SCIENCE CHINA
Information Sciences

• LETTER •

July 2026, Vol. 69, Iss. 7, 179203:1–179203:2
https://doi.org/10.1007/s11432-025-4764-y

Performance-guaranteed finite-time exact-tracking of strict-feedback systems with actuator faults

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Received 13 June 2025/Revised 15 October 2025/Accepted 18 November 2025/Published online 6 February 2026

Citation Kong Y Z, Wu X Q, Mao B, et al. Performance-guaranteed finite-time exact-tracking of strict-feedback systems with actuator faults. Sci China Inf Sci, 2026, 69(7): 179203, https://doi.org/10.1007/s11432-025-4764-y

Trajectory tracking is a fundamental control problem with broad applications [1, 2]. While existing methods typically guarantee only asymptotic convergence [1] or bounded tracking errors [2], finite-time exact tracking exhibits superior convergence precision, speed, and robustness, making it highly suitable for practical applications.

However, tracking control faces several challenges. Among these, actuator faults pose a critical threat to system reliability, thereby prompting the development of fault-tolerant control (FTC) methods [1]. While existing approaches like optimal fuzzy FTC [1] address uncertain nonlinear systems with actuator and sensor faults, they lack transient performance guarantees. Prescribed performance control (PPC) enforces funnel-shaped error convergence, yet integrating FTC and PPC for finite-time exact tracking in strict-feedback systems remains unresolved—a gap this work addresses.

The main contributions of this study are as follows. First, while existing studies such as [1,2] only achieved asymptotic tracking or bounded tracking control for strict-feedback nonlinear systems with actuator faults, this study realizes global performance-guaranteed finite-time exact tracking control via sliding mode control methods for the first time, and successfully addresses the critical issue of unknown virtual control directions. Second, the proposed tan-type error transformation method and novel Lyapunov-like function effectively eliminate the initial condition constraints and error surface effects inherent in conventional control methods, while simultaneously guaranteeing the global boundedness of all closed-loop signals. These innovative results provide a comprehensive solution to the key challenges in exact tracking control.

Notation. Throughout this study, R^n denotes the n-dimensional Euclidean space. $|x|$ denotes the absolute value of scalar x. $||\boldsymbol{x}||$ denotes the Euclidean norm of vector $\boldsymbol{x}$. $\boldsymbol{x}^\top$ represents the transpose of vector $\boldsymbol{x}$. N^+ represents the positive integer set.

Problem formulation and preliminaries. Consider an uncertain strict-feedback nonlinear system:

$$\begin{aligned}\dot{x}_i(t) =& f_i(\bar{\boldsymbol{x}}_i(t)) + g_i(\bar{\boldsymbol{x}}_i(t))x_{i+1}(t) + d_i(t), 1 \leqslant i \leqslant n-1,\\ \dot{x}_n(t) =& f_n(\bar{\boldsymbol{x}}_n(t)) + g_n(\bar{\boldsymbol{x}}_n(t))u(t) + d_n(t),\\ y(t) =& x_1(t),\end{aligned} \tag{1}$$

where $x_i \in R, \bar{\boldsymbol{x}}_i = [x_1, \dots, x_i]^\top \in R^i$ is the system state, $y(t)$ and $d_i(t) \in R$ denote the output and external disturbance of the system, respectively; $f_i(\bar{\boldsymbol{x}}_i)$ and $g_i(\bar{\boldsymbol{x}}_i) : R^i \to R$ are unknown continuous and piecewise continuous nonlinear functions. $u(t) \in R$ denotes the control input. Considering actuator faults, the control input $u(t)$ is modeled as $u(t) = \tau(t)v(t) + \theta(t)$, where $v(t)$ denotes the control signal to be designed; $\tau(t)$ and $\theta(t)$ stand for the partial loss of effectiveness and the float fault of the control signal, respectively. Define the reference signal to be tracked as $y_0(t)$, and $\bar{\boldsymbol{y}}_i(t) = [y_0(t), \dots, y_0(t)]^\top \in R^i$.

The control objectives are to design the control signal $v(t)$ such that: first, the output $y(t)$ tracks the reference signal $y_0(t)$ with prescribed performance; second, exact tracking is guaranteed within a finite time; third, all signals in the closed-loop system are uniformly bounded globally.

Main results. The design begins with the following exponential function:

$$\rho_i(t) = \left(\frac{\pi}{2} - \rho_{i\infty}\right) \mathrm{e}^{-\rho_{i1}t} + \rho_{i\infty},$$

where $\frac{\pi}{2} > \rho_{i\infty} > 0$ and $\rho_{i1} > 0$. Let the tracking error denote $e_1(t) = y(t) - y_0(t)$, and $e_i(t) = x_i(t) - \alpha_{i-1}(t)$, $i = 2, 3, \dots, n$. The virtual control signals $\beta_i(t)$, $i = 1, 2, \dots, n-1$, are passed through first-order filters: $\bar{d}_i\dot{\alpha}_i(t) + \alpha_i(t) = \beta_i(t)$, $\alpha_i(0) = \beta_i(0)$, where $\bar{d}_i > 0$, and $\alpha_i(t)$ is the output of the filter. The corresponding filtering error is defined as $z_i(t) = \beta_i(t) - \alpha_i(t)$. Define the transformed errors $\omega_i(t)$ and the integral sliding mode variables $s_i(t)$ as $\omega_i(t) = \tan\left(\frac{\pi}{2}k_i(t)\right)$, $k_i(t) = \arctan(e_i(t))/\rho_i(t)$, $s_i(t) = \omega_i(t) + \lambda_i\int_0^t \mathrm{sign}(\omega_i(\mu))\mathrm{d}\mu$, and low-pass filters as $\bar{\tau}_i\dot{\zeta}_i(t) + \zeta_i(t) = -\mathrm{sign}(s_i(t))$, where $\lambda_i, \bar{\tau}_i > 0$ are a positive constant and the filter constant, and $\zeta_i(0) = 0$, $i = 1, 2, \dots, n$. Then, the virtual signals and actual controller are designed as follows:

$$\begin{aligned}\beta_1 =& -c_1\mathrm{sign}(\xi_1),\\ \beta_i =& -\frac{c_i\sqrt{s_i + \varrho_i|s_i|}}{\xi_i} - \varrho_i'\xi_i\phi_i\left(\bar{L}_i^2 + \frac{z_{i-1}^2}{\bar{d}_{i-1}^2}\right)\end{aligned} \tag{2}$$

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$$- \frac{|\xi_{i-1}|\phi_{i-1}(z_{i-1}^2 + x_i^2)}{\xi_i \phi_i}, \ i = 2, 3, \dots, n-1, \tag{3}$$

$$v = -\frac{1}{\gamma_n}\left(\frac{c_n\sqrt{s_n + \varrho_n|s_n|}}{\xi_n} + \varrho_n'\xi_n\phi_n\left(\bar{L}_n^2 + \frac{z_{n-1}^2}{\bar{d}_{n-1}^2}\right) + \frac{|\xi_{n-1}|\phi_{n-1}(z_{n-1}^2 + x_n^2)}{\xi_n\phi_n} + \sum_{i=1}^{n}\frac{|\xi_i|\phi_i H_i}{\xi_n\phi_n}\right), \tag{4}$$

where $c_i > 0$ $(i = 1, 2, \dots, n)$, $\varrho_i > 1$, and $\varrho_i' > 0$ $(i = 2, 3, \dots, n)$ are scalars to be designed. Meanwhile, we define $\phi_i = \frac{\pi}{2\rho_i \cos^2(\frac{\pi}{2}k_i)}$, $\gamma_i = \cos^2(k_i\rho_i)$ and $\bar{L}_i = L_i(\bar{\boldsymbol{x}}_i, \bar{\boldsymbol{y}}_i, t)\|\bar{\boldsymbol{x}}_i - \bar{\boldsymbol{y}}_i\|$, where the function $L_i(\bar{\boldsymbol{x}}_i, \bar{\boldsymbol{y}}_i, t)$ is given as in Assumption 4 in Appendix A. Additionally, the variables ξ_i are defined as $\xi_1 = s_1$ and, for $i = 2, 3, \dots, n$, $\xi_i = 1 + \varrho_i \mathrm{sign}(s_i)$. Let $s(t) = \max\{|s_i(t)|, \dots, |s_n(t)|\}$. Then the feedback gain $H_i(t)$ is defined as

$$H_i(t) = h_i(t) + r_i(\|\omega\|_t + |Q_t|)\mathrm{e}^{-\bar{r}_i t}, \tag{5}$$

where $\|\omega\|_t = \max\{\sup_{0\leqslant\mu\leqslant t}|\omega_1(\mu)|, \dots, \sup_{0\leqslant\mu\leqslant t}|\omega_n(\mu)|\}$, $r_i, \bar{r}_i > 0$, $Q_t = \{q \in N^+ | s(t_q) = 0, \mathrm{sign}(s(t_q^-)) \neq 0, t_{q-1} < t_q \leqslant t, t_0 = 0\}$, and $|Q_t|$ is the cardinality of Q_t. Then the switching gain $h_i(t)$ is designed as follows.

- If $s(t) \neq 0$, then $h_i(t)$ evolves according to

$$\dot{h}_i(t) = \bar{h}_{i1}|s_i(t)| + \bar{h}_{i2}, \ h_i(0) > 0;$$

- If $s(t) = 0$, then $h_i(t)$ is switched to

$$h_i(t) = \bar{h}_{i3}|\zeta_i(t)| + \bar{h}_{i4},$$

where $\bar{h}_{ij} > 0$, $j = 1, 2, 4$, and $\bar{h}_{i3} = H_i(\underline{t}_i)$ with $\underline{t}_i$ being the latest instant up to t such that $\mathrm{sign}(s(\underline{t}_i^-)) \neq 0$ and $s(\underline{t}_i) = 0$.

Then the main results are presented in the following theorem.

Theorem 1. Consider system (1) with actuator faults and known control directions. If Assumptions 1–5 in Appendix A all hold, then virtual signals and the control signal, as shown in (2)–(4), can guarantee all three objectives.

Then, we consider the case where the directions of virtual control signals g_i, $i = 1, 2, \dots, n-1$ are unknown. Similarly, the virtual signals and actual controller are designed as

$$\beta_1 = -c_1\frac{\mathrm{sign}(\xi_1)}{b_1}, \tag{6}$$

$$\beta_i = -\frac{c_i\sqrt{s_i + \varrho_i|s_i|}}{b_i\xi_i} - \frac{\mathrm{sign}(\xi_i)}{b_i}\left(\bar{L}_i + \frac{|z_{i-1}|}{\bar{d}_{i-1}}\right) - \frac{|\xi_{i-1}|\phi_{i-1}(z_{i-1}^2 + x_i^2)}{b_i\xi_i\phi_i}, \ i = 2, 3, \dots, n-1, \tag{7}$$

$$v = -\frac{1}{\gamma_n}\left(\frac{c_n\sqrt{s_n + \varrho_n|s_n|}}{\xi_n} + \varrho_n'\left(\bar{L}_n^2 + \frac{z_{n-1}^2}{\bar{d}_{n-1}^2}\right) + \frac{|\xi_{n-1}|\phi_{n-1}(z_{n-1}^2 + x_n^2)}{\xi_n\phi_n} + \sum_{i=1}^{n}\frac{|\xi_i|\phi_i H_i}{\xi_n\phi_n}\right), \tag{8}$$

where $b_i \neq 0$ $(i = 1, 2, \dots, n-1)$. The other variables are set to the same. Then the main results are presented in the following theorem.

Theorem 2. Consider system (1) with actuator faults and unknown virtual control directions. If Assumptions 1–4 and 6 in Appendix A all hold, then virtual signals and the control signal, as shown in (6)–(8), can guarantee all three objectives.

The results of the stability analysis and the proofs of Theorems 1 and 2 are provided in detail in Appendix A. The simulations are performed as shown in Appendix B, demonstrating the effectiveness of the proposed method. Figure 1 shows the schematic diagram of the proposed control mechanism, and concludes Theorems 1 and 2.

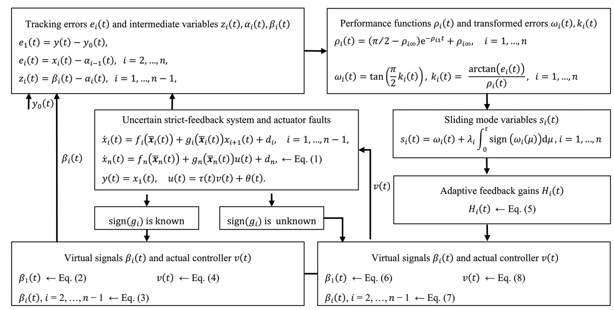

Figure 1 The control mechanism.

Remark 1. Note that the initial value of $\tan(\rho_i(t))$ tends to infinity, which results in a natural satisfaction of the initial PPC condition: $e_i(0) < \tan(\rho_i(0))$, thereby addressing the initial value constraints. The transformed errors ω_i and k_i convert the constrained problem into an unconstrained tracking problem. The sliding mode variable s_i handles unknown nonlinearities and bounded uncertainties, and guarantees finite-time stability of the tracking error. The low-pass filter ζ_i ensures boundedness of the switching gain h_i during sliding motion $(s(t) = 0)$. The parameter $b_i \neq 0$ substitutes the virtual control coefficients g_i, $i = 1, 2, \dots, n-1$.

Conclusion and future work. This study addresses the problem of global performance-guaranteed, finite-time tracking fault-tolerant control for uncertain strict-feedback nonlinear systems, utilizing integral sliding mode techniques. The feasibility and effectiveness of the proposed approach are validated in Appendixes A and B through theoretical analysis and numerical simulations. Future studies will focus on achieving predefined-time exact tracking with guaranteed performance for broader classes of nonlinear systems, including those affected by computational demands, sensor noise, and hardware constraints.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 92267101, 62573301, 62503335, 62473265).

Supporting information Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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