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ADP-based composite learning framework for asynchronous event-triggered control of singularly perturbed systems and its applications

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This study focuses on the event-triggered composite optimal control problem of linear singularly perturbed systems (SPSs) through adaptive dynamic programming (ADP). For more details on the literature review and motivation, see Appendix A. The main contributions are as follows. (i) In terms of ADP, we, for the first time, propose a data-driven composite control methodology without identification for solving the optimal stabilization problem of linear SPSs subject to completely unknown dynamics. Meanwhile, we originally introduced a convergence factor into the performance index for the solvability of the optimal control problem of the fast subsystem. The proposed learning algorithm could serve as an underlying framework to study the relevant data-driven ADP control problems for SPSs with uncertain dynamics. (ii) Based on the developed composite learning algorithm, we further impose the asynchronous event-triggering mechanism associated with decoupled slow and fast modes to save network resources, which is independent of model parameters. In particular, when an event appears in the fast state, only the signal of the fast state is transmitted to the controller, and the control strategy is thus updated; the same applies to the slow state. Furthermore, we employ singular perturbations to analyze the closed-loop stability and show the existence of a positive lower bound of triggering intervals between two triggers to exclude the Zeno behavior.

Problem formulation. Consider a linear standard SPS

$$\dot{x}(t) = \mathcal{A}_{11}x(t) + \mathcal{A}_{12}z(t) + \mathcal{B}_1 u(t), \tag{1a}$$

$$\mu\dot{z}(t) = \mathcal{A}_{21}x(t) + \mathcal{A}_{22}z(t) + \mathcal{B}_2 u(t), \tag{1b}$$

where $x \in \mathbb{R}^{n_x}$, $z \in \mathbb{R}^{n_z}$, $u \in \mathbb{R}^{n_u}$, and $y(t) = \mathcal{C}x(t) \in \mathbb{R}^{n_y}$ represent the slow state, fast state, control input, and output, respectively, $0 < \mu \ll 1$ is a singular perturbation parameter. Define an infinite-horizon performance index

$$J(x(0), u) = \int_0^{\infty} \mathrm{e}^{2\alpha\tau}(y^{\mathrm{T}}\mathcal{Q}y + u^{\mathrm{T}}\mathcal{R}u)d\tau, \tag{2}$$

where $\mathcal{Q} \succ 0$ and $\mathcal{R} \succ 0$ are the weight matrices, and $\alpha > 0$ is a convergence factor; specifically, the control u that minimizes (2) can make the convergence rate of the closed-loop system (1) faster than $\mathrm{e}^{-\alpha t}$ [1].

Assumption 1. The convergence factor α is available such that $\alpha > \max\{0, \max\{\mathrm{Re}(\lambda(-\mathcal{A}_{22}))\}\}$.

Owing to the existence of the singular perturbation parameter μ, the conventional algorithms to solve the full-order optimal problem might face numerical stiffness issues [2–4]. For this reason, Appendix B introduces the composite controller

$$u = K_0 x_s + K_2 z_f \triangleq K_1 x + K_2 z \tag{3}$$

as a near-optimal control input to solve the optimal control problem associated with (1) and (2), where $x_s \in \mathbb{R}^{n_x}$ and $z_f \in \mathbb{R}^{n_z}$ are the so-called slow time-scale and fast time-scale states, and $K_1 = (I + K_2\mathcal{A}_{22}^{-1}\mathcal{B}_2)K_0 + K_2\mathcal{A}_{22}^{-1}\mathcal{A}_{21}$. To reduce the number of controller updates, we want to execute the composite optimal controller (3) to stabilize (1) through event-triggered sampling. Since the fast state z varies much faster than the slow state x, the asynchronous triggering mechanism will be designed in this study. As a result, the control input (3) should be modified as

$$u = K_1\hat{x} + K_2\hat{z}, \tag{4}$$

and the sampled states $\hat{x}, \hat{z}$ are given by $\hat{x}(t) = x(t_x^i)$, $\forall t \in [t_x^i, t_x^{i+1})$ and $\hat{z}(t) = z(t_z^j)$, $\forall t \in [t_z^j, t_z^{j+1})$, where $\{t_x^i\}$ and $\{t_z^j\}$ are the increasing sequences of sampling time instants for slow and fast states with $i, j \in \mathbb{N}$. To facilitate designs of the triggering conditions Φ_s and Φ_f, we further define the errors between the real states x, z and the sampled states $\hat{x}, \hat{z}$ as $e_x(t) = \hat{x}(t) - x(t)$ and $e_z(t) = \hat{z}(t) - z(t)$, which are used to determine when to trigger an event.

Data-driven composite learning. The ADP technique is employed to design gains K_1 and K_2 without any knowledge about matrices $\mathcal{A}_{11}$, $\mathcal{A}_{12}$, $\mathcal{A}_{21}$, $\mathcal{A}_{22}$, $\mathcal{B}_1$, $\mathcal{B}_2$, and $\mathcal{C}$. We first recall the model-based Kleinman algorithm to solve the corresponding algebraic Riccati equation (ARE) of the slow subsystem optimal control problem by $(\mathcal{A}_0 + \alpha I + \mathcal{B}_0 K_0^k)^{\mathrm{T}} P_s^k + P_s^k(\mathcal{A}_0 + \alpha I + \mathcal{B}_0 K_0^k) = -\mathcal{C}^{\mathrm{T}}\mathcal{Q}\mathcal{C} - (K_0^k)^{\mathrm{T}}\mathcal{R}K_0^k$, $k = 0, 1, 2, \ldots$, where $\mathcal{A}_0 = \mathcal{A}_{11} - \mathcal{A}_{12}\mathcal{A}_{22}^{-1}\mathcal{A}_{21}$, $\mathcal{B}_0 = \mathcal{B}_1 - \mathcal{A}_{12}\mathcal{A}_{22}^{-1}\mathcal{B}_2$, $K_0^{k+1} = -\mathcal{R}^{-1}\mathcal{B}_0^{\mathrm{T}} P_s^k$, and $\mathcal{A}_0 + \alpha I + \mathcal{B}_0 K_0^0$ is Hurwitz. For a behavior

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policy u_s^0, the closed-loop slow subsystem can be expressed as $\dot{x}_s = (\mathcal{A}_0 + \mathcal{B}_0 K_0^k)x_s + \mathcal{B}_0(u_s^0 - K_0^k x_s)$. Inspired by [5], along the trajectories of x_s and the above model-based algorithm, we have

$$\begin{aligned}&\mathrm{e}^{2\alpha(t+\delta t)}x_s^{\mathrm{T}}(t+\delta t)P_s^k x_s(t+\delta t) - \mathrm{e}^{2\alpha t}x_s^{\mathrm{T}}(t)P_s^k x_s(t)\\&= -2\int_t^{t+\delta t}\mathrm{e}^{2\alpha\tau}(u_s^0 - K_0^k x_s)\mathcal{R}K_0^{k+1}x_s d\tau\\&\quad - \int_t^{t+\delta t}\mathrm{e}^{2\alpha\tau}(y_s^{\mathrm{T}}\mathcal{Q}y_s + x_s^{\mathrm{T}}(K_0^k)^{\mathrm{T}}\mathcal{R}K_0^k x_s)d\tau,\end{aligned}\tag{5}$$

where $y_s = \mathcal{C}x_s$ and δt is the sampling interval. In what follows, we seek $K_2\mathcal{A}_{22}^{-1}\mathcal{B}_2$, $K_2\mathcal{A}_{22}^{-1}\mathcal{A}_{21}$, and K_2 that appear in the composite controller (4) through system measurements. Likewise, the ARE of the fast subsystem can be solved using the model-based Kleinman algorithm $(\mathcal{A}_{22} + \alpha I + \mathcal{B}_2 K_2^j)^{\mathrm{T}}P_f^j + P_f^j(\mathcal{A}_{22} + \alpha I + \mathcal{B}_2 K_2^j) = -(K_2^j)^{\mathrm{T}}\mathcal{R}K_2^j$, $j = 0, 1, 2, \ldots$, where $K_2^{j+1} = -\mathcal{R}^{-1}\mathcal{B}_2^{\mathrm{T}}P_f^j$, $\mathcal{A}_{22} + \alpha I + \mathcal{B}_2 K_2^0$ is Hurwitz. For a behavior policy u_f^0, the closed-loop fast subsystem is written as $\mu\dot{z}_f = (\mathcal{A}_{22} + \mathcal{B}_2 K_2^j)z_f + \mathcal{B}_2(u_f^0 - K_2^j z_f)$. Along the trajectories of z_f, one has

$$\begin{aligned}&\mu\mathrm{e}^{2\alpha(t+\delta t)}z_f^{\mathrm{T}}(t+\delta t)P_f^j z_f(t+\delta t) - \mu\mathrm{e}^{2\alpha t}z_f^{\mathrm{T}}(t)P_f^j z_f(t)\\&= -2\int_t^{t+\delta t}\mathrm{e}^{2\alpha\tau}(u_f^0 - K_2^j z_f)\mathcal{R}K_2^{j+1}z_f d\tau\\&\quad - \int_t^{t+\delta t}\mathrm{e}^{2\alpha\tau}z_f^{\mathrm{T}}(K_2^j)^{\mathrm{T}}\mathcal{R}K_2^j z_f d\tau.\end{aligned}\tag{6}$$

By singular perturbations (also see Appendix B), the fast time-scale state z_f is identical to $z_f = z + \mathcal{A}_{22}^{-1}\mathcal{A}_{21}x_s + \mathcal{A}_{22}^{-1}\mathcal{B}_2 u_s^0 \triangleq \Lambda\phi_s$ with $\Lambda = [I, \mathcal{A}_{22}^{-1}\mathcal{A}_{21}, \mathcal{A}_{22}^{-1}\mathcal{B}_2]$ and $\phi_s = [z^{\mathrm{T}}, x_s^{\mathrm{T}}, (u_s^0)^{\mathrm{T}}]^{\mathrm{T}}$. Then, Eq. (6) is rearranged as

$$\begin{aligned}&\mu\mathrm{e}^{2\alpha(t+\delta t)}\phi_s^{\mathrm{T}}(t+\delta t)\bar{P}_f^j\phi_s(t+\delta t) - \mu\mathrm{e}^{2\alpha t}\phi_s^{\mathrm{T}}(t)\bar{P}_f^j\phi_s(t)\\&= -2\int_t^{t+\delta t}\mathrm{e}^{2\alpha\tau}(u_f^0 - \bar{K}_2^j\phi_s)\mathcal{R}\bar{K}_2^{j+1}\phi_s d\tau\\&\quad - \int_t^{t+\delta t}\mathrm{e}^{2\alpha\tau}\phi_s^{\mathrm{T}}(\bar{K}_2^j)^{\mathrm{T}}\mathcal{R}\bar{K}_2^j\phi_s d\tau,\end{aligned}\tag{7}$$

where $\bar{P}_f^j = \Lambda^{\mathrm{T}}P_f^j\Lambda$ and $\bar{K}_2^j = [K_2^j, K_2^j\mathcal{A}_{22}^{-1}\mathcal{A}_{21}, K_2^j\mathcal{A}_{22}^{-1}\mathcal{B}_2] \equiv K_2^j\Lambda$. We can see that Eqs. (5) and (7) can be used to compute P_s^k, K_0^{k+1}, and $\bar{P}_f^j, \bar{K}_2^j$ without knowledge of system dynamics, respectively. However, the information of x_s cannot be directly measured from the actual plant (1). Since $x(t) = x_s(t) + \mathcal{O}(\mu)$, we will utilize x instead of x_s during learning. We summarize the data-driven composite ADP learning algorithm in Appendix C, which iteratively learns the composite optimal feedback gains, where $K_1 = (I + K_2^j\mathcal{A}_{22}^{-1}\mathcal{B}_2)K_0^k + K_2^j\mathcal{A}_{22}^{-1}\mathcal{A}_{21}$ and $K_2 = K_2^j$.

Assumption 2. The triple $(\mathcal{A}_0, \mathcal{B}_0, \mathcal{C})$ is controllable and detectable.

Assumption 3. The pair $(\mathcal{A}_{22}, \mathcal{B}_2)$ is controllable.

Theorem 1. Consider system (1) and let Assumptions 1–3 hold. For data-driven ADP learning algorithms (5) and (7), we have

$$\lim_{k\to\infty}P_s^k = P_s + \mathcal{O}(\mu),\ \lim_{k\to\infty}K_0^k = K_0 + \mathcal{O}(\mu),\tag{8a}$$

$$\lim_{j\to\infty}\bar{P}_f^j = \Lambda^{\mathrm{T}}P_f\Lambda + \mathcal{O}(\mu),\ \lim_{j\to\infty}\bar{K}_2^j = K_2\Lambda + \mathcal{O}(\mu).\tag{8b}$$

Proof. Please see Appendix D.

Asynchronous event-triggering mechanism. We present an asynchronous event-triggering mechanism and provide the theoretical analysis. Note that the definitions of relevant parameters are provided in Appendix E.

Theorem 2. Suppose that Assumptions 1–3 hold and $\|\mathcal{B}_a\| \leqslant c$. If either of the following triggering conditions is satisfied,

$$\begin{aligned}\Phi_s = &-\varsigma_s^{\mathrm{T}}(K_0^{\mathrm{T}}\mathcal{R}K_0 + 2\alpha P_s)\varsigma_s - y_{\varsigma_s}^{\mathrm{T}}\mathcal{Q}y_{\varsigma_s} + c\gamma_s^{-1}\|P_s\varsigma_s\|^2\\&+ 2c\|P_s\varsigma_s\|\|K_1 e_x\| + \gamma_f\|\mathcal{R}\|\|K_1 e_x\|^2 \geqslant 0,\end{aligned}\tag{9a}$$

$$\begin{aligned}\Phi_f = &-\varsigma_f^{\mathrm{T}}(K_2^{\mathrm{T}}\mathcal{R}K_2 + 2\alpha P_f)\varsigma_f + 2\varsigma_f^{\mathrm{T}}K_2^{\mathrm{T}}\mathcal{R}K_2 e_z\\&+ \gamma_f^{-1}\|\mathcal{R}\|\left\|K_2\varsigma_f\right\|^2 + c\gamma_s\|K_2 e_z\|^2 \geqslant 0\end{aligned}\tag{9b}$$

with $\gamma_s > \frac{c\|P_s\|^2}{2\alpha\min\{\lambda(P_s)\}}$ and $\gamma_f > \frac{\|\mathcal{R}\|\|K_2\|^2}{2\alpha\min\{\lambda(P_f)\}}$, then, there exists a $\mu^* > 0$ such that for all $\mu \in (0, \mu^*]$, the closed-loop system (1) under (4) with the event-triggering mechanism $t_x^{i+1} = \inf\{t \in \mathbb{R}^+ | t > t_x^i \wedge \Phi_s \geqslant 0\}$ and $t_z^{i+1} = \inf\{t \in \mathbb{R}^+ | t > t_z^i \wedge \Phi_f \geqslant 0\}$ is asymptotically stable at the origin.

Proof. Please see Appendix E.

From Theorem 2, the update of the slow state is only related to measurements of the exact slow mode $\varsigma_s(t)$ and its error $e_x(t)$, and the update of the fast state is only related to measurements of the exact fast mode $\varsigma_f(t)$ and its error $e_z(t)$. Thus, this is referred to as an asynchronous event-triggering mechanism. In the absence of knowledge of the system matrices, we next present a computational way for avoiding ς_s and ς_f. In terms of $\varsigma_s(t) = x(t) + \mathcal{O}(\mu)$ and $\varsigma_f(t) = \Lambda\chi(t) + \mathcal{O}(\mu)$ with $\chi = [z^{\mathrm{T}}, x^{\mathrm{T}}, (K_0 x)^{\mathrm{T}}]^{\mathrm{T}}$ (see Appendix E for more details), we reorganize (9) as

$$\begin{aligned}\Phi_s = &-x^{\mathrm{T}}(K_0^{\mathrm{T}}\mathcal{R}K_0 + 2\alpha P_s)x - y^{\mathrm{T}}\mathcal{Q}y + c\gamma_s^{-1}\|P_s x\|^2\\&+ 2c\|P_s x\|\|K_1 e_x\| + \gamma_f\|\mathcal{R}\|\|K_1 e_x\|^2 \geqslant 0,\end{aligned}\tag{10a}$$

$$\begin{aligned}\Phi_f = &-\chi^{\mathrm{T}}(\bar{K}_2^{\mathrm{T}}\mathcal{R}\bar{K}_2 + 2\alpha\bar{P}_f)\chi + 2\chi^{\mathrm{T}}\bar{K}_2^{\mathrm{T}}\mathcal{R}K_2 e_z\\&+ \gamma_f^{-1}\|\mathcal{R}\|\left\|\bar{K}_2\chi\right\|^2 + c\gamma_s\|K_2 e_z\|^2 \geqslant 0.\end{aligned}\tag{10b}$$

It is noted that K_0, P_s, K_1, $\bar{K}_2$, $\bar{P}_f$, K_2, and P_f can be obtained from data-driven learning algorithm.

Theorem 3. Under the conditions of Theorem 2, considering the system (1) under (4) with the event-triggering mechanism $t_x^{i+1} = \inf\{t \in \mathbb{R}^+ | t > t_x^i \wedge \Phi_s \geqslant 0\}$ and $t_z^{i+1} = \inf\{t \in \mathbb{R}^+ | t > t_z^i \wedge \Phi_f \geqslant 0\}$, both triggering intervals $\mathscr{L}_x^i = t_x^{i+1} - t_x^i$ and $\mathscr{L}_z^j = t_z^{j+1} - t_z^j$ have a positive lower bound.

Proof. Please see Appendix F.

Applications. In Appendix G, we carry out a benchmark example and a two-area power system to support our theoretical results.

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Supporting information Appendixes A–G. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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