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Hypersonic flight discrete-time optimal control with prescribed performance and saturation constraints

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In contrast to optimal control strategies designed for continuous-time systems, discrete-time optimal control has garnered considerable attention due to the widespread use of digital systems in modern control applications [1], particularly in hypersonic flight systems (HFS), which represent a class of long-range strategic aerospace vehicles. Although discrete-time optimal control methodologies have achieved notable progress, few of them guarantee system outputs with predefined transient and steady-state performance characteristics. Prescribed performance control (PPC) [2] has emerged as an effective mathematical framework for enforcing desired dynamic behaviors on system outputs. However, existing research on PPC predominantly focuses on continuous-time systems, leaving a significant gap in its application to discrete-time systems. Despite advances in continuous-time PPC [3], studies on PPC-integrated optimal control remain confined to continuous-time dynamics, with little effort devoted to extending these methods to discrete-time optimal control with prescribed performance guarantees. This raises a critical challenge: How can discrete-time optimal control strategies be developed to ensure prescribed performance? Given that most practical computer-controlled systems operate in discrete time, there is a pressing need to establish discrete-time optimal control frameworks capable of delivering guaranteed transient and steady-state performance, particularly for hypersonic flight systems that require both high precision and optimal operational qualities. Therefore, the primary objective of this study is to develop an ADP-based optimal control framework for discrete-time nonlinear systems with unknown dynamics, aiming to achieve predetermined output behaviors while ensuring prescribed performance.

Inspired by the aforementioned discussions (see Appendix A), the motivation for this study originates from the need for prescribed-time performance optimal control in discrete-time systems subject to input saturation constraints. The proposed design is realized through the integration of value iteration (VI) and adaptive dynamic programming (ADP) (see Appendix B). The methodology presented in this work constitutes a significant step toward establishing an effective framework for prescribed-time performance-driven optimal control under saturation constraints.

System plant. Without loss of generality, we first consider a class of discrete-time systems with control actuators subject to saturation constraints. For these systems, a novel prescribed-time performance optimal control approach will be subsequently proposed and applied to HFS.

$$x_{k+1} = f(x_k) + g(x_k)u_k, \tag{1}$$

where $x_k = [x_{1,k}, x_{2,k}, \ldots, x_{n,k}]^\top \in \Re^n$ is the system state, $u_k = [u_{1,k}, u_{2,k}, \ldots, u_{m,k}]^\top \in \Re^m$ is the control input with saturation constraint $-\bar{u} \leqslant u_k \leqslant \bar{u} = [\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_m]^\top \in \Re^m_{>0}$, $\bar{u}$ is the upper bound of u_k, $f(x_k): \Re^n \mapsto \Re^n$ and $g(x_k): \Re^n \mapsto \Re^{n\times m}$ are continuous functions of x_k satisfying $f(x_k)|_{x_k=0} = 0$, so that $x_k = 0$ is the equilibrium state of (1) under $u_k = 0$, the positive integers $m \in \mathcal{Z}_{>0}$ and $n \in \mathcal{Z}_{>0}$ denote the number of control input and system state, respectively, and $k = 0:1:\infty$ is the time index/step.

The control synthesis for (1) requires the priori knowledge that the system (1) can be stabilizable on a compact set $\Omega_x \subseteq \Re^n$ (at least one admissible control u_k exists that for all $x_k|_{k=0} \in \Omega_x \subseteq \Re^n$, the state $x_k|_{k\to\infty} \to 0$). Though existing studies are capable of optimally stabilizing the system (1), all of them fail to meanwhile achieve prescribed transient and steady-state qualities for x_k. In the following subsection, pioneering work will be presented to guarantee x_k with those predetermined transient and steady-state behaviors, by limiting x_k within a prescribed boundary in the discrete-time domain.

Fixed-time prescribed performance. For the sake of guaranteeing the system state x_k of (1) with a novel type of fixed-time prescribed performance (3), we firstly devise the following discrete-time performance function $\rho_{l,k} \in \Re_{>0}$:

$$\rho_{l,k} = \begin{cases} \rho_l^- \tanh\left(\pi - \dfrac{2\pi}{\mathcal{K}_l}k\right) + \rho_l^+, \ k \leqslant \mathcal{K}_l, \\ \rho_{l,\mathcal{K}}, k > \mathcal{K}_l, \end{cases} \tag{2}$$

where $l = 1:1:n$. The positive integer $\mathcal{K}_l \in \mathcal{Z}_{>0}$ means the required steps for $\rho_{l,k}$ to converge from its initial value $\rho_{l,0} = \rho_{l,k}|_{k=0} \in \Re_{>0}$ to its steady-state value $\rho_{l,\mathcal{K}} = \rho_{l,k}|_{k>\mathcal{K}_l} \in \Re_{>0}$, so that $\rho_{l,0}$ should be larger than $\rho_{l,\mathcal{K}}$, and ρ_l^- and ρ_l^+ are defined as $\rho_l^- = \frac{\rho_{l,0}}{2} - \frac{\rho_{l,\mathcal{K}}}{2}$ and $\rho_l^+ = \frac{\rho_{l,0}}{2} + \frac{\rho_{l,\mathcal{K}}}{2}$.

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To achieve a fixed-time prescribed performance, x_k should always evolve within the boundary

$$-\rho_{l,k} < x_{l,k} < \rho_{l,k},\ l = 1:1:n. \quad (3)$$

To limit x_k within the boundary (3), by the PPC theory [3], we should further define the transformed error $\alpha_{l,k} \in \Re$

$$\alpha_{l,k} = \frac{1}{2}\ln\left(\frac{\rho_{l,k} + x_{l,k}}{\rho_{l,k} - x_{l,k}}\right), l = 1:1:n. \quad (4)$$

We propose the inverse thinking approach, in which a novel asymptotic iteration for $\alpha_{l,k} \in \Re$ is first developed and subsequently employed to derive a new dynamic system that enables the design of discrete-time optimal PPC. The asymptotic iteration for $\alpha_{l,k} \in \Re$ is defined as follows:

$$\alpha_{l,k+1} = \frac{1}{2}\ln\left(\frac{\rho_{l,k+1} + x_{l,k+1}}{\rho_{l,k+1} - x_{l,k+1}}\right) := \eta_l \alpha_{l,k} \quad (5)$$

with $-1 < \eta_l < 1$ and $l = 1:1:n$.

It is observed from (5) that

$$x_{l,k+1} = \frac{\mathrm{e}^{2\eta_l \alpha_{l,k}} - 1}{1 + \mathrm{e}^{2\eta_l \alpha_{l,k}}}\rho_{l,k+1} := \mathcal{X}_{l,k+1}, l = 1:1:n. \quad (6)$$

In view of (1) and (6), we easily construct a new dynamic system

$$\mathcal{X}_{k+1} = f(\mathcal{X}_k) + g(\mathcal{X}_k)u_k \quad (7)$$

with $\mathcal{X}_k = [\mathcal{X}_{1,k}, \mathcal{X}_{2,k}, \ldots, \mathcal{X}_{n,k}]^\top \in \Re^n$, $\mathcal{X}_{l,k} = \frac{\mathrm{e}^{2\eta_l \alpha_{l,k-1}} - 1}{1+\mathrm{e}^{2\eta_l \alpha_{l,k-1}}}\rho_{l,k}$, and $l = 1:1:n$.

Control aim. The control synthesis aims to find a saturated optimal controller u_k^*, which is constrained by $u_k^* \in [-\bar{u}, \bar{u}]$, to maintain the system state x_k, starting from any initial point $x_k|_{k=0} \in \Omega_x \subseteq \Re^n$, at its equilibrium state $x_k = 0$, with the convergence trajectory of x_k satisfying desired prescribed performance (3), and also to minimize the following cost function:

$$\mathcal{J}(\mathcal{X}_k) = \sum_{j=k}^{\infty} (\mathcal{X}_j^\top \mathcal{Q}_{\mathcal{X}} \mathcal{X}_j + \mathcal{W}(u_j)) = r(\mathcal{X}_k, u_k) + \mathcal{J}(\mathcal{X}_{k+1}) \quad (8)$$

with $r(\mathcal{X}_k, u_k) = \mathcal{X}_k^\top \mathcal{Q}_{\mathcal{X}} \mathcal{X}_k + \mathcal{W}(u_k)$, $\mathcal{W}(u_j) = 2\int_0^{u_j} \mathrm{atanh}^\top\left(\frac{\tau}{\bar{u}}\right) \cdot \mathcal{R}_u \bar{u} d\tau$, and $\mathrm{atanh}(\tau/\bar{u}) = [\mathrm{atanh}(\tau/\bar{u}_1), \mathrm{atanh}(\tau/\bar{u}_2), \ldots, \mathrm{atanh}(\tau/\bar{u}_m)]^\top$, where τ is the integration variable, and $\mathcal{Q}_{\mathcal{X}} \in \Re^{n\times n}$ and $\mathcal{R}_u \in \Re^{m\times m}$ are positive definite symmetric matrices.

Control implementation. Inspired by [4], the optimal controller u_k^* and the optimal cost function $\mathcal{J}^*(\mathcal{X}_k)$ are approximately estimated as follows:

$$\hat{u}_k^* = \hat{W}_{u,k}^\top \phi_u(\mathcal{X}_k),\ \hat{\mathcal{J}}^*(\mathcal{X}_k) = \hat{W}_{\mathcal{J},k}^\top \phi_{\mathcal{J}}(\mathcal{X}_k), \quad (9)$$

where $\hat{u}_k^*$ and $\hat{\mathcal{J}}^*(\mathcal{X}_k)$ are the estimations of u_k^* and $\mathcal{J}^*(\mathcal{X}_k)$, respectively, with

$$\begin{cases} \hat{W}_{u,k} = [\hat{\omega}_{u1,k}, \hat{\omega}_{u2,k}, \hat{\omega}_{u3,k}, \hat{\omega}_{u4,k}, \hat{\omega}_{u5,k}]^\top \in \Re^5, \\ \hat{W}_{\mathcal{J},k} = [\hat{\omega}_{\mathcal{J}1,k}, \hat{\omega}_{\mathcal{J}2,k}, \hat{\omega}_{\mathcal{J}3,k}, \hat{\omega}_{\mathcal{J}4,k}, \hat{\omega}_{\mathcal{J}5,k}]^\top \in \Re^5, \\ \phi_u(\mathcal{X}_k) = \phi_{\mathcal{J}}(\mathcal{X}_k) = [\mathcal{X}_k, \mathcal{X}_k^2, \mathcal{X}_k^3, \mathcal{X}_k^4, \mathcal{X}_k^5]^\top. \end{cases}$$

Employing the gradient-based adaptation [5] to minimize $E_{u,k} = \frac{1}{2}e_{u,k}^2$ and $E_{\mathcal{J},k} = \frac{1}{2}e_{\mathcal{J},k}^2$, we develop the following adaptive laws for $\hat{W}_{u,k}$ and $\hat{W}_{\mathcal{J},k}$:

$$\begin{aligned} \hat{W}_{u,k+1} &= \hat{W}_{u,k} - \alpha_u \frac{\partial E_{u,k}}{\partial e_{u,k}} \frac{\partial e_{u,k}}{\partial \hat{W}_{u,k}} \\ &= \hat{W}_{u,k} - \alpha_u e_{u,k} \phi_u(\mathcal{X}_k), \end{aligned} \quad (10)$$

$$\begin{aligned} \hat{W}_{\mathcal{J},k+1} &= \hat{W}_{\mathcal{J},k} - \alpha_{\mathcal{J}} \frac{\partial E_{\mathcal{J},k}}{\partial e_{\mathcal{J},k}} \frac{\partial e_{\mathcal{J},k}}{\partial \hat{W}_{\mathcal{J},k}} \\ &= \hat{W}_{\mathcal{J},k} - \alpha_{\mathcal{J}} e_{\mathcal{J},k} \tilde{\phi}_{\mathcal{J}}(\mathcal{X}_k). \end{aligned} \quad (11)$$

Contributions. (1) This study introduces an asymptotic update mechanism to extend the discrete-time plant into an augmented dynamic system incorporating fixed-time prescribed performance characteristics. This formulation enables the development of an optimal control synthesis framework capable of achieving desired transient and steady-state performance in discrete time. (2) Existing discrete-time PPC methods are predominantly confined within the sliding mode design (SMD) framework, which restricts their compatibility with other control methodologies and limits opportunities for performance enhancement. In contrast, the proposed discrete-time optimal PPC approach overcomes the structural limitations inherent in SMD, thereby broadening its theoretical applicability and potential for integration with advanced control strategies. (3) A novel cost function is formulated to derive discrete-time optimal PPC protocols, specifically tailored for application to hypersonic flight systems. The proposed design explicitly incorporates saturation constraints, addressing a critical limitation observed in existing adaptive dynamic programming (ADP) approaches [1–3], where control signals may exceed actuator limits during computation, leading to implementation failures in practical scenarios.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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