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ISS analysis for asynchronous impulsive stochastic interconnected systems

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The concept of input-to-state stability (ISS), proposed in [1], ensures that the solution's norm asymptotically converges to an origin-centered region with radius scaling proportionally to the input signal's magnitude, independent of the initial state. Subsequent research has significantly expanded ISS theory across diverse dynamical systems, such as impulsive stochastic systems [2], interconnected systems [3], and delayed systems [4]. For example, ISS of a class of impulsive stochastic interconnected systems (ISISs) was investigated by combining the Lyapunov method with graph-theoretic approaches [5]. Existing results on impulsive systems have not adequately addressed the challenges posed by asynchronous impulses in ISS analysis. More detailed review is given in Appendix B.

This work aims to investigate the ISS of asynchronous ISISs under an asynchronous average impulsive interval (AII). The following critical difficulties need to be addressed. First, it is essential to define a suitable Lyapunov function whose estimation can be effectively integrated under asynchronous impulses. Second, the treatment of external inputs in both continuous dynamics and asynchronous impulses requires careful consideration. Finally, existing methods, including synchronous impulses and asynchronous distributed impulses, are not directly applicable to asynchronous impulses in this study. Specifically, distinct approaches must be developed for time-delayed versus delay-free cases, with the transition between them being non-trivial. The main contributions are as follows.

(a) We develop novel and easily verifiable ISS criteria that extend prior work by (i) incorporating asynchronous impulses and (ii) generalizing the AII framework from impulsive control to general impulsive effects (including destabilizing or stabilizing impulses). Specifically, a node Lyapunov function, by introducing an exponential ISS-node-Lyapunov function and an improved auxiliary function regarding the asynchronous impulsive instants, is constructed and then summed with weights dependent on the coupling strength.

(b) For delay-free ISISs, we establish readily implementable ISS conditions through stochastic analysis techniques and the Lyapunov method. For time-delayed ISISs, we derive an applicable ISS criterion by extending the delay-free framework with delay-dependent terms via the Lyapunov-Razumikhin approach, while maintaining verifiability without imposing additional constraints.

(c) Different from existing results on synchronous impulses and asynchronous distributed impulses, the proposed approach can effectively handle the heterogeneity of interconnected systems. Specifically, our results can tolerate higher-frequency impulsive disturbances or require lower-frequency control actions compared to conventional schemes.

Problem formulation. Consider asynchronous ISISs without time delay in a digraph $\mathcal{G}$ with L ($L \geqslant 2$) subsystems (i.e., nodes) described by

$$\begin{cases} \mathrm{d}x_u(t) = G_u\big(x_u(t), y_u(t), t\big)\mathrm{d}W_u(t) \\ \qquad + \left[\displaystyle\sum_{v=1}^{L} A_{uv} H_{uv}\big(x_u(t), x_v(t), t\big) \right. \\ \qquad \left. + F_u\big(x_u(t), y_u(t), t\big)\right]\mathrm{d}t, \ t \in \mathbb{R}_{\geqslant t_0} \backslash \mathcal{T}_u, \\ x_u(t) = T_u(x_u(t-), y_u(t-), t), \ t \in \mathcal{T}_u, \ u \in \mathbb{L}, \end{cases} \tag{1}$$

where $x_u(t) \in \mathbb{R}^{\xi_u}, y_u \in U^{\zeta_u}$ are considered to be the system state and external input, $A_{uv} \geqslant 0$ is coupling strength, $W_u(t) \in \mathbb{R}^{\tilde{\xi}_u}$ is Brownian motion, $\mathcal{T}_u = \{t_n^u\}_{n\in\mathbb{Z}_+}$ is the impulsive sequence, t_n^u is the n-th impulsive instant of the u-th node, $t_0^u = t_0$, $F_u : \mathbb{R}^{\xi_u} \times \mathbb{R}^{\zeta_u} \times \mathbb{R}_+ \to \mathbb{R}^{\xi_u}$, $H_{uv} : \mathbb{R}^{\xi_u} \times \mathbb{R}^{\xi_v} \times \mathbb{R}_+ \to \mathbb{R}^{\xi_u}$, $G_u : \mathbb{R}^{\xi_u} \times \mathbb{R}^{\zeta_u} \times \mathbb{R}_+ \to \mathbb{R}^{\xi_u \times \tilde{\xi}_u}$, $T_u : \mathbb{R}^{\xi_u} \times \mathbb{R}^{\zeta_u} \times \mathbb{R}_+ \to \mathbb{R}^{\xi_u}$, and $x_u(t+) = x_u(t), x_u(t-)$ exists. Assume that F_u, H_{uv}, G_u, T_u satisfy the local Lipschitz condition and linear growth condition to ensure the existence and uniqueness for the global solution of system (1) when there is no Zeno behavior, and $F_u(0,0,t) = 0$, $H_{uv}(0,0,t) = 0$, $G_u(0,0,t) = 0$, $T_u(0,0,t) = 0$ for any $t \in \mathbb{R}_{\geqslant t_0}$. Denote $x = (x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \ldots, x_L^{\mathrm{T}})^{\mathrm{T}}$. A motivational example is collected in Appendix C so as to illustrate the practical motivation of asynchronous ISISs.

Assumption 1. Letting $N_u(t,s)$ be the number of $\mathcal{T}_u$ in $(s,t]$, we call τ_u^* the asynchronous AII of u-th node, if $\frac{t-s}{\tau_u^*} - N_u^{(0)} \leqslant N_u(t,s) \leqslant \frac{t-s}{\tau_u^*} + N_u^{(0)}$ holds, where $N_u^{(0)} \geqslant 1$ is the elasticity

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number.

Assumption 2. There are positive constants $\tau_{\min}^{(u)}, \tau_{\max}^{(u)}$ such that $\tau_{\min}^{(u)} \leqslant t_{n+1}^u - t_n^u \leqslant \tau_{\max}^{(u)}$, for all $u \in \mathbb{L}, n \in \mathbb{Z}_+$.

Assumption 1 defines the asynchronous AII condition, a standard and mild constraint. Assumption 2 adopts a fixed dwell-time form, yet remains mild as stability is independent of the specific values of $\tau_{\min}^{(u)}$ and $\tau_{\max}^{(u)}$, see Remark 3 in Appendix C.

Definition 1. Function $\Phi_u \in C^{2,1}(\mathbb{R}^{\xi_u} \times \mathbb{R}_+; \mathbb{R}_+)$ is said to be an exponential ISS-node-Lyapunov function for system (1), if there exist functions $\alpha_u^{(1)} \in \mathcal{VK}_\infty, \alpha_u^{(2)} \in \mathcal{CK}_\infty, \chi_u^{(1)}, \chi_u^{(2)} \in \mathcal{K}_\infty, \tilde{H}_v : \mathbb{R}^+ \to \mathbb{R}^+$, rate coefficients $C_u, d_u \in \mathbb{R}$, and nonnegative constants $\tilde{A}_{uv}$, $\hat{H}_v$ such that $\tilde{H}_v(t) \leqslant \hat{H}_v t$,

$$\alpha_u^{(1)}(|x_u|^p) \leqslant \Phi_u(x_u, t) \leqslant \alpha_u^{(2)}(|x_u|^p), \tag{2}$$

$$\begin{aligned}\mathcal{L}\Phi_u(x_u(t), t) \leqslant & \sum_{v=1}^{L} \tilde{A}_{uv} \tilde{H}_v\big(\Phi_v(x_v(t), t)\big) \\ & + \chi_u^{(1)}\big(|y_u(t)|\big) - C_u \Phi_u\big(x_u(t), t\big), \\ & t \in \mathbb{R}_{t \geqslant t_0} \backslash \mathcal{T}_u,\end{aligned} \tag{3}$$

$$\Phi_u\big(T_u(x_u, y_u, t_n^u), t_n^u\big) \leqslant \mathrm{e}^{-d_u} \Phi_u(x_u, t_n^u-) + \chi_u^{(2)}\big(|y_u|\big), \tag{4}$$

where $x_u \in \mathbb{R}^{\xi_u}, y_u \in U^{\zeta_u}, u \in \mathbb{L}, n \in \mathbb{Z}_+, p > 0$.

Note that rate coefficients satisfy $C_u, d_u \in \mathbb{R}$ in Definition 1, i.e., $d_u > 0$ is stabilizing impulses while $d_u < 0$ is destabilizing impulses. Moreover, it is not hard to see that if $C_u > 0, d_u > 0$, system (1) is easy to achieve stability, but if $C_u < 0, d_u < 0$, system (1) is difficult to achieve stability. Let $\mathcal{T}_u^*$ be the family of $\mathcal{T}_u$ with asynchronous AII τ_u^* under Assumptions 1 and 2.

Theorem 1. System (1) is ISS and stochastic ISS for all $\mathcal{T}_u \in \mathcal{T}_u^*$, if there exists an exponential ISS-node-Lyapunov function such that

$$\frac{d_u}{\tau_u^*} + C_u - \sum_{v=1}^{L} \tilde{A}_{uv} \hat{H}_v \mathrm{e}^{N_u^{(0)}|d_u|} > 0, \tag{5}$$

and $\big(\mathcal{G}, \big(\tilde{A}_{uv} \hat{H}_v \mathrm{e}^{N_u^{(0)}|d_u|}\big)_{L\times L}\big)$ is strongly connected.

The proof of Theorem 1, along with the analysis of condition (5) and the discussion overcoming the limitation of methods requiring purely stabilizing or destabilizing impulses, is provided in Appendix D.

Next, consider asynchronous ISISs with time delay

$$\begin{cases} \mathrm{d}x_u(t) = G_u(x_u(t), x_u(t-\delta(t)), y_u(t), t)\mathrm{d}W_u(t) \\ \qquad + \Big[F_u\big(x_u(t), x_u(t-\delta(t)), y_u(t), t\big) \\ \qquad + \sum_{v=1}^{L} A_{uv} H_{uv}\big(x_u(t), x_v(t), t\big)\Big]\mathrm{d}t, \ t \in \mathbb{R}_{\geqslant t_0} \backslash \mathcal{T}_u, \\ x_u(t) = T_u\big(x_u(t-), y_u(t-), t\big), \ t \in \mathcal{T}_u, \ u \in \mathbb{L}, \end{cases} \tag{6}$$

where $0 \leqslant \delta(t) \leqslant \tilde{\delta}$ is the time delay. Since the ISS-node-Lyapunov function defined in Definition 1 applies only to the delay-free case, it is not suitable for the time-delayed case. Therefore, we must introduce a corresponding ISS-node-Lyapunov functional tailored for system (6).

Definition 2. Function $\Phi_u \in C^{2,1}(\mathbb{R}^{\xi_u} \times \mathbb{R}_+; \mathbb{R}_+)$ is said to be an exponential ISS-node-Lyapunov functional for system (6), if there exist functions $\alpha_u^{(1)} \in \mathcal{VK}_\infty$, $\alpha_u^{(2)} \in \mathcal{CK}_\infty$, $\chi_u^{(1)}, \chi_u^{(2)} \in \mathcal{K}_\infty$, $\tilde{H}_v : \mathbb{R}^+ \to \mathbb{R}^+$, constants $C_u^{(1)}, d_u \in \mathbb{R}$, nonnegative constants $\tilde{A}_{uv}, \hat{H}_v, C_u^{(2)}$ such that $\tilde{H}_v(t) \leqslant \hat{H}_v t$, (2), (4) and the following (7) hold

$$\begin{aligned} & \mathcal{L}\Phi_u\big((x_u)_t, t\big) \\ & \leqslant C_u^{(2)} \Phi_u\big((x_u)_t(-\delta(t)), t-\delta(t)\big) - C_u^{(1)} \Phi_u\big((x_u)_t(0), t\big) \\ & \quad + \sum_{v=1}^{L} \tilde{A}_{uv} \tilde{H}_v\big(\Phi_v\big((x_v)_t(0), t\big)\big) \\ & \quad + \chi_u^{(1)}(|y_u(t)|), \ t \in \mathbb{R}_{\geqslant t_0} \backslash \mathcal{T}_u. \end{aligned} \tag{7}$$

Theorem 2. System (6) is ISS for all $\mathcal{T}_u \in \mathcal{T}_u^*$, if there is an exponential ISS-node-Lyapunov functional such that

$$\begin{aligned} \check{C}_u \triangleq & \frac{d_u}{\tau_u^*} + C_u^{(1)} - \sum_{v=1}^{L} \tilde{A}_{uv} \hat{H}_v \mathrm{e}^{N_u^{(0)}|d_u|} \\ & - \max_{u \in \mathbb{L}} \Big\{ C_u^{(2)} \mathrm{e}^{N_u^{(0)}|d_u|} \Big\} > 0, \end{aligned} \tag{8}$$

and $(\mathcal{G}, (\tilde{A}_{uv} \hat{H}_v \mathrm{e}^{N_u^{(0)}|d_u|})_{L\times L})$ is strongly connected.

The proof of Theorem 2 and a comparative analysis of conditions (5) and (8) are provided in Appendix E. The proposed theoretical results are validated through interconnected spring-mass damper systems with numerical simulations in Appendix F. Therein, an algorithm for verifying parameter selection in Theorem 2 (stabilizing impulses case), an analysis of the mixed impulsive effects scenario, and discussions of the impact of impulsive parameters on ISS are also provided.

Conclusion. In this study, the ISS of asynchronous ISISs without and with time delay was investigated under an asynchronous AII. This work established a unified framework for ISS analysis of stochastic networked systems subject to asynchronous impulses, effectively accommodating the heterogeneity of interconnected systems, and mixed stabilizing or destabilizing effects. Future studies will focus on relaxing AII condition and strongly connected condition.

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Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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