SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

September 2026, Vol. 69, Iss. 9, 192203:1–192203:16
https://doi.org/10.1007/s11432-025-4998-1

Multi-sensor fusion filtering for 2-D systems with measurement censoring and energy harvesting scheme under an outlier-resistant approach

Yuanyuan LI[1], Jinling LIANG[1*] & Jianlong QIU[2]

[1]*School of Mathematics, Southeast University, Nanjing 210096, China*
[2]*School of Automation and Electrical Engineering, Linyi University, Linyi 276005, China*

Received 19 May 2025/Revised 7 September 2025/Accepted 17 September 2025/Published online 13 August 2026

Abstract This paper intends to address the outlier-resistant fusion filtering problem for a class of nonlinear two-dimensional systems with the energy harvesting sensors under the measurement censoring scheme. The energy harvesting technology is considered to provide the energy required for measurement signal transmission, where the energy harvested at each shifting point is characterized by a random variable, and the signal can be successfully transmitted only when the sensor accumulates sufficient energy. The censored measurements are modeled by the well-known Tobit model and further characterized by a series of Bernoulli random variables with local probability approximation. Additionally, a saturation function is introduced to mitigate the impact of abnormal innovations caused by measurement outliers. The design of the fused filter in this paper follows two main steps: firstly, the local filters are established in the presence of incomplete measurements that are caused by energy constraints and censoring, where upper bounds for the filtering error variances are derived and subsequently minimized at each shifting point by selecting appropriate parameters; and secondly, the obtained local estimates are fused by deploying a suitable fusion scheme, where the fused estimate is shown to be more accurate than the local ones. Finally, a numerical example is provided to verify the effectiveness of the proposed outlier-resistant fusion filtering algorithm.

Keywords two-dimensional system, measurement censoring, Tobit Kalman filtering, multi-sensor fusion filtering, energy harvesting sensor, measurement outlier

Citation Li Y Y, Liang J L, Qiu J L. Multi-sensor fusion filtering for 2-D systems with measurement censoring and energy harvesting scheme under an outlier-resistant approach. Sci China Inf Sci, 2026, 69(9): 192203, https://doi.org/10.1007/s11432-025-4998-1

1 Introduction

As a typical multi-variable system, the state of two-dimensional (2-D) systems evolves along two independent directions, which is the key distinction from the one-dimensional (1-D) ones. On one hand, this characteristic enables the 2-D systems to exhibit a wide range of applications in practical engineering, such as thermal processing, image data processing, and gas absorption [1–3]. On the other hand, the complex dynamical behaviors bring many challenges to the study of 2-D systems, because many important conclusions derived for 1-D models cannot be directly extended to the 2-D systems. Up to now, fruitful achievements have been developed for 2-D systems, including the stability analysis [4], filtering/estimation [5], and controller design [6, 7]. Among them, various filtering design problems (including the set-membership filtering, Kalman-type filtering, and H_∞ filtering) have been of continuous interest to scholars [8–11].

Measurement censoring is a common phenomenon in the engineering field, which is mainly caused by factors such as communication restrictions, sensor saturation and occlusion regions. Censoring leads the measurement output to be nonlinear, which in turn causes the measurement noises to no longer obey the Gaussian distribution. Consequently, the traditional Kalman filtering is no longer applicable. Up to now, there exist two methods to tackle the measurement nonlinearity caused by censoring. One is to regard it as a general measurement nonlinearity and utilize the extended Kalman filtering (EKF), unscented Kalman filtering (UKF), as well as particle filtering (PF) to tackle it. Another is to construct the Tobit model for the censored measurements and employ a Kalman-like scheme called Tobit Kalman filtering to tackle it. This strategy was initially proposed in [12] and later demonstrated in [13] to be more effective than EKF, UKF, and PF, since it fully utilizes the system state information. Based on the idea in [12, 14], a large amount of research has been carried out for networked systems subject to measurement censoring [15–19].

* Corresponding author (email: jinlliang@seu.edu.cn)

With the improvement of information acquisition and transmission technology, the multi-sensor fusion filtering problem has garnered increasing attention from scholars. This technique aims to achieve more accurate estimation than those derived from individual sensors by efficiently integrating the data from multiple sensors. So far, a considerable amount of research has been conducted on multi-sensor fusion filtering issue for both 1-D and 2-D systems [20–22]. For instance, the robust fusion filtering problem has been investigated for uncertain multi-sensor systems under energy harvesting constraints in [23], where the covariance intersection (CI) fusion technique is adopted to fuse the local estimates.

In a multi-sensor system, signal transmission of the multiple sensors consumes a significant amount of energy, which consequently renders the energy supply to become a troublesome concern. The traditional approach is to equip each sensor with an individual power source for charging, but this method cannot be sustained due to the limited energy storage capacity of batteries. Therefore, the energy harvesting sensors have recently received much attention due to their advantage in replenishing energy and thereby extending the lifespan of sensors [23–25]. For example, in [26], the quadratic filtering issue has been studied for a category of stochastic nonlinear non-Gaussian systems, where energy-harvesting sensors are deployed to provide the energy required for signal transmission. Note that the amount of energy harvested by the sensors at each instant may be random due to the influence of uncertain external environmental factors. Consequently, both the quantity of energy stored by the sensors and whether the signal could be successfully transmitted or not at each instant are stochastic, and the random variables governing them are correlated across different time instants, which significantly complicates the filter design and the corresponding performance evaluation. It is worth mentioning that, the existing literature focuses exclusively on 1-D systems, and to date, only scattered studies have addressed the energy harvesting concerns under the 2-D framework.

In the networked environment, measurement outliers frequently occur in the signal transmission processes due to factors such as sensor faults, cyber attacks, and sudden environmental changes [27]. These measurement outliers typically exhibit characteristics including occasional occurrences and unexpectedly large amplitudes, which would lead to a large amplitude of measurement innovation, thereby impairing the filter performance if left unaddressed [28]. Therefore, in recent years, the outlier-resistant filters have been developed for the purpose of improving the filters' robustness to abrupt and abnormal outlier changes [29–31]. For example, in [31], an outlier-resistant filter has been constructed for a type of 1-D multi-rate system, where the exponential boundedness of the filtering error has been analyzed. However, when it comes to the 2-D systems, there is almost no relevant result, not to mention that the measurement censoring and energy harvesting phenomena are simultaneously considered.

To more accurately reflect the system's operational environment and achieve reliable state estimation, this article focuses on addressing the multi-sensor fusion outlier-resistant filtering problem for nonlinear 2-D systems, where the energy harvesting technology and measurement censoring are also taken into account. The main contributions of this brief can be summarized as follows. (1) The fusion Tobit Kalman filtering issue is considered for 2-D systems with measurement outliers for the first time. (2) The local outlier-resistant estimators are developed for the nonlinear 2-D system with energy harvesting sensors, based on which the fusion estimation is derived according to the CI fusion rule. (3) By choosing appropriate filter parameters, the minimal upper bounds (UBs) are obtained for the local as well as the fusion estimation error variances, and simulation results demonstrate that the fusion estimation outperforms the local estimates.

The remainder of this paper is structured as follows. In Section 2, we present the nonlinear 2-D systems and the measurement output model, where the energy harvesting constraints and measurement censoring scheme are incorporated. Section 3 provides the main results, including some fundamental lemmas and filter design. Section 4 gives a simulation example to illustrate the validity of the developed theoretical results. Finally, this paper is concluded in Section 5.

Notations: I denotes the identity matrix with appropriate dimensions. Matrix $\Gamma < 0$ implies that Γ is real, symmetric, and negative definite. $\mathrm{tr}\{\cdot\}$ signifies the trace of some square matrix. For vectors $q_1, q_2, \ldots, q_m \in \mathbb{R}^n$, $\mathrm{col}\{q_1, q_2, \ldots, q_m\}$ denotes the vector $[q_1^{\mathrm{T}}, q_2^{\mathrm{T}}, \ldots, q_m^{\mathrm{T}}]^{\mathrm{T}}$. $\mathbb{E}\{\gamma \,|\, \zeta\}$ and $\mathrm{Var}\{\gamma \,|\, \zeta\}$ denote the conditional expectation and variance of the random variable γ with respect to ζ, respectively. $\mathrm{Prob}\{\cdot\}$ denotes the probability of event '$\cdot$' occurring. $[0\ \hbar]_{\mathbb{Z}}$ denotes the integers set $\{0, 1, 2, \ldots, \hbar\}$ with given nonnegative integer $\hbar$. When $x > 0$, the symbolic function $\mathrm{sign}(x) = 1$; otherwise, it equals -1.

2 Problem formulation

Consider the following nonlinear 2-D shift-varying system constrained within a finite horizon:

$$x(\varrho_t, \varrho_r) = g_1(\varrho_t, \varrho_r - 1, x(\varrho_t, \varrho_r - 1)) + g_2(\varrho_t - 1, \varrho_r, x(\varrho_t - 1, \varrho_r))$$

$$+ B_1(\varrho_t, \varrho_r - 1)\omega(\varrho_t, \varrho_r - 1) + B_2(\varrho_t - 1, \varrho_r)\omega(\varrho_t - 1, \varrho_r), \tag{1}$$

where $\varrho_t, \varrho_r \in [0\ \hbar]_{\mathbb{Z}}$ ($\hbar$ is a known positive integer), $x(\varrho_t, \varrho_r) \in \mathbb{R}^n$ is the system state, $\omega(\varrho_t, \varrho_r)$ represents the zero-mean Gaussian white noise sequence with variance $Q(\varrho_t, \varrho_r) \geqslant 0$, and $B_l(\varrho_t, \varrho_r)$ $(l = 1, 2)$ are known shift-varying matrices with compatible dimensions. For $l = 1, 2$, the nonlinear functions $g_l(\varrho_t, \varrho_r, x(\varrho_t, \varrho_r))$ meet the following constraints for all $\varrho_t,\ \varrho_r \in [0\ \hbar]_{\mathbb{Z}}$:

$$g_l(\varrho_t, \varrho_r, 0) = 0, \tag{2a}$$

$$\begin{aligned} &\|g_l(\varrho_t, \varrho_r, x_1(\varrho_t, \varrho_r)) - g_l(\varrho_t, \varrho_r, x_2(\varrho_t, \varrho_r)) - A_l(\varrho_t, \varrho_r)(x_1(\varrho_t, \varrho_r) - x_2(\varrho_t, \varrho_r))\| \\ &\quad \leqslant a_l(\varrho_t, \varrho_r)\|x_1(\varrho_t, \varrho_r) - x_2(\varrho_t, \varrho_r)\|, \qquad \forall\ x_1(\varrho_t, \varrho_r), x_2(\varrho_t, \varrho_r) \in \mathbb{R}^n, \end{aligned} \tag{2b}$$

where $A_l(\varrho_t, \varrho_r)$ and $a_l(\varrho_t, \varrho_r)$ $(l = 1, 2)$ signify the known shift-varying matrices and nonnegative constants, respectively. The initial states $x(\varrho_t, 0)$ and $x(0, \varrho_r)$ are set as two white-noise sequences satisfying $\mathbb{E}\{x(\varrho_t, 0)\} = \eta_1(\varrho_t)$ and $\mathbb{E}\{x(0, \varrho_r)\} = \eta_2(\varrho_r)$ for $\varrho_t, \varrho_r \in [0\ \hbar]_{\mathbb{Z}}$, where $\eta_1(\varrho_t)$ and $\eta_2(\varrho_r)$ are given vectors with $\eta_1(0) = \eta_2(0)$.

We assume that system (1) is measured by W sensor nodes, and the measurement of the sth ($s \in [1\ W]_{\mathbb{Z}}$) sensor node is modeled as

$$y_s^*(\varrho_t, \varrho_r) = C_s(\varrho_t, \varrho_r)x(\varrho_t, \varrho_r) + \nu_s(\varrho_t, \varrho_r), \tag{3}$$

where $y_s^*(\varrho_t, \varrho_r) = [y_{s1}^*(\varrho_t, \varrho_r), \ldots, y_{sm_s}^*(\varrho_t, \varrho_r)]^{\mathrm{T}} \in \mathbb{R}^{m_s}$ indicates the original uncensored measurement, $\nu_s(\varrho_t, \varrho_r) \in \mathbb{R}^{m_s}$ represents the zero-mean Gaussian white noise with variance $R_s(\varrho_t, \varrho_r) = \mathrm{diag}\{R_{s1}(\varrho_t, \varrho_r), \ldots, R_{sm_s}(\varrho_t, \varrho_r)\} > 0$ and $C_s(\varrho_t, \varrho_r) = [C_{s1}(\varrho_t, \varrho_r), C_{s2}(\varrho_t, \varrho_r), \ldots, C_{sm_s}(\varrho_t, \varrho_r)]^{\mathrm{T}}$ is the known shift-varying matrix with appropriate dimensions.

In practical systems, the occurrence of the measurement censoring phenomenon is unavoidable due to factors such as sensor failures, low device sensitivity, or communication limitations. Therefore, following a similar approach as in [12], we incorporate the measurement censoring mechanism into the model of each sensor, which leads to the following Tobit observation model for the censored measurement:

$$\bar{y}_{sd}(\varrho_t, \varrho_r) = \begin{cases} y_{sd}^*(\varrho_t, \varrho_r), & \text{if } y_{sd}^*(\varrho_t, \varrho_r) > \tau_{sd}, \\ \tau_{sd}, & \text{if } y_{sd}^*(\varrho_t, \varrho_r) \leqslant \tau_{sd}, \end{cases} \tag{4}$$

where $\bar{y}_{sd}(\varrho_t, \varrho_r) \in \mathbb{R}$ is the dth entry of the censored measurement $\bar{y}_s(\varrho_t, \varrho_r)$, and τ_{sd} represents a known censoring threshold for $s \in [1\ W]_{\mathbb{Z}}$ and $d \in [1\ m_s]_{\mathbb{Z}}$.

For the sth sensor, in order to accurately characterize the Tobit observation model (4), a sequence of Bernoulli variables $\xi_{sd}(\varrho_t, \varrho_r)$ $(d \in [1\ m_s]_{\mathbb{Z}})$ is employed to depict the censored measurements $\bar{y}_{sd}(\varrho_t, \varrho_r)$ as follows:

$$\xi_{sd}(\varrho_t, \varrho_r) = \begin{cases} 1, & \text{if } y_{sd}^*(\varrho_t, \varrho_r) > \tau_{sd}, \\ 0, & \text{if } y_{sd}^*(\varrho_t, \varrho_r) \leqslant \tau_{sd}. \end{cases}$$

It is obvious that, when the random variable $\xi_{sd}(\varrho_t, \varrho_r) = 1$, the measurement $y_{sd}^*(\varrho_t, \varrho_r)$ is not censored, which means $\bar{y}_{sd}(\varrho_t, \varrho_r) = y_{sd}^*(\varrho_t, \varrho_r)$; when $\xi_{sd}(\varrho_t, \varrho_r) = 0$, the measurement $y_{sd}^*(\varrho_t, \varrho_r)$ is censored, implying that $\bar{y}_{sd}(\varrho_t, \varrho_r) = \tau_{sd}$. Depending on whether $y_{sd}^*(\varrho_t, \varrho_r)$ is censored or not, the Tobit model (4) can be re-expressed as follows:

$$\bar{y}_{sd}(\varrho_t, \varrho_r) = \xi_{sd}(\varrho_t, \varrho_r)y_{sd}^*(\varrho_t, \varrho_r) + (1 - \xi_{sd}(\varrho_t, \varrho_r))\tau_{sd}. \tag{5}$$

Then, $\bar{y}_s(\varrho_t, \varrho_r) \triangleq [\bar{y}_{s1}(\varrho_t, \varrho_r), \bar{y}_{s2}(\varrho_t, \varrho_r), \ldots, \bar{y}_{sm_s}(\varrho_t, \varrho_r)]^{\mathrm{T}}$ can be written as

$$\bar{y}_s(\varrho_t, \varrho_r) = \Xi_s(\varrho_t, \varrho_r)y_s^*(\varrho_t, \varrho_r) + (I - \Xi_s(\varrho_t, \varrho_r))\tau_s, \tag{6}$$

where $\tau_s \triangleq [\tau_{s1}, \tau_{s2}, \ldots, \tau_{sm_s}]^{\mathrm{T}}$, $\Xi_s(\varrho_t, \varrho_r) \triangleq \mathrm{diag}\{\xi_{s1}(\varrho_t, \varrho_r), \xi_{s2}(\varrho_t, \varrho_r), \ldots, \xi_{sm_s}(\varrho_t, \varrho_r)\}$ with

$$\mathbb{E}\{\Xi_s(\varrho_t, \varrho_r)\} = \bar{\Xi}_s(\varrho_t, \varrho_r) \triangleq \mathrm{diag}\{\bar{\xi}_{s1}(\varrho_t, \varrho_r), \bar{\xi}_{s2}(\varrho_t, \varrho_r), \ldots, \bar{\xi}_{sm_s}(\varrho_t, \varrho_r)\},$$

in which $\bar{\xi}_{sd}(\varrho_t, \varrho_r)$ $(d \in [1\ m_s]_{\mathbb{Z}})$ is a nonnegative constant that will be derived later.

In this paper, we intend to address the fusion estimation problem for the multi-sensor system described by (1) and (6). Here, each sensor adopts the energy harvesting technology, which allows it to accumulate energy from external environment. The measurement $\bar{y}_s(\varrho_t, \varrho_r)$ can be successfully transmitted to the corresponding remote filter only when the stored energy of the sth sensor is sufficient for signal transmission. In this study, we assume that the sensors consume one unit of energy to transmit the measurement at each point.

To start with, we define a two-indexed ordered sequence: the notation $(\varrho_t^{(1)}, \varrho_r^{(1)}) < (\varrho_t^{(2)}, \varrho_r^{(2)})$ signifies that $(\varrho_t^{(1)}, \varrho_r^{(1)}) \in \{(\vec{\varrho}_t, \vec{\varrho}_r) : \vec{\varrho}_t = \varrho_t^{(2)}, \vec{\varrho}_r < \varrho_r^{(2)}\} \cup \{(\vec{\varrho}_t, \vec{\varrho}_r) : \vec{\varrho}_t < \varrho_t^{(2)}, \vec{\varrho}_r \in [0\ \hbar]_{\mathbb{Z}}\}$, and the equality $(\varrho_t^{(1)}, \varrho_r^{(1)}) = (\varrho_t^{(2)}, \varrho_r^{(2)})$ holds if and only if $\varrho_t^{(1)} = \varrho_t^{(2)}$ and $\varrho_r^{(1)} = \varrho_r^{(2)}$. For $(\varrho_t, \varrho_r) \in \{(\vec{\varrho}_t, \vec{\varrho}_r) : \vec{\varrho}_t \in [0\ \hbar]_{\mathbb{Z}}, \vec{\varrho}_r \in [0\ \hbar]_{\mathbb{Z}}\} \setminus \{(0,0)\}$, denoting the previous step of (ϱ_t, ϱ_r) as $(\varrho_t^0, \varrho_r^0)$, it is obvious that $(\varrho_t^0, \varrho_r^0)$ satisfies the following constraint:

$$(\varrho_t^0, \varrho_r^0) = \max\{(\vec{\varrho}_t, \vec{\varrho}_r) : (\vec{\varrho}_t, \vec{\varrho}_r) < (\varrho_t, \varrho_r)\},$$

where $\max\{(\vec{\varrho}_t, \vec{\varrho}_r) : \text{Condition 1}\}$ represents the last shift step in the ordered sequence satisfying 'Condition 1'.

Defining the amount of energy stored by sensor s at point (ϱ_t, ϱ_r) as $z_s(\varrho_t, \varrho_r) \in \mathbb{N}$, for $(\varrho_t, \varrho_r) \in \{(\vec{\varrho}_t, \vec{\varrho}_r) : \vec{\varrho}_t \in [0\ \hbar]_{\mathbb{Z}}, \vec{\varrho}_r \in [0\ \hbar]_{\mathbb{Z}}\} \setminus \{(0,0)\}$, the evolution of $z_s(\varrho_t, \varrho_r)$ satisfies

$$z_s(\varrho_t, \varrho_r) = \min\{z_s(\varrho_t^0, \varrho_r^0) + h_s(\varrho_t^0, \varrho_r^0) - \gamma_s(\varrho_t^0, \varrho_r^0), D_s\}, \tag{7}$$

where $(\varrho_t^0, \varrho_r^0)$ represents the previous step of (ϱ_t, ϱ_r), and $D_s \in \mathbb{N}$ denotes the maximum energy storage capacity of sensor s. The initial condition of $z_s(\varrho_t, \varrho_r)$ is set as $z_s(0,0) = \kappa_s \leqslant D_s$ with κ_s being a known nonnegative value. The variable $\gamma_s(\varrho_t, \varrho_r)$ is defined as

$$\gamma_s(\varrho_t, \varrho_r) \triangleq \begin{cases} 1, & \text{if } z_s(\varrho_t, \varrho_r) > 0, \\ 0, & \text{if } z_s(\varrho_t, \varrho_r) = 0. \end{cases} \tag{8}$$

In (7), $h_s(\varrho_t, \varrho_r)$ represents the amount of energy collected by sensor s at point (ϱ_t, ϱ_r). Additionally, $h_s(\varrho_t, \varrho_r)$ is assumed to be a white random sequence, and its probability distribution is characterized as

$$\text{Prob}\{h_s(\varrho_t, \varrho_r) = k\} = p_{s,k}, \qquad k = 0, 1, 2, \ldots, D_s,$$

where $p_{s,k}$ satisfies $0 \leqslant p_{s,k} \leqslant 1$ and $\sum_{k=0}^{D_s} p_{s,k} = 1$.

Since the energy stored in sensor s is stochastically varying, whether the sensor can successfully broadcast its measurement at each shifting point is inherently probabilistic. Defining the actual measurement received by corresponding local filter s as $y_s(\varrho_t, \varrho_r) = [y_{s1}(\varrho_t, \varrho_r), y_{s2}(\varrho_t, \varrho_r), \ldots, y_{sm_s}(\varrho_t, \varrho_r)]^{\mathrm{T}}$, then $y_s(\varrho_t, \varrho_r)$ can be expressed as follows:

$$y_s(\varrho_t, \varrho_r) = \gamma_s(\varrho_t, \varrho_r)\bar{y}_s(\varrho_t, \varrho_r). \tag{9}$$

Assumption 1. The random sequences $\omega(\varrho_t^{(1)}, \varrho_r^{(1)})$, $\nu_{sd}(\varrho_t^{(2)}, \varrho_r^{(2)})$, $h_s(\varrho_t^{(3)}, \varrho_r^{(3)})$ and the initial conditions $x(\varrho_t^{(4)}, 0)$, $x(0, \varrho_r^{(4)})$ are mutually independent for all $s \in [1\ W]_{\mathbb{Z}}$, $d \in [1\ m_s]_{\mathbb{Z}}$, $\varrho_t^{(\imath)}, \varrho_r^{(\imath)} \in [0\ \hbar]_{\mathbb{Z}}$ with $\imath = 1, 2, 3, 4$.

Remark 1. This article makes an initial attempt to introduce the energy harvesting technology into the nonlinear 2-D system with measurement censoring when investigating the filtering/estimation problem. As shown in (8) and (9), at point (ϱ_t, ϱ_r), if the stored energy of the sth sensor is greater than zero, then the remote filter could successfully receive the censored measurement sent by the sth sensor; otherwise, the input of the filter remains zero until the sensor harvests enough energy from the external environment.

The objective of this paper is to (i) develop local outlier-resistant Tobit Kalman filters for the nonlinear 2-D system (1) in the presence of energy harvesting scheme; (ii) find an UB matrix for each local filtering error variance (FEV) and minimize it by designing appropriate filter parameters; and (iii) fuse the obtained local estimations by means of the CI fusion rule.

3 Main results

In this section, recursive filters are to be constructed for the nonlinear 2-D system (1) with censored measurements under the energy harvesting scheme. To proceed with the derivation, we first calculate the probability distribution of random variable $\gamma_s(\varrho_t, \varrho_r)$ as well as the conditional expectation and variance of the censored measurement $y_s(\varrho_t, \varrho_r)$ regarding the system state. Then, UBs are recursively derived for the local FEVs, and the filter gains are further designed to ensure the obtained UBs always reach their minima at each step shifting point.

To begin with, we start by calculating the probability distribution of the energy level $z_s(\varrho_t,\varrho_r)$ for the sth sensor. Let $p_s(\varrho_t,\varrho_r)\triangleq[p_s^{(0)}(\varrho_t,\varrho_r),p_s^{(1)}(\varrho_t,\varrho_r),\ldots,p_s^{(D_s)}(\varrho_t,\varrho_r)]^{\mathrm{T}}$, where $p_s^{(i)}(\varrho_t,\varrho_r)\triangleq\mathrm{Prob}\{z_s(\varrho_t,\varrho_r)=i\}$ for $i=0,1,2,\ldots,D_s$. In the light of the ideas proposed in [23], the following two lemmas are given, where the detailed proofs are omitted due to page limitations.

Lemma 1. For the energy level presented in (7), its probability distribution $p_s(\varrho_t,\varrho_r)$ satisfies the following recursion:

$$p_s(\varrho_t,\varrho_r)=\ell_s+J_sp_s(\varrho_t^0,\varrho_r^0),\tag{10}$$

where $(\varrho_t,\varrho_r)\in\{(\vec{\varrho}_t,\vec{\varrho}_r):\vec{\varrho}_t\in[0\ \hbar]_{\mathbb{Z}},\vec{\varrho}_r\in[0\ \hbar]_{\mathbb{Z}}\}\setminus\{(0,0)\}$, $(\varrho_t^0,\varrho_r^0)$ represents the previous step of (ϱ_t,ϱ_r),

$$\ell_s\triangleq[\underbrace{0,\ldots,0}_{D_s},1]^{\mathrm{T}},\qquad J_s\triangleq\begin{bmatrix} p_{s,0} & p_{s,0} & 0 & \cdots & 0\\ p_{s,1} & p_{s,1} & p_{s,0} & \cdots & 0\\ p_{s,2} & p_{s,2} & p_{s,1} & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ p_{s,D_s-1} & p_{s,D_s-1} & p_{s,D_s-2} & \cdots & p_{s,0}\\ -\sum_{i=0}^{D_s-1}p_{s,i} & -\sum_{i=0}^{D_s-1}p_{s,i} & -\sum_{i=0}^{D_s-2}p_{s,i} & \cdots & -p_{s,0}\end{bmatrix}.$$

Based on recursion (10), the probability distribution of $\gamma_s(\varrho_t,\varrho_r)$ is presented in the following lemma.

Lemma 2. The random variable $\gamma_s(\varrho_t,\varrho_r)$ defined as in (8) obeys the following probability distribution:

$$\mathrm{Prob}\{\gamma_s(\varrho_t,\varrho_r)=1\}=\bar{\gamma}_s(\varrho_t,\varrho_r),\qquad \mathrm{Prob}\{\gamma_s(\varrho_t,\varrho_r)=0\}=1-\bar{\gamma}_s(\varrho_t,\varrho_r),\tag{11}$$

where

$$\bar{\gamma}_s(\varrho_t,\varrho_r)\triangleq1-\chi_sp_s(\varrho_t,\varrho_r)\ \text{ with }\ \chi_s\triangleq[1,\underbrace{0,\ldots,0}_{D_s}],\tag{12}$$

and $p_s(\varrho_t,\varrho_r)$ is recursively obtained by (10). Moreover, we have

$$\mathbb{E}\{\gamma_s(\varrho_t,\varrho_r)\}=\bar{\gamma}_s(\varrho_t,\varrho_r)\ \text{ and }\ \mathrm{Var}\{\gamma_s(\varrho_t,\varrho_r)\}=\bar{\gamma}_s(\varrho_t,\varrho_r)(1-\bar{\gamma}_s(\varrho_t,\varrho_r))\triangleq\hat{\gamma}_s(\varrho_t,\varrho_r).$$

Lemma 3 ([32])**.** Assume that random variable $\vec{x}$ obeys the normal distribution with expectation as μ and variance as $\tilde{\sigma}^2$, where $\tilde{\sigma}>0$. Let b be a constant. Then the following equalities hold:

$$\mathbb{E}\{\vec{x}\,|\,\vec{x}>b\}=\mu+\tilde{\sigma}\lambda(\beta),\qquad \mathrm{Var}\{\vec{x}\,|\,\vec{x}>b\}=\tilde{\sigma}^2(1-\delta(\beta)),$$

where

$$\beta=\frac{b-\mu}{\tilde{\sigma}},\qquad \lambda(\beta)=\frac{\phi(\beta)}{1-\Phi(\beta)},\qquad \delta(\beta)=\lambda(\beta)(\lambda(\beta)-\beta).\tag{13}$$

Here, $\phi(\cdot)$ and $\Phi(\cdot)$ represent the probability density function and the cumulative distribution function of the standard normal distribution, respectively.

The following lemma is inspired by the work in [14], where the proof details are also omitted because of the consideration of page limitations.

Lemma 4. The conditional mathematical expectation and variance of the actual received measurement $y_{sd}(\varrho_t,\varrho_r)$ ($s\in[1\ W]_{\mathbb{Z}}$, $d\in[1\ m_s]_{\mathbb{Z}}$) with respect to $x(\varrho_t,\varrho_r)$ are calculated as follows:

$$\begin{aligned}\mathbb{E}\{y_{sd}(\varrho_t,\varrho_r)\,|\,x(\varrho_t,\varrho_r)\}=&\ \bar{\gamma}_s(\varrho_t,\varrho_r)\big(\Phi(\beta_{sd}(\varrho_t,\varrho_r))\tau_{sd}+(1-\Phi(\beta_{sd}(\varrho_t,\varrho_r)))\\ &\times(C_{sd}^{\mathrm{T}}(\varrho_t,\varrho_r)x(\varrho_t,\varrho_r)+\sqrt{R_{sd}(\varrho_t,\varrho_r)}\lambda(\beta_{sd}(\varrho_t,\varrho_r)))\big),\end{aligned}\tag{14}$$

$$\begin{aligned}&\mathrm{Var}\{y_{sd}(\varrho_t,\varrho_r)\,|\,x(\varrho_t,\varrho_r)\}\\ &=\hat{\gamma}_s(\varrho_t,\varrho_r)\tau_{sd}^2\Phi(\beta_{sd}(\varrho_t,\varrho_r))+\hat{\gamma}_s(\varrho_t,\varrho_r)(1-\Phi(\beta_{sd}(\varrho_t,\varrho_r)))\big(C_{sd}^{\mathrm{T}}(\varrho_t,\varrho_r)x(\varrho_t,\varrho_r)x^{\mathrm{T}}(\varrho_t,\varrho_r)C_{sd}(\varrho_t,\varrho_r)\\ &\quad+R_{sd}(\varrho_t,\varrho_r)+\sqrt{R_{sd}(\varrho_t,\varrho_r)}\lambda(\beta_{sd}(\varrho_t,\varrho_r))(\tau_{sd}+C_{sd}^{\mathrm{T}}(\varrho_t,\varrho_r)x(\varrho_t,\varrho_r))\big)\end{aligned}$$

$$+ \bar{\gamma}_s^2(\varrho_t, \varrho_r) R_{sd}(\varrho_t, \varrho_r)(1 - \Phi(\beta_{sd}(\varrho_t, \varrho_r)))\big(1 - \lambda(\beta_{sd}(\varrho_t, \varrho_r))(\lambda(\beta_{sd}(\varrho_t, \varrho_r)) - \beta_{sd}(\varrho_t, \varrho_r)) + (\beta_{sd}(\varrho_t, \varrho_r) - \lambda(\beta_{sd}(\varrho_t, \varrho_r)))^2 \Phi(\beta_{sd}(\varrho_t, \varrho_r))\big), \tag{15}$$

where

$$\lambda(\beta_{sd}(\varrho_t, \varrho_r)) = \frac{\phi(\beta_{sd}(\varrho_t, \varrho_r))}{1 - \Phi(\beta_{sd}(\varrho_t, \varrho_r))}, \quad \beta_{sd}(\varrho_t, \varrho_r) = \frac{\tau_{sd} - C_{sd}^{\mathrm{T}}(\varrho_t, \varrho_r) x(\varrho_t, \varrho_r)}{\sqrt{R_{sd}(\varrho_t, \varrho_r)}}.$$

For the sth sensor, the corresponding nonlinear local Tobit Kalman filter for system (1) with measurement (9) is established as follows:

$$\hat{x}_p^{(s)}(\varrho_t, \varrho_r) = g_1((\varrho_t, \varrho_r - 1), \hat{x}_u^{(s)}(\varrho_t, \varrho_r - 1)) + g_2((\varrho_t - 1, \varrho_r), \hat{x}_u^{(s)}(\varrho_t - 1, \varrho_r)), \tag{16a}$$

$$\hat{x}_u^{(s)}(\varrho_t, \varrho_r) = \hat{x}_p^{(s)}(\varrho_t, \varrho_r) + G_s(\varrho_t, \varrho_r)\sigma_s(y_s(\varrho_t, \varrho_r) - \hat{y}_s(\varrho_t, \varrho_r)), \tag{16b}$$

where $\varrho_t, \varrho_r \in [1\ \hbar]_{\mathbb{Z}}$, $\hat{x}_p^{(s)}(\varrho_t, \varrho_r) \in \mathbb{R}^n$ represents the prediction value of the system state, $\hat{x}_u^{(s)}(\varrho_t, \varrho_r) \in \mathbb{R}^n$ is the state estimation of the sth filter; and the related initial conditions are given as $\hat{x}_u^{(s)}(\varrho_t, 0) = \vartheta_{1,s}(\varrho_t)$ and $\hat{x}_u^{(s)}(0, \varrho_r) = \vartheta_{2,s}(\varrho_r)$ with $\vartheta_{1,s}(0) = \vartheta_{2,s}(0)$ for $\varrho_t, \varrho_r \in [0\ \hbar]_{\mathbb{Z}}$. $G_s(\varrho_t, \varrho_r)$ signifies the filter parameter to be determined later. The saturation function $\sigma_s(\cdot) : \mathbb{R}^{m_s} \to \mathbb{R}^{m_s}$ is described as below:

$$\sigma_s(\overleftarrow{l}) \triangleq [\sigma_{s1}(l_1), \sigma_{s2}(l_2), \ldots, \sigma_{sm_s}(l_{m_s})]^{\mathrm{T}},$$

where $\overleftarrow{l} \triangleq [l_1, l_2, \ldots, l_{m_s}]^{\mathrm{T}} \in \mathbb{R}^{m_s}$, $\sigma_{sd}(l_d) \triangleq \mathrm{sign}(l_d)\min\{\bar{\sigma}_{sd}^{\max}, |l_d|\}$ ($d \in [1\ m_s]_{\mathbb{Z}}$) with positive scalar $\bar{\sigma}_{sd}^{\max}$ representing the dth entry of the given threshold vector $\bar{\sigma}_s^{\max} \in \mathbb{R}^{m_s}$. And $\hat{y}_s(\varrho_t, \varrho_r) \in \mathbb{R}^{m_s}$ is defined as follows:

$$\hat{y}_s(\varrho_t, \varrho_r) = \bar{\gamma}_s(\varrho_t, \varrho_r)\big(\bar{\Xi}_s(\varrho_t, \varrho_r)(C_s(\varrho_t, \varrho_r)\hat{x}_p^{(s)}(\varrho_t, \varrho_r) + \overleftarrow{\lambda}(\tau_s, \hat{x}_p^{(s)}(\varrho_t, \varrho_r), R_s(\varrho_t, \varrho_r))\hat{R}_s(\varrho_t, \varrho_r)) + (I - \bar{\Xi}_s(\varrho_t, \varrho_r))\tau_s\big),$$

where $\hat{R}_s(\varrho_t, \varrho_r) \triangleq \big[\sqrt{R_{s1}(\varrho_t, \varrho_r)}, \ldots, \sqrt{R_{sm_s}(\varrho_t, \varrho_r)}\big]^{\mathrm{T}}$, $\overleftarrow{\lambda}(\tau_s, \hat{x}_p^{(s)}(\varrho_t, \varrho_r), R_s(\varrho_t, \varrho_r)) \triangleq \mathrm{diag}\{\lambda(\hat{\beta}_{s1}(\varrho_t, \varrho_r)), \ldots, \lambda(\hat{\beta}_{sm_s}(\varrho_t, \varrho_r))\}$ with

$$\lambda(\hat{\beta}_{sd}(\varrho_t, \varrho_r)) = \frac{\phi(\hat{\beta}_{sd}(\varrho_t, \varrho_r))}{1 - \Phi(\hat{\beta}_{sd}(\varrho_t, \varrho_r))}, \quad \hat{\beta}_{sd}(\varrho_t, \varrho_r) = \frac{\tau_{sd} - C_{sd}^{\mathrm{T}}(\varrho_t, \varrho_r)\hat{x}_p^{(s)}(\varrho_t, \varrho_r)}{\sqrt{R_{sd}(\varrho_t, \varrho_r)}},$$

for $s \in [1\ W]_{\mathbb{Z}}$ and $d \in [1\ m_s]_{\mathbb{Z}}$.

Remark 2. The outlier-resistant filters are constructed for 2-D systems for the first time in (16), from which we can observe that saturation functions are employed to constrain the innovation into a limited range, and thus the possible measurement outliers could be effectively dealt with. It should be noted that the established filter (16) turns into the ordinary one when the saturation level $\bar{\sigma}_s^{\max}$ goes towards infinity.

Assumption 2. Drawing on the idea from [12], when the prediction error is small, the predicted value of the system state could provide a comparatively accurate estimate for the censoring probability. For the sake of convenience, we offer the following reliable approximation for $\bar{\xi}_{sd}(\varrho_t, \varrho_r)$ in this study: for $s \in [1\ W]_{\mathbb{Z}}$ and $d \in [1\ m_s]_{\mathbb{Z}}$,

$$\bar{\xi}_{sd}(\varrho_t, \varrho_r) = \mathbb{E}\{\xi_{sd}(\varrho_t, \varrho_r)\} \doteq \Phi\Big(\frac{C_{sd}^{\mathrm{T}}(\varrho_t, \varrho_r)x(\varrho_t, \varrho_r) - \tau_{sd}}{\sqrt{R_{sd}(\varrho_t, \varrho_r)}}\Big) \doteq \Phi\Big(\frac{C_{sd}^{\mathrm{T}}(\varrho_t, \varrho_r)\hat{x}_p^{(s)}(\varrho_t, \varrho_r) - \tau_{sd}}{\sqrt{R_{sd}(\varrho_t, \varrho_r)}}\Big),$$

which has been tested statistically and shown to be usually reliable.

Based on Assumption 2, we know that $\bar{\xi}_{sd}(\varrho_t, \varrho_r) = 1 - \Phi(\hat{\beta}_{sd}(\varrho_t, \varrho_r))$. Thus, the developed conditional expectation and variance in (14) and (15) can be respectively rewritten as

$$\begin{aligned}
&\mathbb{E}\{y_{sd}(\varrho_t, \varrho_r) \,|\, x(\varrho_t, \varrho_r)\} \\
&\quad \doteq \bar{\gamma}_s(\varrho_t, \varrho_r)\big(\bar{\xi}_{sd}(\varrho_t, \varrho_r)(C_{sd}^{\mathrm{T}}(\varrho_t, \varrho_r)x(\varrho_t, \varrho_r) + \sqrt{R_{sd}(\varrho_t, \varrho_r)}\lambda(\hat{\beta}_{sd}(\varrho_t, \varrho_r))) + (1 - \bar{\xi}_{sd}(\varrho_t, \varrho_r))\tau_{sd}\big), \\
&\mathrm{Var}\{y_{sd}(\varrho_t, \varrho_r) \,|\, x(\varrho_t, \varrho_r)\} \\
&\quad = \hat{\gamma}_s(\varrho_t, \varrho_r)\tau_{sd}^2\Phi(\hat{\beta}_{sd}(\varrho_t, \varrho_r)) + \hat{\gamma}_s(\varrho_t, \varrho_r)(1 - \Phi(\hat{\beta}_{sd}(\varrho_t, \varrho_r)))\big(C_{sd}^{\mathrm{T}}(\varrho_t, \varrho_r)x(\varrho_t, \varrho_r)x^{\mathrm{T}}(\varrho_t, \varrho_r)C_{sd}(\varrho_t, \varrho_r)
\end{aligned}$$

$$
\begin{aligned}
&+ R_{sd}(\varrho_t,\varrho_r) + \sqrt{R_{sd}(\varrho_t,\varrho_r)}\lambda(\hat\beta_{sd}(\varrho_t,\varrho_r))(\tau_{sd} + C_{sd}^{\mathrm{T}}(\varrho_t,\varrho_r)x(\varrho_t,\varrho_r))\big) \\
&+ \bar\gamma_s^2(\varrho_t,\varrho_r)R_{sd}(\varrho_t,\varrho_r)(1-\Phi(\hat\beta_{sd}(\varrho_t,\varrho_r)))\big(1-\lambda(\hat\beta_{sd}(\varrho_t,\varrho_r))(\lambda(\hat\beta_{sd}(\varrho_t,\varrho_r)) - \beta_{sd}(\varrho_t,\varrho_r)) \\
&+ (\beta_{sd}(\varrho_t,\varrho_r) - \lambda(\hat\beta_{sd}(\varrho_t,\varrho_r)))^2\Phi(\hat\beta_{sd}(\varrho_t,\varrho_r))\big).
\end{aligned}
\tag{17}
$$

Furthermore, the measurement innovation $y_s(\varrho_t,\varrho_r) - \hat y_s(\varrho_t,\varrho_r)$ in (16b) can be rewritten as follows:

$$
\begin{aligned}
&y_s(\varrho_t,\varrho_r) - \hat y_s(\varrho_t,\varrho_r) \\
&\quad = y_s(\varrho_t,\varrho_r) - \mathbb{E}\{y_s(\varrho_t,\varrho_r)\,|\,x(\varrho_t,\varrho_r)\} + \bar\gamma_s(\varrho_t,\varrho_r)\bar\Xi_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)(x(\varrho_t,\varrho_r) - \hat x_p^{(s)}(\varrho_t,\varrho_r)).
\end{aligned}
\tag{18}
$$

Define $\tilde y_s(\varrho_t,\varrho_r) = y_s(\varrho_t,\varrho_r) - \hat y_s(\varrho_t,\varrho_r) \triangleq [\tilde y_{s1}(\varrho_t,\varrho_r), \tilde y_{s2}(\varrho_t,\varrho_r), \ldots, \tilde y_{sm_s}(\varrho_t,\varrho_r)]^{\mathrm{T}}$. For the purpose of facilitating the later analysis, we give another expression for the saturation function $\sigma_s(\tilde y_s(\varrho_t,\varrho_r))$. According to the definition of $\sigma_{sd}(\cdot)$, we have

$$
\sigma_{sd}(\tilde y_{sd}(\varrho_t,\varrho_r)) = \begin{cases} \tilde y_{sd}(\varrho_t,\varrho_r), & \text{if } |\tilde y_{sd}(\varrho_t,\varrho_r)| \leqslant \bar\sigma_{sd}^{\max}, \\ \mathrm{sign}(\tilde y_{sd}(\varrho_t,\varrho_r))\bar\sigma_{sd}^{\max}, & \text{otherwise.} \end{cases}
$$

For any $\alpha, \bar\beta \in \mathbb{R}$, let us define a scalar function $f(\cdot,\cdot)$ satisfying

$$
f(\alpha,\beta) = \begin{cases} 0, & \text{if } \alpha \leqslant \bar\beta, \\ 1, & \text{otherwise,} \end{cases}
$$

based on which, $\sigma_{sd}(\tilde y_{sd}(\varrho_t,\varrho_r))$ can be re-expressed as

$$
\sigma_{sd}(\tilde y_{sd}(\varrho_t,\varrho_r)) = \big(1 - f(|\tilde y_{sd}(\varrho_t,\varrho_r)|, \bar\sigma_{sd}^{\max})\big)\tilde y_{sd}(\varrho_t,\varrho_r) + f(|\tilde y_{sd}(\varrho_t,\varrho_r)|, \bar\sigma_{sd}^{\max})\mathrm{sign}(\tilde y_{sd}(\varrho_t,\varrho_r))\bar\sigma_{sd}^{\max}.
$$

Then, the saturation function $\sigma_s(\tilde y_s(\varrho_t,\varrho_r))$ can be rewritten as

$$
\sigma_s(\tilde y_s(\varrho_t,\varrho_r)) = (I - \Lambda_s(\varrho_t,\varrho_r))\tilde y_s(\varrho_t,\varrho_r) + \Lambda_s(\varrho_t,\varrho_r)L_s(\varrho_t,\varrho_r),
\tag{19}
$$

where

$$
\begin{aligned}
\Lambda_s(\varrho_t,\varrho_r) &= \mathrm{diag}\{f(|\tilde y_{s1}(\varrho_t,\varrho_r)|, \bar\sigma_{s1}^{\max}), f(|\tilde y_{s2}(\varrho_t,\varrho_r)|, \bar\sigma_{s2}^{\max}), \ldots, f(|\tilde y_{sm_s}(\varrho_t,\varrho_r)|, \bar\sigma_{sm_s}^{\max})\}, \\
L_s(\varrho_t,\varrho_r) &= \mathrm{col}\{\mathrm{sign}(\tilde y_{s1}(\varrho_t,\varrho_r))\bar\sigma_{s1}^{\max}, \mathrm{sign}(\tilde y_{s2}(\varrho_t,\varrho_r))\bar\sigma_{s2}^{\max}, \ldots, \mathrm{sign}(\tilde y_{sm_s}(\varrho_t,\varrho_r))\bar\sigma_{sm_s}^{\max}\}.
\end{aligned}
$$

From the definition of $\Lambda_s(\varrho_t,\varrho_r)$, it is obvious that $\Lambda_s^2(\varrho_t,\varrho_r) \leqslant I$.

For subsequent study, let us give the following notations:

$$
\begin{aligned}
&X(\varrho_t,\varrho_r) \triangleq \mathbb{E}\{x(\varrho_t,\varrho_r)x^{\mathrm{T}}(\varrho_t,\varrho_r)\}, \\
&\varepsilon_p^{(s)}(\varrho_t,\varrho_r) \triangleq x(\varrho_t,\varrho_r) - \hat x_p^{(s)}(\varrho_t,\varrho_r), \qquad \varepsilon_u^{(s)}(\varrho_t,\varrho_r) \triangleq x(\varrho_t,\varrho_r) - \hat x_u^{(s)}(\varrho_t,\varrho_r), \\
&\Sigma_p^{(s)}(\varrho_t,\varrho_r) \triangleq \mathbb{E}\{\varepsilon_p^{(s)}(\varrho_t,\varrho_r)(\varepsilon_p^{(s)}(\varrho_t,\varrho_r))^{\mathrm{T}}\}, \qquad \Sigma_u^{(s)}(\varrho_t,\varrho_r) \triangleq \mathbb{E}\{\varepsilon_u^{(s)}(\varrho_t,\varrho_r)(\varepsilon_u^{(s)}(\varrho_t,\varrho_r))^{\mathrm{T}}\}.
\end{aligned}
$$

Then, the following error dynamics can be attained from (1) and (16):

$$
\begin{aligned}
\varepsilon_p^{(s)}(\varrho_t,\varrho_r) &= \tilde g_1(\varrho_t,\varrho_r-1,\varepsilon_u^{(s)}(\varrho_t,\varrho_r-1)) + \tilde g_2(\varrho_t-1,\varrho_r,\varepsilon_u^{(s)}(\varrho_t-1,\varrho_r)) \\
&\quad + B_1(\varrho_t,\varrho_r-1)\omega(\varrho_t,\varrho_r-1) + B_2(\varrho_t-1,\varrho_r)\omega(\varrho_t-1,\varrho_r),
\end{aligned}
\tag{20a}
$$

$$
\varepsilon_u^{(s)}(\varrho_t,\varrho_r) = \varepsilon_p^{(s)}(\varrho_t,\varrho_r) - G_s(\varrho_t,\varrho_r)\sigma_s(\tilde y_s(\varrho_t,\varrho_r)),
\tag{20b}
$$

where, for $l=1,2$, $\tilde g_l(\varrho_t,\varrho_r,\varepsilon_u^{(s)}(\varrho_t,\varrho_r)) = g_l(\varrho_t,\varrho_r,x(\varrho_t,\varrho_r)) - g_l(\varrho_t,\varrho_r,\hat x_u^{(s)}(\varrho_t,\varrho_r))$.

Lemma 5 ([33])**.** Let μ_1 and $\breve\varepsilon_1$ be given positive scalars, and then, the second moment $X(\varrho_t,\varrho_r)$ for system (1) is upper bounded by the solution of the following evolutionary equation:

$$
\begin{aligned}
\bar X(\varrho_t,\varrho_r) &= (1+\mu_1)\hat X_1(\varrho_t,\varrho_r-1) + (1+\mu_1^{-1})\hat X_2(\varrho_t-1,\varrho_r) + B_1(\varrho_t,\varrho_r-1)Q(\varrho_t,\varrho_r-1) \\
&\quad \times B_1^{\mathrm{T}}(\varrho_t,\varrho_r-1) + B_2(\varrho_t-1,\varrho_r)Q(\varrho_t-1,\varrho_r)B_2^{\mathrm{T}}(\varrho_t-1,\varrho_r),
\end{aligned}
\tag{21}
$$

where $\varrho_t,\varrho_r \in [1\ \hbar]_{\mathbb{Z}}$, for $l \in \{1,2\}$, $\hat X_l(\varrho_t,\varrho_r)$ is defined as follows:

$$
\hat X_l(\varrho_t,\varrho_r) \triangleq (1+\breve\varepsilon_1)a_l^2(\varrho_t,\varrho_r)\mathrm{tr}\{\bar X(\varrho_t,\varrho_r)\}I + (1+\breve\varepsilon_1^{-1})A_l(\varrho_t,\varrho_r)\bar X(\varrho_t,\varrho_r)A_l^{\mathrm{T}}(\varrho_t,\varrho_r),
$$

and the initial states associated with (21) are given as $\bar X(\varrho_t,0) = X(\varrho_t,0)$, $\bar X(0,\varrho_r) = X(0,\varrho_r)$.

Lemma 6 ([33]). For the prediction error system (20a), the evolutionary dynamic of the prediction error variance $\Sigma_p^{(s)}(\varrho_t,\varrho_r)$ is presented as below:

$$\begin{aligned}\Sigma_p^{(s)}(\varrho_t,\varrho_r)=&\,\Phi_1^{(s)}(\varrho_t,\varrho_r-1)+\Phi_2^{(s)}(\varrho_t-1,\varrho_r)+\Psi^{(s)}(\varrho_t,\varrho_r)+(\Psi^{(s)}(\varrho_t,\varrho_r))^{\mathrm{T}}+B_1(\varrho_t,\varrho_r-1)\\&\times Q(\varrho_t,\varrho_r-1)B_1^{\mathrm{T}}(\varrho_t,\varrho_r-1)+B_2(\varrho_t-1,\varrho_r)Q(\varrho_t-1,\varrho_r)B_2^{\mathrm{T}}(\varrho_t-1,\varrho_r),\end{aligned}\tag{22}$$

where $\varrho_t,\varrho_r\in[1\ \hbar]_{\mathbb{Z}}$, for $l=1,2$,

$$\begin{aligned}\Phi_l^{(s)}(\varrho_t,\varrho_r)&\triangleq\mathbb{E}\{\tilde g_l(\varrho_t,\varrho_r,\varepsilon_u^{(s)}(\varrho_t,\varrho_r))\tilde g_l^{\mathrm{T}}(\varrho_t,\varrho_r,\varepsilon_u^{(s)}(\varrho_t,\varrho_r))\},\\ \Psi^{(s)}(\varrho_t,\varrho_r)&\triangleq\mathbb{E}\{\tilde g_1(\varrho_t,\varrho_r-1,\varepsilon_u^{(s)}(\varrho_t,\varrho_r-1))\tilde g_2^{\mathrm{T}}(\varrho_t-1,\varrho_r,\varepsilon_u^{(s)}(\varrho_t-1,\varrho_r))\}.\end{aligned}$$

Using a method similar to that in [33] for the nonlinearities in (22), we arrive at the following inequality:

$$\begin{aligned}\Sigma_p^{(s)}(\varrho_t,\varrho_r)\leqslant&\,(1+\bar\mu_s)\hat\Sigma_1^{(s)}(\varrho_t,\varrho_r-1)+(1+\bar\mu_s^{-1})\hat\Sigma_2^{(s)}(\varrho_t-1,\varrho_r)+B_1(\varrho_t,\varrho_r-1)Q(\varrho_t,\varrho_r-1)\\&\times B_1^{\mathrm{T}}(\varrho_t,\varrho_r-1)+B_2(\varrho_t-1,\varrho_r)Q(\varrho_t-1,\varrho_r)B_2^{\mathrm{T}}(\varrho_t-1,\varrho_r),\end{aligned}\tag{23}$$

where, for $l=1,2$,

$$\hat\Sigma_l^{(s)}(\varrho_t,\varrho_r)\triangleq(1+\bar\varepsilon_s)a_l^2(\varrho_t,\varrho_r)\mathrm{tr}\{\Sigma_u^{(s)}(\varrho_t,\varrho_r)\}I+(1+\bar\varepsilon_s^{-1})A_l(\varrho_t,\varrho_r)\Sigma_u^{(s)}(\varrho_t,\varrho_r)A_l^{\mathrm{T}}(\varrho_t,\varrho_r)$$

with $\bar\mu_s$ and $\bar\varepsilon_s$ being given positive scalars.

Theorem 1. Under the conditions of Lemma 5, for given positive scalars $\bar\mu_s$ and $\bar\varepsilon_s$, the prediction and estimation error variances of the sth ($s\in[1\ W]_{\mathbb{Z}}$) local filter satisfy the following relationships:

$$\Sigma_p^{(s)}(\varrho_t,\varrho_r)\leqslant\Omega_s(\varrho_t,\varrho_r),\qquad\Sigma_u^{(s)}(\varrho_t,\varrho_r)\leqslant\Theta_s(\varrho_t,\varrho_r),\tag{24}$$

where the matrix sequences $\{\Omega_s(\varrho_t,\varrho_r)\}$ and $\{\Theta_s(\varrho_t,\varrho_r)\}$ ($\forall\varrho_t,\varrho_r\in[1\ \hbar]_{\mathbb{Z}}$) satisfy the following evolutionary equations:

$$\begin{aligned}\Omega_s(\varrho_t,\varrho_r)=&\,(1+\bar\mu_s)\hat\Omega_1^{(s)}(\varrho_t,\varrho_r-1)+(1+\bar\mu_s^{-1})\hat\Omega_2^{(s)}(\varrho_t-1,\varrho_r)+B_1(\varrho_t,\varrho_r-1)Q(\varrho_t,\varrho_r-1)\\&\times B_1^{\mathrm{T}}(\varrho_t,\varrho_r-1)+B_2(\varrho_t-1,\varrho_r)Q(\varrho_t-1,\varrho_r)B_2^{\mathrm{T}}(\varrho_t-1,\varrho_r),\end{aligned}\tag{25}$$

$$\begin{aligned}\Theta_s(\varrho_t,\varrho_r)=&\,\zeta_{s,1}(I-\bar\gamma_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\bar\Xi_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r))\Omega_s(\varrho_t,\varrho_r)\\&\times(I-\bar\gamma_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\bar\Xi_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r))^{\mathrm{T}}+\zeta_{s,2}G_s(\varrho_t,\varrho_r)\big(U_s(\varrho_t,\varrho_r)+\rho\vec V_s(\varrho_t,\varrho_r)\\&\times\vec V_s^{\mathrm{T}}(\varrho_t,\varrho_r)+\bar V_s(\varrho_t,\varrho_r)\circ(C_s(\varrho_t,\varrho_r)X(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r))\\&+\rho^{-1}\bar C_s(\varrho_t,\varrho_r)(I\otimes X(\varrho_t,\varrho_r))\bar C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\big)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)+\zeta_{s,5}\bar\sigma_sG_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\\&+\zeta_{s,3}\bar\gamma_s^2(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\mathrm{tr}\{\bar\Xi_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)\Omega_s(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar\Xi_s(\varrho_t,\varrho_r)\}G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\\&+\zeta_{s,4}G_s(\varrho_t,\varrho_r)\mathrm{tr}\{U_s(\varrho_t,\varrho_r)+\rho\vec V_s(\varrho_t,\varrho_r)\vec V_s^{\mathrm{T}}(\varrho_t,\varrho_r)+\rho^{-1}\bar C_s(\varrho_t,\varrho_r)(I\otimes X(\varrho_t,\varrho_r))\\&\times\bar C_s^{\mathrm{T}}(\varrho_t,\varrho_r)+\bar V_s(\varrho_t,\varrho_r)\circ(C_s(\varrho_t,\varrho_r)X(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r))\}G_s^{\mathrm{T}}(\varrho_t,\varrho_r),\end{aligned}\tag{26}$$

in which the initial conditions meet $\Theta_s(\varrho_t,0)=\Sigma_u^{(s)}(\varrho_t,0)$ and $\Theta_s(0,\varrho_r)=\Sigma_u^{(s)}(0,\varrho_r)$; and for $d\in[1\ m_s]_{\mathbb{Z}}$, $l=1,2$,

$$\begin{aligned}&\zeta_{s,1}=1+\varsigma_{s,1}+\varsigma_{s,2}+\varsigma_{s,3}+\varsigma_{s,4},\qquad\zeta_{s,2}=1+\varsigma_{s,1}^{-1}+\varsigma_{s,5}+\varsigma_{s,6}+\varsigma_{s,7},\\&\zeta_{s,3}=1+\varsigma_{s,2}^{-1}+\varsigma_{s,5}^{-1}+\varsigma_{s,8}+\varsigma_{s,9},\qquad\zeta_{s,4}=1+\varsigma_{s,3}^{-1}+\varsigma_{s,6}^{-1}+\varsigma_{s,8}^{-1}+\varsigma_{s,10},\\&\zeta_{s,5}=1+\varsigma_{s,4}^{-1}+\varsigma_{s,7}^{-1}+\varsigma_{s,9}^{-1}+\varsigma_{s,10}^{-1},\qquad U_s(\varrho_t,\varrho_r)=[U_{s,di}(\varrho_t,\varrho_r)]_{m_s\times m_s},\\&\hat\Omega_l^{(s)}(\varrho_t,\varrho_r)\triangleq(1+\bar\varepsilon_s)a_l^2(\varrho_t,\varrho_r)\mathrm{tr}\{\Theta_s(\varrho_t,\varrho_r)\}I+(1+\bar\varepsilon_s^{-1})A_l(\varrho_t,\varrho_r)\Theta_s(\varrho_t,\varrho_r)A_l^{\mathrm{T}}(\varrho_t,\varrho_r),\\&\bar\sigma_s=\sum_{d=1}^{m_s}(\bar\sigma_{sd}^{\max})^2,\quad\bar C_s(\varrho_t,\varrho_r)=\mathrm{diag}\{C_{s1}^{\mathrm{T}}(\varrho_t,\varrho_r),C_{s2}^{\mathrm{T}}(\varrho_t,\varrho_r),\ldots,C_{sm_s}^{\mathrm{T}}(\varrho_t,\varrho_r)\},\\&\vec V_s(\varrho_t,\varrho_r)=[\vec V_{s,di}(\varrho_t,\varrho_r)]_{m_s\times m_s},\quad\bar V_s(\varrho_t,\varrho_r)=[\bar V_{s,di}(\varrho_t,\varrho_r)]_{m_s\times m_s}.\end{aligned}$$

Here, when $d=i$,

$$U_{s,dd}(\varrho_t,\varrho_r)=\hat\gamma_s(\varrho_t,\varrho_r)\tau_{sd}^2\Phi(\hat\beta_{sd}(\varrho_t,\varrho_r))+\hat\gamma_s(\varrho_t,\varrho_r)(1-\Phi(\hat\beta_{sd}(\varrho_t,\varrho_r)))(R_{sd}(\varrho_t,\varrho_r)$$

$$
\begin{aligned}
&+ \sqrt{R_{sd}(\varrho_t,\varrho_r)}\lambda(\hat{\beta}_{sd}(\varrho_t,\varrho_r))\tau_{sd}) + \bar{\gamma}_s^2(\varrho_t,\varrho_r)R_{sd}(\varrho_t,\varrho_r)(1-\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r)))\\
&\times \big(1-\lambda^2(\hat{\beta}_{sd}(\varrho_t,\varrho_r)) + \lambda^2(\hat{\beta}_{sd}(\varrho_t,\varrho_r))\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r))\big) + \bar{\gamma}_s^2(\varrho_t,\varrho_r)\sqrt{R_{sd}(\varrho_t,\varrho_r)}\\
&\times (1-2\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r)))\phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r))\tau_{sd} + \bar{\gamma}_s^2(\varrho_t,\varrho_r)(1-\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r)))\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r))\tau_{sd}^2,
\end{aligned}
$$

$$
\begin{aligned}
\vec{V}_{s,dd}(\varrho_t,\varrho_r) = &\frac{1}{2}\big(\hat{\gamma}_s(\varrho_t,\varrho_r)(1-\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r)))\sqrt{R_{sd}(\varrho_t,\varrho_r)}\lambda(\hat{\beta}_{sd}(\varrho_t,\varrho_r)) + \bar{\gamma}_s^2(\varrho_t,\varrho_r)\sqrt{R_{sd}(\varrho_t,\varrho_r)}\\
&\times (2\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r))-1)\phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r)) + 2\bar{\gamma}_s^2(\varrho_t,\varrho_r)\tau_{sd}(\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r))-1)\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r))\big),\\
\bar{V}_{s,dd}(\varrho_t,\varrho_r) = &\hat{\gamma}_s(\varrho_t,\varrho_r)(1-\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r))) + \bar{\gamma}_s^2(\varrho_t,\varrho_r)(1-\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r)))\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r)),
\end{aligned}
$$

and when $d \neq i$,

$$
\begin{aligned}
U_{s,di}(\varrho_t,\varrho_r) = &\hat{\gamma}_s(\varrho_t,\varrho_r)\Big(\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r))\Phi(\hat{\beta}_{si}(\varrho_t,\varrho_r))\tau_{sd}\tau_{si} + \Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r))\tau_{sd}(1-\Phi(\hat{\beta}_{si}(\varrho_t,\varrho_r)))\\
&\times \lambda(\hat{\beta}_{si}(\varrho_t,\varrho_r))\sqrt{R_{si}(\varrho_t,\varrho_r)} + (1-\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r)))\lambda(\hat{\beta}_{sd}(\varrho_t,\varrho_r))\sqrt{R_{sd}(\varrho_t,\varrho_r)}\\
&\times \Phi(\hat{\beta}_{si}(\varrho_t,\varrho_r))\tau_{si} + (1-\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r)))\lambda(\hat{\beta}_{sd}(\varrho_t,\varrho_r))\sqrt{R_{sd}(\varrho_t,\varrho_r)}\\
&\times (1-\Phi(\hat{\beta}_{si}(\varrho_t,\varrho_r)))\lambda(\hat{\beta}_{si}(\varrho_t,\varrho_r))\sqrt{R_{si}(\varrho_t,\varrho_r)}\Big),\\
\vec{V}_{s,di}(\varrho_t,\varrho_r) = &\hat{\gamma}_s(\varrho_t,\varrho_r)\Big(\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r))\tau_{sd} + (1-\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r)))\lambda(\hat{\beta}_{sd}(\varrho_t,\varrho_r))\sqrt{R_{sd}(\varrho_t,\varrho_r)}\Big)\\
&\times (1-\Phi(\hat{\beta}_{si}(\varrho_t,\varrho_r))),\\
\bar{V}_{s,di}(\varrho_t,\varrho_r) = &\hat{\gamma}_s(\varrho_t,\varrho_r)(1-\Phi(\hat{\beta}_{sd}(\varrho_t,\varrho_r)))(1-\Phi(\hat{\beta}_{si}(\varrho_t,\varrho_r))),
\end{aligned}
$$

with $\varsigma_{s,\jmath}$ ($\jmath \in [1\ 10]_{\mathbb{Z}}$) and ρ being known positive scalars.

Proof. Combining (18), (19), and (20b), we rewrite the estimation error dynamics $\varepsilon_u^{(s)}(\varrho_t,\varrho_r)$ as follows:

$$
\begin{aligned}
\varepsilon_u^{(s)}(\varrho_t,\varrho_r) = &(I-\bar{\gamma}_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r))\varepsilon_p^{(s)}(\varrho_t,\varrho_r) - G_s(\varrho_t,\varrho_r)H_s(\varrho_t,\varrho_r)\\
&+ \bar{\gamma}_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)\varepsilon_p^{(s)}(\varrho_t,\varrho_r)\\
&+ G_s(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)H_s(\varrho_t,\varrho_r) - G_s(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)L_s(\varrho_t,\varrho_r),
\end{aligned}
\tag{27}
$$

where $H_s(\varrho_t,\varrho_r) = y_s(\varrho_t,\varrho_r) - \mathbb{E}\{y_s(\varrho_t,\varrho_r)\,|\,x(\varrho_t,\varrho_r)\}$. From (27), it is clear that

$$
\begin{aligned}
\Sigma_u^{(s)}(\varrho_t,\varrho_r) = &\big(I-\bar{\gamma}_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)\big)\Sigma_p^{(s)}(\varrho_t,\varrho_r)\big(I-\bar{\gamma}_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)\\
&\times C_s(\varrho_t,\varrho_r)\big)^{\mathrm{T}} + G_s(\varrho_t,\varrho_r)\mathbb{E}\{H_s(\varrho_t,\varrho_r)H_s^{\mathrm{T}}(\varrho_t,\varrho_r)\}G_s^{\mathrm{T}}(\varrho_t,\varrho_r) + \mathbb{E}\big\{\bar{\gamma}_s^2(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\\
&\times \Lambda_s(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)\varepsilon_p^{(s)}(\varrho_t,\varrho_r)(\varepsilon_p^{(s)}(\varrho_t,\varrho_r))^{\mathrm{T}}C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)\\
&\times G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\big\} + \mathbb{E}\big\{G_s(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)H_s(\varrho_t,\varrho_r)H_s^{\mathrm{T}}(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\big\}\\
&+ \mathbb{E}\big\{G_s(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)L_s(\varrho_t,\varrho_r)L_s^{\mathrm{T}}(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\big\} - \Upsilon_{1,s}(\varrho_t,\varrho_r)\\
&- \Upsilon_{1,s}^{\mathrm{T}}(\varrho_t,\varrho_r) + \Upsilon_{2,s}(\varrho_t,\varrho_r) + \Upsilon_{2,s}^{\mathrm{T}}(\varrho_t,\varrho_r) + \Upsilon_{3,s}(\varrho_t,\varrho_r) + \Upsilon_{3,s}^{\mathrm{T}}(\varrho_t,\varrho_r) - \Upsilon_{4,s}(\varrho_t,\varrho_r)\\
&- \Upsilon_{4,s}^{\mathrm{T}}(\varrho_t,\varrho_r) - \Upsilon_{5,s}(\varrho_t,\varrho_r) - \Upsilon_{5,s}^{\mathrm{T}}(\varrho_t,\varrho_r) - \Upsilon_{6,s}(\varrho_t,\varrho_r) - \Upsilon_{6,s}^{\mathrm{T}}(\varrho_t,\varrho_r)\\
&+ \Upsilon_{7,s}(\varrho_t,\varrho_r) + \Upsilon_{7,s}^{\mathrm{T}}(\varrho_t,\varrho_r) + \Upsilon_{8,s}(\varrho_t,\varrho_r) + \Upsilon_{8,s}^{\mathrm{T}}(\varrho_t,\varrho_r) - \Upsilon_{9,s}(\varrho_t,\varrho_r) - \Upsilon_{9,s}^{\mathrm{T}}(\varrho_t,\varrho_r)\\
&- \Upsilon_{10,s}(\varrho_t,\varrho_r) - \Upsilon_{10,s}^{\mathrm{T}}(\varrho_t,\varrho_r),
\end{aligned}
\tag{28}
$$

where

$$
\begin{aligned}
\Upsilon_{1,s}(\varrho_t,\varrho_r) = &\mathbb{E}\{(I-\bar{\gamma}_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r))\varepsilon_p^{(s)}(\varrho_t,\varrho_r)H_s^{\mathrm{T}}(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\},\\
\Upsilon_{2,s}(\varrho_t,\varrho_r) = &\mathbb{E}\{(I-\bar{\gamma}_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r))\varepsilon_p^{(s)}(\varrho_t,\varrho_r)\\
&\times (\varepsilon_p^{(s)}(\varrho_t,\varrho_r))^{\mathrm{T}}C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\gamma}_s(\varrho_t,\varrho_r)\},\\
\Upsilon_{3,s}(\varrho_t,\varrho_r) = &\mathbb{E}\{(I-\bar{\gamma}_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r))\varepsilon_p^{(s)}(\varrho_t,\varrho_r)H_s^{\mathrm{T}}(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\},\\
\Upsilon_{4,s}(\varrho_t,\varrho_r) = &\mathbb{E}\{(I-\bar{\gamma}_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r))\varepsilon_p^{(s)}(\varrho_t,\varrho_r)L_s^{\mathrm{T}}(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\},\\
\Upsilon_{5,s}(\varrho_t,\varrho_r) = &\mathbb{E}\{G_s(\varrho_t,\varrho_r)H_s(\varrho_t,\varrho_r)(\varepsilon_p^{(s)}(\varrho_t,\varrho_r))^{\mathrm{T}}C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\gamma}_s(\varrho_t,\varrho_r)\},
\end{aligned}
$$

$$
\begin{aligned}
\Upsilon_{6,s}(\varrho_t,\varrho_r) &= \mathbb{E}\{G_s(\varrho_t,\varrho_r)H_s(\varrho_t,\varrho_r)H_s^{\mathrm{T}}(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\},\\
\Upsilon_{7,s}(\varrho_t,\varrho_r) &= \mathbb{E}\{G_s(\varrho_t,\varrho_r)H_s(\varrho_t,\varrho_r)L_s^{\mathrm{T}}(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\},\\
\Upsilon_{8,s}(\varrho_t,\varrho_r) &= \mathbb{E}\{\bar{\gamma}_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)\varepsilon_p^{(s)}(\varrho_t,\varrho_r)\\
&\quad\times H_s^{\mathrm{T}}(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\},\\
\Upsilon_{9,s}(\varrho_t,\varrho_r) &= \mathbb{E}\{\bar{\gamma}_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)\varepsilon_p^{(s)}(\varrho_t,\varrho_r)\\
&\quad\times L_s^{\mathrm{T}}(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\},\\
\Upsilon_{10,s}(\varrho_t,\varrho_r) &= \mathbb{E}\{G_s(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)H_s(\varrho_t,\varrho_r)L_s^{\mathrm{T}}(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\}.
\end{aligned}
$$

Based on some matrix calculations and the properties of the matrix trace, we get that

$$
\begin{aligned}
&\mathbb{E}\big\{\bar{\gamma}_s^2(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)\varepsilon_p^{(s)}(\varrho_t,\varrho_r)\\
&\quad\times(\varepsilon_p^{(s)}(\varrho_t,\varrho_r))^{\mathrm{T}}C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\big\}\\
&\quad\leqslant \bar{\gamma}_s^2(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\mathrm{tr}\{\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)\Sigma_p^{(s)}(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)\}G_s^{\mathrm{T}}(\varrho_t,\varrho_r)
\end{aligned}\tag{29}
$$

and

$$
\begin{aligned}
&\mathbb{E}\{G_s(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)H_s(\varrho_t,\varrho_r)H_s^{\mathrm{T}}(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\}\\
&\quad\leqslant G_s(\varrho_t,\varrho_r)\mathrm{tr}\{\mathbb{E}\{H_s(\varrho_t,\varrho_r)H_s^{\mathrm{T}}(\varrho_t,\varrho_r)\}\}G_s^{\mathrm{T}}(\varrho_t,\varrho_r).
\end{aligned}\tag{30}
$$

Besides, it follows from the definition of $L_s(\varrho_t,\varrho_r)$ that $\mathrm{tr}\{L_s(\varrho_t,\varrho_r)L_s^{\mathrm{T}}(\varrho_t,\varrho_r)\}=\bar{\sigma}_s$, and accordingly, we have

$$
\begin{aligned}
&\mathbb{E}\{G_s(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)L_s(\varrho_t,\varrho_r)L_s^{\mathrm{T}}(\varrho_t,\varrho_r)\Lambda_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\}\\
&\quad\leqslant G_s(\varrho_t,\varrho_r)\mathbb{E}\{\mathrm{tr}\{L_s(\varrho_t,\varrho_r)L_s^{\mathrm{T}}(\varrho_t,\varrho_r)\}\}G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\\
&\quad= \bar{\sigma}_sG_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r).
\end{aligned}\tag{31}
$$

For $H_s(\varrho_t,\varrho_r)$ in (28), according to the properties of the conditional expectation and variance, we have

$$
\begin{aligned}
&\mathbb{E}\{H_s(\varrho_t,\varrho_r)H_s^{\mathrm{T}}(\varrho_t,\varrho_r)\}\\
&\quad=\mathbb{E}\big\{\mathbb{E}\{(y_s(\varrho_t,\varrho_r)-\mathbb{E}\{y_s(\varrho_t,\varrho_r)\,|\,x(\varrho_t,\varrho_r)\})(y_s(\varrho_t,\varrho_r)-\mathbb{E}\{y_s(\varrho_t,\varrho_r)\,|\,x(\varrho_t,\varrho_r)\})^{\mathrm{T}}\,\big|\,x(\varrho_t,\varrho_r)\}\big\}\\
&\quad=\mathbb{E}\{\mathrm{Var}\{y_s(\varrho_t,\varrho_r)\,|\,x(\varrho_t,\varrho_r)\}\}.
\end{aligned}\tag{32}
$$

On the basis of (17), we can derive that

$$
\begin{aligned}
&\mathrm{Var}\{y_s(\varrho_t,\varrho_r)\,|\,x(\varrho_t,\varrho_r)\}\\
&\quad=U_s(\varrho_t,\varrho_r)+\vec{V}_s(\varrho_t,\varrho_r)(I\otimes x^{\mathrm{T}}(\varrho_t,\varrho_r))\bar{C}_s^{\mathrm{T}}(\varrho_t,\varrho_r)+\bar{C}_s(\varrho_t,\varrho_r)(I\otimes x(\varrho_t,\varrho_r))\\
&\qquad\times\vec{V}_s^{\mathrm{T}}(\varrho_t,\varrho_r)+\bar{V}_s(\varrho_t,\varrho_r)\circ\big(C_s(\varrho_t,\varrho_r)x(\varrho_t,\varrho_r)x(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\big).
\end{aligned}\tag{33}
$$

According to (32) and (33), we have

$$
\begin{aligned}
&\mathbb{E}\{H_s(\varrho_t,\varrho_r)H_s^{\mathrm{T}}(\varrho_t,\varrho_r)\}\\
&\quad\leqslant U_s(\varrho_t,\varrho_r)+\rho\vec{V}_s(\varrho_t,\varrho_r)\vec{V}_s^{\mathrm{T}}(\varrho_t,\varrho_r)+\rho^{-1}\bar{C}_s(\varrho_t,\varrho_r)(I\otimes X(\varrho_t,\varrho_r))\bar{C}_s^{\mathrm{T}}(\varrho_t,\varrho_r)\\
&\qquad+\bar{V}_s(\varrho_t,\varrho_r)\circ(C_s(\varrho_t,\varrho_r)X(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r)).
\end{aligned}\tag{34}
$$

According to (28)–(31), (34) and the elementary inequality, we derive that

$$
\begin{aligned}
\Sigma_u^{(s)}(\varrho_t,\varrho_r)\leqslant{}&\zeta_{s,1}\big(I-\bar{\gamma}_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)\big)\Sigma_p^{(s)}(\varrho_t,\varrho_r)\big(I-\bar{\gamma}_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\\
&\times\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)\big)^{\mathrm{T}}+\zeta_{s,2}G_s(\varrho_t,\varrho_r)\big(U_s(\varrho_t,\varrho_r)+\rho\vec{V}_s(\varrho_t,\varrho_r)\vec{V}_s^{\mathrm{T}}(\varrho_t,\varrho_r)\\
&+\rho^{-1}\bar{C}_s(\varrho_t,\varrho_r)(I\otimes X(\varrho_t,\varrho_r))\bar{C}_s^{\mathrm{T}}(\varrho_t,\varrho_r)+\bar{V}_s(\varrho_t,\varrho_r)\circ(C_s(\varrho_t,\varrho_r)X(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r))\big)\\
&\times G_s^{\mathrm{T}}(\varrho_t,\varrho_r)+\zeta_{s,5}\bar{\sigma}_sG_s(\varrho_t,\varrho_r)G_s^{\mathrm{T}}(\varrho_t,\varrho_r)+\zeta_{s,3}\bar{\gamma}_s^2(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\\
&\times\mathrm{tr}\{\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)\Sigma_p^{(s)}(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)\}G_s^{\mathrm{T}}(\varrho_t,\varrho_r)
\end{aligned}
$$

$$+ \zeta_{s,4}G_s(\varrho_t,\varrho_r)\mathrm{tr}\big\{U_s(\varrho_t,\varrho_r) + \rho\vec{V}_s(\varrho_t,\varrho_r)\vec{V}_s^{\mathrm{T}}(\varrho_t,\varrho_r) + \rho^{-1}\bar{C}_s(\varrho_t,\varrho_r)(I\otimes X(\varrho_t,\varrho_r))$$
$$\times \bar{C}_s^{\mathrm{T}}(\varrho_t,\varrho_r) + \bar{V}_s(\varrho_t,\varrho_r)\circ(C_s(\varrho_t,\varrho_r)X(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r))\big\}G_s^{\mathrm{T}}(\varrho_t,\varrho_r). \tag{35}$$

On one hand, it follows from (23) and (25) that

$$\begin{aligned}\Sigma_p^{(s)}(\varrho_t,\varrho_r) - \Omega_s(\varrho_t,\varrho_r) \leqslant &(1+\bar{\mu}_s)(\hat{\Sigma}_1^{(s)}(\varrho_t,\varrho_r-1) - \hat{\Omega}_1^{(s)}(\varrho_t,\varrho_r-1))\\ &+ (1+\bar{\mu}_s^{-1})(\hat{\Sigma}_2^{(s)}(\varrho_t-1,\varrho_r) - \hat{\Omega}_2^{(s)}(\varrho_t-1,\varrho_r)).\end{aligned} \tag{36}$$

On the other hand, from (26) and (35), we obtain

$$\begin{aligned}&\Sigma_u^{(s)}(\varrho_t,\varrho_r) - \Theta_s(\varrho_t,\varrho_r)\\ &\leqslant \zeta_{s,1}(I - \bar{\gamma}_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r))(\Sigma_p^{(s)}(\varrho_t,\varrho_r) - \Omega_s(\varrho_t,\varrho_r))(I - \bar{\gamma}_s(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\\ &\quad\times \bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r))^{\mathrm{T}} + \zeta_{s,2}G_s(\varrho_t,\varrho_r)\big(\rho^{-1}\bar{C}_s(\varrho_t,\varrho_r)(I\otimes(X(\varrho_t,\varrho_r) - \bar{X}(\varrho_t,\varrho_r)))\bar{C}_s^{\mathrm{T}}(\varrho_t,\varrho_r)\\ &\quad+ \bar{V}_s(\varrho_t,\varrho_r)\circ(C_s(\varrho_t,\varrho_r)(X(\varrho_t,\varrho_r) - \bar{X}(\varrho_t,\varrho_r))C_s^{\mathrm{T}}(\varrho_t,\varrho_r))\big)G_s^{\mathrm{T}}(\varrho_t,\varrho_r) + \zeta_{s,3}\bar{\gamma}_s^2(\varrho_t,\varrho_r)G_s(\varrho_t,\varrho_r)\\ &\quad\times \mathrm{tr}\{\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)(\Sigma_p^{(s)}(\varrho_t,\varrho_r) - \Omega_s(\varrho_t,\varrho_r))C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)\}G_s^{\mathrm{T}}(\varrho_t,\varrho_r)\\ &\quad+ \zeta_{s,4}G_s(\varrho_t,\varrho_r)\mathrm{tr}\{\rho^{-1}\bar{C}_s(\varrho_t,\varrho_r)(I\otimes(X(\varrho_t,\varrho_r) - \bar{X}(\varrho_t,\varrho_r)))\bar{C}_s^{\mathrm{T}}(\varrho_t,\varrho_r)\\ &\quad+ \bar{V}_s(\varrho_t,\varrho_r)\circ(C_s(\varrho_t,\varrho_r)(X(\varrho_t,\varrho_r) - \bar{X}(\varrho_t,\varrho_r))C_s^{\mathrm{T}}(\varrho_t,\varrho_r))\}G_s^{\mathrm{T}}(\varrho_t,\varrho_r).\end{aligned} \tag{37}$$

Finally, we will use the mathematical induction to prove validity of (24). According to the definition of $\hat{\Sigma}_l^{(s)}(\varrho_t,\varrho_r)$ and $\hat{\Omega}_l^{(s)}(\varrho_t,\varrho_r)$, we have $\hat{\Sigma}_l^{(s)}(1,0) - \hat{\Omega}_l^{(s)}(1,0) \leqslant 0$ and $\hat{\Sigma}_l^{(s)}(0,1) - \hat{\Omega}_l^{(s)}(0,1) \leqslant 0$. Thus, when $(\varrho_t,\varrho_r) \in \{(\vec{\varrho}_t,\vec{\varrho}_r) \in \mathbb{N}_+\times\mathbb{N}_+ : \vec{\varrho}_t + \vec{\varrho}_r = 2\}$, in the context of (36), (42), and Lemma 5, one has $\Sigma_p^{(s)}(1,1) - \Omega_s(1,1) \leqslant 0$ and $\Sigma_u^{(s)}(1,1) - \Theta_s(1,1) \leqslant 0$. We assume that Eq. (24) holds for all $(\varrho_t,\varrho_r) \in \{(\vec{\varrho}_t,\vec{\varrho}_r) \in \mathbb{N}_+\times\mathbb{N}_+ : \vec{\varrho}_t,\vec{\varrho}_r \leqslant \hbar; \vec{\varrho}_t + \vec{\varrho}_r = q\}$, where $q \in [2\ 2\hbar-1]_{\mathbb{Z}}$ is a given integer. Then, for $(\varrho_t,\varrho_r) \in \{(\vec{\varrho}_t,\vec{\varrho}_r) \in \mathbb{N}_+\times\mathbb{N}_+ : \vec{\varrho}_t,\vec{\varrho}_r \leqslant \hbar; \vec{\varrho}_t + \vec{\varrho}_r = q+1\}$, one has

$$\begin{aligned}&\hat{\Sigma}_1^{(s)}(\varrho_t,\varrho_r-1) - \hat{\Omega}_1^{(s)}(\varrho_t,\varrho_r-1)\\ &= (1+\bar{\varepsilon}_s)a_1^2(\varrho_t,\varrho_r-1)\mathrm{tr}\{\Sigma_u^{(s)}(\varrho_t,\varrho_r-1) - \Theta_s(\varrho_t,\varrho_r-1)\}I\\ &\quad+ (1+\bar{\varepsilon}_s^{-1})A_1(\varrho_t,\varrho_r-1)(\Sigma_u^{(s)}(\varrho_t,\varrho_r-1) - \Theta_s(\varrho_t,\varrho_r-1))A_1^{\mathrm{T}}(\varrho_t,\varrho_r-1)\\ &\leqslant 0\end{aligned}$$

and

$$\begin{aligned}&\hat{\Sigma}_2^{(s)}(\varrho_t-1,\varrho_r) - \hat{\Omega}_2^{(s)}(\varrho_t-1,\varrho_r)\\ &= (1+\bar{\varepsilon}_s)a_2^2(\varrho_t-1,\varrho_r)\mathrm{tr}\{\Sigma_u^{(s)}(\varrho_t-1,\varrho_r) - \Theta_s(\varrho_t-1,\varrho_r)\}I\\ &\quad+ (1+\bar{\varepsilon}_s^{-1})A_2(\varrho_t-1,\varrho_r)(\Sigma_u^{(s)}(\varrho_t-1,\varrho_r) - \Theta_s(\varrho_t-1,\varrho_r))A_2^{\mathrm{T}}(\varrho_t-1,\varrho_r)\\ &\leqslant 0,\end{aligned}$$

which further combining (36) with (42) as well as Lemma 5 assure that $\Sigma_p^{(s)}(\varrho_t,\varrho_r) - \Omega_s(\varrho_t,\varrho_r) \leqslant 0$, and thereby $\Sigma_u^{(s)}(\varrho_t,\varrho_r) - \Theta_s(\varrho_t,\varrho_r) \leqslant 0$. As a result, Eq. (24) is valid for all $\varrho_t,\varrho_r \in [1\ \hbar]_{\mathbb{Z}}$ and the proof is completed.

In what follows, the local filter parameter $G_s(\varrho_t,\varrho_r)$ will be designed so that the provided UB $\Theta_s(\varrho_t,\varrho_r)$ reaches its minimum at each shifting point.

Theorem 2. Under the conditions of Theorem 1, the UB $\Theta_s(\varrho_t,\varrho_r)$ of the FEV can be minimized with its minimum satisfying the following equation:

$$\begin{aligned}\Theta_s(\varrho_t,\varrho_r) = &\zeta_{s,1}\Omega_s(\varrho_t,\varrho_r) - \zeta_{s,1}^2\bar{\gamma}_s^2(\varrho_t,\varrho_r)\Omega_s(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)\\ &\times \Pi_s^{-1}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)\Omega_s(\varrho_t,\varrho_r),\end{aligned} \tag{38}$$

when the local filter gain $G_s(\varrho_t,\varrho_r)$ is designed as

$$G_s(\varrho_t,\varrho_r) = \zeta_{s,1}\bar{\gamma}_s(\varrho_t,\varrho_r)\Omega_s(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)\Pi_s^{-1}(\varrho_t,\varrho_r), \tag{39}$$

where

$$\begin{aligned}\Pi_s(\varrho_t,\varrho_r) &= \zeta_{s,1}\bar{\gamma}_s^2(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)\Omega_s(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)\\
&\quad + \zeta_{s,2}\big(U_s(\varrho_t,\varrho_r) + \rho\vec{V}_s(\varrho_t,\varrho_r)\vec{V}_s^{\mathrm{T}}(\varrho_t,\varrho_r) + \rho^{-1}\bar{C}_s(\varrho_t,\varrho_r)(I\otimes X(\varrho_t,\varrho_r))\bar{C}_s^{\mathrm{T}}(\varrho_t,\varrho_r)\\
&\quad + \bar{V}_s(\varrho_t,\varrho_r)\circ(C_s(\varrho_t,\varrho_r)X(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r))\big)\\
&\quad + \zeta_{s,3}\bar{\gamma}_s^2(\varrho_t,\varrho_r)\mathrm{tr}\{\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)\Omega_s(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)\}I\\
&\quad + \zeta_{s,4}\mathrm{tr}\{U_s(\varrho_t,\varrho_r) + \rho\vec{V}_s(\varrho_t,\varrho_r)\vec{V}_s^{\mathrm{T}}(\varrho_t,\varrho_r) + \rho^{-1}\bar{C}_s(\varrho_t,\varrho_r)(I\otimes X(\varrho_t,\varrho_r))\bar{C}_s^{\mathrm{T}}(\varrho_t,\varrho_r)\\
&\quad + \bar{V}_s(\varrho_t,\varrho_r)\circ(C_s(\varrho_t,\varrho_r)X(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r))\}I + \zeta_{s,5}\bar{\sigma}_s I.\end{aligned}$$

Proof. From (26), the UB matrix $\Theta_s(\varrho_t,\varrho_r)$ can be rewritten as

$$\begin{aligned}\Theta_s(\varrho_t,\varrho_r) &= \big(G_s(\varrho_t,\varrho_r) - \zeta_{s,1}\bar{\gamma}_s(\varrho_t,\varrho_r)\Omega_s(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)\Pi_s^{-1}(\varrho_t,\varrho_r)\big)\Pi_s(\varrho_t,\varrho_r)\\
&\quad\times\big(G_s(\varrho_t,\varrho_r) - \zeta_{s,1}\bar{\gamma}_s(\varrho_t,\varrho_r)\Omega_s(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)\Pi_s^{-1}(\varrho_t,\varrho_r)\big)^{\mathrm{T}}\\
&\quad - \zeta_{s,1}^2\bar{\gamma}_s^2(\varrho_t,\varrho_r)\Omega_s(\varrho_t,\varrho_r)C_s^{\mathrm{T}}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)\Pi_s^{-1}(\varrho_t,\varrho_r)\bar{\Xi}_s(\varrho_t,\varrho_r)C_s(\varrho_t,\varrho_r)\Omega_s(\varrho_t,\varrho_r)\\
&\quad + \zeta_{s,1}\Omega_s(\varrho_t,\varrho_r).\end{aligned}$$

Then, from the fact $\Pi_s(\varrho_t,\varrho_r) > 0$, we know that the minimum of $\Theta_s(\varrho_t,\varrho_r)$ is acquired when the filter gain is designed as in (39), and the minimum of $\Theta_s(\varrho_t,\varrho_r)$ satisfies (38).

According to the CI fusion rules given in [34], the fusion Tobit Kalman filter for system (1) can be constructed as follows:

$$\hat{x}(\varrho_t,\varrho_r) = \Theta(\varrho_t,\varrho_r)\sum_{s=1}^{W}\alpha_s(\varrho_t,\varrho_r)\Theta_s^{-1}(\varrho_t,\varrho_r)\hat{x}_u^{(s)}(\varrho_t,\varrho_r), \tag{40a}$$

$$\Theta(\varrho_t,\varrho_r) \triangleq \left(\sum_{s=1}^{W}\alpha_s(\varrho_t,\varrho_r)\Theta_s^{-1}(\varrho_t,\varrho_r)\right)^{-1}, \tag{40b}$$

where $\alpha_s(\varrho_t,\varrho_r)$ ($s\in[1\ W]_{\mathbb{Z}}$) are weighting coefficients that can be acquired by solving the following optimization problem:

$$\min_{\alpha_1(\varrho_t,\varrho_r),\alpha_2(\varrho_t,\varrho_r),\ldots,\alpha_W(\varrho_t,\varrho_r)} \mathrm{tr}\{\Theta(\varrho_t,\varrho_r)\}, \tag{41}$$

subject to $\sum_{s=1}^{W}\alpha_s(\varrho_t,\varrho_r) = 1$, $0 \leqslant \alpha_s(\varrho_t,\varrho_r) \leqslant 1$.

Remark 3. For the original 2-D system (1) and local filter (16), the fusion rule (40) is verified to be consistent in [34]; i.e., $\mathbb{E}\{(x(\varrho_t,\varrho_r)-\hat{x}(\varrho_t,\varrho_r))(x(\varrho_t,\varrho_r)-\hat{x}(\varrho_t,\varrho_r))^{\mathrm{T}}\} \leqslant \Theta(\varrho_t,\varrho_r)$ and $\Theta(\varrho_t,\varrho_r) \leqslant \Theta_s(\varrho_t,\varrho_r)$ always hold. Besides, the optimization problem in (41) can be solved by using the function 'fmincon' in MATLAB optimization toolbox.

4 Numerical example

In this section, we intend to present a numerical example to verify the effectiveness of the constructed filter.

In practical engineering, the evolution processes of many continuous physical systems, such as the air drying, gas absorption, and water stream heating systems, can be characterized by the following Darboux equation:

$$\frac{\partial^2 u(z,t)}{\partial z\partial t} = a_1\frac{\partial u(z,t)}{\partial t} + a_2\frac{\partial u(z,t)}{\partial z} + a_0 u(z,t) + f(z,t),$$

where $u(z,t)$ is an unknown function at space $z\in[0\ Z]_{\mathbb{Z}}$ and time $t\in[0\ T]_{\mathbb{Z}}$, and $f(z,t)$ is a known forcing function with $f(z,t) = b_0\cos(b_1 r(z,t)) - b_2 b_1\sin(b_1 u(z,t))(\partial u(z,t)/\partial z)$. Here,

$$r(z,t) = \frac{\partial u(z,t)}{\partial t} - a_2 u(z,t) - b_2\cos(b_1 u(z,t)),$$

and a_0, a_1, a_2, b_0, b_1 and b_2 are known real coefficients.

According to the definition of $r(z,t)$, we have

$$\frac{\partial r(z,t)}{\partial z} = a_1\frac{\partial u(z,t)}{\partial t} + a_0u(z,t) + b_0\cos(b_1r(z,t)), \tag{42a}$$

$$\frac{\partial u(z,t)}{\partial t} = r(z,t) + a_2u(z,t) + b_2\cos(b_1u(z,t)). \tag{42b}$$

To discretize (42), the following approximations are utilized:

$$\frac{\partial r(z,t)}{\partial z} \doteq \frac{r((\varrho_t+1)\Delta z, \varrho_r\Delta t) - r(\varrho_t\Delta z, \varrho_r\Delta t)}{\Delta z}, \quad \frac{\partial u(z,t)}{\partial t} \doteq \frac{u(\varrho_t\Delta z, (\varrho_r+1)\Delta t) - r(\varrho_t\Delta z, \varrho_r\Delta t)}{\Delta t},$$

where Δz and Δt are the step sizes. Let $r(\varrho_t,\varrho_r) \triangleq r(\varrho_t\Delta z,\varrho_r\Delta t)$, $u(\varrho_t,\varrho_r) \triangleq u(\varrho_t\Delta z,\varrho_r\Delta t)$, $x(\varrho_t,\varrho_r) \triangleq [r^{\mathrm{T}}(\varrho_t,\varrho_r), u^{\mathrm{T}}(\varrho_t,\varrho_r)]^{\mathrm{T}}$. The partial differential equation (42) can be cast into the following form:

$$x(\varrho_t,\varrho_r) = g_1(\varrho_t,\varrho_r-1,x(\varrho_t,\varrho_r-1)) + g_2(\varrho_t-1,\varrho_r,x(\varrho_t-1,\varrho_r)),$$

where $g_l(\varrho_t,\varrho_r,x(\varrho_t,\varrho_r)) = D_l(\varrho_t,\varrho_r)x(\varrho_t,\varrho_r) + E_l(\varrho_t,\varrho_r)\cos(b_1x(\varrho_t,\varrho_r))$ for $l=1,2$, and

$$D_1(\varrho_t,\varrho_r) = \begin{bmatrix} 0 & 0 \\ \Delta t & 1+a_2\Delta t \end{bmatrix}, D_2(\varrho_t,\varrho_r) = \begin{bmatrix} 1+a_1\Delta z & \bar{a}_0\Delta z \\ 0 & 0 \end{bmatrix}, E_1 = \begin{bmatrix} 0 & 0 \\ 0 & b_2\Delta t \end{bmatrix}, E_2 = \begin{bmatrix} b_0\Delta z & a_1b_2\Delta z \\ 0 & 0 \end{bmatrix},$$

with $\bar{a}_0 = a_0 + a_1a_2$.

In this example, the shift-varying system can be described by (1) with the following parameters: $\Delta z = \Delta t = 0.2$, $a_0 = -2.6825 + 1.1\cos(0.25(\varrho_t+\varrho_r))$, $a_1 = -2.15$, $a_2 = -2.55$, $b_0 = 1.1$, $b_1 = 0.11$, $b_2 = -0.05$. It is evident that the nonlinear functions $g_l(\varrho_t,\varrho_r,x(\varrho_t,\varrho_r))$ given above satisfy condition (2) with $A_l(\varrho_t,\varrho_r) = D_l(\varrho_t,\varrho_r)$ and $a_l(\varrho_t,\varrho_r) = b_1\|E_l(\varrho_t,\varrho_r)\|$ for $l=1,2$. Other system parameters are chosen as follows:

$$\begin{aligned}
&B_1(\varrho_t,\varrho_r) = [0.16, 0.18+\exp(-3\varrho_t)]^{\mathrm{T}}, \quad B_2(\varrho_t,\varrho_r) = [0.17+0.1\sin(\varrho_r), 0.17]^{\mathrm{T}},\\
&C_1(\varrho_t,\varrho_r) = [1+0.1\sin(\varrho_t+\varrho_r), -0.6], \quad C_2(\varrho_t,\varrho_r) = [0.99, -0.63+\exp(-2\varrho_r)],\\
&C_3(\varrho_t,\varrho_r) = [1.1+0.1\cos(\varrho_t+\varrho_r), -0.62], \quad C_4(\varrho_t,\varrho_r) = [1.13, -0.58], \quad C_5(\varrho_t,\varrho_r) = [0.98, -0.65].
\end{aligned}$$

For simulation purpose, the initial conditions involved in system (1) and the filter (16) are chosen as

$$\begin{aligned}
&\eta_1(\varrho_t) = \eta_2(\varrho_r) \equiv [0,0]^{\mathrm{T}}, \quad X(\varrho_t,0) = X(0,\varrho_r) \equiv 0.01I,\\
&\vartheta_{1,1}(\varrho_t) = \vartheta_{2,1}(\varrho_r) \equiv [0.05,0.05]^{\mathrm{T}}, \quad \Sigma_u^{(1)}(\varrho_t,0) = \Sigma_u^{(1)}(0,\varrho_r) \equiv [0.05,0.05]^{\mathrm{T}}[0.05,0.05] + 0.01I,\\
&\vartheta_{1,2}(\varrho_t) = \vartheta_{2,2}(\varrho_r) \equiv [0.3,0.3]^{\mathrm{T}}, \quad \Sigma_u^{(2)}(\varrho_t,0) = \Sigma_u^{(2)}(0,\varrho_r) \equiv [0.3,0.3]^{\mathrm{T}}[0.3,0.3] + 0.01I,\\
&\vartheta_{1,3}(\varrho_t) = \vartheta_{2,3}(\varrho_r) \equiv [0.2,0.2]^{\mathrm{T}}, \quad \Sigma_u^{(3)}(\varrho_t,0) = \Sigma_u^{(3)}(0,\varrho_r) \equiv [0.2,0.2]^{\mathrm{T}}[0.2,0.2] + 0.01I,\\
&\vartheta_{1,4}(\varrho_t) = \vartheta_{2,4}(\varrho_r) \equiv [0.12,0.12]^{\mathrm{T}}, \quad \Sigma_u^{(4)}(\varrho_t,0) = \Sigma_u^{(4)}(0,\varrho_r) \equiv [0.12,0.12]^{\mathrm{T}}[0.12,0.12] + 0.01I,\\
&\vartheta_{1,5}(\varrho_t) = \vartheta_{2,5}(\varrho_r) \equiv [0.25,0.25]^{\mathrm{T}}, \quad \Sigma_u^{(5)}(\varrho_t,0) = \Sigma_u^{(5)}(0,\varrho_r) \equiv [0.25,0.25]^{\mathrm{T}}[0.25,0.25] + 0.01I.
\end{aligned}$$

Variances of the process noise $\omega(\varrho_t,\varrho_r)$ and the measurement noise $\nu_s(\varrho_t,\varrho_r)$ are selected as $Q(\varrho_t,\varrho_r) = 0.012I$, $R_1(\varrho_t,\varrho_r) = 0.38$, $R_2(\varrho_t,\varrho_r) = 0.12$, $R_3(\varrho_t,\varrho_r) = 0.64$, $R_4(\varrho_t,\varrho_r) = 0.25$, and $R_5(\varrho_t,\varrho_r) = 0.46$, respectively.

In addition, the censoring thresholds are set to be $\tau_1 = 0$, $\tau_2 = 0.2$, $\tau_3 = -0.2$, $\tau_4 = 0.12$, $\tau_5 = -0.12$, and the saturation level is assigned as $\bar{\sigma}_s^{\max} = 3$ for $s \in [1\ 3]_{\mathbb{Z}}$. The maximum energy storage capacities of the sensors in the energy harvesting process are respectively assumed as $D_1 = 3$, $D_2 = 4$, $D_3 = 5$, $D_4 = 5$, $D_5 = 3$, and the initially stored energies in the sensors are set as $\kappa_1 = 3$, $\kappa_2 = 4$, $\kappa_3 = 5$, $\kappa_4 = 5$, and $\kappa_5 = 3$. At each shifting point, it is assumed that the energy $h_s(\varrho_t,\varrho_r)$ harvested by each sensor follows the following distribution:

$$\begin{aligned}
&p_{1,0} = 0.1, \quad p_{1,1} = 0.2, \quad p_{1,2} = 0.3, \quad p_{1,3} = 0.4;\\
&p_{2,0} = 0.1, \quad p_{2,1} = 0.1, \quad p_{2,2} = 0.2, \quad p_{2,3} = 0.3, \quad p_{2,4} = 0.3;\\
&p_{3,0} = 0.1, \quad p_{3,1} = 0.1, \quad p_{3,2} = 0.1, \quad p_{3,3} = 0.1, \quad p_{3,4} = 0.3, \quad p_{3,5} = 0.3;\\
&p_{4,0} = 0.1, \quad p_{4,1} = 0.1, \quad p_{4,2} = 0.1, \quad p_{4,3} = 0.1, \quad p_{4,4} = 0.1, \quad p_{4,5} = 0.5;\\
&p_{5,0} = 0.2, \quad p_{5,1} = 0.1, \quad p_{5,2} = 0.3, \quad p_{5,3} = 0.4.
\end{aligned}$$

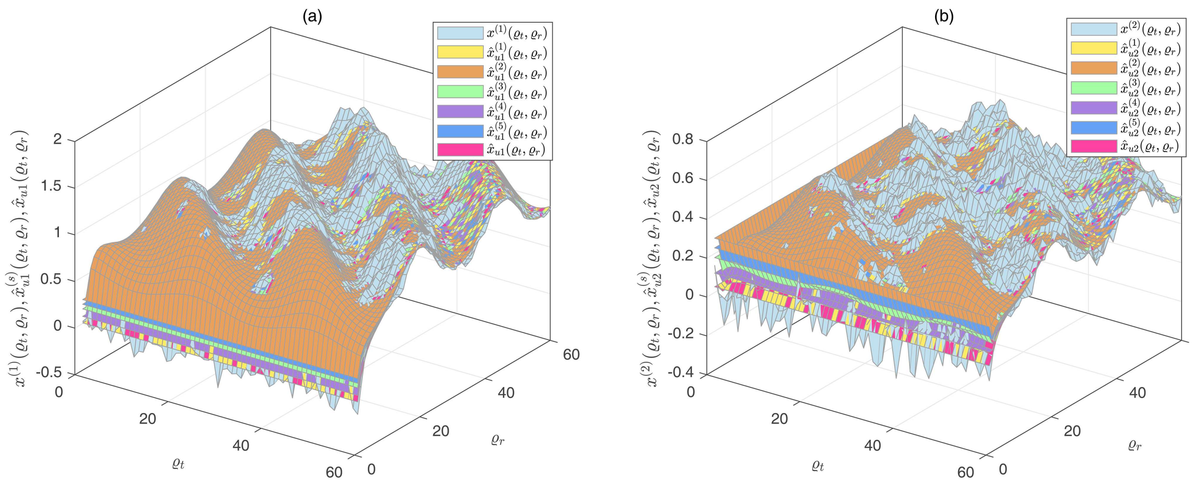

Figure 1 (Color online) Trajectories of the original system and its local filters as well as the fusion filter, where their first components are shown in (a) and the second components are shown in (b).

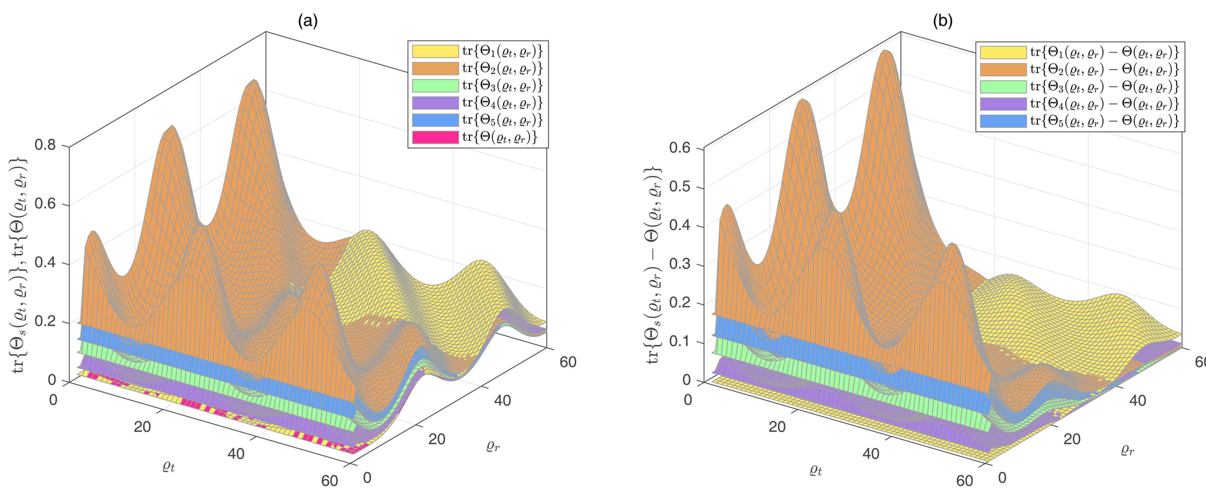

Figure 2 (Color online) The evolutionary trajectories of (a) $\mathrm{tr}\{\Theta(\varrho_t,\varrho_r)\}$ and $\mathrm{tr}\{\Theta_s(\varrho_t,\varrho_r)\}$ as well as (b) $\mathrm{tr}\{\Theta_s(\varrho_t,\varrho_r)-\Theta(\varrho_t,\varrho_r)\}$ for $s\in[1\ 5]_{\mathbb{Z}}$.

According to (10) and (12), $\bar{\gamma}_s(\varrho_t,\varrho_r)$ can be calculated. The measurement outliers are described by a zero-mean Gaussian random variable with covariance 3×10^5. Moreover, we assume that the measurement outliers occur only at points (ϱ_t,ϱ_r) that satisfy $\mathrm{mod}(\varrho_t+\varrho_r,5)=0$. The tunable parameters in Theorem 1 are set as $\rho=1$, $\mu_1=\bar{\mu}_s=1$, $\varepsilon_1=\bar{\varepsilon}_s=33$, $\varsigma_{1,1}=0.032$, $\varsigma_{2,1}=0.01$, $\varsigma_{3,1}=0.013$, $\varsigma_{4,1}=0.02$, $\varsigma_{5,1}=0.01$, $\varsigma_{s,2}=\varsigma_{s,3}=\varsigma_{s,4}=0.01$, $\varsigma_{s,k}=1$ for $s\in[1\ 5]_{\mathbb{Z}}$, $k\in[5\ 10]_{\mathbb{Z}}$.

Under the parameter settings given above, the filter gains $G_s(\varrho_t,\varrho_r)$ can be designed in light of (36). Based on the obtained estimate $\hat{x}^{(s)}_u(\varrho_t,\varrho_r)$ in (16b) and the minimized FEV UB $\Theta_s(\varrho_t,\varrho_r)$ in (38), the fused estimation $\hat{x}(\varrho_t,\varrho_r)$ and error variance UB $\Theta(\varrho_t,\varrho_r)$ can be derived according to the CI fusion rule (40), where parameters $\alpha_s(\varrho_t,\varrho_r)$ are acquired by solving the optimization problem (41) for $s\in[1\ 5]_{\mathbb{Z}}$.

The simulation results are presented in Figures 1 and 2 over a limited horizon $[0\ \hbar]_{\mathbb{Z}}\times[0\ \hbar]_{\mathbb{Z}}$ with $\hbar=60$. To be more specific, Figure 1 represents the evolutionary trajectories of the first and second components of the original system state, the estimation of each local filter and the fused estimation, where $x^{(l)}(\varrho_t,\varrho_r)$, $\hat{x}^{(s)}_{ul}(\varrho_t,\varrho_r)$ and $\hat{x}_{ul}(\varrho_t,\varrho_r)$ represent respectively the lth entry of $x(\varrho_t,\varrho_r)$, $\hat{x}^{(s)}_u(\varrho_t,\varrho_r)$ and $\hat{x}_u(\varrho_t,\varrho_r)$ for $l=1,2$ and $s\in[1\ 5]_{\mathbb{Z}}$. Figure 2 portrays the evolutionary trajectories of $\mathrm{tr}\{\Theta_s(\varrho_t,\varrho_r)\}$, $\mathrm{tr}\{\Theta(\varrho_t,\varrho_r)\}$, and $\mathrm{tr}\{\Theta_s(\varrho_t,\varrho_r)-\Theta(\varrho_t,\varrho_r)\}$ $(s\in[1\ 5]_{\mathbb{Z}})$, from which we can see that the fusion error variance UB $\Theta(\varrho_t,\varrho_r)$ has the minimum values.

To illustrate impact of the censoring thresholds on the proposed filtering performance, we compare the obtained minimal UB $\Theta_1(\varrho_t,\varrho_r)$ by selecting different censoring thresholds. Besides the previously provided value $\tau_{11}=0$, we select two additional values for τ_1: -0.5 and 0.5, and define the corresponding minimal UBs as $\Theta_1^{\hat{\tau}_1}(\varrho t,\varrho_r)$ and $\Theta_1^{\check{\tau}_1}(\varrho t,\varrho_r)$, respectively. Figure 3 depicts the evolutionary trajectories of $\mathrm{tr}\{\Theta_1(\varrho_t,\varrho_r)-\Theta_1^{\hat{\tau}_1}(\varrho_t,\varrho_r)\}$ and

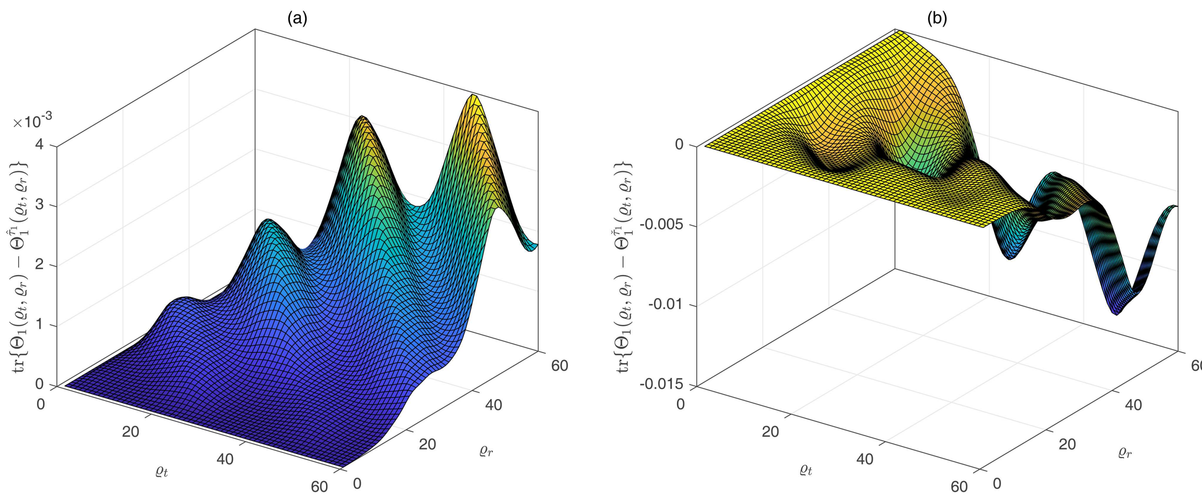

Figure 3 (Color online) The evolutionary trajectories of (a) $\mathrm{tr}\{\Theta_1(\varrho_t,\varrho_r)-\Theta_1^{\hat{\tau}_1}(\varrho_t,\varrho_r)\}$ and (b) $\mathrm{tr}\{\Theta_1(\varrho_t,\varrho_r)-\Theta_1^{\check{\tau}_1}(\varrho_t,\varrho_r)\}$.

$\mathrm{tr}\{\Theta_1(\varrho_t,\varrho_r)-\Theta_1^{\check{\tau}_1}(\varrho_t,\varrho_r)\}$, from which we can see that the trace of the minimal UB $\Theta_1(\varrho_t,\varrho_r)$ increases as the censoring threshold τ_{11} increases, i.e., the filtering performance deteriorates as the censoring threshold increases, which is consistent with the practical engineering perspective.

5 Conclusion

In this paper, we have dealt with the outlier-resistant fusion Tobit Kalman filtering issue for a class of nonlinear 2-D systems with energy harvesting sensors. Firstly, under the energy harvesting constraints, we have derived the conditional mathematical expectation and variance of the censored measurement regarding the system state. Secondly, a local outlier-resistant Tobit Kalman filter has been constructed for the addressed nonlinear 2-D system, where an UB for the local FEV is constructed and subsequently minimized by designing proper filter parameters. Additionally, the obtained local estimates have been fused by utilizing the CI fusion mechanism. Finally, an illustrative example has been provided to show the validity of the proposed filtering algorithm. In the future, we aim to extend the findings of this work to other 2-D models, such as those with parameter uncertainties and multiplicative noises under cyber attacks [35, 36], or to tackle other issues for the addressed 2-D model such as the guaranteed cost control problems [37, 38].

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 62373103, 62573219), Natural Science Foundation of Jiangsu Province (Grant No. BK20240009), and Jiangsu Provincial Scientific Research Center of Applied Mathematics (Grant No. BK20233002).

References

1 Lu W S, Antoniou A. Two-Dimensional Digital Filters. New York: Marcel Dekker, 1992
2 Galkowski K, Rogers E. Control systems analysis for the Fornasini-Marchesini 2D systems model-progress after four decades. Int J Control, 2018, 91: 2801–2822
3 Fornasini E, Marchesini G. State-space realization theory of two-dimensional filters. IEEE Trans Automat Contr, 1976, 21: 484–492
4 Ooba T, Funahashi Y. Stability analysis of two-dimensional systems by means of finitely constructed bilateral quadratic forms. IEEE Trans Automat Contr, 2004, 49: 2068–2073
5 Li L L, Yang R N, Feng Z G, et al. Resilient sliding mode control for 2D CPSs under denial-of-service attacks. Sci China Inf Sci, 2024, 67: 162202
6 Dong S, Zhou X, Tang J, et al. Cross-coupling indirect iterative learning control method for batch processes with time-varying uncertainties. Syst Sci Control Eng, 2024, 12: 2291406
7 Cheng P, Wu D, He S P, et al. Asynchronous control for 2-D Markov jump cyber-physical systems against aperiodic denial-of-service attacks. Sci China Inf Sci, 2023, 66: 172204
8 Wang Y, Zhao D, Li Y, et al. Unbiased minimum variance fault and state estimation for linear discrete time-varying two-dimensional systems. IEEE Trans Automat Contr, 2017, 62: 5463–5469
9 Li M, Liang J. Set-membership estimation for nonlinear 2-D systems with missing measurements. IEEE Trans Circuits Syst II, 2023, 70: 146–150
10 Zhao D, Ding S X, Karimi H R, et al. Robust H_∞ filtering for two-dimensional uncertain linear discrete time-varying systems: a krein space-based method. IEEE Trans Automat Contr, 2019, 64: 5124–5131
11 Zhu K, Wang Z, Ding D, et al. Encryption-decryption-based set-membership filtering for two-dimensional systems: on security and boundedness. Automatica, 2025, 173: 112091
12 Allik B, Miller C, Piovoso M J, et al. The Tobit Kalman filter: an estimator for censored measurements. IEEE Trans Contr Syst Technol, 2016, 24: 365–371
13 Allik B, Miller C, Piovoso M J, et al. Nonlinear estimators for censored data: a comparison of the EKF, the UKF and the Tobit Kalman filter. In: Proceedings of the American Control Conference, Chicago, 2015. 5146–5151

14 Geng H, Wang Z, Chen Y, et al. Multi-sensor filtering fusion with parametric uncertainties and measurement censoring: monotonicity and boundedness. IEEE Trans Signal Process, 2021, 69: 5875–5890
15 Zhang J, Zhao S. Distributed adaptive Tobit Kalman filter for networked systems under sensor delays and censored measurements. IEEE Trans Signal Inf Process over Netws, 2022, 8: 445–458
16 Loumponias K. Colored Tobit Kalman filter. Commun Stat-Theor Meth, 2023, 52: 1533–1547
17 Huang J, He X. Detection of intermittent fault for discrete-time systems with output dead-zone: a variant Tobit Kalman filtering approach. J Control Sci Eng, 2017, 2017: 1–9
18 Geng H, Wang Z, Ma L, et al. Distributed filter design over sensor networks under try-once-discard protocol: dealing with sensor-bias-corrupted measurement censoring. IEEE Trans Syst Man Cybern Syst, 2024, 54: 3032–3043
19 Li Y, Liang J. Distributed Tobit Kalman filtering for random parameter 2-D system: dealing with amplify-and-forward relay and stochastic communication protocol. Inf Sci, 2025, 718: 122355
20 Wang W, Kan X, Ding D, et al. Distributed correntropy Kalman filtering over sensor networks with FlexRay-based protocols. Int J Syst Sci, 2025, 56: 1347–1359
21 Wang Y A, Shen B, Zou L, et al. A survey on recent advances in distributed filtering over sensor networks subject to communication constraints. Int J Network Dyn Intell, 2023, 2: 100007
22 Shen Y, Wang Z, Shen B, et al. Fusion estimation for multi-rate linear repetitive processes under weighted try-once-discard protocol. Inf Fusion, 2020, 55: 281–291
23 Shen B, Wang Z, Tan H, et al. Robust fusion filtering over multisensor systems with energy harvesting constraints. Automatica, 2021, 131: 109782
24 Xiao B, Wu Q M J, Yan L P. Multisensor fusion estimation of nonlinear systems with intermittent observations and heavy-tailed noises. Sci China Inf Sci, 2022, 65: 192203
25 Seifullaev R, Knorn S, Ahlén A, et al. Reinforcement learning-based transmission policies for energy harvesting powered sensors. IEEE Trans Green Commun Netw, 2024, 8: 1564–1573
26 Wang S, Wang Z, Dong H, et al. Recursive state estimation for stochastic nonlinear non-Gaussian systems using energy-harvesting sensors: a quadratic estimation approach. Automatica, 2023, 147: 110671
27 Bhuvana V P, Preissl C, Tonello A M, et al. Multi-sensor information filtering with information-based sensor selection and outlier rejection. IEEE Sens J, 2018, 18: 2442–2452
28 Gandhi M A, Mili L. Robust Kalman filter based on a generalized maximum-likelihood-type estimator. IEEE Trans Signal Process, 2010, 58: 2509–2520
29 Jiang B, Gao H Y, Han F, et al. Recursive filtering for nonlinear systems subject to measurement outliers. Sci China Inf Sci, 2021, 64: 172206
30 Qu B, Peng D, Shen Y, et al. A survey on recent advances on dynamic state estimation for power systems. Int J Syst Sci, 2024, 55: 3305–3321
31 Shen Y, Wang Z, Shen B, et al. Outlier-resistant recursive filtering for multisensor multirate networked systems under weighted try-once-discard protocol. IEEE Trans Cybern, 2021, 51: 4897–4908
32 Greene W H. Econometric Analysis. 7th ed. Upper Saddle River: Prentice Hall, 2011
33 Wang F, Wang Z, Liang J, et al. Locally minimum-variance filtering of 2-D systems over sensor networks with measurement degradations: a distributed recursive algorithm. IEEE Trans Cybern, 2022, 52: 996–1008
34 Niehsen W. Information fusion based on fast covariance intersection filtering. In: Proceedings of the 5th International Conference on Information Fusion, Annapolis, 2002. 901–904
35 Li X, Zhang P, Dong H. A robust covert attack strategy for a class of uncertain cyber-physical systems. IEEE Trans Automat Contr, 2024, 69: 1983–1990
36 Tong L Y, Liang J L, Liu Y. Critical observability of partially observed discrete event systems under cyber attacks. Sci China Inf Sci, 2024, 67: 129203
37 Zou Y, Tian E. Guaranteed cost intermittent control for discrete-time system: a data-driven method. Int J Network Dyn Intell, 2024, 3: 100015
38 Zhu K, Wang Z, Ding D, et al. Privacy-preserving control for 2-D systems with guaranteed probability. IEEE Trans Syst Man Cybern Syst, 2024, 54: 4999–5011