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Predefined-time adaptive neural network optimal output feedback control for nonlinear systems

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Over the past decade, intelligent optimal control for nonlinear systems has made great progress, and numerous intelligent optimal control policies have been developed. For example, Ref. [1] proposed an adaptive neural network (NN) optimal control scheme for unknown nonlinear systems. Ref. [2] developed a Laval Nozzle-based NN optimal intermittent control of nonlinear systems. Although the above mentioned intelligent optimal control schemes can achieve optimal control performance, they require controlled systems to reach their equilibrium points over an infinite time. To overcome this problem, the finite-time NN optimal control [3] was developed.

An inherent drawback of finite time optimal control is that the settling time function is not uniformly bounded, the convergent time to the equilibrium point may increase as the vector norm of the initial condition increases. Alternatively, Ref. [4] proposed a predefined-time NN optimal control policy for nonlinear systems. The presented predefined-time optimal controller is able to guarantee that the closed-loop system is stable and minimize the performance index over a given finite time. However, the optimal policy in [4] requires that the controlled system states must be available for measurement.

This study investigates the predefined-time adaptive NN optimal output feedback control problem for nonlinear systems with unmeasurable states and the input saturation. An NN state observer is first established to estimate the unmeasurable states. Then, a predefined-time adaptive NN optimal output feedback control scheme is proposed, where an NN is used as a critic network to learn the solutions of Hamilton-Jacobi-Bellman (HJB) equation. The proposed NN optimal control scheme can ensure that the controlled system is predefined-time stable and control inputs do not exceed the given saturation bounds.

Problem formulation. Consider the following nonlinear dynamical system with the input saturation:

$$\begin{cases}\dot{x}(t) = f(x(t)) + G(x(t))v(u(t)),\\ y(t) = Cx(t),\end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^s$ are the system state and output vector. $f(x(t))$ is an unknown nonlinear function. $G(x(t))$ is a known invertible matrix function and $C \in \mathbb{R}^{s\times m}$ is the system output matrix. $v(u(t)) = [\mathrm{sat}(u_1), \ldots, \mathrm{sat}(u_m)]^\top \in \mathbb{R}^m$ is the control input with actuators saturation. $\mathrm{sat}(u_i(t))$ is defined with saturation bounds $\bar{u} > 0$.

Assumption 1 ([1]). Assume that f and G are Lipschitz continuous with $f(0) = 0$.

Control objective. This article will develop an NN optimal output feedback control, which ensures that the nonlinear system (1) is predefined-time stable and optimized.

Main results. Let $BM(x(t)) = f(x(t)) - Ax(t)$, then Eq. (1) is

$$\dot{x} = Ax + BM(x) + G(x)v(u(t)), \tag{2}$$

where $A \in \mathbb{R}^{m\times m}$, $B \in \mathbb{R}^{m\times s}$ are known matrices and assume that $\mathrm{rank}(B) = s$. Similar to [1], by using an NN $\Lambda^{*\top}\psi(x(t))$ to approximate the unknown function $M(x(t))$, and assume that

$$M(x) = \Lambda^{*\top}\psi(x) + \epsilon, \tag{3}$$

where Λ^* is an ideal weight matrix, $\psi(\cdot)$ is the NN activation function and ϵ is the approximation error. From (2) and (3), one has

$$\dot{x}(t) = Ax(t) + B(\Lambda^{*\top}\psi(x) + \epsilon) + G(x(t))v(u(t)). \tag{4}$$

An NN state observer is constructed as

$$\begin{cases}\dot{\hat{x}}(t) = A\hat{x}(t) + B(\hat{\Lambda}^\top\psi(\hat{x}(t)) + \hat{\epsilon}) + G(\hat{x}(t))v(u(t)) + L\tilde{y},\\ \hat{y}(t) = C\hat{x}(t),\end{cases} \tag{5}$$

where $\tilde{y} = y - \hat{y}$, $L \in \mathbb{R}^{m\times s}$ is the observer gain. $\hat{\Lambda}$ and $\hat{\epsilon}$ are the estimates of Λ^* and ϵ. Select the observer gain L such that $A - LC$ is a stable matrix. There exists a positive definite matrix $P > 0$ satisfying the following equations:

$$(A - LC)^\top P + P(A - LC) = -c_0 I \quad \text{and} \quad B^\top P = C, \tag{6}$$

where $c_0 > 0$ and I is a identity matrix.

The adaptive laws of weight $\hat{\Lambda}$ and $\hat{\epsilon}$ are designed as

$$\dot{\hat{\Lambda}} = \mathrm{Proj}(\eta_1\psi(\hat{x})\tilde{y}^\top, \hat{\Lambda}) \quad \text{and} \quad \dot{\hat{\epsilon}} = \mathrm{Proj}(\eta_2\tilde{y}, \hat{\epsilon}), \tag{7}$$

where $\eta_1 > 0$ and $\eta_2 > 0$ are the learning rates of $\hat{\Lambda}$ and $\hat{\epsilon}$.

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Theorem 1. Consider the system (1) under Assumption 1, the designed observer (5) with weight adaptive laws (7) can ensure that the observer error $\tilde{x}(t)$ converges to zero.

Proof. Please see Appendix A.

Inspired by [4], define cost function $J(\hat{x})$ and value function $V_1(\hat{x}(t))$ as follows:

$$J(\hat{x}(t)) = \int_0^\infty \mathcal{K}(\hat{x}, u)\, dt, \quad V_1(\hat{x}(t)) = \int_t^\infty \mathcal{K}(\hat{x}, u)\, d\tau. \tag{8}$$

The optimal value function $V_1^*(\hat{x}(t))$ is defined as

$$V_1^*(\hat{x}(t)) = \min_u \int_t^\infty \mathcal{K}(\hat{x}, u)\, d\tau, \tag{9}$$

where $\mathcal{K}(\hat{x}, u) = \mathcal{K}_1(\hat{x}) + \mathcal{K}_2(\hat{x})u + U(u)$, $\mathcal{K}_1(\hat{x}) \geqslant 0$ and $\mathcal{K}_2(\hat{x})u \geqslant 0$ are continuous on $\mathbb{R}^m$. $U(u) = 2\int_0^u \left(\bar{u}\varphi^{-1}(s/\bar{u})\right)^\top \mathcal{R}(\hat{x})\, ds$, $\mathcal{R}(\hat{x}) > 0$ and $\varphi = \tanh(\cdot)$. Select $\mathcal{K}_1(\hat{x})$ and $\mathcal{K}_2(\hat{x})$ to satisfy

$$\begin{aligned} &\nabla V_1^{*\top}(\hat{x}) \left[Z(\hat{x}, y) - \frac{1}{2} G \mathcal{R}^{-1} (\mathcal{K}_2(\hat{x}) + \nabla V_1^{*\top}(\hat{x}) G)^\top \right] \\ &\leqslant -\frac{\gamma}{T_c} \left(\alpha V_1^{*\frac{p+1}{2}}(\hat{x}) + \beta V_1^{*\frac{q+1}{2}}(\hat{x}) \right), \end{aligned} \tag{10}$$

$$\begin{aligned} &\mathcal{K}_1(\hat{x}) + \nabla V_1^{*\top}(\hat{x}) Z(\hat{x}, y) - 2\bar{u} D^{*\top} \mathcal{R} \varphi(D^*/\bar{u}) \\ &+ U\big(-\bar{u}\varphi(D^*/\bar{u})\big) = 0, \end{aligned} \tag{11}$$

where $D^* = \frac{1}{2}\mathcal{R}^{-1}(\mathcal{K}_2(\hat{x}) + \nabla V_1^{*\top}(\hat{x}) G)^\top$, $\alpha > 0$, $\beta > 0$, $0 < p < 1$, $q > 1$ and $\gamma > 0$.

The Hamiltonian function is as follows:

$$\mathcal{H}(\hat{x}, u, \nabla V_1^\top) = \mathcal{K}(\hat{x}, u) + \nabla V_1^\top (Z(\hat{x}, y) + G(\hat{x})u). \tag{12}$$

Therefore, design the optimal control law $u^*(t)$ as

$$u^*(t) = -\bar{u} \tanh(D^*/\bar{u}). \tag{13}$$

By using a critic NN $\Lambda_c^{*\top}\phi(\hat{x})$ to simultaneously approximate the optimal value function $V_1^*(\hat{x})$ and its gradient $\nabla V_1^*(\hat{x})$, one has

$$V_1^*(\hat{x}) = \Lambda_c^{*\top}\phi(\hat{x}) + \epsilon_c, \nabla V_1^*(\hat{x}) = \nabla\phi^\top(\hat{x})\Lambda_c^* + \nabla\epsilon_c, \tag{14}$$

where Λ_c^* is an ideal weight vector, $\phi(\cdot)$ is the NN activation function and ϵ_c is the approximation error. Then, the estimations of $V_1^*(\hat{x})$ and $\nabla V_1^*(\hat{x})$ are

$$\hat{V}_1(\hat{x}) = \hat{\Lambda}_c^\top \phi(\hat{x}), \nabla\hat{V}_1(\hat{x}) = \nabla\phi(\hat{x})^\top \hat{\Lambda}_c. \tag{15}$$

Then, Eq. (13) is estimated by

$$\hat{u}(t) = -(\bar{u} - \xi)\tanh(\hat{D}/(\bar{u} - \xi)) - \xi, \tag{16}$$

where $\hat{D} = \frac{1}{2}\mathcal{R}^{-1}(\mathcal{K}_2(\hat{x}) + \hat{\Lambda}_c^\top \nabla\phi(\hat{x}) G)^\top$, $\xi = \frac{a\|\hat{x}\|^2}{b + \|\hat{x}\|^2} 1_m$ is a robustifying term to reduce the effect of the approximation errors. $1_m = [1, 1, \ldots, 1]^\top \in \mathbb{R}^m$ and b is a given positive constant. $a > 0$ is a design parameter.

Similar to [3], we design the weight adaptive law as

$$\begin{aligned} \dot{\hat{\Lambda}}_c = \Bigg(&-\varrho \frac{\Xi(t)}{\bar{\Psi}(t)} \left\lceil \frac{\varpi(t)}{\bar{\Psi}(t)} \right\rfloor^{\gamma_1} - \varrho \sum_{k=1}^d \frac{\Xi(t_k)}{\bar{\Psi}(t_k)} \left\lceil \frac{\varpi(t_k)}{\bar{\Psi}(t_k)} \right\rfloor^{\gamma_1} \\ &- \varrho \frac{\Xi(t)}{\bar{\Psi}(t)} \left\lceil \frac{\varpi(t)}{\bar{\Psi}(t)} \right\rfloor^{\gamma_2} - \varrho \sum_{k=1}^d \frac{\Xi(t_k)}{\bar{\Psi}(t_k)} \left\lceil \frac{\varpi(t_k)}{\bar{\Psi}(t_k)} \right\rfloor^{\gamma_2} \Bigg), \end{aligned} \tag{17}$$

where $\varpi(t) = \mathcal{H}(\hat{x}(t), \hat{u}(t), \hat{\Lambda}_c^\top(t)\nabla\phi(\hat{x}))$, $\varpi(t_k) = \mathcal{H}(\hat{x}(t_k), \hat{u}(t_k), \hat{\Lambda}_c^\top(t)\nabla\phi(\hat{x}))$, $\Xi(t) = \nabla\phi(\hat{x})\dot{\hat{x}}$ and $\bar{\Psi}(t) = \Xi(t)^\top \Xi(t) + 1$. $0 < \gamma_1 < 1$, $\gamma_2 > 1$, $\varrho > 0$ is the learning rate. $\lceil \cdot \rfloor^h = |\cdot|^h \mathrm{sign}(\cdot)$, where $h > 0$. Let $H = [\Xi(t_1)/\bar{\Psi}(t_1), \ldots, \Xi(t_d)/\ \bar{\Psi}(t_d)]$.

Assumption 2 ([4]). Assume that H is sufficiently rich, i.e., the matrix H satisfies $\mathrm{rank}(H) = N'$, N' is NN hidden layer number.

Theorem 2. Consider the system (1) under Assumptions 1 and 2. The optimal controller (16), with the NN weight updating law (17) can make the controlled system predefined-time stable within the predefined time T_c.

Proof. Please see Appendix B.

Simulation. Take the spacecraft system as an example to illustrate the effectiveness of the presented optimal control approach. The simulation results are shown in Figure 1. The simulation results indicate that the proposed optimal controller can guarantee the system is predefined-time stable within $T_c = 5.8097$ s and does not exceed the input saturation bound $\bar{u} = 0.5$. In addition, we make a simulation comparison with the existing NN output feedback control method. The detailed simulation and comparison results are provided in Appendix C.

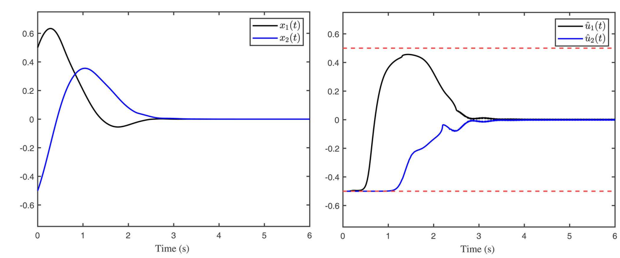

Figure 1 (Color online) The system states $x_1(t)$, $x_2(t)$ and controllers $\hat{u}_1(t)$, $\hat{u}_2(t)$.

Conclusion. This study has investigated the predefined-time optimal output feedback control problem for the nonlinear systems with unmeasured states and input saturation. An NN state observer has been established to estimate the unmeasured states. Then, based on the NN state observer, a predefined-time optimal output feedback scheme has been developed by using a critic network to learn the solutions of HJB equation and the optimal controller. The presented adaptive NN optimal control method can ensure that the controlled system is predefined-time stable and does not exceed the saturation bounds. Simulation results and comparisons validate the effectiveness of the proposed approach.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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