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Asynchronous control of Markov jump systems with actuator saturation and denial-of-service attacks

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Abstract This paper investigates the stochastic mean-square stability of Markov jump systems (MJSs) in the presence of uncertain transition probabilities, denial-of-service (DoS) attacks, and actuator saturation. The timing of DoS attacks is modeled as a Bernoulli process, with constraints on the total number and frequency of attacks due to limited network resources. By employing multiple Lyapunov functions and iterative methods in conjunction with linear matrix inequality (LMI) techniques, we establish sufficient conditions to guarantee stochastic mean-square stability. This approach extends the existing research on the stability of MJSs under various uncertainties and attacks. Two optimization problems are formulated to maximize the domain of attraction in the mean-square sense. The effectiveness of the proposed method is demonstrated through the simulation of an RLC circuit. Additionally, a numerical case study investigates the influence of the amount of known information in the transition probability matrix and the asynchronous situation between the system and the controller on the optimal estimation of the domain of attraction in the mean-square sense. This work provides a comprehensive framework for analyzing and enhancing the stability of MJSs under complex real-world conditions.

Keywords Markov jump systems, denial-of-service attacks, hidden Markov chains, actuator saturation, linear matrix inequality

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1 Introduction

Markov jump systems (MJSs) have been extensively studied across various fields due to their capability to model dynamic parameter variations. Typical applications include tunnel diode circuits [1], autonomous ground vehicles [2], and DC motor models [3]. Significant progress has been made in MJS research over the past decade [4–10]. A prevalent assumption in many existing studies is the synchronization between the system mode and the controller mode. Many existing approaches such as sampled-data control [11], $\mathcal{H}_\infty$ control [12], and mean-square stability control [13] assume synchronization between system and controller modes. Time-delay issues in Markov systems are also well-documented [14, 15]. In practice, however, synchronization may be disrupted by network congestion or malicious attacks. To address this, asynchronous controller designs have emerged as a flexible solution that subsumes both mode-dependent and mode-independent strategies [16]. For example, Ref. [17] addressed asynchronous state feedback control for MJSs, while Ref. [18] showed through passivity-based analysis that such methods can often achieve reduced conservatism. Ref. [19] addressed the composite $\mathcal{H}_\infty$ control problem for hidden MJSs subject to replay attacks. Meanwhile, Ref. [20] proposed a novel hybrid reinforcement Q-learning control method for adaptive fuzzy $\mathcal{H}_\infty$ control of discrete-time nonlinear MJSs using the Takagi-Sugeno fuzzy model.

Actuator saturation is a critical factor in studying system nonlinearity. As a prevalent nonlinear issue, actuator saturation limits the system control input to the peak output of the actuator, which can significantly impact the stability of the closed-loop system. Consequently, actuator saturation has garnered considerable attention from researchers [21–28]. However, to date, there has been no scholarly research addressing the Markov system's response to DoS attacks and physical asynchronous control issues under actuator saturation. This gap in the literature underscores the importance of our research, which aims to address these unresolved issues.

The primary types of cyber-attacks include DoS attacks [29–35], deception attacks [36–39], and replay attacks [40, 41]. Among these, DoS attacks are the most common and damaging. These attacks involve malicious actors

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overwhelming a server with frequent requests, rendering it incapable of responding to control signals and eventually causing the controller to become nonfunctional for a period. For instance, Ref. [32] employed a sliding control strategy to achieve robust control of Markov systems under DoS attacks, while Ref. [33] utilized multiple Lyapunov functions to address asynchronous control issues under DoS attacks. Additionally, Ref. [34] investigated DoS attacks on the communication channels of semi-MJSs. Deception attacks destabilize systems via misleading controller signals. Stabilization under such attacks is studied in [38], while Ref. [39] proposed a switching filter-based method for finite-time $\mathcal{H}_\infty$ filtering control of delayed MJSs. Replay attacks disrupt stability by re-injecting historical sensor data; Refs. [40, 41] analyzed MJSs under such attacks, with Ref. [41] introduced a detection/identification framework and a corresponding stability-ensuring controller.

Based on the aforementioned considerations, this paper investigates actuator saturation in MJSs under DoS attacks. This paper proposes a novel framework for analyzing the stochastic mean-square stability of MJSs under integrated challenges of asynchronous control, actuator saturation, and DoS attacks. Key contributions include:

(1) Establishing LMI-based stability conditions by modeling resource-constrained DoS attacks as a Bernoulli process;

(2) Developing optimization problems to quantify conservatism reduction and evaluate framework effectiveness;

(3) Validating the approach through RLC circuit simulations and numerical case studies on the impact of transition probability matrix information completeness and system-controller synchronization.

Notations. $\mathbb{R}^n$ denotes the n dimensional Euclidean space and $\mathbb{R}^{n\times m}$ stands for the set of $n \times m$ real matrices. $\mathbb{R} \geqslant 0$ denotes the set of reals greater than (greater than or equal to 0). For $r(k) = p$, $\bar{A}(r(k))$ is simplified as $\bar{A}_p$. A^{T} denotes the transpose of matrix A, and the "$*$" denotes a symmetric element in the same matrix. The notation $S > 0$ (respectively, $S \geqslant 0$) means that the matrix S is positive definite (respectively, semi-positive definite) and the sign "$<$" and "$\leqslant$" are used in "negative definite" and "semi-negative definite". The notation $\mathbf{diag}\{P_1, \ldots, P_n\}$ stands for a block-diagonal matrix, "P_n" represents the internal elements of the block-diagonal matrix and "n" represents the number of "P_n". $\mathbb{E}$ stands for the mathematical expectation, $\|\cdot\|$ stands for Euclidean norm.

2 Problem formulation

Actuator saturation is a common phenomenon in practical control systems, which can lead to performance degradation and even instability. For example, in aerospace applications, oversized actuators can increase the aircraft's weight, thereby increasing fuel costs. Therefore, studying how to design effective control strategies under actuator saturation to enhance the robustness and performance of systems is of great theoretical and practical significance. In this paper, we consider the following MJSs subject to actuator saturation

$$x(k+1) = \bar{A}(r(k))x(k) + \bar{B}(r(k))\sigma(u(k)). \tag{1}$$

The considered system is defined on a probability space $(\mathscr{U}, \mathscr{F}, \mathscr{P})$. For any system mode $r(k) = p$, system matrices $\bar{A}_p$, $\bar{B}_p$ are defined as $\bar{A}_p = A_p + \Delta A_p$, $\bar{B}_p = B_p + \Delta B_p$, respectively, where ΔA_p, ΔB_p are parameter uncertainties satisfying $[\Delta A_p, \Delta B_p] = U_p \Delta_p [V_{ap}, V_{bp}]$ and $\Delta_p^{\mathrm{T}} \Delta_p \leqslant I$. Moreover, matrices A_p, B_p, U_p, V_{ap}, and V_{bp} are all pre-known constant matrices. System state $x(k) \in \mathbb{R}^{n_s}$ and system control input $u(k) \in \mathbb{R}^{m_u}$. Standard saturation function $\sigma(u(k)) = [\sigma(u_1(k)), \sigma(u_2(k)), \ldots, \sigma(u_m(k))]^{\mathrm{T}}$, where $\sigma(u_p(k))$ is defined as $\sigma(u_p(k)) = \mathrm{sign}(u_p(k)) \min\{1, |u_p(k)|\}$, and

$$\mathrm{sign}(u_p(k)) = \begin{cases} -1, & \text{if} \quad u_p(k) < 0, \\ 0, & \text{if} \quad u_p(k) = 0, \\ 1, & \text{if} \quad u_p(k) > 0. \end{cases}$$

All indices of system modes are contained in a pre-given finite set $\mathscr{N} = \{1, 2, \ldots, N\}$, where N is a pre-known positive integer. The evolution of system modes is represented by a homogeneous Markov chain $\{r(k),\ k \geqslant 0\}$. The transition probability matrix is given by $\Pi = \{\pi_{pq}\}$. The associated transition probability is given as follows:

$$\pi_{pq} = \Pr\{r(k+1) = q \mid r(k) = p\}, \tag{2}$$

where $\pi_{pq} \in [0, 1]$ and $\sum_{q=1}^{N} \pi_{pq} = 1,\ \forall p, q \in \mathscr{N}$.

Then, an asynchronous state feedback controller in the following form will be considered

$$u(k) = \delta(k) K(\theta(k)) x(k), \tag{3}$$

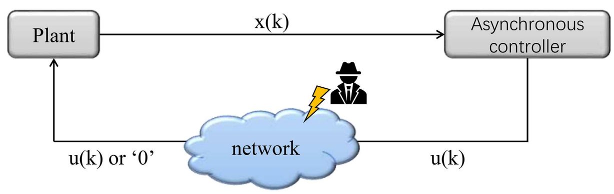

Figure 1 (Color online) System structure suffering from DoS attacks.

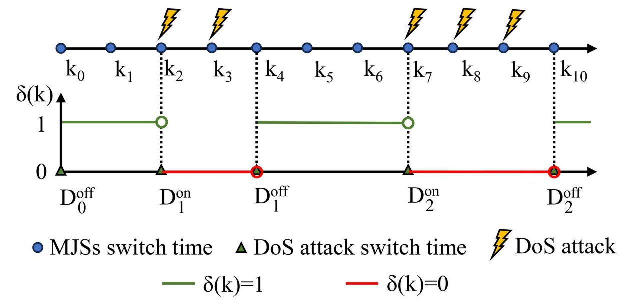

Figure 2 (Color online) An example of the signal transmission.

where $\{\theta(k),\ k \geqslant 0\}$ is controller mode and is contained in a finite set $\mathscr{M} = \{1, 2, \ldots, M\}$, where M is pre-known positive integer. The DoS attack indicator function is denoted by $\delta(k)$. The transition of controller modes is governed by a stochastic process defined by the conditional probability matrix $\Psi = \{\mu_{ps}\}$, where

$$\mu_{ps} = \Pr\{\theta(k) = s \mid r(k) = p\}, \tag{4}$$

and μ_{ps} satisfy $\mu_{ps} \in [0, 1]$, $\sum_{s=1}^{M} \mu_{ps} = 1$ for all $p \in \mathscr{N}$ and $s \in \mathscr{M}$.

We assume that the entries of Π and Ψ are partially accessible, i.e., Π and Ψ may have following form:

$$\begin{bmatrix} \pi_{11} & ? & \pi_{13} & ? \\ ? & ? & ? & \pi_{24} \\ \pi_{31} & ? & \pi_{33} & ? \\ ? & ? & \pi_{43} & \pi_{44} \end{bmatrix}, \quad \begin{bmatrix} \mu_{11} & ? & \mu_{13} & ? \\ ? & ? & ? & \mu_{24} \\ ? & \mu_{32} & \mu_{33} & ? \\ ? & ? & \mu_{43} & \mu_{44} \end{bmatrix}.$$

Assuming $r(k) = p$, we denote set $\mathscr{N} = \mathscr{N}_p^k + \mathscr{N}_p^u$ and $\mathscr{M} = \mathscr{M}_p^k + \mathscr{M}_p^u$ with

$$\begin{aligned} \mathscr{N}_p^k &\triangleq \{q : \pi_{pq} \text{ is known}\},\ \mathscr{N}_p^u \triangleq \{q : \pi_{pq} \text{ is unknown}\}, \\ \mathscr{M}_p^k &\triangleq \{s : \mu_{ps} \text{ is known}\},\ \mathscr{M}_p^u \triangleq \{s : \mu_{ps} \text{ is unknown}\}. \end{aligned}$$

When the above set is not $\varnothing$, by defining $a^{(p)} + c^{(p)} = N$ and $b^{(p)} + d^{(p)} = M$, we obtain the following sets:

$$\begin{aligned} \mathscr{N}_p^k &= \{\bar{n}_1, \ldots, \bar{n}_{a^{(p)}}\},\ 1 \leqslant a^{(p)} \leqslant N,\ \mathscr{N}_p^u = \{\hat{n}_1, \ldots, \hat{n}_{c^{(p)}}\},\ 1 \leqslant c^{(p)} \leqslant N, \\ \mathscr{M}_p^k &= \{\bar{m}_1, \ldots, \bar{m}_{b^{(p)}}\},\ 1 \leqslant b^{(p)} \leqslant M,\ \mathscr{M}_p^u = \{\hat{m}_1, \ldots, \hat{m}_{d^{(p)}}\},\ 1 \leqslant d^{(p)} \leqslant M. \end{aligned}$$

Remark 1. As described in [18, 28, 33], the asynchronous controller can be transformed into a mode-independent controller and a mode-dependent controller. When the controller has only one mode (i.e., $\mathscr{M} = \{1\}$), the controller uses the same parameters for different system modes, and the asynchronous controller is equivalent to the mode-independent controller. When the mode of the controller is always consistent with the system mode (i.e., $\mathscr{N} = \mathscr{M}$, $\mu_{ps} = 1$ for $p = s$), the asynchronous controller is equivalent to the synchronous controller.

As shown in Figure 1, MJSs fail to respond when the switch time is due to the influence of DoS attacks. In this paper, a DoS attack follows a Bernoulli distribution with i (i.e., $\Pr\{\delta(k) = 0\} = i,\ i \in [0, 1]$). $\delta(k) = 0$ characterizes the situation in which an attack happens, and $\delta(k) = 1$ stands for that MJSs were not affected by DoS attacks. Furthermore, the indicator function is defined as

$$\delta(k) = \begin{cases} 1,\ k \in [D_n^{\text{off}}, D_{n+1}^{\text{on}}), \\ 0,\ k \in [D_n^{\text{on}}, D_n^{\text{off}}). \end{cases}$$

Refer to Figure 2, the indicator function $\delta(k) = 1$, $u(k) = u(k)$, when the MJSs are not under DoS attacks (e.g., $k \in [k_0, k_2)$). On the other hand, the MJSs suffer from DoS attacks at time k_2 and at time k_3, causing the MJSs to be in a temporary open-loop state (i.e., $\delta(k) = 0,\ u(k) = 0,\ k \in [k_2, k_4)$). According to the definition of the standard saturation function, we get $x(k+1) = \bar{A}(r(k))x(k),\ k \in [k_2, k_4)$.

To further develop the research in this paper, some crucial concepts are listed below.

Symbol h_{sq} expresses the elements in row q of symmetric polyhedron set $\mathscr{L}(L_s)$ and set $\mathscr{L}(L_s)$ is defined as follows:

$$\mathscr{L}\big(L_s\big) = \big\{x(k) \in \mathbb{R}^{n_s} : \ \big|h_{sq}x(k)\big| \leqslant 1\big\}. \tag{5}$$

Ellipsoid set $\mathscr{R}(P_{pl})$ contains all system states satisfying $x^{\mathrm{T}}(k)P_{pl}x(k) \leqslant 1$, where P_{pl} is positive-definite matrices, precisely,

$$\mathscr{R}(P_{pl}) = \big\{x(k) \in \mathbb{R}^{n_s} : \ x^{\mathrm{T}}(k)P_{pl}x(k) \leqslant 1\big\}. \tag{6}$$

Definition 1 ([27])**.** For any initial state $x(0) \in \mathscr{J}$, if the following statement holds:

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}\|x(k)\|^2\middle| x(0),\ r(0)\right\} < \infty, \tag{7}$$

then, the set $\mathscr{J}$ is called the domain of attraction in mean-square sense of system (1).

Lemma 1 ([26])**.** For two given matrices $K_s \in \mathbb{R}^{m_u\times n_s}$ and $L_s \in \mathbb{R}^{m_u\times n_s}$, system state $x(k) \in \mathscr{L}(L_s)$ implies that the standard saturation function $\sigma(K_s x(k))$ can be expressed as follows:

$$\sigma\big(K_s x(k)\big) = \sum_{l=1}^{2^m}\eta_l(k)\big(R_l K_s + R_l^- L_s\big)x(k), \tag{8}$$

where symbol l represents the saturated mode at the k-th time instant, $0 \leqslant \eta_l(k) \leqslant 1$ and $\sum_{l=1}^{2^m}\eta_l(k) = 1$. $R_l \in \mathbb{R}^{m_u\times m_u}$ are diagonal matrices satisfying that every diagonal element is either 1 or 0, and the symbol R_l^- is defined as $R_l^- = I - R_l$.

Lemma 2 ([28])**.** For any given matrices U, V, and Δ with appropriate dimensions. If $\Delta^{\mathrm{T}}\Delta \leqslant I$ holds, then for any sufficiently small positive parameter ε, the following equation holds:

$$U\Delta V + (U\Delta V)^{\mathrm{T}} \leqslant \varepsilon UU^{\mathrm{T}} + \varepsilon^{-1}V^{\mathrm{T}}V. \tag{9}$$

In order to implement a more progressive control strategy in an environment susceptible to aperiodic DoS attacks, this study proposes reasonable descriptions and assumptions regarding DoS attacks, as referred to in [30]. During time $[0,k)$, the symbol $\Psi_d(k)$ stands for the total number of DoS attacks, and the symbol $\Psi_f(k)$ stands for the total number of DoS attacks off/on transition instants. Hence,

$$\Psi_d(k) = \sum_{s=1}^{k-1}(1-\delta_s), \tag{10}$$

$$\Psi_f(k) = (1-\delta_0) + \sum_{s=1}^{k-1}\delta_{s-1}(1-\delta_s). \tag{11}$$

From the perspective of energy consumption, the energy of the attacker is limited, and a sleep period is necessary for the next subsequent attack. Consequently, we establish the following reasonable assumptions regarding the maximum number and frequency of attacks.

Assumption 1. Following [30], we define parameters $\varphi_d,\ \varphi_f \in \mathbb{R} \geqslant 0$ and $\tau_d,\ \tau_f \in [0,1]$ such that:

(1) The attack count $\Psi_d(k)$ satisfies $\Psi_d(k) \leqslant \varphi_d + \tau_d k$;

(2) The attack frequency $\Psi_f(k)$ satisfies $\Psi_f(k) \leqslant \varphi_f + \tau_f k$, for all relevant k, where φ_d and φ_f characterize initial attack intensities and τ_d and τ_f represent attack growth rates.

3 Main results

Initially, conditions are established through multiple Lyapunov functions and iterative methods to ensure the mean-square stability of MJSs (1) under DoS attacks, with the set $\bigcap_{p=1}^{N}\bigcap_{l=1}^{2^m}\mathscr{R}(P_{pl})$ serving as an estimate of the domain of attraction in the mean-square sense (DoA-MSS).

Theorem 1. Under the considered DoS attack, if there exist scalars $\xi \geqslant 1$, $\lambda_1 \in [0,1)$, $\lambda_2 > 0$, such that the following condition holds:

$$\rho = \ln(1-\lambda_1) + \tau_d\ln((1+\lambda_2)/(1-\lambda_1)) + 2\tau_f\ln\xi < 0, \tag{12}$$

and if there exist matrices $P_{pl} \in \mathbb{R}^{n_s \times n_s} > 0$, $S_p \in \mathbb{R}^{n_s \times n_s} > 0$, such that the following conditions hold:

$$\sum_{s=1}^{M} \mu_{ps} \left(\hat{A}_{pls}^{\mathrm{T}} \sum_{q=1}^{N} \pi_{pq} P_{qt} \hat{A}_{pls} \right) - (1-\lambda_1) P_{pl} < 0, \tag{13}$$

$$\bar{A}_p^{\mathrm{T}} \sum_{q=1}^{N} \pi_{pq} S_q \bar{A}_p - (1+\lambda_2) S_p < 0, \tag{14}$$

$$P_{pl} \leqslant \xi S_p,\ \ S_p \leqslant \xi P_{pl}, \tag{15}$$

$$\mathscr{R}(P_{pl}) \subset \mathscr{L}(L_s), \tag{16}$$

where $\hat{A}_{pls} = \bar{A}_p + \bar{B}_p R_l K_s + \bar{B}_p R_l^- L_s$, then, the system (1) with the asynchronous controller (3) is mean-square stable and the set $\bigcap_{p=1}^{N} \bigcap_{l=1}^{2^m} \mathscr{R}(P_{pl})$ is an estimation of DoA-MSS.

Proof. Without loss of generality, we assume $r(k) = p$ and $\theta(k) = s$. When the MJSs (1) is not under DoS attacks (i.e., $k \in [D_n^{\mathrm{off}}, D_{n+1}^{\mathrm{on}})$), it means that $\delta(k) = 1$ and $u(k) = K_s x(k)$. In this situation, the system can be described as

$$x(k+1) = \bar{A}_p x(k) + \bar{B}_p \sigma(K_s x(k)).$$

Assume that initial state $x(0)$ is an element of DoA-MSS of system (1) (i.e., $x(k) \in \bigcap_{p=1}^{N} \bigcap_{l=1}^{2^m} \mathscr{R}(P_{pl})$). Then, by condition (16), one has $x(k) \in \mathscr{L}(L_s)$. According to Lemma 1, Eq. (17) can be obtained as

$$\begin{aligned} x(k+1) &= \bar{A}_p x(k) + \bar{B}_p \sigma(K_s x(k)) \\ &= \sum_{l=1}^{2^m} \eta_l(k) [\bar{A}_p + \bar{B}_p R_l K_s + \bar{B}_p R_l^- L_s] x(k) \\ &= \sum_{l=1}^{2^m} \eta_l(k) \hat{A}_{pls} x(k). \end{aligned} \tag{17}$$

Define the Lyapunov function as follows:

$$\mathscr{V}_1(k) = x^{\mathrm{T}}(k) \left(\sum_{l=1}^{2^m} \eta_l(k) P_{pl} \right) x(k).$$

And, the expectation of $\Delta \mathscr{V}_1(k)$ can be calculated as

$$\begin{aligned} \mathbb{E}\{\Delta \mathscr{V}_1(k)\} &= \mathbb{E}\{\mathscr{V}_1(k+1) | x(k),\ r(k) = p\} - \mathscr{V}_1(k) \\ &= \sum_{q=1}^{N} \pi_{pq} \mathbb{E}\{\mathscr{V}_1(k+1) | x(k),\ r(k) = p,\ r(k+1) = q\} - \mathscr{V}_1(k) \\ &= x(k+1)^{\mathrm{T}} \sum_{t=1}^{2^m} \eta_t(k) \sum_{q=1}^{N} \pi_{pq} P_{qt} x(k+1) - x^{\mathrm{T}}(k) \sum_{l=1}^{2^m} \eta_l(k) P_{pl} x(k). \end{aligned} \tag{18}$$

Based on (17) and (18), we have

$$\mathbb{E}\{\Delta \mathscr{V}_1(k)\} + \lambda_1 \mathscr{V}_1(k) = x^{\mathrm{T}}(k) \sum_{l=1}^{2^m} \eta_l(k) \left\{ \sum_{s=1}^{M} \mu_{ps} \left(\hat{A}_{pls}^{\mathrm{T}} \sum_{t=1}^{2^m} \eta_t(k) \sum_{q=1}^{N} \pi_{pq} P_{qt} \hat{A}_{pls} \right) - (1-\lambda_1) P_{pl} \right\} x(k).$$

Due to $0 \leqslant \eta_l(k) \leqslant 1$, $\lambda_1 \in [0, 1)$ and $\sum_{l=1}^{2^m} \eta_l(k) = 1$, $\mathbb{E}\{\Delta \mathscr{V}_1(k)\} + \lambda_1 \mathscr{V}_1(k) < 0$ can be satisfied from (13). However, in contrast to the previously discussed scenario, when the system (1) suffers from DoS attacks (i.e., $\delta(k) = 0$ for $k \in [D_n^{\mathrm{on}}, D_n^{\mathrm{off}})$), the system (1) can be described as

$$x(k+1) = \bar{A}_p x(k). \tag{19}$$

In this case, the system (1) degenerates to an open-loop system (19). Then, define the other Lyapunov function as follows:

$$\mathscr{V}_2(k) = x^{\mathrm{T}}(k) S_p x(k). \tag{20}$$

The expectation of $\Delta\mathscr{V}_2(k)$ is defined as $\mathbb{E}\{\mathscr{V}_2(k+1)|x(k),\ r(k)=p\}-\mathscr{V}_2(k)$. Further, one has

$$\mathbb{E}\{\Delta\mathscr{V}_2(k)\}-\lambda_2\mathscr{V}_2(k)=x^{\mathrm{T}}(k)\left\{\bar{A}_p^{\mathrm{T}}\sum_{q=1}^{N}\pi_{pq}S_q\bar{A}_p-(1+\lambda_2)S_p\right\}x(k). \tag{21}$$

Condition $\mathbb{E}\{\Delta\mathscr{V}_2(k)\}-\lambda_2\mathscr{V}_2(k)<0$ can be satisfied based on (14). According to (13) and (14), the inequalities $\mathbb{E}\{\Delta\mathscr{V}_1(k)\}+\lambda_1\mathscr{V}_1(k)<0$ and $\mathbb{E}\{\Delta\mathscr{V}_2(k)\}-\lambda_2\mathscr{V}_2(k)<0$ can be obtained. Hence, we can conclude the following based on (13) and (14)

$$\begin{cases}\mathbb{E}\{\mathscr{V}_1(k+1)\}<(1-\lambda_1)\mathbb{E}\{\mathscr{V}_1(k)\},\ k\in[D_n^{\mathrm{off}},D_{n+1}^{\mathrm{on}}),\\ \mathbb{E}\{\mathscr{V}_2(k+1)\}<(1+\lambda_2)\mathbb{E}\{\mathscr{V}_2(k)\},\ k\in[D_n^{\mathrm{on}},D_n^{\mathrm{off}}).\end{cases}$$

Moreover, condition (15) implies that

$$\mathbb{E}\{\mathscr{V}_1(k)\}<\xi\mathbb{E}\{\mathscr{V}_2(k)\},\ \mathbb{E}\{\mathscr{V}_2(k)\}<\xi\mathbb{E}\{\mathscr{V}_1(k)\}.$$

For any switching point, it can be obtained that $\mathbb{E}\{\mathscr{V}_1(D_n^{\mathrm{off}^+})\}\leqslant\xi\mathbb{E}\{\mathscr{V}_2(D_n^{\mathrm{off}^-})\}$, $\mathbb{E}\{\mathscr{V}_2(D_n^{\mathrm{on}^+})\}\leqslant\xi\mathbb{E}\{\mathscr{V}_1(D_n^{\mathrm{on}^-})\}$. The symbol $D_n^{\mathrm{off}^-}$ (or $D_n^{\mathrm{off}^+}$) stands for the left (or right) limit at the time point D_n^{off}. The Lyapunov function $\mathscr{V}(k)$ is switching between $\mathscr{V}_1(k)$ and $\mathscr{V}_2(k)$ because of the impact of DoS attacks. By the iteration method, one has

$$\begin{aligned}\mathbb{E}\{\mathscr{V}(k)\}&<(1-\lambda_1)\ \mathbb{E}\{\mathscr{V}_1(k-1)\}\\&<(1-\lambda_1)^2\ \mathbb{E}\{\mathscr{V}_1(k-2)\}\\&<(1-\lambda_1)^2\ \xi\mathbb{E}\{\mathscr{V}_2(k-2)\}\\&<\cdots\\&<(1-\lambda_1)^{k-\Psi_d(k)}(1+\lambda_2)^{\Psi_d(k)}\xi^{2\Psi_f(k)}\mathscr{V}(0)\\&<\mathrm{e}^{(k-\Psi_d(k))\ln(1-\lambda_1)+\Psi_d(k)\ln(1+\lambda_2)+2\Psi_f(k)\ln\xi}\mathscr{V}(0).\end{aligned}$$

Then, combining Assumption 1, one can get

$$\begin{aligned}\mathbb{E}\{\mathscr{V}(k)\}&<\mathrm{e}^{(k-\Psi_d(k))\ln(1-\lambda_1)+\Psi_d(k)\ln(1+\lambda_2)+2\Psi_f(k)\ln\xi}\mathscr{V}(0)\\&<\mu\mathrm{e}^{k(\ln(1-\lambda_1)+\tau_d\ln((1+\lambda_2)/(1-\lambda_1))+2\tau_f\ln\xi)}\mathscr{V}(0),\end{aligned}$$

where $\mu=\mathrm{e}^{\varphi_d\ln((1+\lambda_2)/(1-\lambda_1))+2\varphi_f\ln\xi}$. Letting $\rho=\ln(1-\lambda_1)+\tau_d\ln((1+\lambda_2)/(1-\lambda_1))+2\tau_f\ln\xi$, one has

$$\mathbb{E}\{\mathscr{V}(k)\}<\mu\mathrm{e}^{\rho k}\mathscr{V}(0).$$

Furthermore, by condition (12), one has

$$\mathbb{E}\|x(k)\|^2<\frac{\mathscr{K}_1}{\mathscr{K}_2}\mu\mathrm{e}^{\rho k}x(0)^2<\infty, \tag{22}$$

where $\mathscr{K}_1=\max\{\lambda(P_{1p}),\ \lambda(P_{2p})\}$, $\mathscr{K}_2=\min\{\lambda(P_{1p}),\ \lambda(P_{2p})\}$. Since $x(k)\in\bigcap_{p=1}^{N}\bigcap_{l=1}^{2^m}\mathscr{R}(P_{pl})$, inequality (22) implies that the system (1) under DoS attacks is mean-square stable and $\bigcap_{p=1}^{N}\bigcap_{l=1}^{2^m}\mathscr{R}(P_{pl})$ is a DoA-MSS of systems (19). The proof is completed.

Remark 2. Due to external environmental influences and limited computational resources, obtaining precise values for all elements of the transition probability matrix (TPM) at any instant incurs high costs. In Theorem 2, the sufficient condition from Theorem 1 for guaranteeing system stability is extended to a more general scenario with partially unknown transition probabilities through the application of LMI techniques.

Theorem 2. Under the considered DoS attack, if there exist scalars $\xi\geqslant1$, $\lambda_1\in[0,1)$, $\lambda_2>0$ such that the parameters of the DoS attack's frequency and duration satisfy (12), and if there exist matrices $P_{pl}\in\mathbb{R}^{n_s\times n_s}>0$, $\bar{S}_p\in\mathbb{R}^{n_s\times n_s}>0$, $R_{ps}\in\mathbb{R}^{n_s\times n_s}>0$, such that (15), (16) and the following conditions hold:

$$\begin{bmatrix}-(1-\lambda_1)P_{pl} & \varpi_{12}\\ * & \varpi_{22}\end{bmatrix}<0, \tag{23}$$

$$\begin{bmatrix} R_{ps} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix} < 0, \tag{24}$$

$$\begin{bmatrix} -(1+\lambda_2)\bar{S}_p & \bar{\Lambda}_{12} \\ * & \bar{\Lambda}_{22} \end{bmatrix} < 0, \tag{25}$$

where

$$\varpi_{12} = [\sqrt{\mu_{p\bar{m}_1}}I, \ldots, \sqrt{\mu_{p\bar{m}_{b(p)}}}I, \underbrace{\sqrt{g}I, \ldots, \sqrt{g}I}_{d^{(p)}}],\ \varpi_{22} = \mathbf{diag}\left\{-R^{-1}_{p\bar{m}_1}, \ldots, -R^{-1}_{p\bar{m}_{b(p)}}, -R^{-1}_{p\hat{m}_1}, \ldots, -R^{-1}_{p\hat{m}_{d(p)}}\right\},$$

$$\Theta_{12} = [\sqrt{\pi_{p\bar{n}_1}}\hat{A}^{\mathrm{T}}_{pls}, \ldots, \sqrt{\pi_{p\bar{n}_{a(p)}}}\hat{A}^{\mathrm{T}}_{pls}, \underbrace{\sqrt{h}\hat{A}^{\mathrm{T}}_{pls}, \ldots, \sqrt{h}\hat{A}^{\mathrm{T}}_{pls}}_{c^{(p)}}],\ \Theta_{22} = \mathbf{diag}\left\{\bar{P}_{\bar{n}_1 t}, \ldots, \bar{P}_{\bar{n}_{a(p)} t}, \bar{P}_{\hat{n}_1 t}, \ldots, \bar{P}_{\hat{n}_{c(p)} t}\right\},$$

$$\Lambda_{12} = [\sqrt{\pi_{p\bar{n}_1}}\bar{A}^{\mathrm{T}}_{p}, \ldots, \sqrt{\pi_{p\bar{n}_{a(p)}}}\bar{A}^{\mathrm{T}}_{p}, \underbrace{\sqrt{h}\bar{A}^{\mathrm{T}}_{p}, \ldots, \sqrt{h}\bar{A}^{\mathrm{T}}_{p}}_{c^{(p)}}],\ \bar{\Lambda}_{22} = \mathbf{diag}\left\{-\bar{S}_{\bar{n}_1}, \ldots, -\bar{S}_{\bar{n}_{a(p)}}, -\bar{S}_{\hat{n}_1}, \ldots, -\bar{S}_{\hat{n}_{c(p)}}\right\},$$

$$\bar{\Lambda}_{12} = \bar{S}_p\Lambda_{12},\ \bar{S}_p = S_p^{-1},\ \bar{P}_{pl} = P_{pl}^{-1}.$$

Then, the system (1) with the asynchronous controller (3) is mean-square stable under DoS attacks and $\bigcap_{p=1}^{N}\bigcap_{l=1}^{2^m}\mathscr{R}(P_{pl})$ is an estimation of DoA-MSS.

Proof. Let $R_{ps} \in \mathbb{R}^{n_s \times n_s} > 0$, the following two inequalities hold when condition (13) is true:

$$\sum_{s=1}^{M}\mu_{ps}R_{ps} - (1-\lambda_1)P_{pl} < 0, \tag{26}$$

$$R_{ps} - \hat{A}^{\mathrm{T}}_{pls}\sum_{q=1}^{N}\pi_{pq}P_{qt}\hat{A}_{pls} < 0. \tag{27}$$

For (26), one has

$$\begin{aligned}\sum_{s=1}^{M}\mu_{ps}R_{ps} - (1-\lambda_1)P_{pl} &= \sum_{s=\bar{m}_1, s\in\mathscr{M}_p^k}^{\bar{m}_{b(p)}}\mu_{ps}R_{ps} + \sum_{s=\hat{m}_1, s\in\mathscr{M}_p^u}^{\hat{m}_{d(p)}}\mu_{ps}R_{ps} - (1-\lambda_1)P_{pl} \\ &\leqslant \sum_{s=\bar{m}_1, s\in\mathscr{M}_p^k}^{\bar{m}_{b(p)}}\mu_{ps}R_{ps} + \left(1 - \sum_{s=\bar{m}_1, s\in\mathscr{M}_p^k}^{\bar{m}_{b(p)}}\mu_{ps}\right)\sum_{s=\hat{m}_1, s\in\mathscr{M}_p^u}^{\hat{m}_{d(p)}}R_{ps} - (1-\lambda_1)P_{pl}.\end{aligned} \tag{28}$$

According to the Schur complement, inequality (28) $\leqslant 0$ holds under the sufficient condition (23). In the same way, when the entries of Π are partially accessible, condition (27) can be rewritten as

$$R_{ps} - \hat{A}^{\mathrm{T}}_{pls}\sum_{q=\bar{n}_1, q\in\mathscr{N}_p^k}^{\bar{n}_{a(p)}}\pi_{pq}P_{qt}\hat{A}_{pls} - \hat{A}^{\mathrm{T}}_{pls}\left(1 - \sum_{q=\bar{n}_1, q\in\mathscr{N}_p^k}^{\bar{n}_{a(p)}}\pi_{pq}\right)\sum_{q=\hat{n}_1, q\in\mathscr{N}_p^u}^{\hat{n}_{c(p)}}P_{qt}\hat{A}_{pls} < 0.$$

Setting $h = 1 - \sum_{q=\bar{n}_1, q\in\mathscr{N}_p^k}^{\bar{n}_{a(p)}}\pi_{pq}$ and $g = 1 - \sum_{s=\bar{m}_1, s\in\mathscr{M}_p^k}^{\bar{m}_{b(p)}}\mu_{ps}$, the above inequality can be rewritten as (24) by using the Schur complement.

Using the same technique as above, condition (14) can be transformed as

$$\bar{A}_p^{\mathrm{T}}\sum_{q=1}^{N}\pi_{pq}S_q\bar{A}_p - (1+\lambda_2)S_p \leqslant \bar{A}_p^{\mathrm{T}}\sum_{q=\bar{n}_1, q\in\mathscr{N}_p^k}^{\bar{n}_{a(p)}}\pi_{pq}S_q\bar{A}_p + h\bar{A}_p^{\mathrm{T}}S_q\bar{A}_p - (1+\lambda_2)S_p. \tag{29}$$

Based on the Schur complement, the inequality (29) < 0 holds under the condition that

$$\begin{bmatrix} -(1+\lambda_2)S_p & \Lambda_{12} \\ * & \bar{\Lambda}_{22} \end{bmatrix} < 0. \tag{30}$$

Through $\mathbf{diag}\{\bar{S}_p,\ I\}$ and its transpose, we conduct congruent transformation of (30) to get (25). The proof is completed.

Remark 3. The conventional methods in [42, 43] for addressing unknown probabilities in inequalities possess certain strengths; however, our novel approach for inequalities (27) and (26) provides a distinctive benefit. By tolerating a marginal increase in conservatism, we successfully decrease the number of LMIs required. Consequently, this reduction significantly lowers the complexity of the theoretical framework, enhancing its efficiency and practicality.

Remark 4. Unlike the prior work presented in [33], this study incorporates system parameter errors, which enhances the resilience and adaptability of our findings to more challenging environments. When $\Delta A_p = 0$ (i.e., $\bar{A}_p = A_p$), the stability proof for the open-loop system from [33] is recovered as a special case in our work. In Theorem 3, this paper further addresses the system parameters by utilizing Lemma 2 for the scenarios where $\Delta A_p \neq 0$. This enhancement enables our conclusions to apply to more complex external environments and constrained conditions. Next, the coupling issue among K_s, L_s, and P_{pl} in Theorem 2 will be addressed in Theorem 3 by introducing new matrix variables and applying a congruent transformation.

Theorem 3. Under the considered DoS attack, if there exist scalars $\xi \geqslant 1$, $\lambda_1 \in [0, 1)$, $\lambda_2 > 0$ such that the parameters of DoS attacks' frequency and duration satisfy (12), and if there exist matrices $\bar{P}_{pl} \in \mathbb{R}^{n_s \times n_s} > 0$, $\bar{S}_p \in \mathbb{R}^{n_s \times n_s} > 0$, $\bar{R}_{ps} \in \mathbb{R}^{n_s \times n_s} > 0$, $G \in \mathbb{R}^{n_s \times n_s} > 0$ and exist scalars $\varepsilon_p > 0$, $\tau_p > 0$ such that

$$\begin{bmatrix} -(1-\lambda_1)\bar{P}_{pl} & \hat{\varpi}_{12} \\ * & \hat{\varpi}_{22} \end{bmatrix} < 0, \tag{31}$$

$$\begin{bmatrix} \bar{R}_{ps} - G^{\mathrm{T}} - G & \hat{\Theta}_{12} & 0 & \hat{\Theta}_{14} \\ * & \hat{\Theta}_{22} & \Theta_{23} & 0 \\ * & * & -\varepsilon_p^{-1} I & 0 \\ * & * & * & -\varepsilon_p I \end{bmatrix} < 0, \tag{32}$$

$$\begin{bmatrix} -(1+\lambda_2)\bar{S}_p & \bar{S}_p\hat{\Lambda}_{12} & 0 & V_{ap} \\ * & \bar{\Lambda}_{22} & \Theta_{23} & 0 \\ * & * & -\tau_p^{-1} I & 0 \\ * & * & * & -\tau_p I \end{bmatrix} < 0, \tag{33}$$

$$\begin{bmatrix} -\xi\bar{S}_p & \bar{S}_p \\ * & -\bar{P}_{pl} \end{bmatrix} \leqslant 0, \quad \begin{bmatrix} -\xi\bar{P}_{pl} & \bar{P}_{pl} \\ * & -\bar{S}_p \end{bmatrix} \leqslant 0, \tag{34}$$

$$\begin{bmatrix} -G^{\mathrm{T}} - G + \bar{P}_{pl} & {z_{sq}}^{\mathrm{T}} \\ * & -1 \end{bmatrix} \leqslant 0, \tag{35}$$

where

$$\hat{\varpi}_{12} = [\sqrt{\mu_{p\bar{m}_1}}\bar{P}_{pl}, \ldots, \sqrt{\mu_{p\bar{m}_{b(p)}}}\bar{P}_{pl}, \underbrace{\sqrt{g}\bar{P}_{pl}, \ldots, \sqrt{g}\bar{P}_{pl}}_{d^{(p)}}],\ \hat{\Lambda}_{12} = [\sqrt{\pi_{p\bar{n}_1}}A_p^{\mathrm{T}}, \ldots, \sqrt{\pi_{p\bar{n}_{a(p)}}}A_p^{\mathrm{T}}, \underbrace{\sqrt{h}A_p^{\mathrm{T}}, \ldots, \sqrt{h}A_p^{\mathrm{T}}}_{c^{(p)}}],$$

$$\hat{\Theta}_{12} = [\sqrt{\pi_{p\bar{n}_1}}A_{pls}^{\mathrm{T}}, \ldots, \sqrt{\pi_{p\bar{n}_{a(p)}}}A_{pls}^{\mathrm{T}}, \underbrace{\sqrt{h}A_{pls}^{\mathrm{T}}, \ldots, \sqrt{h}A_{pls}^{\mathrm{T}}}_{c^{(p)}}],\ \hat{\Theta}_{22} = \mathbf{diag}\left\{\bar{P}_{\bar{n}_1 t}, \ldots, \bar{P}_{\bar{n}_{a(p)} t}, \bar{P}_{\hat{n}_1 t}, \ldots, \bar{P}_{\hat{n}_{c(p)} t}\right\},$$

$$\Theta_{23} = [\sqrt{\pi_{p\bar{n}_1}}U_p, \ldots, \sqrt{\pi_{p\bar{n}_{a(p)}}}U_p, \underbrace{\sqrt{h}U_p, \ldots, \sqrt{h}U_p}_{c^{(p)}}]^{\mathrm{T}},\ \hat{\Theta}_{14} = [V_{ap} + V_{bp}R_l Y_s + V_{bp}R_l^- Z_s]^{\mathrm{T}},$$

$$A_{pls} = A_p G + B_p R_l Y_s + B_p R_l^- Z_s,\ \hat{\varpi}_{22} = \varpi_{22},\ \bar{P}_{pl} = P_{pl}^{-1},\ Y_s = K_s G,\ Z_s = L_s G.$$

Then, the system (1) with the asynchronous controller (3) is mean-square stable and $\bigcap_{p=1}^{N}\bigcap_{l=1}^{2^m}\mathscr{R}(P_{pl})$ is an estimation of DoA-MSS. In addition, the parameters of controller gains can be obtained by $K_s = X_s G^{-1}$ and $L_s = Y_s G^{-1}$.

Proof. Initially, we perform a congruence transformation on inequality (23) by employing the matrix $\mathbf{diag}\{\bar{P}_{pl}^{\mathrm{T}}, I\}$ and its transpose. Therefore, we obtain the equivalent condition (31) for the inequality (23). Subsequently, leveraging the relationships $\bar{A}_p = A_p + \triangle A_p$, $\bar{B}_p = B_p + \triangle B_p$, $\left[\triangle A_p, \triangle B_p\right] = U_p \triangle_p \left[V_{ap}, V_{bp}\right]$ and Lemma 2, we derive

(36) for inequality (24).

$$\begin{aligned}\begin{bmatrix} R_{ps} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix} &= \begin{bmatrix} R_{ps} & \bar{\Theta}_{12} \\ * & \hat{\Theta}_{22} \end{bmatrix} + U \triangle_p V + (U \triangle_p V)^{\mathrm{T}} \\ &\leqslant \begin{bmatrix} R_{ps} & \bar{\Theta}_{12} \\ * & \hat{\Theta}_{22} \end{bmatrix} + \varepsilon_p U U^{\mathrm{T}} + \varepsilon_p^{-1} V^{\mathrm{T}} V, \end{aligned} \tag{36}$$

where

$$U^{\mathrm{T}} = \begin{bmatrix} 0 & \Theta_{23}^{\mathrm{T}} \end{bmatrix}, \; V = \begin{bmatrix} \Theta_{14}^{\mathrm{T}} & 0 \end{bmatrix}, \; \ddot{A}_{pls} = A_p + B_p R_l K_s + B_p R_l^- L_s,$$

$$\bar{\Theta}_{12} = \left[\sqrt{\pi_{p\bar{n}_1}} \ddot{A}_{pls}^{\mathrm{T}}, \ldots, \sqrt{\pi_{p\bar{n}_{a(p)}}} \ddot{A}_{pls}^{\mathrm{T}}, \underbrace{\sqrt{h} \ddot{A}_{pls}^{\mathrm{T}}, \ldots, \sqrt{h} \ddot{A}_{pls}^{\mathrm{T}}}_{c^{(p)}} \right].$$

According to Schur complement, Eq. (36) $\leqslant 0$ can be obtained by (37)

$$\begin{bmatrix} R_{ps} & \bar{\Theta}_{12} & 0 & \Theta_{14} \\ * & \hat{\Theta}_{22} & \Theta_{23} & 0 \\ * & * & -\varepsilon_p^{-1} I & 0 \\ * & * & * & -\varepsilon_p I \end{bmatrix} \leqslant 0, \tag{37}$$

where $\Theta_{14} = [V_{ap} + V_{bp} R_l K_s + V_{bp} R_l^- L_s]^{\mathrm{T}}$. Further, by left- and right-multiplying with the diagonal matrix $\mathbf{diag}\{G^{\mathrm{T}} \; I \; \cdots \; I\}$ and $G^{\mathrm{T}} R_{ps} G \leqslant \bar{R}_{ps} - G^{\mathrm{T}} - G$, one has

$$\begin{bmatrix} G^{\mathrm{T}} R_{ps} G & G^{\mathrm{T}} \bar{\Theta}_{12} & 0 & G^{\mathrm{T}} \Theta_{14} \\ * & \hat{\Theta}_{22} & \Theta_{23} & 0 \\ * & * & -\varepsilon_p^{-1} I & 0 \\ * & * & * & -\varepsilon_p I \end{bmatrix} \leqslant \begin{bmatrix} \bar{R}_{ps} - G^{\mathrm{T}} - G & \hat{\Theta}_{12} & 0 & \hat{\Theta}_{14} \\ * & \hat{\Theta}_{22} & \Theta_{23} & 0 \\ * & * & -\varepsilon_p^{-1} I & 0 \\ * & * & * & -\varepsilon_p I \end{bmatrix}. \tag{38}$$

Inequality (38) < 0 can be obtained by (32). On the other hand, we let $\mathscr{F}_p = \begin{bmatrix} V_{ap}^{\mathrm{T}}, 0 \end{bmatrix}$. For the condition (25), one has

$$\begin{aligned}\begin{bmatrix} -(1+\lambda_2)\bar{S}_p & \bar{\Lambda}_{12} \\ * & \bar{\Lambda}_{22} \end{bmatrix} &= \begin{bmatrix} -(1+\lambda_2)\bar{S}_p & \bar{S}_p \hat{\Lambda}_{12} \\ * & \bar{\Lambda}_{22} \end{bmatrix} + U \triangle_p \mathscr{F}_p + (U \triangle_p \mathscr{F}_p)^{\mathrm{T}} \\ &\leqslant \begin{bmatrix} -(1+\lambda_2)\bar{S}_p & \bar{S}_p \hat{\Lambda}_{12} \\ * & \bar{\Lambda}_{22} \end{bmatrix} + \tau_p U U^{\mathrm{T}} + \tau_p^{-1} \mathscr{F}_p^{\mathrm{T}} \mathscr{F}_p \\ &\leqslant \begin{bmatrix} -(1+\lambda_2)\bar{S}_p & \bar{S}_p \hat{\Lambda}_{12} & 0 & V_{ap} \\ * & \bar{\Lambda}_{22} & \Theta_{23} & 0 \\ * & * & -\tau_p^{-1} I & 0 \\ * & * & * & -\tau_p I \end{bmatrix}. \end{aligned} \tag{39}$$

Then, inequality (39) $\leqslant 0$ can be satisfied by the condition (33). By leveraging the Schur complement, one has effortlessly derived (40) through (15).

$$\begin{bmatrix} -\xi S_p & S_p \\ * & -\bar{P}_{pl} \end{bmatrix} \leqslant 0, \; \begin{bmatrix} -\xi P_{pl} & P_{pl} \\ * & -\bar{S}_p \end{bmatrix} \leqslant 0. \tag{40}$$

Subsequently, by applying the congruent transformation to (40) using the matrices $\mathbf{diag}\{\bar{S}_p, \; I\}$ and $\mathbf{diag}\{\bar{P}_{pl}, \; I\}$, we have successfully derived (34).

In addition, Eq. (16) is equivalent to the following equation:

$$|h_{sq}x(k)| \leqslant 1,\ \forall x(k) \in \bigcap_{p=1}^{N}\bigcap_{l=1}^{2^m} \mathscr{R}\left(P_{pl}\right). \tag{41}$$

Moreover, notice that condition (41) can be transformed as $h_{sq}^{\mathrm{T}}h_{sq} \leqslant P_{pl}$. By Schur complement, $h_{sq}^{\mathrm{T}}h_{sq} \leqslant P_{pl}$ can be further rewritten as

$$\begin{bmatrix} -P_{pl} & h_{sq}^{\mathrm{T}} \\ * & -1 \end{bmatrix} \leqslant 0. \tag{42}$$

By left- and right-multiplying with the diagonal matrix $\mathbf{diag}\{G^{\mathrm{T}},\ I\}$ and using $G^{\mathrm{T}}P_{pl}G \leqslant \bar{P}_{pl} - G^{\mathrm{T}} - G$, the sufficient condition (35) of (42) can be derived.

Remark 5. Compared to Theorem 2, the decoupling of K_s and L_s in Theorem 3 enhances the flexibility in controller design and solution from a system design perspective. Furthermore, the utilization of Lemma 2 and the introduction of scalar variables ε_p and τ_p enable the systematic consideration and mitigation of the impacts arising from uncertainties in system parameters. By LMI technology, an effective framework is established to tackle the uncertainties and ensure robust performance of the MJS system under DoS attacks.

Before conducting simulations, we introduce two optimal control problems to intuitively ascertain whether $x(0)$ is encompassed within the domain of attraction in DoA-MSS, and to further underscore the superiority of our result as outlined below.

$$\begin{aligned} &\text{OP1: } \mathbf{sup}\,\alpha \\ &\qquad \bar{P}_{pl} > 0,\ S_p > 0,\ \bar{R}_{ps} > 0,\ G > 0,\ Y_s,\ Z_s,\ \varepsilon_p > 0,\ \tau_p > 0, \\ &\qquad \text{s.t.} \quad \text{(a) } \alpha x(0) \in \bigcap_{p=1}^{N}\bigcap_{l=1}^{2^m} \mathscr{R}(P_{pl}),\ \text{(b) LMIs (31)–(35).} \end{aligned}$$

Notice that Condition $\alpha x(0) \in \bigcap_{p=1}^{N}\bigcap_{l=1}^{2^m} \mathscr{R}(P_{pl})$ can be rewritten by $\alpha^2 x(0)^{\mathrm{T}}P_{pl}x(0) \leqslant 1$. As a result, we define $\rho = \alpha^{-2}$. In this way, $x(0)^{\mathrm{T}}P_{pl}x(0) \leqslant \rho$ holds. Further, by the Schur complement, the following statement can be obtained

$$\begin{bmatrix} -\rho & x(0)^{\mathrm{T}} \\ * & -\bar{P}_{pl} \end{bmatrix} \leqslant 0. \tag{43}$$

Now, OP1 is changed to the following optimization problem.

$$\begin{aligned} &\text{OP2}: \mathbf{inf}\,\rho \\ &\qquad \bar{P}_{pl} > 0,\ S_p > 0,\ \bar{R}_{ps} > 0,\ G > 0,\ Y_s,\ Z_s,\ \omega_p > 0,\ \tau_p > 0, \\ &\qquad \xi \geqslant 1,\ \lambda_1 \in [0,1),\ \lambda_2 > 0, \\ &\qquad \text{s.t.} \quad \text{LMIs (31)–(35) and (43).} \end{aligned}$$

For the case that $\rho_{\min} < 1$, it implies that $x(0)$ is in the DoA-MSS. Furthermore, the smaller the value of $\rho_{\min}$, the closer our estimated DoA-MSS approximates the actual one, rendering our estimation less conservative and more precise.

4 Simulations

In this part, two simulation examples are provided to demonstrate the effectiveness of our proposed results. An RLC circuit example from [44] is shown in Experiment 1 to illustrate the effectiveness of the obtained results.

Experiment 1. As shown in Figure 3, the RLC circuit consists of a power source $U(t)$, a resistor R, an inductor L, and a collection of capacitors C_s $(s = 1,\ 2)$. Different capacitors are switchable to alter the system parameters. Denote $U_C(t)$ as the voltage across the capacitor, $i_L(t)$ as the current through the inductor L, $i_R(t)$ as the current through the resistor R, and $i_c(t)$ as the current through the capacitor. Define the system state $x(t) = \begin{bmatrix} U_C(t) \\ i_L(t) \end{bmatrix}$ and the control input $u(t) = U(t)$. The system matrices are given by

$$A_1 = \begin{bmatrix} -\frac{1}{C_1R} & \frac{1}{C_1} \\ -\frac{1}{L} & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} -\frac{1}{C_2R} & \frac{1}{C_2} \\ -\frac{1}{L} & 0 \end{bmatrix},\ B_1 = B_2 = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}.$$

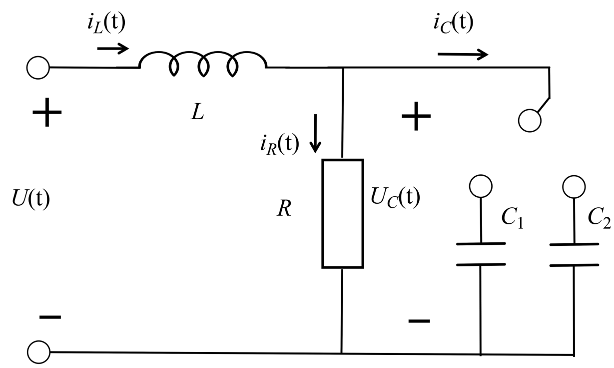

Figure 3 RLC circuit.

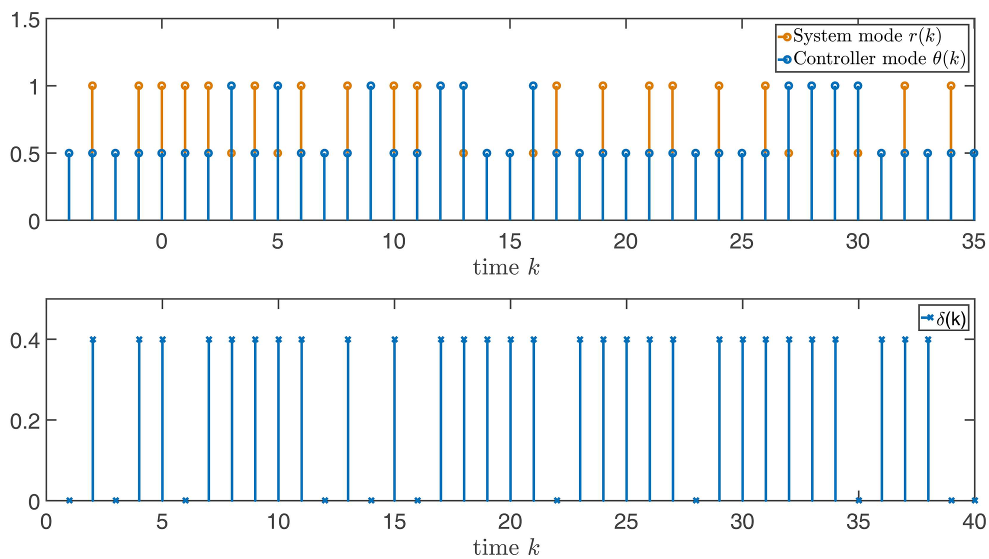

Figure 4 (Color online) The evolution of system mode, control mode, and DoS attack indicator function $\delta(k)$ ($i = 0.4$).

The parameters are chosen as $L = 2$ H, $R = 200$ Ω, $C_1 = 0.8$ F, $C_2 = 0.5$ F, with initial state $x_0 = [\begin{smallmatrix} 0.2 \\ 0.25 \end{smallmatrix}]$. The matrices describing the error of system parameters are considered as

$$U_1 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \ U_2 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \ V_{a1} = V_{a2} = \begin{bmatrix} 0.5, \ 0.5 \end{bmatrix}, \ V_{b1} = V_{b2} = 0.5.$$

The transition matrices are given by

$$\Pi = \begin{bmatrix} 0.3 & 0.7 \\ 0.6 & 0.4 \end{bmatrix}, \ \Psi = \begin{bmatrix} ? & ? \\ 0.8 & 0.2 \end{bmatrix}.$$

In addition, referring to [33], we choose $\xi = 4, \ \lambda_1 = 0.7, \ \lambda_2 = 3, \ \varphi_f = 4, \ \varphi_d = 3, \tau_d = 0.30, \ \tau_f = 0.20, \ p = 0.4$. The optimal value $\rho_{\min} = 0.6374$ is obtained by solving OP2 with the computed feedback gain matrices: $K_1 = \begin{bmatrix} -0.330, -1.7194 \end{bmatrix}$, $L_1 = \begin{bmatrix} 0.8435, 0.1085 \end{bmatrix}$, $K_2 = \begin{bmatrix} -0.5531, -0.9376 \end{bmatrix}$, $L_2 = \begin{bmatrix} 0.8531, 0.1049 \end{bmatrix}$.

The asynchronous control of MJSs with partially unknown parameter uncertainties and transition probabilities was investigated through fifty repeated experiments. Figure 4 illustrates the evolution of mode changes in both the system and controller, along with the DoS attack indicator function $\delta(k)$. Figure 5 depicts the trajectory of the system state during the experimental process.

Remark 6. The system state without an asynchronous nous controller throughout the process eventually becomes unstable. The asynchronous controller design method presented in this paper is correct and effective in achieving robust control of the MJSs. On the other hand, the asynchronous controller enables the system to reach stable state more quickly in the absence of DoS attacks.

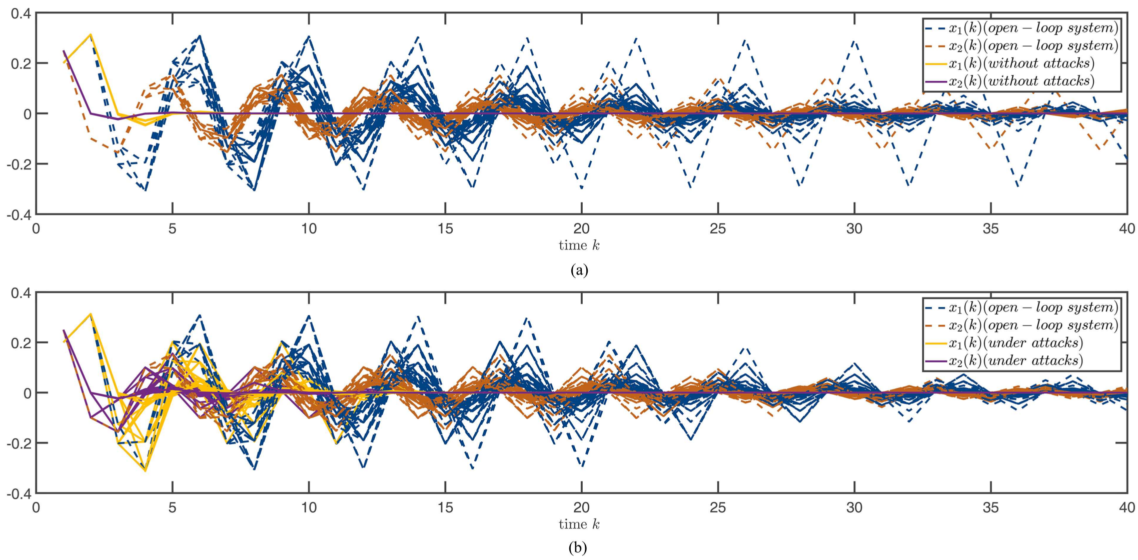

Figure 5 (Color online) (a) The system state evolution in the absence of DoS attacks; (b) the system state evolution under DoS attacks.

Table 1 The optimal value $\gamma_{\min}$ under different TPMs.

$\gamma_{\min}$	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5
Π_1	0.5247	0.5289	0.5097	0.5305	0.5331
Π_2	0.5747	0.6153	0.6038	0.6112	0.6282
Π_3	0.5634	0.6122	0.6167	0.6179	0.6230

Experiment 2. In this section, we conduct a numerical case study to investigate the impact of the amount of known information in the TPM on the optimal estimation of DoA-MSS. The new parameters are considered as follows:

$$A_1 = \begin{bmatrix} 0.5 & 1 \\ 0 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} 1 & 1.5 \\ 0.5 & 1 \end{bmatrix},\ B_1 = B_2 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix},\ \Pi_1 = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix},\ \Pi_2 = \begin{bmatrix} 0.6 & 0.4 \\ ? & ? \end{bmatrix},\ \Pi_3 = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix},$$
$$\Phi_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \Phi_2 = \begin{bmatrix} 0.4 & 0.6 \\ 0 & 1 \end{bmatrix},\ \Phi_3 = \begin{bmatrix} 0 & 1 \\ 0.8 & 0.2 \end{bmatrix},\ \Phi_4 = \begin{bmatrix} 0.4 & 0.6 \\ 0.8 & 0.2 \end{bmatrix},\ \Phi_5 = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}.$$

The other parameters are kept the same as those in Experiment 1. By combining these values of TPM and using OP2, we obtained optimal estimates of the system's domain of attraction, as shown in Table 1.

Remark 7. Table 1 not only validates that asynchronous control (Φ_4) achieves reduced conservatism (significantly lower performance indices) compared to mode-independent control (Φ_5) under identical system parameters (Π_3), but also implies the system's potential to handle incomplete model knowledge under real-world uncertainties. Specifically, asynchronous control reduces conservatism by incorporating mode-dependent features while maintaining linear matrix inequality (LMI) feasibility. Furthermore, the amount of mode transition information determined in the TPM affects the synchronization between system modes and the controller: more complete TPM information leads to higher synchronization, and less conservative estimation of OP2 in the DoA-MSS.

5 Conclusion

This paper has addressed the asynchronous control of MJSs under actuator saturation and DoS attacks. Through the integration of multiple Lyapunov functions, iterative methods, and LMI techniques, we have developed an asynchronous controller that ensures mean-square stability while estimating the domain of attraction. Critically, the proposed design has accommodated both parameter uncertainties and partially unknown transition probability matrices, significantly enhancing practical applicability. Simulation studies have provided fundamental validation, demonstrating that systems without asynchronous control inevitably become unstable, whereas our controller not only achieves robust stabilization but also enables faster convergence during attack-free periods. Furthermore, the analysis has confirmed that increasing knowledge of transition probabilities while improving mode synchronization can reduce conservatism in the DoA-MSS estimation, as quantitatively evidenced in Table 1. Future research should leverage deep reinforcement learning frameworks to autonomously optimize transition probability estimation under high-dimensional uncertainties, develop attack-tolerant controllers against multi-modal attacks (e.g., zero-dynamics attacks, replay attacks), and design transfer learning mechanisms for cross-domain defense strategy adaptation.

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