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• Supplementary File •

Event-based prescribed-time control for uncertain nonlinear systems with unknown time-varying powers: a non-adaptive control scheme

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Appendix A Lemma

Lemma A1. [1] If $0 < p(t) \leqslant 1$, the following inequality holds for $\forall z_1, z_2 \in \mathbb{R}$,

$$|z_1 + z_2|^{p(t)} \leqslant |z_1|^{p(t)} + |z_2|^{p(t)}, \tag{A1}$$

$$|z_1|^{p(t)} + |z_2|^{p(t)} \leqslant 2^{1-p(t)}(|z_1| + |z_2|)^{p(t)}, \tag{A2}$$

$$\left|\lceil z_1 \rfloor^{p(t)} - \lceil z_2 \rfloor^{p(t)}\right| \leqslant 2^{1-p(t)}|z_1 - z_2|^{p(t)}. \tag{A3}$$

Lemma A2. [1] If $x(t) > 0$, $y(t) > 0$ are continuous functions, and $\pi > 0$, for $\forall a, b \in \mathbb{R}$, the following inequality is satisfied,

$$|a^x b^y| \leqslant \pi \frac{x}{x+y}|a|^{x+y} + \pi^{-\frac{x}{y}} \frac{y}{x+y}|b|^{x+y}. \tag{A4}$$

Lemma A3. [2] For any positive functions $p(t)$ satisfies $0 < a \leqslant p(t) \leqslant b$, the following inequality is satisfied for $\forall z \in \mathbb{R}$,

$$|z|^p \leqslant |z|^a + |z|^b. \tag{A5}$$

Appendix B Controller design

This section designs an event-triggered, time-prescribed controller with time-varying gain, aimed at driving the system state to zero within a prescribed time. Before proceeding with the controller design, several propositions are provided below. By defining adjustable parameters d_{ij}, $i \in N_{\overline{2,n}}$, $j \in N_{\overline{1,4}}$, it can be readily drawn from Young's inequality and *Lemmas 1-3*.

Proposition B1. For $i \in N_{\overline{2,n}}$, there exist positive constants φ_{i1} satisfy the inequality

$$\begin{aligned}
&b_{i-1}z_{i-1}(\lceil x_i \rfloor^{p_{i-1}} - \lceil \chi_{i-1} \rfloor^{p_{i-1}}) \\
&\leqslant b_{i-1}2^{1-\underline{p}}z_{i-1}z_i^{p_{i-1}} \\
&\leqslant \frac{1}{4}|z_{i-1}|^{1+p_{i-1}} + \delta_{i1}\varphi_{i1}|z_i|^{1+p_{i-1}} \\
&\leqslant \frac{1}{4}|z_{i-1}|^{1+p_{i-1}} + \frac{\delta_{i1}^2}{d_{i1}}|z_i|^{1+p_{i-1}} + \frac{d_{i1}\varphi_{i1}^2}{4}|z_i|^{1+p_{i-1}},
\end{aligned} \tag{B1}$$

where δ_{i1} are unknown constants.

Proposition B2. For $\forall p(t)$ such as $0 < p(t) \leqslant 1$ and $i \in N_{\overline{2,n}}$, there has

$$\begin{aligned}
|x_i|^p &= \left|z_i - \gamma_{i-1}^{\frac{1}{\underline{p}}}(z_{i-1} + \lceil z_{i-1} \rfloor^{\frac{\bar{p}}{\underline{p}}})\right|^p \\
&\leqslant |z_i|^p + \gamma_{i-1}^{\frac{p}{\underline{p}}}(1 + |z_{i-1}|^{\frac{\bar{p}}{\underline{p}}-1})^p|z_{i-1}|^p.
\end{aligned} \tag{B2}$$

Then, It follows from *Assumption 2* that

$$z_i f_i \leqslant \theta h_i|z_i|\Big(\sum_{j=1}^{i}|z_j|^{\bar{p}} + \sum_{j=1}^{i-1}\gamma_j^{\frac{\bar{p}}{\underline{p}}}(1 + |z_j|^{\frac{\bar{p}}{\underline{p}}-1})^{\bar{p}}|z_j|^{\bar{p}}$$

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$$\leqslant \frac{1}{4}\sum_{j=1}^{i-1}|z_j|^{1+\bar{p}}+\delta_{i2}\varphi_{i2}|z_i|^{1+\bar{p}}$$
$$\leqslant \frac{1}{4}\sum_{j=1}^{i-1}|z_j|^{1+\bar{p}}+\frac{\delta_{i2}^2}{d_{i2}}|z_i|^{1+\bar{p}}+\frac{d_{i2}\varphi_{i2}^2}{4}|z_i|^{1+\bar{p}}, \tag{B3}$$

where φ_{i2} are nonnegative smooth functions and δ_{i2} are unknown constants.

Proposition B3. It can be calculated that

$$\begin{aligned}
&z_i\Big(\sum_{j=1}^{i-1}\frac{-\partial\chi_{i-1}}{\partial x_j}\dot{x}_j+\frac{-\partial\chi_{i-1}}{\partial\mu^{\frac{1}{l}}}\dot{\mu}\Big)\\
&\leqslant\sum_{j=1}^{i-1}\frac{-\partial\chi_{i-1}}{\partial x_j}\Big(b_jz_i(|z_{j+1}|^{p_j}+\gamma_j^{\frac{p_j}{\bar{p}}}(1+|z_j|^{\frac{\bar{p}}{p}-1})^{p_j}|z_j|^{p_j})\\
&\quad+\theta h_jz_i(\sum_{k=1}^{j}|z_k|^{\bar{p}}+\sum_{k=1}^{j-1}\gamma_k^{\frac{\bar{p}}{p}}(1+|z_k|^{\frac{\bar{p}}{p}-1})^{\bar{p}}|z_k|^{\bar{p}})\Big)\\
&\quad+\dot{\mu}\frac{\partial(\mu^{\frac{2}{p}}\gamma_{i-1}^{\frac{1}{p}})}{\partial\mu^{\frac{1}{l}}}(1+\lceil z_{i-1}\rceil^{\frac{\bar{p}-p}{p}})z_{i-1}z_i\\
&\leqslant\frac{1}{6}\sum_{j=1}^{i-1}(|z_j|^{1+p_j}+|z_j|^{1+p_{j-1}}+|z_j|^{1+\bar{p}})+\delta_{i3}\varphi_{i3}|z_i|^{1+\bar{p}}\\
&\quad+\delta_{i4}\varphi_{i4}\sum_{j=1}^{i-1}|z_i|^{1+p_j}\\
&\leqslant\frac{1}{6}\sum_{j=1}^{i-1}(|z_j|^{1+p_j}+|z_j|^{1+p_{j-1}}+|z_j|^{1+\bar{p}})+\frac{\delta_{i3}^2}{d_{i3}}|z_i|^{1+\bar{p}}\\
&\quad+\frac{d_{i3}\varphi_{i3}^2}{4}|z_i|^{1+\bar{p}}+\frac{\delta_{i4}^2}{d_{i4}}\sum_{j=1}^{i-1}|z_i|^{1+p_j}+\frac{d_{i4}\varphi_{i4}^2}{4}\sum_{j=1}^{i-1}|z_i|^{1+p_j},
\end{aligned}\tag{B4}$$

where $\varphi_{i3}(\bar{x}_i,t)$ and $\varphi_{i4}(\bar{x}_i,t)$ are positive smooth functions, δ_{i3} and δ_{i4} are unknown constants.

Step 1: First, the Lyapunov function is devised to $W_1=\frac{1}{2}z_1^2$. Differentiating W_1 yields

$$\begin{aligned}
\dot{W}_1&\leqslant z_1(b_1\lceil x_2\rceil^{p_1}+f_1)\\
&\leqslant b_1z_1\lceil\chi_1\rceil^{p_1}+f_1z_1+b_1z_1(\lceil x_2\rceil^{p_1}-\lceil\chi_1\rceil^{p_1}).
\end{aligned}\tag{B5}$$

Under *Assumptions 2*, there exists

$$z_1f_1\leqslant\frac{\theta^2}{d_{11}}|z_1|^{1+\bar{p}}+\frac{d_{11}}{4}h_1^2|z_1|^{1+\bar{p}}. \tag{B6}$$

Then, $\gamma_1=c_1+\frac{d_{11}}{4}h_1^2$ is designed. (B5) is transformed to

$$\dot{W}_1\leqslant-(\beta_{11}+\beta_{12})(|z_1|^{1+p_1}+|z_1|^{1+\frac{\bar{p}p_1}{p}})+b_1z_1(\lceil x_2\rceil^{p_1}-\lceil\chi_1\rceil^{p_1}), \tag{B7}$$

where $\beta_{11}=\mu c_1-\delta_1$ and $\beta_{12}=(2^{p_1-1}b_1\mu^{\frac{2p_1-p}{p}}-1)\mu\gamma_1$ are smooth functions, $\delta_1=\frac{\theta^2}{d_{11}}$ is an unknown constant.

Step i: For $i\in N_{\overline{2,n-1}}$, the Lyapunov function is defined as $W_i=W_{i-1}+\frac{1}{2}z_i^2$ and differentiating W_i yields

$$\dot{W}_i\leqslant\dot{W}_{i-1}+b_iz_i\lceil x_{i+1}\rceil^{p_i}+f_iz_i-z_i\sum_{j=1}^{i-1}\frac{\partial\chi_{i-1}}{\partial x_j}\dot{x}_j-z_i\frac{\partial\chi_{i-1}}{\partial\mu^{\frac{1}{l}}}\dot{\mu}. \tag{B8}$$

Based on *Propositions 1-3*, we design $\gamma_i=c_i+\varphi_{i1}+\varphi_{i2}+\varphi_{i3}+(i-1)\varphi_{i4}$, one obtains

$$\dot{W}_i\leqslant-\sum_{j=1}^{i}(\beta_{i1}+\beta_{j2})(|z_j|^{1+p_j}+|z_j|^{1+\frac{\bar{p}p_j}{p}})+b_iz_i(\lceil x_{i+1}\rceil^{p_i}-\lceil\chi_i\rceil^{p_i}), \tag{B9}$$

where $\beta_{i1}=\mu c_i-\delta_i$ and $\beta_{j2}=(2^{p_j-1}b_j\mu^{\frac{2p_j-p}{p}}-1)\mu\gamma_j$ are smooth functions, $\delta_i=max\{\delta_{i-1},\hat{\delta}_i\}$ is an unknown constant with $\hat{\delta}_i=\frac{\delta_{i1}^2}{d_{i1}}+\frac{\delta_{i2}^2}{d_{i2}}+\frac{\delta_{i3}^2}{d_{i3}}+\frac{(i-1)\delta_{i4}^2}{d_{i4}}$.

Step n: The Lyapunov function is constructed as $W_n = W_{n-1} + \frac{1}{2}z_n^2$ and design $\gamma_n = c_n + \varphi_{n1} + \varphi_{n2} + \varphi_{n3} + (n-1)\varphi_{n4}$, By differentiating W_n and repeating the above operation yields

$$\dot{W}_n \leqslant -\sum_{i=1}^{n}(\beta_{n1} + \beta_{i2})(|z_i|^{1+p_i} + |z_i|^{1+\frac{\bar{p}p_i}{\underline{p}}}) + b_n z_n(\lceil u \rfloor^{p_n} - \lceil v \rfloor^{p_n}), \tag{B10}$$

where $\beta_{n1} = c\mu - \delta$ and $\beta_{i2} = (2^{p_i-1}b_i\mu^{\frac{2p_i-\underline{p}}{\underline{p}}} - 1)\mu\gamma_i$ are smooth functions, $\delta = max\{\delta_i\}$, $i \in N_{\overline{1,n}}$, is an unknown constant with $\delta_n = \frac{\delta_{n1}^2}{d_{n1}} + \frac{\delta_{n2}^2}{d_{n2}} + \frac{\delta_{n3}^2}{d_{n3}} + \frac{(n-1)\delta_{n4}^2}{d_{n4}}$.

Appendix C Proof of theorem 1

By the properties of the $\lim_{t\to T}\mu(t) = +\infty$, we know that there definitely exist a moment t_d so that inequality

$$\begin{cases} c\mu(1-\sigma) - \delta > 0, \\ \underline{b}\mu - 2 > 0, \\ \mu - 2^{1-\bar{p}}\bar{b} > 0, \end{cases} \tag{C1}$$

hold. Then, for $t \in [t_d, T)$, there has $\dot{W}_n \leqslant -\beta_{n1}(t)W_n^{\frac{1+\bar{p}}{2}} + b_n z_n(\lceil u \rfloor^{p_n} - \lceil v \rfloor^{p_n})$. Moreover, there exists a moment t_w such that $W_n^{\frac{1+\bar{p}}{2}} \geqslant W_n$ holds. Therefore, inequality (B10) is rewritten as

$$\dot{W}_n \leqslant -\beta_{n1}(t)W_n + b_n z_n(\lceil u \rfloor^{p_n} - \lceil v \rfloor^{p_n}). \tag{C2}$$

On the other hand, according to the trigger mechanism, there has

$$z_n(\lceil u \rfloor^{p_n} - \lceil v \rfloor^{p_n}) \leqslant |z_n||\lceil u \rfloor^{p_n} - \lceil v \rfloor^{p_n}| \leqslant |z_n e|. \tag{C3}$$

Based on the above discussion, we define $t_c = max\{t_w, t_d\}$. Defining $V = W_n + \aleph$, from *Lemma 1*, (C2) and (C3) one has

$$\dot{V} \leqslant -\beta(t)V, \tag{C4}$$

where $\beta(t) = c\mu(1-\sigma) - \delta$ is a positive function. Then, by applying Gronwall's Lemma, it can be shown that $V(t) \leqslant V(t_c)e^{-\int_{t_c}^{t}\beta(s)ds}$ and $\lim_{t\to T^-} z_i^2 = 0$. It follows that $\lim_{t\to T^-} x_1 = 0$ and $\lim_{t\to T^-} \chi_1 = 0 \Rightarrow \lim_{t\to T^-} x_2 = 0 \Rightarrow \cdots \Rightarrow \lim_{t\to T^-} \chi_n = 0 \Rightarrow \lim_{t\to T^-} u = 0$, which indicates that the system states reach the origin within the prescribed time T.

Finally, we demonstrate that Zeno's phenomenon does not occur through a counterfactual argument. For $\forall t \in [t_s, t_{s+1})$, it follows that

$$\epsilon(t) = u(t) - v(t) = v(t_s) - v(t). \tag{C5}$$

Next, by defining the trigger interval $t^* = t_{s+1} - t_s$, the following result can be established:

$$\lim_{t^*\to 0}|z_n(t_s + t^*)((v(t_s) - v(t_s + t^*))^{\underline{p}} + (v(t_s) - v(t_s + t^*)))| = 0. \tag{C6}$$

From event trigger condition, it follows that

$$|z_n(t_s + t^*)((v(t_s) - v(t_s + t^*))^{\underline{p}} + (v(t_s) - v(t_s + t^*)))| > \rho\aleph(t_s + t^*) + \sigma c\sum_{j=1}^{n} z_j^{1+\bar{p}}(t_s + t^*) \geqslant 0. \tag{C7}$$

It is obvious to discover (C6) and (C7) are always contradictory. Therefore, it can be concluded that the premise $t^* \to 0$ is invalid, implying that $t^* > 0$ and does not tend to zero. Consequently, the Zeno phenomenon is avoided.

Remark 1. Since the system powers are unknown in this paper, the event-trigger mechanism designed in the existing conclusions [3] are no longer applicable. This paper proposes new dynamic event triggering rules. The effect of unknown time-varying power of the system is compensated by designing a trigger error signal with low power. In contrast, the triggering mechanism developed above has broader applicability, encompassing nonlinear systems subject to unknown time-varying powers.

Remark 2. The work [4–6] adopt adaptive control schemes to construct error systems that compensate for unknown parameters. This paper proposes a unified novel control method that utilizes the concept of switching to design a time-varying controller gain, effectively addressing the problem of uncertain system parameters while simplifying the control structure. This approach eliminates the reliance on the complex design procedures typical of traditional adaptive control schemes. In contrast, the proposed approach omits the design of an adaptive control law while still guaranteeing system convergence within a prescribed time T, thereby significantly reducing computational complexity. Moreover, the advantage of the PT control framework lies in its ability to eliminate the dependency of finite-time and fixed-time control methods on design parameters and initial conditions, allowing the convergence time to be arbitrarily specified.

Appendix D Simulation example

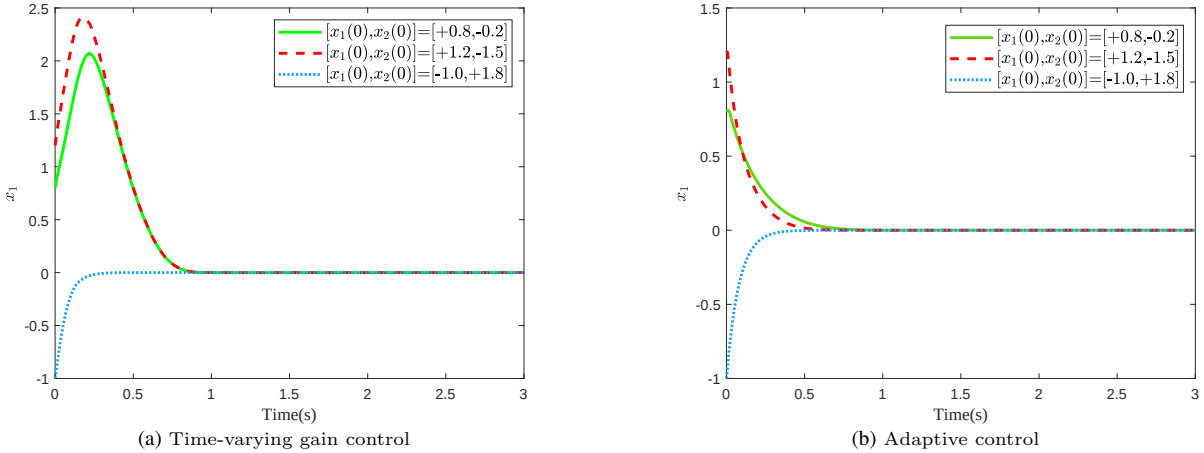
This section evaluates the designed control algorithm through simulation examples based on second-order systems.

$$\begin{cases} \dot{x}_1 = b_1 \lceil x_2 \rfloor^{p_1(t)} + f_1, \\ \dot{x}_2 = b_2 \lceil u \rfloor^{p_2(t)} + f_2, \end{cases} \tag{D1}$$

where $b_1 = 0.5 + 0.1sin(t)$, $b_2 = 1.5 + 0.5sin(x_2)$, $p_1(t) = 0.9 + 0.1sin(t)$, $p_2(t) = 0.5$, $f_1 = (1.5 + 0.5sin(x_1))x_1$ and $f_2 = 1.5x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$. Next, the system is initialized with the values $[x_1(0), x_2(0)] = [+0.8, -0.2], [+1.2, -1.5], [-1.0, +1.8]$ and set the prescribed time to $T = 1$. The parameter of the event triggering mechanism is chosen as $\rho = 0.8$. Based on the conditions described above, the proposed control algorithms are simulated and verified, leading to the results presented below.

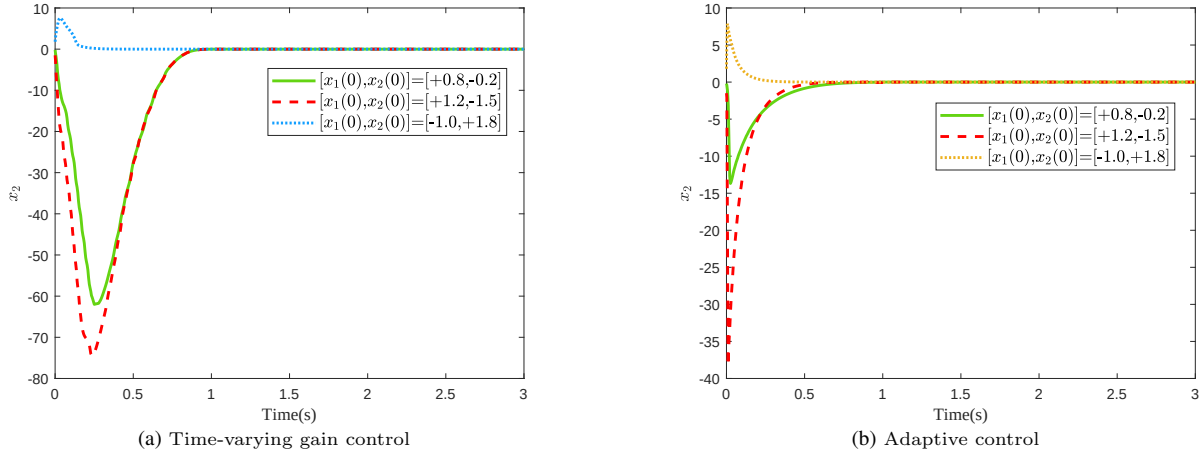
A comparative simulation between the conventional adaptive control algorithm and the time-varying gain control approach proposed herein is conducted to more clearly demonstrate the benefits of the proposed method. Figs.1-5 illustrate the system states, control inputs, and event-triggered responses under varying initial conditions.

Simulation results indicate that both control schemes ensure the system signal reaches zero within the prescribed time. Of particular interest is that although the time-varying gain control scheme introduces a compensation phase during which the system state exhibits a degree of oscillatory behavior in the early stages, the amplitude of these oscillations in the controller's response is significantly smaller compared to that observed under the adaptive control scheme. Furthermore, the number of triggering events under the proposed control scheme is significantly reduced, thereby demonstrating its effectiveness in conserving communication resources.



(a) Time-varying gain control (b) Adaptive control

Figure D1 The responses of x_1.



(a) Time-varying gain control (b) Adaptive control

Figure D2 The responses of x_2.

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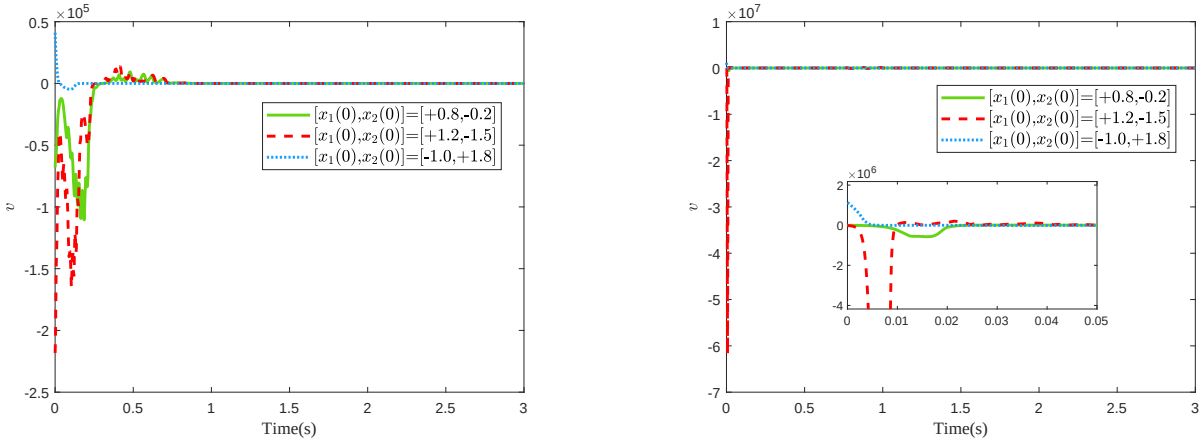
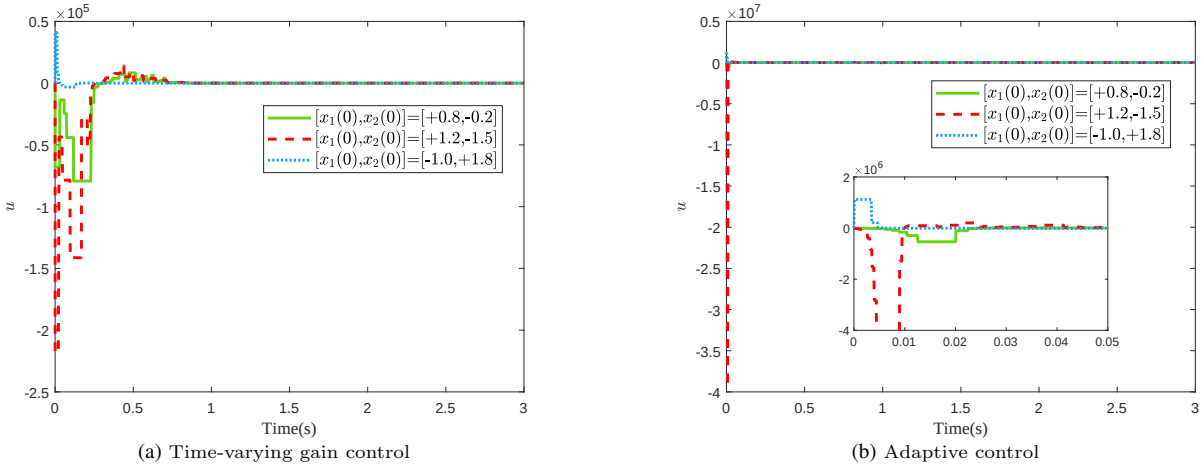


(a) Time-varying gain control (b) Adaptive control

Figure D3 The responses of v.



(a) Time-varying gain control (b) Adaptive control

Figure D4 The responses of u.

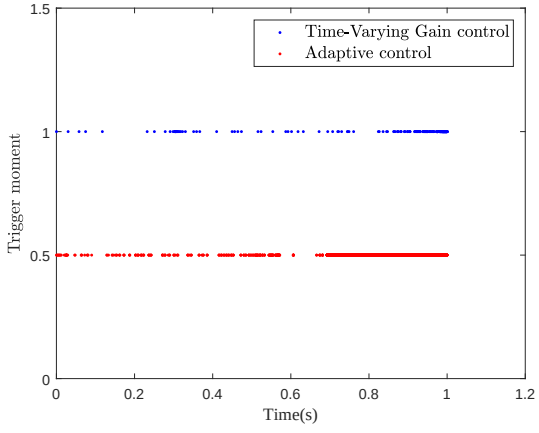


Figure D5 The responses of trigger moment.

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