SCIENCE CHINA
Information Sciences

• Supplementary File •

Deterministic distribution of Gaussian entanglement and quantum steering through fiber channels

Yanru YAN[1], Siyu REN[1], Mengyu ZHAO[1], Dongmei HAN[1], Yalin LI[1],
Meihong WANG[1,2] & Xiaolong SU[1,2*]

[1]*State Key Laboratory of Quantum Optics Technologies and Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan, 030006, China*
[2]*Collaborative Innovation Center of Extreme Optics, Shanxi University,Taiyuan, Shanxi 030006, China*

Appendix A The covariance matrix of distributed modes

The schematic of quantum entanglement and steering distribution is shown in Figure 1 in the main text. The Einstein-Podolsky-Rosen (EPR) state are prepared at the server by coupling two amplitude-squeezed states $\hat{a}_1$ and $\hat{a}_2$ and locking their relative phase to $\pi/2$. The EPR entangled modes $\hat{b}_1$ and $\hat{b}_2$ are given by

$$\begin{aligned}\hat{b}_1 &= \sqrt{\frac{1}{2}}\hat{a}_1 + i\sqrt{\frac{1}{2}}\hat{a}_2,\\ \hat{b}_2 &= \sqrt{\frac{1}{2}}\hat{a}_1 - i\sqrt{\frac{1}{2}}\hat{a}_2.\end{aligned} \tag{A1}$$

After the entangled modes ($\hat{b}_1$ and $\hat{b}_2$) are transmitted through fiber channels to Alice and Bob, respectively, the distributed modes $\hat{b}_1'$ and $\hat{b}_2'$ are given by

$$\begin{aligned}\hat{b}_1' &= \sqrt{\eta_1}\hat{b}_1 + \sqrt{1-\eta_1}(\hat{\nu}_1 + \hat{N}_1),\\ \hat{b}_2' &= \sqrt{\eta_2}\hat{b}_2 + \sqrt{1-\eta_2}(\hat{\nu}_2 + \hat{N}_2),\end{aligned} \tag{A2}$$

where $\eta_{1(2)}$ is the total efficiency from server to Alice (Bob), $\nu_{1(2)}$ represents the vacuum state, and $\hat{N}_{1(2)}$ denotes the excess noise in the optical fiber. The variances of the amplitude and phase quadratures of modes $\hat{b}_1'$ and $\hat{b}_2'$ are

$$\begin{aligned}V(\hat{x}_{\hat{b}_1'}) &= \eta_1 V(\hat{x}_{\hat{b}_1}) + (1-\eta_1)V(\hat{x}_{\hat{\nu}_1}) + (1-\eta_1)V(\hat{x}_{\hat{N}_1}),\\ V(\hat{p}_{\hat{b}_1'}) &= \eta_1 V(\hat{p}_{\hat{b}_1}) + (1-\eta_1)V(\hat{p}_{\hat{\nu}_1}) + (1-\eta_1)V(\hat{p}_{\hat{N}_1}),\\ V(\hat{x}_{\hat{b}_2'}) &= \eta_2 V(\hat{x}_{\hat{b}_2}) + (1-\eta_2)V(\hat{x}_{\hat{\nu}_2}) + (1-\eta_2)V(\hat{x}_{\hat{N}_2}),\\ V(\hat{p}_{\hat{b}_2'}) &= \eta_2 V(\hat{p}_{\hat{b}_2}) + (1-\eta_2)V(\hat{p}_{\hat{\nu}_2}) + (1-\eta_2)V(\hat{p}_{\hat{N}_2}).\end{aligned} \tag{A3}$$

Additionally, when considering the phase fluctuation $\Delta\theta$ during the locking process of the balanced homodyne detection system, the variance of the two distributed modes transmitted over optical fiber channels are given by [1]

$$\begin{aligned}V(\hat{x}_{\hat{b}_1''}) &= V(\hat{x}_{\hat{b}_1'})\cos^2(\Delta\theta_1) + V(\hat{p}_{\hat{b}_1'})\sin^2(\Delta\theta_1),\\ V(\hat{p}_{\hat{b}_1''}) &= V(\hat{p}_{\hat{b}_1'})\cos^2(\Delta\theta_1) + V(\hat{x}_{\hat{b}_1'})\sin^2(\Delta\theta_1),\\ V(\hat{x}_{\hat{b}_2''}) &= V(\hat{x}_{\hat{b}_2'})\cos^2(\Delta\theta_2) + V(\hat{p}_{\hat{b}_2'})\sin^2(\Delta\theta_2),\\ V(\hat{p}_{\hat{b}_2''}) &= V(\hat{p}_{\hat{b}_2'})\cos^2(\Delta\theta_2) + V(\hat{x}_{\hat{b}_2'})\sin^2(\Delta\theta_2).\end{aligned} \tag{A4}$$

The CV entangled state shared between Alice and Bob after the fiber transmission can be described by its covariance matrix, which is given by

$$\boldsymbol{\sigma} = \begin{pmatrix}\boldsymbol{A} & \boldsymbol{C}\\ \boldsymbol{C}^T & \boldsymbol{B}\end{pmatrix} = \begin{bmatrix} V(\hat{x}_{\hat{b}_1''}) & 0 & \sigma(\hat{x}_{\hat{b}_1''},\hat{x}_{\hat{b}_2''}) & 0\\ 0 & V(\hat{p}_{\hat{b}_1''}) & 0 & \sigma(\hat{p}_{\hat{b}_1''},\hat{p}_{\hat{b}_2''})\\ \sigma(\hat{x}_{\hat{b}_1''},\hat{x}_{\hat{b}_2''}) & 0 & V(\hat{x}_{\hat{b}_2''}) & 0\\ 0 & \sigma(\hat{p}_{\hat{b}_1''},\hat{p}_{\hat{b}_2''}) & 0 & V(\hat{p}_{\hat{b}_2''})\end{bmatrix}, \tag{A5}$$

* Corresponding author (email: suxl@sxu.edu.cn)

where the covariance matrix elements can be expressed as $V(\hat{x}_{\hat{b}_1''}) = V(\hat{p}_{\hat{b}_1''}) = \frac{1}{2}\eta_1(V_s + V_{as}) + (1-\eta_1)(1+\delta)$, $V(\hat{x}_{\hat{b}_2''}) = V(\hat{p}_{\hat{b}_2''}) = \frac{1}{2}\eta_2(V_s + V_{as}) + (1-\eta_2)(1+\delta)$, and $\sigma(\hat{x}_{\hat{b}_1''}, \hat{x}_{\hat{b}_2''}) = -\sigma(\hat{p}_{\hat{b}_1''}, \hat{p}_{\hat{b}_2''}) = -\sqrt{\eta_1\eta_2}(V_{as} - V_s)[cos(\Delta\theta_1)cos(\Delta\theta_2) - sin(\Delta\theta_1)sin(\Delta\theta_2)]$. Where $\delta = V(\hat{x}_{\hat{N}_{1(2)}}) = V(\hat{p}_{\hat{N}_{1(2)}})$, and $\Delta\theta_1$ and $\Delta\theta_2$ represent the phase fluctuation of homodyne detectors respectively. Here, we denote the variances of squeezed and anti-squeezed noise as V_s and V_{as}, respectively. The squeezed and anti-squeezed noise levels of the squeezed state are quantified by 10 log_{10} V_s dB and 10 log_{10} V_{as} dB, respectively.

Appendix B Details of experiment

The laser source employed in our experiment is a 1550 nm fiber laser (NKT Photonics). The output beam at 1550 nm, after passing through two mode cleaners for noise filtering, serves as the seed beam of the degenerate optical parametric amplifier (DOPA) cavity. Meanwhile, the light at 775 nm is produced via a second-harmonic generation (SHG) cavity and serves as the pump beam of the DOPA cavity.

In the experiment, the 6 µW squeezed state of light at 1550 nm is prepared by the DOPA cavity. The DOPA cavity adopts a semi-monolithic structure, which consisting of a 1 mm×2 mm×10 mm type-0 periodical poled potassium titanyl phosphate (PPKTP) crystal and a concave mirror. The convex face of the PPKTP crystal with a curvature radius of 12 mm is coated with high reflectivity at 1550 nm and transmissivity of 20% at 775 nm and serves as the input coupler of the DOPA. The plane of the PPKTP crystal is coated with antireflectivity for both wavelengths. The concave mirror, with a curvature radius of 25 mm, is mounted on a piezoelectric transducer serving as the output mirror. The concave mirror is coated with a transmissivity of 12.5% at 1550 nm and high reflectivity at 775 nm. The seed and pump beams of the DOPA cavity are combined on a dichroic mirror and then injectted into the DOPA. When the relative phase between the seed and pump beams of the DOPA is locked to π, the DOPA is operated in the de-amplification status and an amplitude-squeezed state is thereby prepared.

To achieve high-efficiency fiber coupling, we implemented an approach consisting of three main steps. Firstly, we selected a suitable matching lens based on the mode field diameter of the SMF-28E single-mode fiber used, and shaped the input beam to match the mode field diameter of the fiber. Secondly, we used a five-axis fiber aligner for fiber coupling, which enables precise fine-tuning of the positions of the fiber's FC/APC connector and the coupling lens in five degrees of freedom: X, Y, Z, θ_X and θ_Y . Finally, we coated the single-mode fiber with an anti-reflective (AR) coating of R < 0.2% @1550 nm. Ultimately, we achieved a coupling efficiency of 95%.

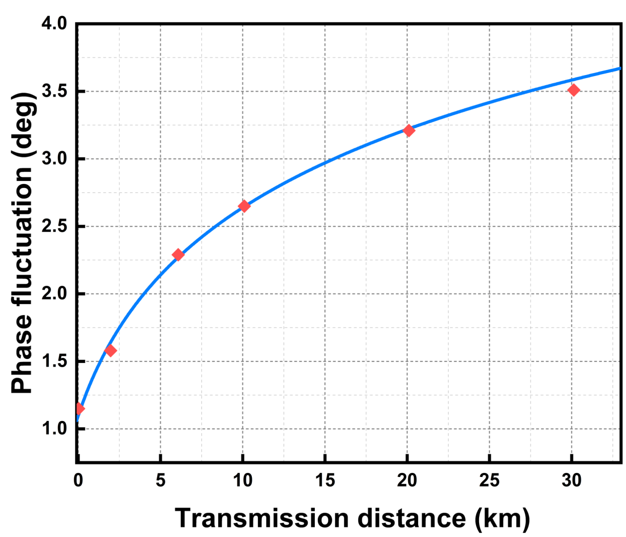

Figure B1 Phase fluctuation of the locking system at different transmission distances. The red diamonds represent the measured data, and the blue curve is the fitting of the experimental data.

Owing to imperfections in the phase-locking system, the measured phase fluctuation of our stabilization system under different transmission distances is summarized in Table I in the main text. The blue curve in Figure B1 is a fitting of the data from Table I in the main text, with its expression given by $\Delta\theta = ln(3 + 1.1 \times L)$, where $\Delta\theta$ is the phase fluctuation and L is the optical fiber length.

Appendix C Measured covariance matrices

At users' station, the demodulated signals are recorded simultaneously by a digital storage oscilloscope with the sampling rate of 1 MS/s. When Alice and Bob lock the relative phase between the signal light and local oscillator (LO) at 0, they can simultaneously measure the amplitude quadrature of both modes. Similarly, when the relative phase between the signal beam and LO beam is locked at $\pi/2$, they can accomplish simultaneous measurement of the phase quadrature for the EPR entangled modes. The relative phase between the distributed modes and LO beam is locked by PDH locking technology [2].

Additionally, in this experiment, we replace the entangled state with the coherent state that has the same power, and the total efficiency and excess noise are estimated via measuring the power and variance of the coherent state before and after the transmission through the fiber channel, respectively. The estimated excess noise of the fiber channel for the transmission of EPR entangled state at different transmission distances are about 0.01 SNU.

We measured the covariance matrixes $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2$,..., and $\boldsymbol{\sigma}_6$ at transmission distances of 16.00 m, 3.90 km, 10.06 km, 20.10 km, 40.18 km and 60.26 km, respectively. The reconstructed covariance matrixes are as follows

$$\boldsymbol{\sigma}_1 = \begin{pmatrix} 6.65 & 0 & -6.43 & 0 \\ 0 & 6.69 & 0 & 6.46 \\ -6.43 & 0 & 7.16 & 0 \\ 0 & 6.46 & 0 & 6.61 \end{pmatrix}; \qquad \boldsymbol{\sigma}_2 = \begin{pmatrix} 6.12 & 0 & -5.26 & 0 \\ 0 & 7.11 & 0 & 6.88 \\ -5.26 & 0 & 5.68 & 0 \\ 0 & 6.88 & 0 & 7.05 \end{pmatrix};$$

$$\boldsymbol{\sigma}_3 = \begin{pmatrix} 5.54 & 0 & -4.80 & 0 \\ 0 & 6.73 & 0 & 6.44 \\ -4.80 & 0 & 5.29 & 0 \\ 0 & 6.44 & 0 & 6.82 \end{pmatrix}; \qquad \boldsymbol{\sigma}_4 = \begin{pmatrix} 4.47 & 0 & -4.27 & 0 \\ 0 & 5.37 & 0 & 4.73 \\ -4.27 & 0 & 5.34 & 0 \\ 0 & 4.73 & 0 & 5.09 \end{pmatrix};$$

$$\boldsymbol{\sigma}_5 = \begin{pmatrix} 3.08 & 0 & -2.35 & 0 \\ 0 & 3.29 & 0 & 2.58 \\ -2.35 & 0 & 3.33 & 0 \\ 0 & 2.58 & 0 & 3.09 \end{pmatrix}; \qquad \boldsymbol{\sigma}_6 = \begin{pmatrix} 2.53 & 0 & -1.69 & 0 \\ 0 & 2.96 & 0 & 2.14 \\ -1.69 & 0 & 2.79 & 0 \\ 0 & 2.14 & 0 & 2.73 \end{pmatrix}.$$

References

1 Takeno Y, Yukawa M, Yonezawa H, et al. Observation of -9 dB quadrature squeezing with improvement of phase stability in homodyne measurement. Opt Express, 2007, 15: 4321

2 Drever R W P, Hall J L, Kowalski F V, et al. Laser phase and frequency stabilization using an optical resonator. Appl Phys B, 1983, 31: 97