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Adaptive event-triggered online learning for tracking control via Gaussian processes

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In recent years, event-triggered online learning (ETOL) control strategies have gained growing attention due to their ability to enhance data efficiency and reduce unnecessary computational costs in real-time control systems [1–5]. By dynamically adjusting the frequency of model updates, ETOL algorithms achieve a favorable balance between learning accuracy and computational complexity. In [1], an ETOL feedback linearizing control law based on GP models was introduced. Jiao et al. [2] developed a trajectory tracking control law for a class of partially unknown nonlinear systems, integrating backstepping with ETOL. In [3], a tunable GP-based event-triggered cascaded control framework was proposed for agile trajectory tracking of quadrotors. In [4], a GP-based distributed learning consensus controller with auxiliary dynamics was developed for multi-agent systems with unknown dynamics. Dai et al. [5] introduced an online cooperative learning strategy for GP-based multi-agent control. However, it is important to note that the ETOL mechanisms in [1–5] rely on the precise knowledge of the Lipschitz constants or the upper bound of the RKHS norm of the system dynamics. Unfortunately, obtaining accurate information about the Lipschitz constants or the upper bounds of the RKHS norm is often not feasible in many practical systems, which severely limits the applicability of these methods.

Motivated by the above observations, in this study, we propose an adaptive ETOL control method for uncertain nonlinear systems. The main contributions are summarized as follows. (1) Unlike existing ETOL control algorithms [1–5], which rely on accurate knowledge of the Lipschitz constants or the upper bound of the RKHS norm of the system dynamics, resulting in it being difficult to obtain in practical implementations. The proposed method eliminates this requirement by integrating BF, GPR, and an adaptive event-triggered mechanism (AETM). Specifically, a BF is introduced to constrain the system states, defining the operating region for GP learning. Within this region, an adaptive law is developed to estimate, in real-time, the parameter related to the Lipschitz constants of the system dynamics, and an AETM is introduced. The GP model updates its training data only when the triggering condition is met, ensuring high data efficiency and reducing computational complexity. (2) In contrast to ETOL control methods in [1–3], which assume that the unknown nonlinear dynamics is globally bounded, the proposed method in this study only requires the unknown nonlinear dynamics to be locally Lipschitz. This relaxation significantly broadens the applicability of the control method to a wider class of nonlinear systems.

Problem statement. Consider a class of uncertain nonlinear systems in the following form:

$$\begin{aligned} \dot{\xi}_i &= \xi_{i+1}, i = 1, ..., n-1, \\ \dot{\xi}_n &= \varphi_n(\boldsymbol{\xi})u + \psi_n(\boldsymbol{\xi}), \end{aligned} \tag{1}$$

where the state vector is defined as $\boldsymbol{\xi} = [\xi_1, \ldots, \xi_n]^{\mathrm{T}} \in \mathbb{X} \subseteq \mathbb{R}^n$ and $\mathbb{X}$ is a compact set. The control signal is represented by $u \subseteq \mathbb{R}$. The nonlinear function $\varphi_n(\boldsymbol{\xi}) : \mathbb{R}^n \to \mathbb{R}$ is known, whereas the function $\psi_n(\boldsymbol{\xi}) : \mathbb{R}^n \to \mathbb{R}$ is unknown. Both $\varphi_n(\boldsymbol{\xi})$ and $\psi_n(\boldsymbol{\xi})$ satisfy locally Lipschitz condition with respect to $\boldsymbol{\xi}$.

The *primary objective* is to construct a GP-based online learning state feedback control law u for any initial condition $\boldsymbol{\xi}(0) = [\xi_1(0), ..., \xi_n(0)]^{\mathrm{T}}$ such that the state tracking error $\boldsymbol{e}$ converges to a small neighborhood around the origin, while ensuring that all signals of the closed-loop system remain uniformly bounded. Here, $\boldsymbol{e} = \boldsymbol{\xi} - \bar{\boldsymbol{y}}_d$ with $\bar{\boldsymbol{y}}_d = [y_d, y_d^{(1)}, ..., y_d^{(n-1)}]^{\mathrm{T}}$ representing the desired tracking trajectory.

Assumption 1 ([1]). For any $\boldsymbol{\xi} \in \mathbb{X}$, it has $\varphi_n(\boldsymbol{\xi}) > 0$.

Assumption 2 ([1]). For any time instant t_κ, where $\kappa \in \mathbb{N}_0$, the state vector $\boldsymbol{\xi}^{(\kappa)} = \boldsymbol{\xi}(t_\kappa)$ and the measurement data of $\psi_n(\boldsymbol{\xi}^{(\kappa)})$, denoted as $\boldsymbol{y}^{(\kappa)} = \psi_n(\boldsymbol{\xi}^{(\kappa)}) + \epsilon^{(\kappa)}$, can be collected. Here, the measurement noise $\epsilon^{(\kappa)} \sim \mathcal{N}(0, \sigma_{on}^2)$ follows a Gaussian distribution, with $\sigma_{on} > 0$ representing the noise variance.

Assumption 3 ([4]). The continuous function $\psi_n(\boldsymbol{\xi})$ is a sample from a GP with a Lipschitz kernel $\kappa(|\xi - \xi'|) = \kappa(\xi, \xi')$ w.r.t. $|\xi - \xi'|$.

Assumption 4. The desired trajectory y_d and its n-th time derivatives are continuous and bounded.

Assumption 5. The measurement data $\dot{\xi}_n$ are available noise free. Thus, $\sigma_{on}^2 = 0$ in Assumption 2.

Lemma 1 ([1]). For the GPR model, selecting the squared exponential kernel as the kernel function, the unknown function $\psi_\kappa(\boldsymbol{\xi})$ and variance functions $\sigma_\kappa(\boldsymbol{\xi})$ are bounded and differentiable.

Lemma 2 ([1]). For Assumption 3 holding and $\delta \in (0, 1)$ being chosen, the following inequality holds with probability at least

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$1-\delta$:

$$|\psi_n(\boldsymbol{\xi}) - \hat{\psi}_\kappa(\boldsymbol{\xi})| \leqslant \beta_\kappa \sigma_\kappa(\boldsymbol{\xi}), \forall \boldsymbol{\xi} \in \mathbb{X}, \kappa \in \mathbb{N}_0, \tag{2}$$

where $\sigma_\kappa(\boldsymbol{\xi})$ represents the posterior standard deviation of the GP, and $\beta_\kappa = \sqrt{2\bar{\psi} + 300\gamma_\kappa \log(\frac{1+\kappa}{\delta})}$ with γ_κ being the maximum mutual information obtained from the training dataset of the GPR model and $\bar{\psi}$ being an unknown constant that satisfies $||\psi_n(\boldsymbol{\xi})||^2_{\mathcal{H}_k} \leqslant \bar{\psi}$.

Control law development. To start, we employ coordinate transformations as follows:

$$e_1 = \xi_1 - y_d, \tag{3}$$

$$e_i = \xi_i - y_d^{(i-1)}, i = 2, ..., n, \tag{4}$$

and the state filtering variable is defined as

$$r = e_n + \lambda_{n-1} e_{n-1} + \cdots + \lambda_1 e_1, \tag{5}$$

where λ_i, $i = 1, ..., n-1$ are the coefficients of the Hurwitz polynomial $H(p) = p^{n-1} + \lambda_{n-1} p^{n-2} + \cdots + \lambda_1$. The online learning controller u is designed as

$$u = \frac{1}{\varphi_n(\boldsymbol{\xi})}(-\hat{\psi}_\kappa(\boldsymbol{\xi}) - \eta_1 r - \rho) - \frac{\eta_2 r}{\zeta^2 - r^2}, \ t \in [t_\kappa \ t_{\kappa+1}), \tag{6}$$

where ζ and η_i $(i = 1, 2)$ are positive design constants, satisfying $\zeta > |r(0)|$. The GP mean functions $\hat{\psi}_\kappa(\boldsymbol{\xi})$ estimate unknown function $\psi_n(\boldsymbol{\xi})$ based on the time-varying dataset $\mathbb{D}_\kappa$ with $\kappa \in \mathbb{N}_0$.

According to Lemma 2, after each model update, the posterior variance function of the GP can reliably quantify the modeling error of the GPR model with high probability. To this end, an event-triggered mechanism, as proposed in [1], is designed as follows:

$$t_{\kappa+1} = \inf\Big\{t > t_\kappa | \beta_\kappa \sigma_\kappa(\boldsymbol{\xi}) \geqslant \eta_1 |r|\Big\}, \tag{7}$$

where the triggering time $t_{\kappa+1}$ is defined as the first time after t_κ when $\beta_\kappa \sigma_\kappa(\boldsymbol{\xi})$ becomes larger than or equal to $\eta_1|r|$. However, as shown in Lemma 2, β_κ depends on the unknown parameter $\bar{\psi}$, making it difficult to determine accurately. This uncertainty may significantly compromise the practical applicability of the resulting GP-based control algorithm. To address this issue, an adaptive estimator $\hat{\beta}_\kappa$ is introduced in this study, designed as follows:

$$\dot{\hat{\beta}}_\kappa = |r|\sigma_\kappa(\boldsymbol{\xi}) - \tau_\kappa \hat{\beta}_\kappa, \hat{\beta}_\kappa(0) = \hat{\beta}_{\kappa 0}, \tag{8}$$

where $\hat{\beta}_{\kappa 0} \in \mathbb{R}_{\geqslant 0}$ and τ_κ is a positive constant. The AETM without and with noise measurements are, respectively, designed as follows:

$$t_{\kappa+1} = \inf\Big\{t > t_\kappa | \hat{\beta}_\kappa \sigma_\kappa(\boldsymbol{\xi}) \geqslant \eta_1 |r|\Big\}, \tag{9}$$

$$t_{\kappa+1} = \inf\Big\{t > t_\kappa | \hat{\beta}_\kappa \sigma_\kappa(\boldsymbol{\xi}) \geqslant \eta_1 |r| \cap r \notin \mathbb{B}_{\sigma_{on}}\Big\}, \tag{10}$$

where

$$\mathbb{B}_{\sigma_{on}} = \Big\{r \in \Upsilon_1 | |r| \leqslant \frac{\hat{\beta}_\kappa \sigma_{on}}{\eta_1}\Big\}. \tag{11}$$

Remark 1. In contrast to the results in [1–5], this study estimates the unknown parameter β_κ by introducing an adaptive estimator $\hat{\beta}_\kappa$, as shown in (9). The updated value of $\hat{\beta}_\kappa$ is determined by the current system state r and $\sigma_\kappa(\boldsymbol{\xi})$, allowing for real-time adjustments that more directly reflect the system dynamics. This approach effectively controls the amount of training data, enhances data utilization efficiency, reduces computational burden, and improves the real-time performance of the system.

Theorem 1. Consider the uncertain nonlinear system (1) under Assumptions 1–5, and apply the online learning controller (6), where $\varphi_n(\boldsymbol{\xi})$ is modeled by a GP mean function $\hat{\psi}_\kappa(\boldsymbol{\xi})$, which is updated according to the AETM (9). Then, for any initial condition $\boldsymbol{\xi}(0)$, all signals of the closed-loop system remain uniformly bounded, and the interevent time $\Delta t_\kappa = t_{\kappa+1} - t_\kappa$ is lower bounded by a positive constant t_{lb}, for all $\kappa \in \mathbb{N}$ with probability $1-\delta$.

Theorem 2. Consider the uncertain nonlinear system (1) under Assumptions 1–4, and apply the online learning controller (6), where $\psi_n(\boldsymbol{\xi})$ is modeled by a GP mean function $\hat{\psi}_\kappa(\boldsymbol{\xi})$, which is updated according to the AETM (10). Then, for any initial condition $\boldsymbol{\xi}(0)$, all signals of the closed-loop system are uniformly bounded to the set $\mathbb{B}_{\sigma_{on}}$ in (11), and the interevent time $\triangle t'_\kappa$ is lower bounded by a positive constant t'_{lb}, for all $\kappa \in \mathbb{N}$, with probability $1-\delta$.

Conclusion. We proposed an adaptive ETOL control strategy for uncertain nonlinear systems. Specifically, we introduced a BF to constrain the system states, effectively defining the operational region for GP learning. Within this region, an adaptive law was developed to estimate parameters related to the Lipschitz constant of the system. The proposed AETM ensures that the GP model updates its training data only when the trigger condition is violated, thereby enhancing data efficiency and reducing computational complexity. Unlike existing event-triggered learning mechanisms, our approach does not rely on precise knowledge of the Lipschitz constants of the system dynamics. Using Lyapunov stability theory, we proved that the proposed control algorithm guarantees the boundedness of all signals in the closed-loop system. Simulation experiments confirmed the effectiveness and advantages of the proposed method, demonstrating its potential for real-time applications in GP-based online learning control.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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