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• **Supplementary File** •

$H\infty$ control of switched system under slow and fast switching

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Appendix A

Consider a linear switched system:

$$\begin{aligned} x(k+1) &= A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k) + D_{\sigma(k)}\omega(k), \\ y(k) &= C_{\sigma(k)}x(k). \end{aligned} \tag{A1}$$

Some necessary definitions are presented for developing the subsequent results.

Definition 1. [1] For switched linear system (A1) with $\omega \equiv 0$ under certain switching signal $\sigma(k)$, if there exist constants $c \geqslant 0$ and $\lambda \in [0,1)$, such that $\|x(k)\| \leqslant c\lambda^k \|x(k_0)\|$, $\forall k \geqslant k_0$ holds, then system (A1) is said to be globally uniformly exponentially stable (GUES).

Definition 2. [2] For any $k_2 > k_1 > 0, s \in \Xi, u \in \Theta$, let $N_{\sigma s}(k_1,k_2)$ and $N_{\sigma u}(k_1,k_2)$ denote the number of switchings as the s^{th} subsystem and the u^{th} subsystem is active within the interval $[k_1,k_2]$, respectively. And let $T_s(k_1,k_2)$ and $T_u(k_1,k_2)$ denote the total running time of the s^{th} slow switching subsystem and the u^{th} fast switching subsystem over the interval $[k_1,k_2]$, respectively. If there exist constants $\tau_{\sigma s}$, $\tau_{\sigma u}$, N_{0s} and N_{0u} such that $N_{\sigma s}(k_1,k_2) \leqslant N_{0s} + \frac{T_s(k_1,k_2)}{\tau_{\sigma s}}$, $N_{\sigma u}(k_1,k_2) \geqslant N_{0u} + \frac{T_u(k_1,k_2)}{\tau_{\sigma u}}$, where $\tau_{\sigma s}$, $\tau_{\sigma u}$ are called Mode-Dependent Dwell Time (MDDT) of stable subsystem, unstable subsystem, respectively. N_{0s} and N_{0u} are called the mode-dependent chatter bound of stable subsystem, unstable subsystem, respectively.

Definition 3. [3] For system (A1), given scalars $\gamma > 0$, $0 < \alpha < 1$. If, under zero initial conditions, for all nonzero $\omega(k) \in l_2[0,\infty)$, the inequality $\sum_{\tau=k_0}^{\infty} \alpha^{(\tau-k_0)} y_\tau^T y_\tau < \gamma^2 \sum_{\tau=k_0}^{\infty} \omega_\tau^T \omega_\tau$ holds, then system (A1) is GUES with an weighted H_∞ performance γ.

Lemma A1. [4] For the given matrix $P > 0$, any matrix W with appropriate dimensions, the following inequality holds $-W^{\mathrm{T}}P^{-1}W \leqslant P - W^{\mathrm{T}} - W$.

Appendix A.1 Stability analysis

We construct a Lyapunov function in the following form:

$$V(k) = \begin{cases} V_s(k), \forall s \in \Xi, \\ V_u^j(k), \forall u \in \Theta, \end{cases} \tag{A2}$$

where $V_s(k)$, $V_u^j(k) \in \mathcal{C}^1$, $j \in \Re_u = \{0,1,\cdots,G_u-1\}$. The matrix function $P_s(k)$ is assumed to be piecewise linear in the minimal dwell time interval $[k_s, k_s+\tau_{min})$, we choose discretized Lyapunov functions

$$V_s(k) = x^{\mathrm{T}}(k)P_s(k)x(k), s \in \Xi, \tag{A3}$$

where

$$P_s(k) = \begin{cases} (1-\alpha)P_s^q + \alpha P_s^{q+1}, k \in N_{s,q}, q = 0,1,\ldots,L-1, \\ P_s^L, k \in [k_s+\tau_{\min}, k_l), \end{cases} \tag{A4}$$

and $\alpha = \frac{(k-k_s-\theta^q)}{h}$. We assume the u^{th} subsystem is activated during $[k_u,k_l)$. With the definition of G_u, $J_{\sigma(k_u)}^j$ and $H_u^g(k_u,k_l)$, we select the MDLF candidate in the form:

$$V_u(k) = x^{\mathrm{T}}(k)P_u^j x(k), u \in \Theta, k \in L_{\sigma(k_u),j}, \tag{A5}$$

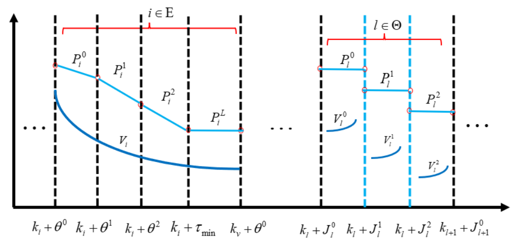

Figure A1 Sketch of Lyapunov matrices$(L = 3, G = 3)$

where P_u^j are positive definite matrices. The diagram of the Lyapunov function and Lyapunov matrices are shown in Fig.A1 Based on the Lyapunov function constructed in (A2), general exponential stability conditions of system (A1) are given in the following theorem.

Theorem A1. Consider the switched linear system (A1) and given scalars $-1 < \lambda_s < 0$, $\mu_s > 1$, $s \in \Xi$, $\lambda_u > 0$, $\mu_u > 1$, $0 < \eta_u \leqslant 1$, $0 < \mu_u {\eta_u}^{G_u - 1} < 1$, $u \in \Theta$. Suppose that there exist $\mathcal{C}^1$ nonnegative functions $V_s(x(k)) : \mathbb{R}^n \to \mathbb{R}$, $V_u^j(x(k)) : \mathbb{R}^n \to \mathbb{R}$, $j = 1, 2, ..., G_u$, and two class $\mathcal{K}_\infty$ functions κ_1 and κ_2, such that

1.

$$\kappa_1(\|x(k)\|) \leqslant V_s(x(k)) \leqslant \kappa_2(\|x(k)\|)\ , \tag{A6}$$

$$\kappa_1(\|x(k)\|) \leqslant V_u^j(x(k)) \leqslant \kappa_2(\|x(k)\|)\ , \tag{A7}$$

2.

$$V_s(x(k+1)) \leqslant (\lambda_s + 1) V_s(x(k)), \tag{A8}$$

$$V_u^j(x(k+1)) \leqslant (\lambda_u + 1) V_u^j(x(k)), \tag{A9}$$

3. $\forall(\sigma(k_i) = s, \sigma(k_i^-) = v) \in \Xi \times \Xi, s \neq v$ and $\forall(\sigma(k_i) = u), \sigma(k_i^-) = s) \in \Theta \times \Xi$,

$$V_s(x(k_i)) \leqslant \mu_s V_v(x(k_i^-)), \tag{A10}$$

$$V_s(x(k_i)) \leqslant \mu_s V_u(x(k_i^-)), \tag{A11}$$

$$V_u^0(x(k_i)) \leqslant \mu_u V_s(x(k_i^-)), \tag{A12}$$

$$V_u^j(x(k_i + J_u^j)) \leqslant \eta_u V_u^{j-1}(x(k_i^- + J_u^j)), \tag{A13}$$

then the system is GUES under switching signals with MDDT satisfying:

$$\tau_{\sigma s} \geqslant -\frac{\ln \mu_s}{\ln(\lambda_s + 1)}, \forall s \in \Xi, \tag{A14}$$

$$\tau_{\sigma u} \leqslant -\frac{\ln \mu_u + (G_u - 1)\ln \eta_u}{\ln(\lambda_u + 1)}, \forall u \in \Theta. \tag{A15}$$

Proof. Without loss of generality, we use $k_1, k_2, \cdots, k_i, k_{i+1}, \cdots, k_{N_\sigma(k_0, K)}$ to denote the switching time on time interval

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$[k_0, K)$. We assume that the first m subsystems are stable, and the last n subsystems are unstable, that is, $M = m + n$.

$$
\begin{aligned}
& V_{\sigma(k_i)}^{G_{\sigma(k_i)}-1}(x(k)) \\
& \leqslant (1+\lambda_{\sigma(k_i)})^{k-k_i} {\eta_{\sigma(k_i)}}^{G_{\sigma(k_i)}-1} V_{\sigma(k_i)}^{0}(x(k_i)) \\
& \leqslant \mu_{\sigma(k_i)} (1+\lambda_{\sigma(k_i)})^{k-k_i} {\eta_{\sigma(k_i)}}^{G_{\sigma(k_i)}-1} V_{\sigma(k_i)-1}^{G_{\sigma(k_i)}-1}(x(k_i^-)) \\
& \leqslant \mu_{\sigma(k_i)} {\eta_{\sigma(k_i)}}^{G_{\sigma(k_i)}-1} {\eta_{\sigma(k_{i-1})}}^{G_{\sigma(k_{i-1})}-1} (1+\lambda_{\sigma(k_i)})^{k-k_i} (1+\lambda_{\sigma(k_{i-1})})^{k_i-k_{i-1}} V_{\sigma(k_{i-1})}^{0}(x(k_{i-1})) \\
& \vdots \\
& \leqslant \prod_{r=n}^{N_\sigma} \left[\mu_{\sigma(k_r)} {\eta_{\sigma(k_r)}}^{G_{\sigma(k_r)}-1} (1+\lambda_{\sigma(k_r)})^{k_{r+1}-k_r} \right] V_{\sigma(k_{n-1})}(x(k_n^-)) \\
& \vdots \\
& \leqslant \prod_{s=1}^{m} \mu_s^{N_{\sigma s}} \prod_{s=1}^{m} (1+\lambda_s)^{T_s(k_0,k)} \prod_{u=m+1}^{M} (\mu_u \eta_u^{G_u-1})^{N_{\sigma u}} \prod_{u=m+1}^{M} (1+\lambda_u)^{T_u(k_0,k)} V_{\sigma(k_0)}(x(k_0)) \\
& \leqslant \prod_{s=1}^{m} \mu_s^{\frac{T_s(k_0,k)}{\tau_{\sigma s}}} \prod_{s=1}^{m} (1+\lambda_s)^{T_s(k_0,k)} \prod_{u=m+1}^{M} (\mu_u \eta_u^{G_u-1})^{\frac{T_u(k_0,k)}{\tau_{\sigma u}}} \prod_{u=m+1}^{M} (1+\lambda_u)^{T_u(k_0,k)} V_{\sigma(k_0)}(x(k_0)) \\
& \leqslant \prod_{s=1}^{m} \left[\mu_s^{\frac{1}{\tau_{\sigma s}}} (1+\lambda_s) \right]^{T_s(k_0,k)} \prod_{u=m+1}^{M} \left[(\mu_u \eta_u^{G_u-1})^{\frac{1}{\tau_{\sigma u}}} (1+\lambda_u) \right]^{T_u(k_0,k)} V_{\sigma(k_0)}(x(k_0)).
\end{aligned}
$$

With (A14), (A15) and $0 < 1+\lambda_s < 1$, $1+\lambda_u > 1$, $\mu_s, \mu_u > 1$, $0 < \eta_u \leqslant 1$, $0 < \mu_u {\eta_u}^{G_u-1} < 1$, one has that $0 < \mu_s^{\frac{1}{\tau_{\sigma s}}}(1+\lambda_s) < 1$, $0 < \left(\mu_u \eta_u^{G_u-1}\right)^{\frac{1}{\tau_{\sigma u}}}(1+\lambda_u) < 1$. Set $\varsigma^2 = \max_{s\in\Xi, u\in\Theta}\{\mu_s^{\frac{1}{\tau_{\sigma s}}}(1+\lambda_s), \left(\mu_u \eta_u^{G_u-1}\right)^{\frac{1}{\tau_{\sigma u}}}(1+\lambda_u)\}$. One gets that $\|x(k)\| \leqslant \sqrt{\frac{\kappa_2}{\kappa_1}} \varsigma^{(k-k_0)} \|x(k_0)\|$, where $\kappa_1 = \min_{s\in\Xi, u\in\Theta}\{\lambda_{min}(P_s^q), \lambda_{min}(P_u^j)\}$, $\kappa_2 = \max_{s\in\Xi, u\in\Theta}\{\lambda_{max}(P_s^j), \lambda_{max}(P_u^j)\}$, $q = 0, 1, 2, ..., L$, $j = 0, 1, 2, ..., G_u - 1$. Therefore, the GUES can be guaranteed. The proof is completed.

Remark 1. It has been observed that mode-dependent average dwell time method provides greater practical value than average dwell time method, as it allows each subsystem to be assigned a distinct average dwell time based on the proposed switching rule [5]. It is worth noting that the mode-dependent dwell-time switching signal typically considers an upper bound for each mode, as shown in equation (A15). However, our proposed slow and fast switching strategy is different from the above mentioned one, which facilitate one to find the upper and lower bounds of admissible mode- dependent dwell time bounds by equation (A14)-(A15).

Appendix A.2 H_∞ performance analysis

In this section, based on Theorem A1, the weighted H_∞ performance for system (A1) is analyzed in the following theorem.

Theorem A2. Consider the switched system (A1) and let $-1 < \lambda_s < 0$, $\mu_s > 1$, $s \in \Xi$, $\lambda_u > 0$, $\mu_u > 1$, $0 < \eta_u \leqslant 1$, $0 < \mu_u {\eta_u}^{G_u-1} < 1$, $u \in \Theta$ and $\gamma_i > 0, \forall i \in \mathcal{S}$ are given constants. Suppose that there exist $\mathcal{C}^1$ nonnegative functions $V_s(x(k)) : \mathbb{R}^n \to \mathbb{R}$, $V_u^j(x(k)) : \mathbb{R}^n \to \mathbb{R}$, $j = 1, 2, ..., G_u$, with $V_{\sigma(k_0)}(x(k_0)) \equiv 0$. With condition of (A6), (A7) and (A10)-(A13), such that

$$V_s(x(k+1)) \leqslant (\lambda_s + 1) V_s(x(k)) - \Gamma(k), \tag{A16}$$

$$V_u^j(x(k+1)) \leqslant (\lambda_u + 1) V_u^j(x(k)) - \Gamma(k), \tag{A17}$$

under switching signals with MDDT (A14)-(A15). The system is GUES and has an weighted l_2-gain no greater than $\gamma_m = \sqrt{\prod_{s=1}^{m} \mu_s^{N_{0s}} \prod_{u=m+1}^{M} \eta_u^{-(G_u-1)}} \times \max_{i\in\mathcal{S}}\{\gamma_i\}$, where $\Gamma(k) \triangleq y_k^T y_k - \gamma_i^2 w_k^T w_k$,

Proof. Based on the Theorem A1 and the Lyapunov function (A2), for $i \in \Theta$, to represent the number of segments between the time interval $[k_i, k_l)$, $j \in \{0, 1, \cdots, G_i\}$, an index function is defined as follows:

$$\mathcal{L}_{L(\sigma(k_i),j)}(k) = \begin{cases} 1, & k \in L(\sigma(k_i), j), \\ 0, & k \notin L(\sigma(k_i), j). \end{cases} \tag{A18}$$

For $k \in L(\sigma(k_i), j)$, we consider a Lyapunov function in the form given by (A2). It can be derived from (A13) and (A17) that

$$V_{\sigma(k_i)}^{G_i-1}(x(k))$$

$$\leqslant (1+\lambda_{\sigma(k_i)})^{k-k_i}\eta_{\sigma(k_i)}{}^{G_i-1}V^0_{\sigma(k_i)}\left(x(k_i+J^0_{\sigma(k_i)})\right)-\sum_{\tau=k_i+J^0_{\sigma(k_i)}}^{k-1}(1+\lambda_{\sigma(k_i)})^{k-1-\tau}\eta_{\sigma(k_i)}{}^{N_{\sigma(k_i)}(\tau,k-1)}\Gamma(\tau), \quad \text{(A19)}$$

where $\Gamma(k)\triangleq y_k^Ty_k-\gamma_i^2w_k^Tw_k$. Let $U_{\sigma(k_i)}(k+1,k_i)\triangleq G_i-1-\sum_{i=0}^{G_i-1}i\mathcal{L}_{L(\sigma(k_i),j)}(k)$, whicn stands for the number of segments between time interval $[k_i,k_l)$ whose values can be chosen as $\{0,1,...,G_i-1\}$. Then, from (A10)-(A13) and (A16)-(A17), one has

$$\begin{aligned}
&V_{\sigma(k_i)}^{G_{\sigma(k_i)}-1}(x(k))\\
&\leqslant\prod_{s=1}^m\mu_s^{N_{\sigma s}}\prod_{s=1}^m(1+\lambda_s)^{T_s(k_0,k)}\prod_{u=m+1}^M(\mu_u\eta_u^{G_u-1})^{N_{\sigma u}}\prod_{u=m+1}^M(1+\lambda_u)^{T_u(k_0,k)}\eta_{\sigma(k_{n-1})}^{U_{\sigma u}(\tau+1,k)}V_{\sigma(k_0)}(x(k_0))\\
&-\sum_{\tau=k_0}^{k-1}\prod_{s=1}^m\mu_s^{N_{\sigma s}(\tau,k)}\prod_{u=m+1}^M(\mu_u\eta_u{}^{G_u-1})^{N_{\sigma u}(\tau,k)}\eta_u^{U_{\sigma u}(\tau+1,k)}\times\prod_{s=1}^m(1+\lambda_s)^{T_s(\tau+1,k)}\prod_{u=m+1}^M(1+\lambda_u)^{T_u(\tau+1,k)}\Gamma(\tau).
\end{aligned}\quad \text{(A20)}$$

Then, let $V_{\sigma(k_0)}(x(k_0))=0$ and from $V_{\sigma(k_i)}^{G_i-1}(x(k))\geqslant 0$. One has

$$\sum_{\tau=k_0}^{k-1}\prod_{s=1}^m\mu_s^{N_{\sigma s}(\tau,k)}\prod_{u=m+1}^M(\mu_u\eta_u{}^{G_u-1})^{N_{\sigma u}(\tau,k)}\eta_u^{U_{\sigma u}(\tau+1,k)}\times\prod_{s=1}^m(1+\lambda_s)^{T_s(\tau+1,k)}\prod_{u=m+1}^M(1+\lambda_u)^{T_u(\tau+1,k)}\Gamma(\tau)\leqslant 0. \quad \text{(A21)}$$

One has

$$\begin{aligned}
&\sum_{\tau=k_0}^{k-1}\prod_{s=1}^m\mu_s^{N_{\sigma s}(\tau,k)}\prod_{u=m+1}^M(\mu_u\eta_u{}^{G_u-1})^{N_{\sigma u}(\tau,k)}\eta_u^{U_{\sigma u}(\tau+1,k)}\times\prod_{s=1}^m(1+\lambda_s)^{T_s(\tau+1,k)}\prod_{u=m+1}^M(1+\lambda_u)^{T_u(\tau+1,k)}y_\tau^Ty_\tau\\
&\leqslant\sum_{\tau=k_0}^{k-1}\prod_{s=1}^m\mu_s^{N_{\sigma s}(\tau,k)}\prod_{u=m+1}^M(\mu_u\eta_u{}^{G_u-1})^{N_{\sigma u}(\tau,k)}\eta_u^{U_{\sigma u}(\tau+1,k)}\times\prod_{s=1}^m(1+\lambda_s)^{T_s(\tau+1,k)}\prod_{u=m+1}^M(1+\lambda_u)^{T_u(\tau+1,k)}\gamma_i^2w_\tau^Tw_\tau.
\end{aligned}\quad \text{(A22)}$$

With the definition of $\Gamma(k)$ and multiplying both sides of (A22) by $\prod_{s=1}^m\mu_s^{-N_{\sigma_s}(k_0,k)}\prod_{u=m+1}^M\eta_u^{-U_{\sigma u}(k_0+1,k)}$, one has

$$\begin{aligned}
&\sum_{\tau=k_0}^{k-1}\prod_{s=1}^m\mu_s^{-N_{\sigma s}(k_0,\tau)}\prod_{u=m+1}^M\eta_u^{-U_{\sigma u}(k_0+1,\tau)}\prod_{u=m+1}^M(\mu_u\eta_u{}^{G_u-1})^{N_{\sigma u}(\tau,k)}\times\prod_{s=1}^m(1+\lambda_s)^{T_s(\tau+1,k)}\prod_{u=m+1}^M\lambda_u^{T_u(\tau+1,k)}y_\tau^Ty_\tau\\
&\leqslant\sum_{\tau=k_0}^{k-1}\prod_{s=1}^m\mu_s^{-N_{\sigma s}(k_0,\tau)}\prod_{u=m+1}^M\eta_u^{-U_{\sigma u}(k_0+1,\tau)}\prod_{u=m+1}^M(\mu_u\eta_u{}^{G_u-1})^{N_{\sigma u}(\tau,k)}\times\prod_{s=1}^m(1+\lambda_s)^{T_s(\tau+1,k)}\prod_{u=m+1}^M\lambda_u^{T_u(\tau+1,k)}\gamma_i^2w_\tau^Tw_\tau.
\end{aligned}\quad \text{(A23)}$$

From (A15), one has $0<\mu_u\eta_u^{G_u-1}<1$ and $U_{\sigma u}(k_0,\tau)\leqslant G_u-1$, with $-1<\lambda_s<0$, $\lambda_u>0$, by Definition 2 and (A23),

$$\begin{aligned}
&\sum_{\tau=k_0}^{k-1}\prod_{s=1}^m\mu_s^{-N_{\sigma s}(k_0,\tau)}\prod_{u=m+1}^M\eta_u^{-U_{\sigma u}(k_0+1,\tau)}\prod_{u=m+1}^M(\mu_u\eta_u{}^{G_u-1})^{N_{\sigma u}(\tau,k)}\times\prod_{s=1}^m(1+\lambda_s)^{T_s(\tau+1,k)}\prod_{u=m+1}^M(1+\lambda_u)^{T_u(\tau+1,k)}y_\tau^Ty_\tau\\
&\leqslant\sum_{\tau=k_0}^{k-1}\prod_{s=1}^m\mu_s^{-N_{\sigma s}(k_0,\tau)}\prod_{u=m+1}^M\eta_u^{-U_{\sigma u}(k_0+1,\tau)}\prod_{u=m+1}^M(\mu_u\eta_u{}^{G_u-1})^{N_{\sigma u}(\tau,k)}\times\prod_{s=1}^m(1+\lambda_s)^{T_s(\tau+1,k)}\prod_{u=m+1}^M(1+\lambda_u)^{T_u(\tau+1,k)}\gamma_i^2w_\tau^Tw_\tau,\\
&\to\sum_{\tau=k_0}^{k-1}\prod_{s=1}^m\mu_s^{-N_{\sigma s}(k_0,\tau)}\prod_{u=m+1}^M(\mu_u\eta_u{}^{G_u-1})^{N_{\sigma u}(\tau,k)}\times\prod_{s=1}^m(1+\lambda_s)^{T_s(\tau+1,k)}\prod_{u=m+1}^M(1+\lambda_u)^{T_u(\tau+1,k)}y_\tau^Ty_\tau\\
&\leqslant\sum_{\tau=k_0}^{k-1}\prod_{s=1}^m\mu_s^{-N_{\sigma s}(k_0,\tau)}\prod_{u=m+1}^M\eta_u^{-(G_u-1)}\prod_{u=m+1}^M(\mu_u\eta_u{}^{G_u-1})^{N_{\sigma u}(\tau,k)}\times\prod_{s=1}^m(1+\lambda_s)^{T_s(\tau+1,k)}\prod_{u=m+1}^M(1+\lambda_u)^{T_u(\tau+1,k)}\gamma_i^2w_\tau^Tw_\tau,\\
&\to\sum_{\tau=k_0}^{k-1}\prod_{s=1}^m\mu_s^{-N_{\sigma s}(k_0,\tau)}y_\tau^Ty_\tau\leqslant\sum_{\tau=k_0}^{k-1}\prod_{s=1}^m\mu_s^{-N_{\sigma s}(k_0,\tau)}\prod_{u=m+1}^M\eta_u^{-(G_u-1)}\gamma_i^2w_\tau^Tw_\tau.
\end{aligned}\quad \text{(A24)}$$

One has

$$\sum_{\tau=k_0}^{\infty}\prod_{s=1}^m\mu_s^{-N_{0s}}\mu_s^{-\frac{\tau-k_0}{\tau_{\sigma s}}}y_\tau^Ty_\tau<\sum_{\tau=k_0}^{\infty}\prod_{u=m+1}^M\eta_u^{-(G_u-1)}\gamma_i^2w_\tau^{\mathrm{T}}w_\tau. \quad \text{(A25)}$$

Then, by (A25), we can obtain an weighted H_∞ performance as follows:

$$\sum_{\tau=k_0}^{\infty}\prod_{s=1}^{m}\left(\mu_s^{-\frac{1}{\tau_{\sigma s}}}\right)^{\tau-k_0} y_\tau^T y_\tau < \sum_{\tau=k_0}^{\infty}\prod_{s=1}^{m}\mu_s^{N_{0s}}\prod_{u=m+1}^{M}\eta_u^{-(G_u-1)}\gamma_i^2 w_\tau^T w_\tau. \tag{A26}$$

Then, by Definition 3, the system (A1) is conclude to be GUES and satisfies $\sum_{\tau=k_0}^{\infty}\prod_{s=1}^{m}\left(\mu_s^{-\frac{1}{\tau_{\sigma s}}}\right)^{(\tau-k_0)} y_\tau^T y_\tau < \gamma_m^2\sum_{\tau=k_0}^{\infty}\omega_\tau^T\omega_\tau$. The weighted H_∞ performance is $\gamma_m = \sqrt{\prod_{s=1}^{m}\mu_s^{N_{0s}}\prod_{u=m+1}^{M}\eta_u^{-(G_u-1)}} \times \max_{i\in\mathcal{S}}\{\gamma_i\}$.

Appendix B

Proof. Based on Lemma A1 and the Lyapunov function (A2), for $i\in\Xi$ and $k\in N_{\sigma(k_i),q}$, one has

$$\begin{aligned}
&\Delta V_i(k) - \lambda_i V_i(k) + \Gamma(k)\\
&= V_i(k+1) - (1+\lambda_i)V_i(k) + \Gamma(k)\\
&= x^T(k+1)P_i(k+1)x(k+1) - \bar{\lambda}_i x^T(k)P_i(k)x(k) + y^{\mathrm{T}}(k)y(k) - \gamma_i^2 w^{\mathrm{T}}(k)\omega(k)\\
&= \Big((\bar{A}_i x(k))^{\mathrm{T}} + (D_i\omega(k))^{\mathrm{T}}\Big)P_i(k+1)\Big(\bar{A}_i x(k) + D_i\omega(k)\Big) - \bar{\lambda}_i x^T(k)P_i(k)x(k)\\
&+ x^{\mathrm{T}}(k)C_i^{\mathrm{T}}C_i x(k) - \gamma_i^2\omega^{\mathrm{T}}(k)\omega(k)\\
&= \zeta^T(k)\begin{bmatrix} \bar{A}_i^{\mathrm{T}}P_i(k+1)\bar{A}_i - \bar{\lambda}_i P_i(k) + C_i^{\mathrm{T}}C_i & \bar{A}_i^{\mathrm{T}}P_i(k+1)D_i\\ * & -\gamma_i^2 I + D_i^{\mathrm{T}}P_i(k+1)D_i\end{bmatrix}\zeta(k)\\
&= \zeta^T(k)\begin{bmatrix} (1-\varphi_i(k))(\Omega_{i,1}+C_i^{\mathrm{T}}C_i) + \varphi_i(k)(\Omega_{i,2}+C_i^{\mathrm{T}}C_i) & \bar{A}_i^{\mathrm{T}}((1-\varphi_i(k))P_i^q + \varphi_i(k)P_i^{q+1})D_i\\ * & -\gamma_i^2 I + D_i^{\mathrm{T}}((1-\varphi_i(k))P_i^q + \varphi_i(k)P_i^{q+1})D_i\end{bmatrix}\zeta(k)\\
&= \zeta^{\mathrm{T}}(k)\Big((1-\varphi_i(k))\bar{\Omega}_1 + \varphi_i(k)\bar{\Omega}_2\Big)\zeta(k),
\end{aligned}$$

where $\zeta(k) \triangleq [x^T(k)\ w^T(k)]^T$, $\varphi_i(k) \triangleq \frac{k+1-k_i-\theta^q}{h} \in (0,1)$,
$\Omega_{i,1} \triangleq A_i^{\mathrm{T}}P_i^q A_i - \bar{\lambda}_i P_i^q + \frac{\bar{\lambda}_i}{h}(P_i^{q+1} - P_i^q)$, $\Omega_{i,2} \triangleq A_i^{\mathrm{T}}P_i^{q+1}A_i - \bar{\lambda}_i P_i^{q+1} + \frac{\bar{\lambda}_i}{h}(P_i^{q+1} - P_i^q)$,

$$\Omega_1 \triangleq \begin{bmatrix}\Omega_{i,1} + C_i^{\mathrm{T}}C_i & A_i^{\mathrm{T}}P_i^q D_i\\ * & -\gamma_i^2 I + D_i^{\mathrm{T}}P_i^q D_i\end{bmatrix},\ \Omega_2 \triangleq \begin{bmatrix}\Omega_{i,2} + C_i^{\mathrm{T}}C_i & A_i^{\mathrm{T}}P_i^{q+1}D_i\\ * & -\gamma_i^2 I + D_i^{\mathrm{T}}P_i^{q+1}D_i\end{bmatrix}.$$

From Lemma A1, We assume that there exists a matrix W_i with appropriate dimensions. One has

$$\begin{aligned}
&\begin{bmatrix} W_i^{\mathrm{T}} &&&&\\ & I &&&\\ && I &&\\ &&& I &\\ &&&& I\end{bmatrix}
\begin{bmatrix} -\frac{h_i+1}{h_i}\bar{\lambda}_i P_i^q & * & * & * & *\\ 0 & -\gamma_i^2 I & * & * & *\\ \bar{A}_i & D_i & -(P_i^q)^{-1} & * & *\\ I & 0 & 0 & -\frac{h_i}{\bar{\lambda}_i}(P_i^{q+1})^{-1} & *\\ C_i & 0 & 0 & 0 & -I\end{bmatrix}
\begin{bmatrix} W_i &&&&\\ & I &&&\\ && I &&\\ &&& I &\\ &&&& I\end{bmatrix}\\
&= \begin{bmatrix} -\frac{h_i+1}{h_i}\bar{\lambda}_i W_i^{\mathrm{T}}P_i^q W_i & * & * & * & *\\ 0 & -\gamma_i^2 I & * & * & *\\ \bar{A}_i W_i & D_i & -(P_i^q)^{-1} & * & *\\ W_i & 0 & 0 & -\frac{h_i}{\bar{\lambda}_i}(P_i^{q+1})^{-1} & *\\ C_i W_i & 0 & 0 & 0 & -I\end{bmatrix}\\
&\leqslant \begin{bmatrix} \frac{h_i+1}{h_i}\bar{\lambda}_i((P_i^q)^{-1} - W_i^{\mathrm{T}} - W_i) & * & * & * & *\\ 0 & -\gamma_i^2 I & * & * & *\\ \bar{A}_i W_i & D_i & -(P_i^q)^{-1} & * & *\\ W_i & 0 & 0 & -\frac{h_i}{\bar{\lambda}_i}(P_i^{q+1})^{-1} & *\\ C_i W_i & 0 & 0 & 0 & -I\end{bmatrix} < 0,
\end{aligned} \tag{B1}$$

and

$$\begin{bmatrix} W_i^{\mathrm{T}} &&&\\ & I &&\\ && I &\\ &&& I\end{bmatrix}
\begin{bmatrix} -\frac{h_i-1}{h_i}\bar{\lambda}_i P_i^{q+1} - \frac{\bar{\lambda}_i}{h_i}P_i^q & * & * & *\\ 0 & -\gamma_i^2 I & * & *\\ \bar{A}_i & D_i & -(P_i^{q+1})^{-1} & *\\ C_i & 0 & 0 & -I\end{bmatrix}
\begin{bmatrix} W_i &&&\\ & I &&\\ && I &\\ &&& I\end{bmatrix}$$

$$= \begin{bmatrix} -\frac{h_i-1}{h_i}\bar{\lambda}_i W_i^{\mathrm{T}} P_i^{q+1} W_i - \frac{\bar{\lambda}_i}{h_i} W_i^{\mathrm{T}} P_i^q W_i & * & * & * \\ 0 & -\gamma_i^2 I & * & * \\ \bar{A}_i W_i & D_i & -(P_i^{q+1})^{-1} & * \\ C_i W_i & 0 & 0 & -I \end{bmatrix}$$

$$\leqslant \begin{bmatrix} \frac{h_i-1}{h_i}\bar{\lambda}_i (P_i^{q+1})^{-1} + \frac{\bar{\lambda}_i}{h_i}(P_i^q)^{-1} - \bar{\lambda}_i(W_i^{\mathrm{T}} + W_i) & * & * & * \\ 0 & -\gamma_i^2 I & * & * \\ \bar{A}_i W_i & D_i & -(P_i^{q+1})^{-1} & * \\ C_i W_i & 0 & 0 & -I \end{bmatrix} < 0. \tag{B2}$$

And W_i is nonsingular, one can obtain

$$\begin{bmatrix} -\frac{h_i+1}{h_i}\bar{\lambda}_i P_i^q & * & * & * & * \\ 0 & -\gamma_i^2 I & * & * & * \\ \bar{A}_i & D_i & -(P_i^q)^{-1} & * & * \\ I & 0 & 0 & -\frac{h_i}{\bar{\lambda}_i}(P_i^{q+1})^{-1} & * \\ C_i & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{B3}$$

$$\begin{bmatrix} -\frac{h_i-1}{h_i}\bar{\lambda}_i P_i^{q+1} - \frac{\bar{\lambda}_i}{h_i} P_i^q & * & * & * \\ 0 & -\gamma_i^2 I & * & * \\ \bar{A}_i & D_i & -(P_i^{q+1})^{-1} & * \\ C_i & 0 & 0 & -I \end{bmatrix} < 0. \tag{B4}$$

Applying the Schur Complement, (B3) and (B4) are equivalent to

$$\bar{\Omega}_1 = \begin{bmatrix} \Omega_{i,1} + C_i^{\mathrm{T}} C_i & \bar{A}_i^{\mathrm{T}} P_i^q D_i \\ * & -\gamma_i^2 I + D_i^{\mathrm{T}} P_i^q D_i \end{bmatrix} < 0, \tag{B5}$$

$$\bar{\Omega}_2 = \begin{bmatrix} \Omega_{i,2} + C_i^{\mathrm{T}} C_i & \bar{A}_i^{\mathrm{T}} P_i^{q+1} D_i \\ * & -\gamma_i^2 I + D_i^{\mathrm{T}} P_i^{q+1} D_i \end{bmatrix} < 0. \tag{B6}$$

Hence, one can obtain $V_i(k+1) \leqslant (1+\lambda_i)V_i(k) - \Gamma(k)$. with $U_i^q = (P_i^q)^{-1}$ and $\bar{A}_i = A_i + B_i K_i,\ T_i = K_i W_i$, rewritten (B1) and (B2) as:

$$\begin{bmatrix} \frac{h_i+1}{h_i}\bar{\lambda}_i(U_i^q - W_i^{\mathrm{T}} - W_i) & * & * & * & * \\ 0 & -\gamma_i^2 I & * & * & * \\ A_i W_i + B_i T_i & D_i & -U_i^q & * & * \\ W_i & 0 & 0 & -\frac{h_i}{\bar{\lambda}_i} U_i^{q+1} & * \\ C_i W_i & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{B7}$$

$$\begin{bmatrix} \frac{h_i-1}{h_i}\bar{\lambda}_i U_i^{q+1} + \frac{\bar{\lambda}_i}{h_i} U_i^q - \bar{\lambda}_i(W_i^{\mathrm{T}} + W_i) & * & * & * \\ 0 & -\gamma_i^2 I & * & * \\ A_i W_i + B_i T_i & D_i & -U_i^{q+1} & * \\ C_i W_i & 0 & 0 & -I \end{bmatrix} < 0. \tag{B8}$$

For $i \in \Xi$ and $k \in [k_i + \tau_{\min}, k_l)$, one has

$$\begin{aligned} &\Delta V_i(k) - \lambda_i V_i(k) + \Gamma(k) \\ &= V_i(k+1) - (1+\lambda_i)V_i(k) + \Gamma(k) \\ &= x^T(k+1)P_i^L x(k+1) - \bar{\lambda}_i x^T(k) P_i^L x(k) + y^{\mathrm{T}}(k) y(k) - \gamma_i^2 \omega^{\mathrm{T}}(k)\omega(k) \\ &= \zeta^T(k) \begin{bmatrix} \bar{A}_i^{\mathrm{T}} P_i^L \bar{A}_i - \bar{\lambda}_i P_i^L + C_i^{\mathrm{T}} C_i & \bar{A}_i^{\mathrm{T}} P_i^L D_i \\ * & -\gamma_i^2 I + D_i^{\mathrm{T}} P_i^L D_i \end{bmatrix} \zeta(k) \\ &= \zeta^{\mathrm{T}}(k)\bar{\Omega}_3 \zeta(k). \end{aligned}$$

From Lemma A1, We assume that there exists a matrix W_i with appropriate dimensions. One has,

$$\begin{bmatrix} W_i^{\mathrm{T}} & & & \\ & I & & \\ & & I & \\ & & & I \end{bmatrix} \begin{bmatrix} -\bar{\lambda}_i P_i^L & * & * & * \\ 0 & -\gamma_i^2 I & * & * \\ \bar{A}_i & D_i & -(P_i^L)^{-1} & * \\ C_i & 0 & 0 & -I \end{bmatrix} \begin{bmatrix} W_i & & & \\ & I & & \\ & & I & \\ & & & I \end{bmatrix}$$

$$
\begin{aligned}
&= \begin{bmatrix} -\bar{\lambda}_i W_i^{\mathrm{T}} P_i^L W_i & * & * & * \\ 0 & -\gamma_i^2 I & * & * \\ \bar{A}_i W_i & D_i & -(P_i^L)^{-1} & * \\ C_i W_i & 0 & 0 & -I \end{bmatrix} \\
&\leqslant \begin{bmatrix} \bar{\lambda}_i((P_i^L)^{-1} - W_i^{\mathrm{T}} - W_i) & * & * & * \\ 0 & -\gamma_i^2 I & * & * \\ \bar{A}_i W_i & D_i & -(P_i^L)^{-1} & * \\ C_i W_i & 0 & 0 & -I \end{bmatrix} < 0.
\end{aligned} \tag{B9}
$$

Applying the Schur Complement, (B9) are equivalent to

$$
\bar{\Omega}_3 = \begin{bmatrix} \bar{A}_i^{\mathrm{T}} P_i^L \bar{A}_i - \bar{\lambda}_i P_i^L + C_i^{\mathrm{T}} C_i & \bar{A}_i^{\mathrm{T}} P_i^L D_i \\ * & -\gamma_i^2 I + D_i^{\mathrm{T}} P_i^L D_i \end{bmatrix} < 0. \tag{B10}
$$

Hence, one can obtain $V_i(k+1) \leqslant (1+\lambda_i)V_i(k) - \Gamma(k)$.
With $U_i^L = (P_i^L)^{-1}$ and $\bar{A}_i = A_i + B_i K_i,\ T_i = K_i W_i$, rewritten (B9) as:

$$
\begin{bmatrix} \bar{\lambda}_i(U_i^L - W_i^{\mathrm{T}} - W_i) & * & * & * \\ 0 & -\gamma_i^2 I & * & * \\ A_i W_i + B_i T_i & D_i & -U_i^L & * \\ C_i W_i & 0 & 0 & -I \end{bmatrix} < 0. \tag{B11}
$$

For $i \in \Theta$, the proof is similar to (B9) - (B11),we can easily get

$$
\begin{aligned}
&\Delta V_i^q(k) - \lambda_i V_i^q(k) + \Gamma(k) \\
&= \zeta^T(k) \begin{bmatrix} \bar{A}_i^{\mathrm{T}} P_i^q \bar{A}_i - \bar{\lambda}_i P_i^q + C_i^{\mathrm{T}} C_i & \bar{A}_i^{\mathrm{T}} P_i^q D_i \\ * & -\gamma_i^2 I + D_i^{\mathrm{T}} P_i^q D_i \end{bmatrix} \zeta(k) \\
&= \zeta^{\mathrm{T}}(k) \bar{\Omega}_4 \zeta(k).
\end{aligned}
$$

From Lemma A1, we assume that there exists a matrix W_i with appropriate dimensions. One has,

$$
\begin{aligned}
&\begin{bmatrix} W_i^{\mathrm{T}} & & & \\ & I & & \\ & & I & \\ & & & I \end{bmatrix} \begin{bmatrix} -\bar{\lambda}_i P_i^q & * & * & * \\ 0 & -\gamma_i^2 I & * & * \\ \bar{A}_i & D_i & -(P_i^q)^{-1} & * \\ C_i & 0 & 0 & -I \end{bmatrix} \begin{bmatrix} W_i & & & \\ & I & & \\ & & I & \\ & & & I \end{bmatrix} \\
&= \begin{bmatrix} -\bar{\lambda}_i W_i^{\mathrm{T}} P_i^q W_i & * & * & * \\ 0 & -\gamma_i^2 I & * & * \\ \bar{A}_i W_i & D_i & -(P_i^q)^{-1} & * \\ C_i W_i & 0 & 0 & -I \end{bmatrix} \\
&\leqslant \begin{bmatrix} \bar{\lambda}_i((P_i^q)^{-1} - W_i^{\mathrm{T}} - W_i) & * & * & * \\ 0 & -\gamma_i^2 I & * & * \\ \bar{A}_i W_i & D_i & -(P_i^q)^{-1} & * \\ C_i W_i & 0 & 0 & -I \end{bmatrix} < 0.
\end{aligned} \tag{B12}
$$

Applying the Schur Complement, (B12) are equivalent to

$$
\bar{\Omega}_4 = \begin{bmatrix} \bar{A}_i^{\mathrm{T}} P_i^q \bar{A}_i - \bar{\lambda}_i P_i^q + C_i^{\mathrm{T}} C_i & \bar{A}_i^{\mathrm{T}} P_i^q D_i \\ * & -\gamma_i^2 I + D_i^{\mathrm{T}} P_i^q D_i \end{bmatrix} < 0. \tag{B13}
$$

Hence, one can obtain $\Delta V_i^q(k+1) \leqslant \lambda_i V_i^q(k) - \Gamma(k)$.
With $U_i^q = (P_i^q)^{-1}$ and $\bar{A}_i = A_i + B_i K_i,\ T_i = K_i W_i$, rewritten (B12) as:

$$
\begin{bmatrix} \bar{\lambda}_i(U_i^q - W_i^{\mathrm{T}} - W_i) & * & * & * \\ 0 & -\gamma_i^2 I & * & * \\ A_i W_i + B_i T_i & D_i & -U_i^q & * \\ C_i W_i & 0 & 0 & -I \end{bmatrix} < 0. \tag{B14}
$$

Then there exists a set of mode-dependent state-feedback controllers such that system (A1) is GUES for any switching signal with MDDT satisfying (A14)-(A15) and has an weighted H_∞ performance index γ_m. We can derive that if the inequalities (B7), (B8), (B11) and (B14) have feasible solutions, the admissible controller gains can be given by $K_i = T_i(W_i)^{-1}$ since $T_i = K_i W_i$, then the proof is completed.

Appendix C

In this section, we consider the following linear switched system (A1), which consists of four subsystems with the disturbance input $\omega(k) = 1.8e^{(-0.12k)}sin(0.5\pi k)$:

$$A_1 = \begin{bmatrix} 0.9 & 0.1 \\ 0 & 0.9 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 2.8 \\ 2.5 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.3 & 0.4 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 \\ 2.5 \end{bmatrix}.$$

$$A_2 = \begin{bmatrix} 0.85 & 0.2 \\ -0.1 & 0.85 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2.5 \\ 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.1 & 0.4 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

$$A_3 = \begin{bmatrix} 1.15 & 0.2 \\ 0.1 & 1.15 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 4 \\ 3.5 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix}, \quad D_3 = \begin{bmatrix} 1.5 \\ 2 \end{bmatrix}.$$

$$A_4 = \begin{bmatrix} 1.2 & -0.2 \\ 0.2 & 1.25 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 1.5 \\ 2.5 \end{bmatrix}, \quad C_4 = \begin{bmatrix} 0.2 & 0.5 \end{bmatrix}, \quad D_4 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

It can easily get both A_1 and A_2 are stable, but A_3 and A_4 are unstable.

Next, for given parameters $\lambda_1 = -0.1$, $\lambda_2 = -0.12$, $\lambda_3 = 0.1$, $\lambda_4 = 0.12$, $\mu_1 = 1.2$, $\mu_2 = 1.25$, $\mu_3 = 1.06$, $\mu_4 = 1.08$, $\eta_3 = 0.9$, $\eta_4 = 0.88$, $q = 2$, $G_3 = G_4 = 3$. Tight bounds on the MDDT can be acquired: $\tau_1 = 1.7305$, $\tau_2 = 1.7456$, $\tau_3 = 1.5995$, $\tau_4 = 1.5769$, from (A14) and (A15), we choose $\tau_{\sigma s} = 2$, $\tau_{\sigma u} = 1$, $\tau_{min} = 1.7305$. The disturbance input is shown in Fig.C1.

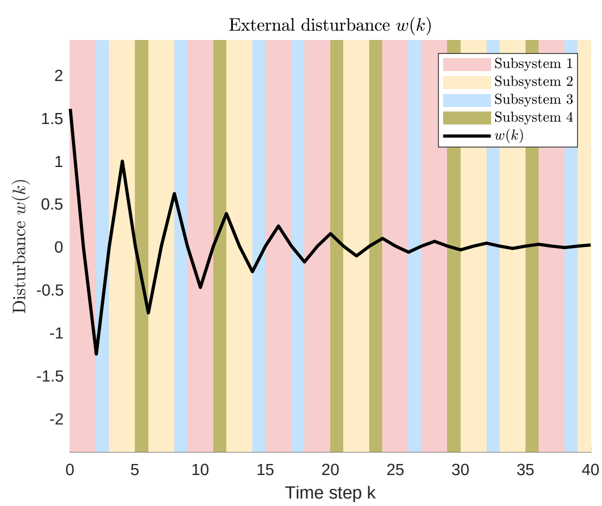


Figure C1 The disturbance input

By solving the optimization problem in Theorem A2 (minimizing γ_i in the criteria), one can obtain $\gamma_1 = 4.8237$, $\gamma_2 = 3.6659$, $\gamma_3 = 3.1285$, $\gamma_4 = 3.5523$. One has $\gamma_m = 6.0905$ and the controllers gains sovled by YALMIP as:

$$K_1 = \begin{bmatrix} -0.1925 & -0.1682 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -0.5321 & 0.2060 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -0.1984 & -0.1479 \end{bmatrix}, \quad K_4 = \begin{bmatrix} -0.3844 & -0.9414 \end{bmatrix}.$$

Then, applying the controllers, giving initial state condition $x(0) = [0, 0]^T$, the state responses of the system are depicted in Fig.C2(b). Fig.C2(a)-C2(d) demonstrate the validity of obtained Theorem 1.

Moreover, results based on previous method, that is, average dwell time with conventional Lyapunov functions $V_i = x^T(k)P_i x(k)$, $\tau_a = 2s$ (τ_a is defined in [6], Lemma 1)

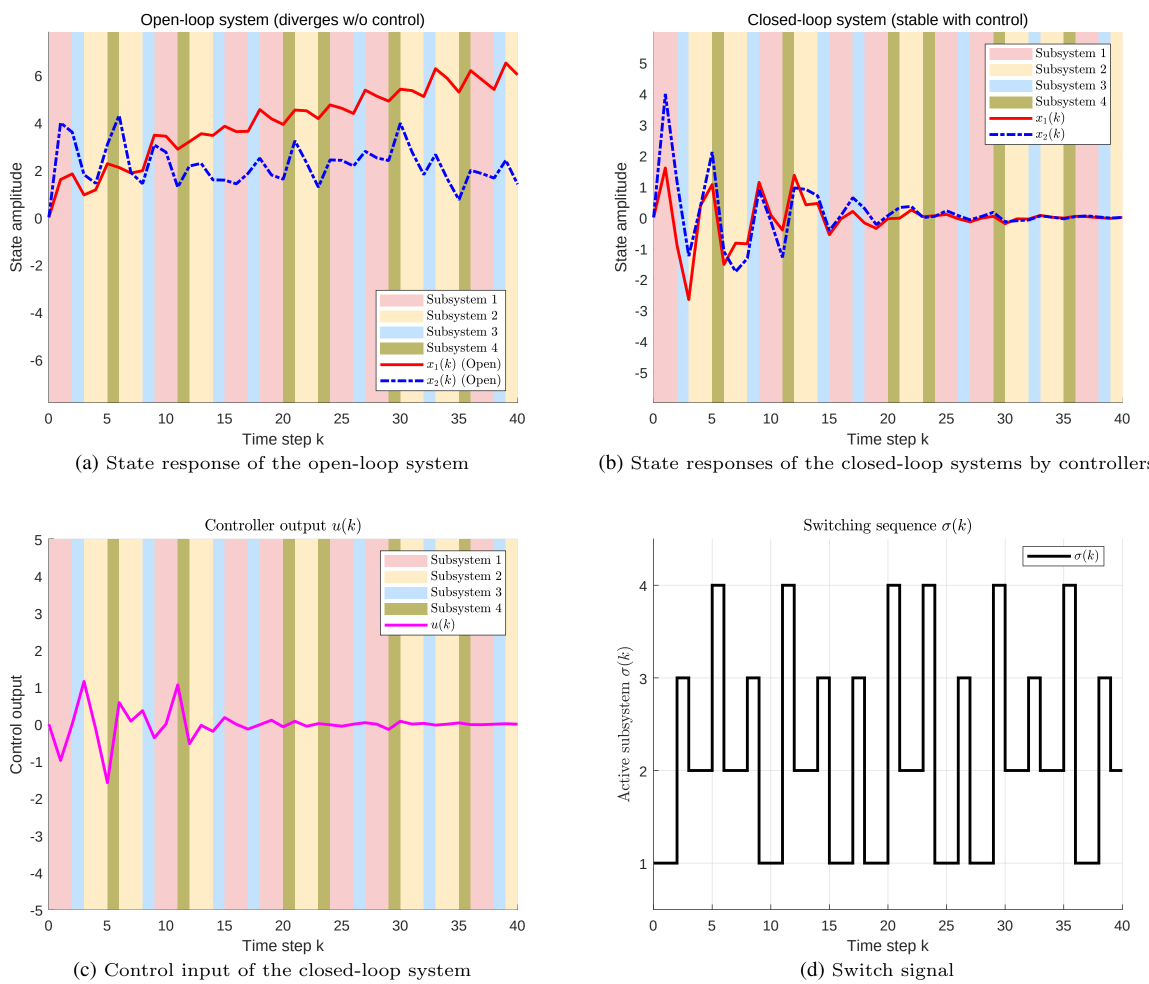
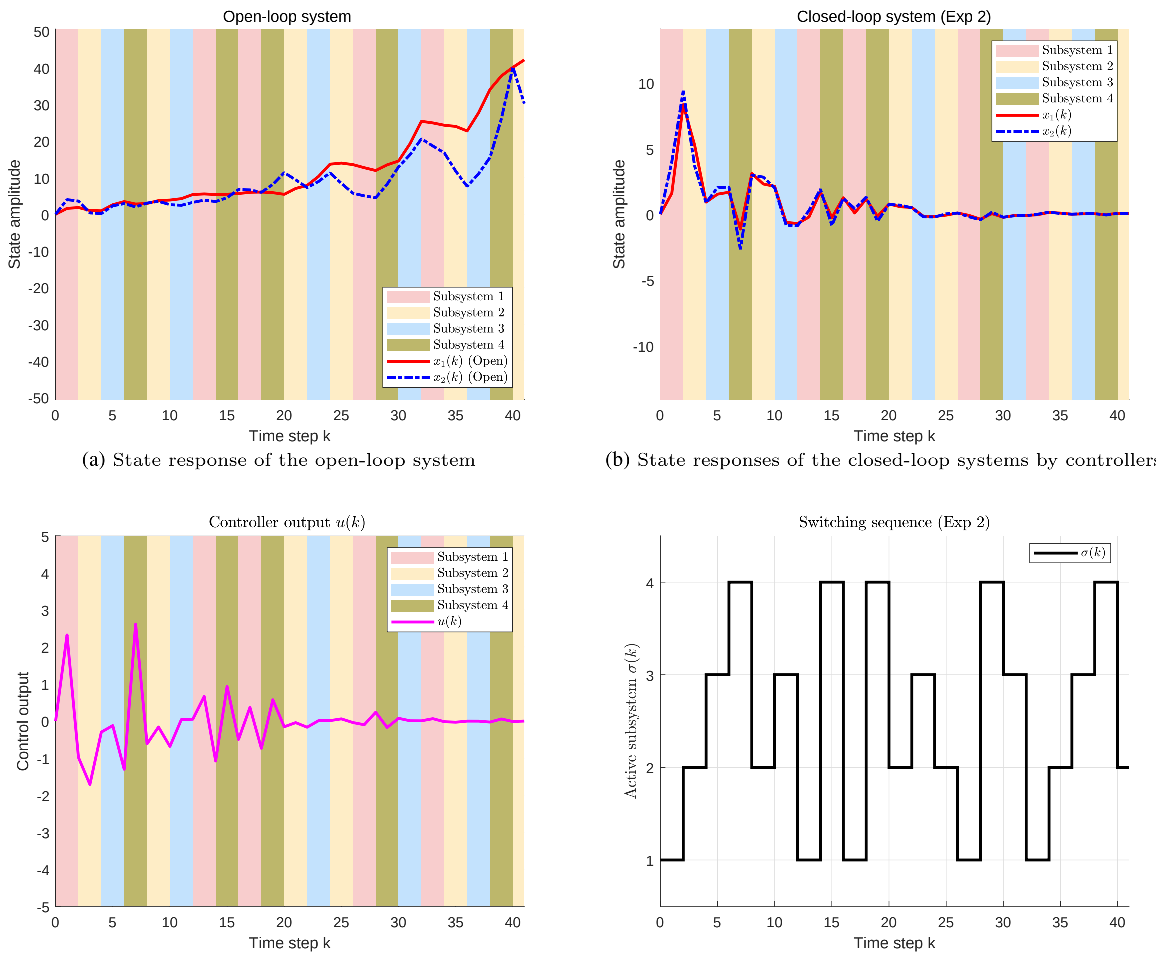


Figure C2 H_∞ control under slow and fast switching



(a) State response of the open-loop system

(b) State responses of the closed-loop systems by controllers

(c) Control input of the closed-loop system

(d) Switch signal

Figure C3 H_∞ control under ADT switching

One can obtain $\gamma_1 = 10.1591$, $\gamma_2 = 4.8211$, $\gamma_3 = 10.3815$, $\gamma_4 = 9.3411$ and $\gamma_m = 10.3815$. And the controllers gains sovled by YALMIP as:

$$K_1 = \begin{bmatrix} -1.5485 & -1.2016 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -0.6613 & 0.4823 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -0.9228 & 0.6338 \end{bmatrix}, \quad K_4 = \begin{bmatrix} 0.8976 & -1.3599 \end{bmatrix}.$$

By the same steps, giving initial state condition $x(0) = [0, 0]^T$. The results are shown in Fig.C3(a)-C3(d). It can be seen that the obtained index γ_m is greater than that of the proposed method. The above simulation examples illustrate that the proposed method gives a tighter bound on MDDT, which achieves a smaller H_∞ performance index γ_m and reduces the conservatism of the switched system.

Remark 2. A multiple discontinuous Lyapunov function is developed for both stable and unstable subsystems in [2], where the associated Lyapunov matrix is time-invariant during each operating interval, as shown in equation (A5) . In addition, the discretized Lyapunov function approach has been recognized as an effective means of mitigating conservatism within the Multiple Lyapunov Function framework, as reported in [5]. The slow and fast switching strategy is first introduced in [2], where more general stability and stabilization conditions are established under time-dependent switching, yielding a tighter bound on the MDADT or ADT. However, the disturbance attenuation performance under this strategy is not been studied. It is essential to require that the control system is robust to external disturbance.

Inspired by these prior studies, we develop a novel MLF structure aimed at further decreasing conservatism. In particular, the MDLF is applied to the unstable subsystems, whereas the discretized Lyapunov function is introduced for the stable subsystems, as shown in equation (A2)-(A5). Compared with existing methods, the proposed approach yields a less restrictive bound on the mode-dependent dwell time and leads to a smaller H_∞ performance level γ_m. Moreover, the incorporation of the discretized Lyapunov function increases the degrees of freedom in the LMI formulation, which contributes to reducing the overall conservatism of the switched system.

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