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$H\infty$ control of switched system under slow and fast switching

Jie ZHANG[1,2], Li YUAN[1,2] & Panshuo LI[1,2*]

[1] *School of Automation, Guangdong University of Technology, Guangzhou 510006, China*
[2] *Guangdong Provincial Key Laboratory of Intelligent Decision and Cooperative Control, Guangzhou 510006, China*

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Switched systems have been extensively studied over the past few decades [1–3]. The common Lyapunov function (CLF) has been widely recognized as a fundamental tool. However, it is difficult and sometimes impractical to require all subsystems to continuously share a common Lyapunov function. The multiple Lyapunov function (MLF) approach [4] is widely adopted due to its greater flexibility in verifying system stability (more details can be found in [5]). Moreover, the discretized Lyapunov function technique serves as a useful tool for reducing conservativeness under MLF in [6]. The piecewise quadratic form of MLFs is utilized in [7] to establish sufficient conditions for ensuring asymptotic stability with reduced conservatism. Based on the MLF approach, switching strategies have primarily focused on state-dependent switching, time-dependent switching, and their combination in [8]. Typical time-dependent switching signals comprise the dwell time (DT) switching signal, the average DT (ADT) switching signal and the mode-dependent ADT (MDADT) switching signal. It has been observed that the MDADT method provides greater practical value than the ADT method, as it allows each subsystem to be assigned a distinct ADT based on the proposed switching rule [9]. The slow and fast switching strategy is first introduced in [10], where more general stability and stabilization conditions are established under time-dependent switching, yielding a tighter bound on the MDADT or ADT. However, the disturbance attenuation performance under this strategy has not been studied. It is essential to require that the control system is robust to external disturbance. Motivated by the above-mentioned work, this study considers a discrete-time linear switched system, where H_∞ control problems under slow and fast switching are investigated. We propose a new MLF, where the MDLF is adopted for the unstable subsystems, and the discretized Lyapunov function is used for the stable subsystems. Then H_∞ performance analysis is given, and a set of mode-dependent H_∞ state-feedback controllers is designed.

Problem formulation. Consider the linear switched system

$$\begin{aligned} &x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k) + D_{\sigma(k)}\omega(k),\\ &y(k) = C_{\sigma(k)}x(k), \end{aligned} \tag{1}$$

where $x(k)$, $u(k)$, $y(k)$ and $\omega(k)$ are the state vector, the control input, the output and the disturbance input, respectively. $\sigma(k)$ is called switching signal, and $\sigma(k) : [0,\infty) \to \mathcal{S} \triangleq \{1,2,\dots,M\}$, where $M > 1$ denotes the number of all subsystems. To facilitate the following development, we assume that the first m subsystems are stable, and the last n subsystems are unstable, that is, $M = m + n$. $A_{\sigma(k)}, B_{\sigma(k)}, C_{\sigma(k)}, D_{\sigma(k)}$ are real matrices with appropriate dimensions, $\sigma(k) \in \mathcal{S}$, $\mathcal{S} = \Xi \bigcup \Theta$, $\Xi \triangleq \{1,2,\dots,m\}$, $\Theta \triangleq \{m+1, m+2,\dots,M\}$, where Ξ and Θ denote the subsets of stable and unstable subsystems, respectively. Inspired by discretized Lyapunov functions in [6] and multiple discontinuous Lyapunov functions in [10], we use k_i to represent the starting time instant of the ith subsystem and the time interval $[k_i, k_l)$ to represent the dwell time of the ith subsystem. For $i \in \Theta$, time interval $[k_i, k_l)$ is partitioned into G_i segments. G_i denotes the number of segments of the Lyapunov function in the ith unstable subsystem. $H_i^g(k_i, k_l)$, $\forall g \in \{1,2,\dots,G_i\}$ denotes the length of each segment. Define $J^j_{\sigma(k_i)} = \sum_{g=1}^{j} H_i^g(k_i,k_l)$, where $J^0_{\sigma(k_i)} = 0$, $\forall j \in \{0,1,\dots,G_i\}$, and let $L_{\sigma(k_i),j} = [k_i + J^j_{\sigma(k_i)}, k_i + J^{j+1}_{\sigma(k_i)})$, $j \in \Re_i = \{0,1,2,\dots,G_i - 1\}$. The time interval $[k_i,k_l)$ can be described as $[k_i,k_l) = \bigcup_j L_{\sigma(k_i),j}$, $j \in \Re_i$. We construct a Lyapunov function in the following form:

$$V(k) = \begin{cases} V_s(k), & \forall s \in \Xi,\\ V_u^j(k), & \forall u \in \Theta, \end{cases} \tag{2}$$

where $V_s(k)$, $V_u^j(k) \in \mathcal{C}^1$, $j \in \Re_u = \{0,1,\dots,G_u - 1\}$. Then, for $s, v \in \Xi$, assume that $V_s(x(k)) < \mu_s V_v(x(k))$, where $\mu_s > 1$. We assume the sth subsystem is activated during $[k_s, k_l)$. Let $\tau_{\min} = \min_{s\in\Xi}\{\tau_{\sigma s}\}$. Then we divide the interval $[k_s, k_s + \tau_{\min})$ into L segments. Let $N_{\sigma(k_s),q} = [k_s + \theta^q, k_s + \theta^{q+1})$, equal length $h = \frac{\tau_{\min}}{L}$, and then $\theta^q = qh = q(\frac{\tau_{\min}}{L})$, $q = 0,1,\dots,L-1$. The time interval $[k_s, k_s + \tau_{\min})$ can be described as $[k_s, k_s + \tau_{\min}) = \bigcup_q N_{\sigma(k_s),q}$, $q = 0,1,\dots,L-1$.

We assume the matrix function $P_s(k)$ is piecewise linear in the minimal dwell time interval $[k_s, k_s + \tau_{\min})$, and choose discretized Lyapunov functions

$$V_s(k) = x^{\mathrm{T}}(k)P_s(k)x(k), s \in \Xi, \tag{3}$$

* Corresponding author (email: panshuoli812@gdut.edu.cn)

where

$$P_s(k) = \begin{cases} (1-\alpha)P_s^q + \alpha P_s^{q+1}, & k \in N_{s,q}, q = 0, 1, \ldots, L-1, \\ P_s^L, & k \in [k_s + \tau_{\min}, k_l), \end{cases} \tag{4}$$

and $\alpha = \frac{(k-k_s-\theta^q)}{h}$.

Then, for $u, z \in \Theta$, assume that $V_u^{G_u-1}(x(k)) < \mu_u V_z^0(x(k))$, $V_u^j(x(k_u + J_u^j)) \leqslant \eta_u V_u^{j-1}(x(k_u^- + J_u^j))$, where $\mu_u > 1$, $0 < \eta_u \leqslant 1$, $V_s(x(k)) < \mu_s V_u(x(k))$. We assume the uth subsystem is activated during $[k_u, k_l)$. With the definition of G_u, $J_{\sigma(k_u)}^j$ and $H_u^g(k_u, k_l)$, we select the MDLF candidate in the form

$$V_u(k) = x^{\mathrm{T}}(k) P_u^j x(k), u \in \Theta, k \in L_{\sigma(k_u),j}, \tag{5}$$

where P_u^j are positive definite matrices. The diagram of the Lyapunov function and Lyapunov matrices is shown in Appendix A. The stability analysis and the weighted H_∞ performance analysis of the switched system (1) are given in Appendix A. Based on it, the H_∞ controller design of the switched system (1) is given in Theorem 1.

Theorem 1. Consider the linear switched system (1). Given scalars $-1 < \lambda_s < 0$, $\mu_s > 1$, $s \in \Xi$ and $\lambda_u > 0$, $\mu_u > 1$, $0 < \eta_u \leqslant 1$, $0 < \mu_u \eta_u^{G_u-1} < 1$, $u \in \Theta$, $\bar{\lambda}_i = 1 + \lambda_i$, $\gamma_i > 0$, $i \in \mathcal{S}$. With conditions (10)–(12), if there exist a set of matrices U_s^q, $U_u^j > 0$, $q = 0, 1, \ldots, L$, $j = 0, 1, \ldots, G_{u-1}$, any matrices W_i and T_i with appropriate dimensions, satisfying

$$\begin{bmatrix} \frac{h+1}{h}\bar{\lambda}_s(U_s^q - W_s^{\mathrm{T}} - W_s) & * & * & * & * \\ 0 & -\gamma_s^2 I & * & * & * \\ A_s W_s + B_s T_s & D_s & -U_s^q & * & * \\ W_s & 0 & 0 & -\frac{h}{\bar{\lambda}_s} U_s^{q+1} & * \\ C_s W_s & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{6}$$

$$\begin{bmatrix} \frac{h-1}{h}\bar{\lambda}_s U_s^{q+1} + \frac{\bar{\lambda}_s}{h} U_s^q - \bar{\lambda}_s(W_s^{\mathrm{T}} + W_s) & * & * & * \\ 0 & -\gamma_s^2 I & * & * \\ A_s W_s + B_s T_s & D_s & -U_s^{q+1} & * \\ C_s W_s & 0 & 0 & -I \end{bmatrix} < 0, \tag{7}$$

$$\begin{bmatrix} \bar{\lambda}_s(U_s^L - W_s^{\mathrm{T}} - W_s) & * & * & * \\ 0 & -\gamma_s^2 I & * & * \\ A_s W_s + B_s T_s & D_s & -U_s^L & * \\ C_s W_s & 0 & 0 & -I \end{bmatrix} < 0, \tag{8}$$

$$\begin{bmatrix} \bar{\lambda}_u(U_u^j - W_u^{\mathrm{T}} - W_u) & * & * & * \\ 0 & -\gamma_u^2 I & * & * \\ A_u W_u + B_u T_u & D_u & -U_u^j & * \\ C_u W_u & 0 & 0 & -I \end{bmatrix} < 0, \tag{9}$$

$$U_u^{j-1} \leqslant \eta_u U_u^j, \ u \in \Theta, \ j \neq 0, \tag{10}$$

$$U_s^L \leqslant \mu_v U_v^0, \quad (s, v) \in \Xi \times \mathcal{S}, \tag{11}$$

$$U_u^{G_u-1} \leqslant \mu_s U_s^0, \quad (u, s) \in \Theta \times \Xi, \tag{12}$$

then there exists a set of mode-dependent H_∞ state-feedback controllers such that system (1) is GUES for any MDDT switching signals satisfying

$$\tau_{\sigma s} \geqslant -\frac{\ln \mu_s}{\ln(\lambda_s + 1)}, \ \forall s \in \Xi, \tag{13}$$

$$\tau_{\sigma u} \leqslant -\frac{\ln \mu_u + (G_u - 1)\ln \eta_u}{\ln(\lambda_u + 1)}, \ \forall u \in \Theta, \tag{14}$$

and satisfies $\sum_{\tau=k_0}^{\infty} \prod_{s=1}^{m} (\mu_s^{-\frac{1}{\tau_{\sigma s}}})^{(\tau-k_0)} y_\tau^{\mathrm{T}} y_\tau < \gamma_m^2 \sum_{\tau=k_0}^{\infty} \omega_\tau^{\mathrm{T}} \omega_\tau$ for all nonzero $\omega(k) \in l_2[0, \infty)$. The weighted H_∞ performance is $\gamma_m = \sqrt{\prod_{s=0}^{m} \mu_s^{N_{0s}} \prod_{u=m+1}^{n} \eta_u^{-(G_u-1)}} \times \max_{i \in \mathcal{S}}\{\gamma_i\}$. Moreover, if a feasible solution exists, the admissible controller gains K_i are given by $K_i = T_i W_i^{-1}$.

Proof. See Appendix B for the proof of Theorem 1.

Remark 1. The simulations are provided in Appendix C to illustrate the efficiency of the proposed results.

Conclusion. The H_∞ control problem for discrete-time linear switched systems consisting of stable and unstable subsystems has been investigated by proposing a new MLF approach. Based on the MDDT approach, fast switching is applied to unstable subsystems, whereas slow switching is adopted for stable subsystems. Then, the MDLF and the discretized Lyapunov function are used during the running time of unstable subsystems and stable subsystems, respectively. Our approach provides a method to compute the upper and lower bounds of admissible mode-dependent dwell time. In terms of the linear matrix inequalities formulation, we have derived the existence conditions for H_∞ controllers. Numerical examples illustrate the effectiveness of the proposed results.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Sun Y, Tian Y, Xie X J. Stabilization of positive switched linear systems and its application in consensus of multiagent systems. IEEE Trans Automat Contr, 2017, 62: 6608–6613
2 Ma R C, An S, Fu J. Dwell-time-based stabilization of switched positive systems with only unstable subsystems. Sci China Inf Sci, 2021, 64: 119205
3 Wan L, Li P, Li P, et al. Stabilization of networked periodic piecewise linear systems under asynchronous switching with packet dropouts. J Franklin Inst, 2022, 359: 7733–7752
4 Branicky M S. Multiple Lyapunov functions and other analysis tools for switched and hybrid systems. IEEE Trans Automat Contr, 1998, 43: 475–482
5 Lin H, Antsaklis P J. Stability and stabilizability of switched linear systems: a survey of recent results. IEEE Trans Automat Contr, 2009, 54: 308–322
6 Xiang W, Xiao J. Stabilization of switched continuous-time systems with all modes unstable via dwell time switching. Automatica, 2014, 50: 940–945
7 Zhu Y, Zheng W X. Multiple Lyapunov functions analysis approach for discrete-time-switched piecewise-affine systems under dwell-time constraints. IEEE Trans Automat Contr, 2020, 65: 2177–2184
8 Alur R, Henzinger T A, Sontag E D. Hybrid Systems III: Verification and Control. Berlin: Springer, 1996
9 Zhang J, Han Z, Zhu F, et al. Stability and stabilization of positive switched systems with mode-dependent average dwell time. Nonlinear Anal-Hybrid Syst, 2013, 9: 42–55
10 Zhao X, Shi P, Yin Y, et al. New results on stability of slowly switched systems: a multiple discontinuous Lyapunov function approach. IEEE Trans Automat Contr, 2017, 62: 3502–3509