC-NSGA-II-B | - | - | - | - | - | - |
| | MLDCMOA | **126398** | **17066.1** | **127481** | **16916.0** | **126518** | **16981.7** |
| T=20 | mEDCMOA | 133058 | 19664.9 | 133699 | 19207.6 | 133073 | 19650.7 |
| | dMOEA | 132638 | 19126.5 | 134827 | 18810.9 | 132659 | 18967.6 |
| | DC-NSGA-II-A | - | - | - | - | - | - |
| | DC-NSGA-II-B | - | - | - | - | - | - |
| | MLDCMOA | **131244** | **18532.8** | **131466** | **18392.9** | **131387** | **18393.6** |

To further testify the capability of tracking changes, we pick out compromise allocation solutions obtained by all algorithms at early, interim, and late stages during 24 cycles, i.e., $T = 1$, $T = 14$, and $T = 20$, shown as Tab. F3. In terms of optimal $FC$, where only the best $FC$ results are investigated, MLDCMOA can achieve the smallest $FC$ values in all three cycles. It is interesting to point out that the corresponding EM results are also the best compared to other algorithms. The same conclusion can be drawn from results of the optimal $EM$. Apart from these, the best compromise solutions of MLDCMOA exhibit outstanding among selected cycles, indicating its great adaptability with load demand switches.

Fig. F2 illustrates power allocation results for 10 units versus time cycles with the best compromise solutions obtained by MLDCMOA. Herein, output power consists of the power demand and network loss in the corresponding allocation plan. The assigned output power for each unit is plotted with a certain colored rectangle, which are stacked together to generate the whole histogram. The two broken lines indicate the optimized output power with different load demand during 24 cycles. As seen in Fig. F2, MLDCMOA responds well to the dynamic load demand. It should be emphasized that results for P1 to P5 show obvious changes during all cycles, since the first 5 units are large-capacity with wide and adjustable ranges in short time. Thus, they make up a higher proportion of the final allocation plan. This is also consistent with the potential assumption in DEED, suggesting the efficiency of MLDCMOA in keep trace of changing load demand and providing balanced allocation plans.

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