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Disturbance rejection trajectory tracking control of Mars coaxial-rotor UAVs via differential flatness

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The Mars coaxial-rotor unmanned aerial vehicles (UAVs) play a crucial role in planetary exploration. However, a coaxial-rotor UAV is an underactuated system with four control inputs but six degrees of freedom to regulate, which significantly affects the control performance and increases the complexity of trajectory tracking [1].

To address the underactuation problem, several fully actuated control methods have been proposed, thereby enhancing controllability [2, 3]. However, for Mars coaxial-rotor UAVs, the underactuation problem has not been addressed. In addition, the gust disturbances in the thin Martian atmosphere further increase the difficulty of achieving precise trajectory tracking control.

To address the impact of disturbances, the active disturbance rejection control (ADRC) [4] has attracted widespread attention. The core of ADRC is the extended state observer (ESO), which estimates the internal and external disturbances as an extended state, and compensates them in the control law to achieve active disturbance rejection. However, conventional ESOs typically achieve asymptotic convergence, which leads to slow response. To address this issue, some methods ensuring convergence within a bounded time are investigated [5]. Nevertheless, environmental disturbances and structural uncertainties can still degrade the dynamic response, posing a particularly severe challenge for Mars coaxial-rotor UAVs. Therefore, it is crucial to develop effective methods that mitigate coupling interference while ensuring both the disturbance rejection capability and the rapid response of the system.

Motivated by the above challenges, this study proposes a control framework for a Mars coaxial-rotor UAV. The main contributions are summarized as follows. (1) The dynamics of Mars coaxial-rotor UAV is transformed into a fully actuated system via the differential flatness approach, enabling simplified controller design and reducing coupling effects. (2) A finite-time disturbance rejection control framework is proposed for the fully actuated system to achieve stable flight, precise trajectory tracking and fast response.

Notation: $\mathbb{R}$ is the set of real numbers. $i_x, i_y, i_z \in \mathbb{R}^3$ are the standard basis vectors. $\|x\|$ is the the Euclidean norm. $\lceil x \rfloor^{\alpha} = |x|^{\alpha}\mathrm{sign}(x)$ for any $x \in \mathbb{R}$ with $\alpha \geqslant 0$.

Model description. The structure of the Mars coaxial-rotor UAV is considered a symmetrical rigid body, and the center of gravity on Mars is coincided with the origin of the body-fixed frame. Let $\xi = [x, y, z]^{\top} \in \mathbb{R}^3$ denote the position vector, and let $\eta = [\phi, \theta, \psi]^{\top} \in \mathbb{R}^3$ as the attitude angle vector contains three Euler angles, namely roll $\phi \in (-\pi/2, \pi/2)$, pitch $\theta \in (-\pi/2, \pi/2)$ and yaw $\psi \in (-\pi, \pi)$. The dynamics of the Mars coaxial-rotor UAV are described as follows:

$$\begin{cases} \ddot{x} = (\cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi)u_1 + k_x\dot{x} + d_x, \\ \ddot{y} = (\sin\psi \sin\theta \cos\phi - \cos\psi \sin\phi)u_1 + k_y\dot{y} + d_y, \\ \ddot{z} = -g_m + \cos\theta \cos\phi u_1 + k_z\dot{z} + d_z, \\ \ddot{\phi} = u_2 + k_{\phi}\dot{\phi}, \\ \ddot{\theta} = u_3 + k_{\theta}\dot{\theta}, \\ \ddot{\psi} = u_4 + k_{\psi}\dot{\psi}, \end{cases} \tag{1}$$

where g_m is the gravity on Mars, d_x, d_y and d_z are the unknown disturbances, $k_i, i = x, y, z, \phi, \theta, \psi$ are the positive coefficients, and $u_i, i = 1, 2, 3, 4$, are the control thrusts. More details are provided in Appendixes A and B.

Assumption 1. The additive disturbances d_x, d_y and d_z are continuously third-order differentiable, and their third-order derivatives are bounded.

Definition 1 ([3]). For system $\dot{x} = f(x, u)$, where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the states and inputs, respectively. It is called differentially flat system, if there exists a set of outputs $y = [y_1, ..., y_m]^{\top} \in \mathbb{R}^m$, such that both the system states x and inputs u can be expressed as algebraic functions of y and a finite number of its time derivatives, namely $x = \delta_x(y, \dot{y}, ..., y^{(k)})$ and $x = \delta_u(y, \dot{y}, ..., y^{(k+1)})$, where $\delta_x(\cdot)$ and $\delta_u(\cdot)$ are smooth functions, and k is a finite number.

Differential flatness dynamics. Considering the UAV system (1), let the virtual input $\tau_z = u_1 \cos\theta \cos\phi - g_m$. Then it can be obtained that $u_1 = \frac{\tau_z + g_{Mars}}{\cos\theta\cos\phi}$. Let $w_x = k_x\dot{x} + d_x$, $w_y = k_y\dot{y} + d_y$, and $w_z = k_z\dot{z} + d_z$ denote total disturbances, and let $\epsilon_1 = tan\theta$ and $\epsilon_2 = \frac{tan\phi}{cos\theta}$. As a fact that $\cos\theta\cos\phi \neq 0$, the position dynamics is described as

$$\begin{cases} \ddot{x} = (\tau_z + g_m)(\epsilon_1 \cos\psi + \epsilon_2 \sin\psi) + w_x, \\ \ddot{y} = (\tau_z + g_m)(\epsilon_1 \sin\psi - \epsilon_2 \cos\psi) + w_y, \\ \ddot{z} = \tau_z + w_z. \end{cases} \tag{2}$$

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Thus ϵ_1 and ϵ_2 can be obtained as

$$\begin{cases} \epsilon_1 = \dfrac{(\ddot{x}-w_x)\cos\psi + (\ddot{y}-w_y)\sin\psi}{\tau_z+g_m}, \\ \epsilon_2 = \dfrac{(\ddot{x}-w_x)\sin\psi - (\ddot{y}-w_y)\cos\psi}{\tau_z+g_m}. \end{cases} \tag{3}$$

During the stable flight, it can be reasonably considered that the yaw angle ψ is small. Thus $\ddot{\epsilon}_1$ and $\ddot{\epsilon}_2$ can be obtained as

$$\begin{cases} \ddot{\epsilon}_1 = \dfrac{x^{(4)}-h_x}{\tau_z+g_m} - \dfrac{(\ddot{x}-w_x)\ddot{\tau}_z}{(\tau_z+g_m)^2} - \dfrac{2\dot{\epsilon}_1\dot{\tau}_z}{\tau_z+g_m}, \\ \ddot{\epsilon}_2 = -\dfrac{y^{(4)}-h_y}{\tau_z+g_m} + \dfrac{(\ddot{y}-w_y)\ddot{\tau}_z}{(\tau_z+g_m)^2} - \dfrac{2\dot{\epsilon}_2\dot{\tau}_z}{\tau_z+g_m}, \end{cases} \tag{4}$$

where $h_x = \ddot{w}_x$ and $h_y = \ddot{w}_y$.

Furthermore, it follows from (1) that ϕ and θ can be obtained as $\theta = \arctan(\frac{(\ddot{x}-w_x)\cos\psi+(\ddot{y}-w_y)\sin\psi}{\ddot{z}+g_m})$, $\phi = \arcsin(\frac{(\ddot{x}-w_x)\sin\psi-(\ddot{y}-w_y)\cos\psi}{\sqrt{(\ddot{x}-w_x)^2+(\ddot{y}-w_y)^2+(\ddot{z}+g_m)^2}})$, respectively.

Let $\tau_1 = \ddot{\epsilon}_1$ and $\tau_2 = \ddot{\epsilon}_2$. Then it follows from (1), (2) and (4) that the flat UAV system is expressed as follows:

$$\begin{cases} x^{(4)} = (\tau_z+g_m)\tau_1 + 2\dot{\epsilon}_1\dot{\tau}_z + \dfrac{(\ddot{x}-w_x)\ddot{\tau}_z}{\tau_z+g_m} + w_x, \\ y^{(4)} = -(\tau_z+g_m)\tau_2 + 2\dot{\epsilon}_2\dot{\tau}_z - \dfrac{(\ddot{y}-w_y)\ddot{\tau}_z}{\tau_z+g_m} + w_y, \\ \ddot{z} = \tau_z + w_z, \\ \ddot{\psi} = u_4 + k_\psi\dot{\psi}. \end{cases} \tag{5}$$

Note that the system has four control inputs τ_1, τ_2, τ_z and u_4, and four outputs x, y, z and ψ. Based on Definition 1, system (5) is fully actuated. The properties analysis is provided in Appendix C.

Controller design. Considering the translational position dynamics, let $p_1 = [x,y]^\top$, $p_2 = [\dot{x},\dot{y}]^\top$, $p_3 = [\ddot{x},\ddot{y}]^\top$, $p_4 = [x^{(3)},y^{(3)}]^\top$. One can obtain

$$\begin{cases} \dot{p}_1 = p_2, \\ \dot{p}_2 = p_3, \\ \dot{p}_3 = p_4, \\ \dot{p}_4 = B_0\tau_p + d_p, \end{cases} \tag{6}$$

where $\tau_p = [\tau_1,\tau_2]^\top$,

$$B_0 = \begin{bmatrix} \tau_z+g_m & 0 \\ 0 & -(\tau_z+g_m) \end{bmatrix}, d_p = \begin{bmatrix} 2\dot{\epsilon}_1\dot{\tau}_z + \frac{(\ddot{x}-w_x)\ddot{\tau}_z}{\tau_z+g_m} + w_x \\ 2\dot{\epsilon}_2\dot{\tau}_z - \frac{(\ddot{y}-w_y)\ddot{\tau}_z}{\tau_z+g_m} + w_y \end{bmatrix},$$

and d_p is the total disturbances. It follows from Assumption 1 that d_p is differential and its derivative is bounded.

First, the finite-time ESO (FESO) is developed. Let $p_5 = d_p$ and $\dot{p}_5 = h_1(t)$. Then the FESO is given as

$$\begin{cases} \dot{\hat{p}}_1 = \hat{p}_2 + l_{11}\tilde{p}_1 + l_{12}\lceil\tilde{p}_1\rfloor^\alpha, \\ \dot{\hat{p}}_2 = \hat{p}_3 + l_{21}\tilde{p}_1 + l_{22}\lceil\tilde{p}_1\rfloor^\alpha, \\ \dot{\hat{p}}_3 = \hat{p}_4 + l_{31}\tilde{p}_1 + l_{32}\lceil\tilde{p}_1\rfloor^\alpha, \\ \dot{\hat{p}}_4 = \hat{p}_5 + l_{41}\tilde{p}_1 + l_{42}\lceil\tilde{p}_1\rfloor^\alpha + B_0\tau_p, \\ \dot{\hat{p}}_5 = l_{51}\tilde{p}_1 + l_{52}\lceil\tilde{p}_1\rfloor^\alpha, \end{cases} \tag{7}$$

where $\hat{p}_i, i = 1,\ldots,5$ are the estimates of p_i, $\tilde{p}_1 = p_1 - \hat{p}_1$, and l_{i1}, l_{i2} are positive parameters, $\alpha \in (0,1)$.

Second, the backstepping control law is developed. Let $p_r = [x_r, y_r]^\top$. The tracking error is defined as $w_1 = p_r - p_1$. Let $w_i = \alpha_{i-1} - p_i, i = 2,3,4$ denote the virtual errors, where α_i is the virtual control law.

Step 1: Choose $V_1 = \frac{1}{2}w_1^\top w_1$. One can obtain $\dot{V}_1 = w_1^\top \dot{p}_r + w_1^\top w_2 - w_1^\top \alpha_1$. Let $\alpha_1 = c_1 w_1 + \dot{p}_r$, where c_1 is positive constant. Then

$$\dot{V}_1 \leqslant -c_1\|w_1\|^2 + w_1^\top w_2. \tag{8}$$

Step 2: Choose $V_2 = \frac{1}{2}w_2^\top w_2$ and $\alpha_2 = w_1 + c_2 w_2 + \dot{\alpha}_1$, where c_2 is a positive constant. One can obtain

$$\dot{V}_2 \leqslant -c_2\|w_2\|^2 - w_2^\top w_1 + w_2^\top w_3. \tag{9}$$

Step 3: Choose $V_3 = \frac{1}{2}w_3^\top w_3$ and $\alpha_3 = w_2 + c_3 w_3 + \dot{\alpha}_2$, where $c_3 > 0$. One has

$$\dot{V}_3 \leqslant -c_3\|w_3\|^2 - w_3^\top w_2 + w_3^\top w_4. \tag{10}$$

Step 4: Choose $V_4 = \frac{1}{2}w_4^\top w_4$. One has

$$\dot{V}_4 \leqslant w_4^\top \left(\sum_i^4 k_i w_i + p_r^{(4)} - d_p - B_0\tau_p\right), \tag{11}$$

where $k_1 = -c_1^4 + 3c_1^2 + c_2^2 + 2c_1c_2 - 2$, $k_2 = c_1^3 + c_2^3 + c_1^2c_2 + c_1c_2^2 - 3c_1 - 4c_2 - c_3$, $k_3 = 3 - c_1^2 - c_2^2 - c_3^2 - c_1c_2 - c_2c_3 - c_1c_3$, and $k_4 = c_1 + c_2 + c_3$. Then the controller is designed as

$$\tau_p = B_0^{-1}\left(\sum_{i=1}^3 k_i w_i + (k_4+k_5)w_4 + k_s\lceil s_1\rfloor^\gamma - \hat{p}_5 + p_r^{(4)}\right), \tag{12}$$

where k_4, k_{s_1}, $\gamma \in (0,1)$ are positive constants. Let $s_1 = w_4$ denote the sliding mode. It follows from (11) and (12) that

$$\dot{V}_4 \leqslant -k_5\|w_4\|^2 - k_{s_1}\|s_1\|^{\gamma+1} - w_4^\top \tilde{p}_5. \tag{13}$$

Theorem 1. Considering the system (6) under the FESO (7) and the controller (12), if $\|h_1(t)\|$ is bounded, the system can be guaranteed uniformly ultimately bounded (UUB) in finite time.

The proof of Theorem 1 is provided in Appendix D. Furthermore, the designs of z-channel and ψ-channel controllers follow the same procedure as described above; the detailed analysis and discussions are provided in Appendixes E–G.

Conclusion. In this study, the differential flatness approach is exploited to transform the Mars coaxial-rotor UAV from an underactuated to a fully actuated system, enabling more effective controller design and reducing coupling effects. A finite-time disturbance-rejection control scheme is developed to handle Martian gust disturbances and dynamic uncertainties, ensuring robustness and fast response. The effectiveness of the proposed method is verified by the simulation results in Appendix H. The proposed control framework exhibits dependence of the convergence time on initial states and parameter settings, which introduces uncertainties. Future work will focus on developing prescribed-time control schemes to guarantee fixed convergence time.

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Supporting information Appendixes A–H. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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