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On distributed privacy-preserving k-WTA networks

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Based on the inputs of all individuals, a k-winner-take-all (k-WTA) network can generate k winners corresponding to individuals with the largest k inputs [1]. A k-WTA network makes a distinction by setting the outputs corresponding to winners and losers to two different states. Recently, inspired by the operation of k-WTA networks based on competitive behavior, k-WTA networks have been extended to multi-robot coordination. Specifically, the k-WTA network is able to activate suitable robots to perform tasks according to states of a multi-robot system, and such allocation is to complete a given task while reducing the computing and executing burdens of the system. Furthermore, such multi-robot coordination based on the k-WTA network also raises the demand for decentralization.

The consensus-based method is one of the important ways to build a distributed system. Its essence is that all individuals in a system take advantage of the local communication and the connectivity of the communication topology to reach a consensus so that the coordination of the whole system is able to be realized. In particular, some distributed k-WTA networks have already been constructed, using the method based on average consensus, where the consensus reached is the average value of the states of all individuals [2]. However, there are still two unresolved problems with them. On the one hand, for a multi-robot system, the information inside the k-WTA network changes over time as the task is executed, leading to the changes in winners generated by the k-WTA network. This is not considered in most k-WTA studies, which leads to the low performance of these k-WTA networks in handling lagging errors. On the other hand, in the distributed k-WTA networks constructed according to the methods based on average consensus, all individuals exchange information directly with each other, which may lead to the risk of privacy leakage when a multi-robot system faces an external privacy attacker. The existing privacy protection average consensus methods have various problems such as being unable to handle dynamic signals, unable to achieve accurate average consensus, and unable to effectively protect the outputs of the k-WTA network [3], which are explained and elaborated in greater detail in Appendix A. In view of the above discussions, in our study, the main contributions are summarized as follows. (i) A dynamic privacy-preserving consensus filter suitable for k-WTA networks is designed, which can protect the output of the k-WTA network while ensuring that the accuracy of the average consensus is not affected. (ii) A lagging-error-free and distributable k-WTA network is constructed by introducing a class of exponential-type penalty functions. And by combining with the designed dynamic privacy-preserving consensus filter, a distributed privacy-preserving k-WTA (DPP-kWTA) network is proposed, which demonstrates the potential for applications in the competitive coordination of multi-robot systems.

Theoretical results. From n inputs, the k-WTA operation is able to select the k largest ones. In particular, it has the following mathematical formulation:

$$\boldsymbol{h}_i=\zeta(\boldsymbol{w}_i)=\begin{cases}1, & \text{if } \boldsymbol{w}_i\in\mathbb{W},\\ 0, & \text{otherwise},\end{cases}\tag{1}$$

where $\zeta(\cdot)$ is a mapping function; $\boldsymbol{w}_i$ and $\boldsymbol{h}_i$ are the input and the corresponding output of the i-th individual, respectively; set $\mathbb{W}$ contains the largest k inputs among all inputs. Liu et al. [4] describe (1) as a quadratic programming problem with equality and inequality constraints

$$\begin{aligned}&\min_{\boldsymbol{h}(t)}\ \varpi\boldsymbol{h}^{\mathrm{T}}(t)\boldsymbol{h}(t)-\boldsymbol{h}^{\mathrm{T}}(t)\boldsymbol{w}(t)\\ &\text{s.t. } \mathbf{1}_n^{\mathrm{T}}\boldsymbol{h}(t)=k,\\ &\qquad 0\leqslant\boldsymbol{h}_i(t)\leqslant 1,\ \ i=1,2,\ldots,n,\end{aligned}\tag{2}$$

where ϖ is a positive constant; superscript T is the transpose symbol; $\boldsymbol{h}(t)=[\boldsymbol{h}_1(t);\boldsymbol{h}_2(t);\cdots;\boldsymbol{h}_n(t)]$; $\boldsymbol{w}(t)=[\boldsymbol{w}_1(t);\boldsymbol{w}_2(t);\cdots;\boldsymbol{w}_n(t)]$. To ensure that the problem has a unique stable solution, $\varpi<(\widetilde{\boldsymbol{w}}_k(t)-\widetilde{\boldsymbol{w}}_{k+1}(t))/2$ should be satisfied, where $\widetilde{\boldsymbol{w}}_k(t)$ and $\widetilde{\boldsymbol{w}}_{k+1}(t)$ are the k-th largest and the $(k+1)$-th largest inputs at time instant t, respectively.

In our work, to facilitate the construction of the k-WTA network, an exponential penalty function is introduced into (2) to handle the inequality constraints in the problem. Specifically, it can be rewritten as follows:

$$\begin{aligned}&\min_{\boldsymbol{h}(t)}\ \varpi\boldsymbol{h}^{\mathrm{T}}(t)\boldsymbol{h}(t)-\boldsymbol{h}^{\mathrm{T}}(t)\boldsymbol{w}(t)+\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{f}(t)\\ &\text{s.t. } \mathbf{1}_n^{\mathrm{T}}\boldsymbol{h}(t)=k,\end{aligned}\tag{3}$$

where $\boldsymbol{\beta}=[\boldsymbol{\beta}_1;\boldsymbol{\beta}_2;\cdots;\boldsymbol{\beta}_{2n}]$ is a constant vector. In particular, for the i-th element of $\boldsymbol{f}(t)\in\mathbb{R}^{2n}$, when $1\leqslant i\leqslant n$,

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$\boldsymbol{f}_i(t) = \mathrm{e}^{-\kappa(1-\boldsymbol{h_i}(t))}$ and when $n+1 \leqslant i \leqslant 2n$, $\boldsymbol{f}_i(t) = \mathrm{e}^{-\kappa \boldsymbol{h}_{i-n}(t)}$ with κ being a positive constant. Furthermore, using the Lagrange multiplier method, the Karush-Kuhn-Tucker conditions are able to be obtained as follows:

$$\begin{cases} 2\varpi \boldsymbol{h}(t) - \boldsymbol{w}(t) + \alpha(t)\mathbf{1}_n + \kappa P^{\mathrm{T}}(\boldsymbol{\beta} \circ \boldsymbol{f}(t)) = \mathbf{0}, \\ \mathbf{1}_n^{\mathrm{T}} \boldsymbol{h}(t) - k = 0, \end{cases} \tag{4}$$

where $\alpha(t)$ is a Lagrange multiplier; $\circ$ is the Hadamard product and $P = [S_n; -S_n]$ with $S_n \in \mathbb{R}^{n\times n}$ being an identity matrix. To obtain the optimal solution $\boldsymbol{h}^*(t)$ of the k-WTA problem, a vector-form error function $\boldsymbol{\gamma}(t)$ is constructed. Specifically, $\boldsymbol{\gamma}(t)$ and its derivative with respect to time can be expressed as

$$\boldsymbol{\gamma}(t) = Q\boldsymbol{b}(t) - \boldsymbol{c}(t),\ \dot{\boldsymbol{\gamma}}(t) = E(t)\dot{\boldsymbol{b}}(t) - \boldsymbol{d}(t), \tag{5}$$

where $Q = [2\varpi S_n\, \mathbf{1}_n; \mathbf{1}_n^{\mathrm{T}}\, 0]$; $\boldsymbol{b}(t) = [\boldsymbol{h}(t); \alpha(t)]$; $\boldsymbol{c}(t) = [\boldsymbol{w}(t) - \kappa P^{\mathrm{T}}(\boldsymbol{\beta} \circ \boldsymbol{f}(t)); k]$; $E(t) = [2\varpi S_n + \kappa^2 P^{\mathrm{T}} D(\boldsymbol{\beta} \circ \boldsymbol{f}(t)) P\, \mathbf{1}_n; \mathbf{1}_n^{\mathrm{T}}\, 0]$; $\dot{\boldsymbol{b}}(t) = [\dot{\boldsymbol{h}}(t); \dot{\alpha}(t)]$; $\boldsymbol{d}(t) = [\dot{\boldsymbol{w}}(t); 0]$. Then, based on a neural dynamics method: $\dot{\boldsymbol{\gamma}}(t) = -\theta\boldsymbol{\gamma}(t)$ with $\theta > 0$ being the convergence parameter, a k-WTA network is designed as

$$E(t)\dot{\boldsymbol{b}}(t) = -\theta(Q\boldsymbol{b}(t) - \boldsymbol{c}(t)) + \boldsymbol{d}(t). \tag{6}$$

In particular, note that Eq. (6) does not take into account the constraint of the communication topology, thereby not being able to be applied to a distributed communication network. Therefore, in order to construct a distributed k-WTA network, a consensus filter needs to be introduced.

For the i-th individual of an undirected connected network, the privacy-preserving information interaction strategy via random number switching encryption is designed as

$$\begin{aligned} &\dot{\boldsymbol{q}}_i(t) = \dot{\boldsymbol{h}}_i(t) - \iota \textstyle\sum_{j\in\mathbb{N}_i} (\boldsymbol{p}_i(t) - \boldsymbol{p}_j(t)), \\ &\boldsymbol{p}_i(t) = \boldsymbol{q}_i(t) - \textstyle\sum_{j\in\mathbb{N}_i} \boldsymbol{\delta}_j^i(\epsilon) + \textstyle\sum_{j\in\mathbb{N}_i} \boldsymbol{\delta}_i^j(\epsilon), \\ &\boldsymbol{q}_i(0) = \boldsymbol{h}_i(0),\ \ i = 1,\, 2,\, \ldots,\, n, \end{aligned} \tag{7}$$

where $\mathbb{N}_i$ represents the set of neighbors of the i-th individual; $\boldsymbol{\delta}_j^i$ and $\boldsymbol{\delta}_i^j$ are sequences of random numbers transmitted to the i-th individual by the j-th individual and to the j-th individual by the i-th individual, respectively, with the j-th individual being a neighbor of the i-th one. ϵ is the index to the sequences of random numbers. Note that the update of index ϵ depends on the time points in a random time series $\boldsymbol{\tau}$, which is initialized and sent to all individuals before the task is executed. Meanwhile, due to the symmetric nature of undirected graphs, the corollary $\mathbf{1}_n^{\mathrm{T}}\boldsymbol{q}(t) \equiv \mathbf{1}_n^{\mathrm{T}}\boldsymbol{p}(t)$ is able to be derived, which means that the privacy-preserving method does not affect the dynamic averaging of the information. Besides, this encryption strategy mainly focuses on the attack method where attackers infer the states of individuals through communication among individuals in the dynamic average consensus approach, which is difficult to compare with the privacy protection work in other fields [5].

Furthermore, combined (7) with (6), the DPP-kWTA network is able to be written as

$$E(t)\dot{\boldsymbol{b}}(t) = -\theta(W\boldsymbol{b}(t) - \boldsymbol{g}(t)) + \boldsymbol{d}(t), \tag{8}$$

where $\boldsymbol{g}(t) = [-\boldsymbol{w}(t) - \kappa P^{\mathrm{T}}(\boldsymbol{\beta} \circ \boldsymbol{f}(t));\ k - \sum_{i=1}^{n} \boldsymbol{p}_i(t)]$ and $W = [\varpi S_n,\ \mathbf{1}_n; \mathbf{0}_n,\ 0]$ with $\mathbf{0}_n \in \mathbb{R}^n$ consisting of zeros. Besides, the following three theorems are provided to prove the convergence and privacy protection capabilities of the DPP-kWTA network (see Appendix B.1 for details).

Theorem 1. For the proposed DPP-kWTA network (8), variable $\boldsymbol{b}(t)$, that is, $[\boldsymbol{h}(t); \alpha(t)]$, globally converges to the theoretical solution. In other words, the output $\boldsymbol{h}(t)$ of the proposed DPP-kWTA network (8) converges to the optimal solution of the k-WTA problem (3).

Theorem 2. The proposed DPP-kWTA network (8) enables $\boldsymbol{b}(t)$ to converge to the theoretical solution $\boldsymbol{b}^*(t)$ in an exponential form with rate θ.

Theorem 3. In an undirected communication topology, the key information $\boldsymbol{h}_i(t)$ of the DPP-kWTA network (8) cannot be obtained by an external eavesdropper. Specifically, the privacy attack model is only able to obtain the encrypted information from the privacy-preserving consensus filter (7).

Simulation and experiment. To demonstrate the superiority and feasibility of the DPP-kWTA network, numerical simulations based on the Matlab platform, multi-robot target capture task simulations, and experiments based on the multiple E-puck2 robot experimental platform are respectively carried out (see Appendix C). In numerical simulations, a communication topology consisting of four individuals ($n = 4$) is considered (see Figure A1(a)). Four sinusoidal signals with different phases serve as their inputs. The two individuals ($k = 2$) with the maximum inputs at the current moment will be selected as the winners, and their corresponding output states will be 1. As shown in Figure C1, the DPP-kWTA network can quickly complete the task and successfully protect the network's outputs when facing the privacy attack model. Meanwhile, compared with an existing work, the DPP-kWTA network has obvious advantages in lagging errors and privacy protection. In the simulation of the multi-robot target capture task, a communication topology consisting of eight robots ($n = 8$) is considered (see Figure A1(b)). The DPP-kWTA network can successfully select the three robots ($k = 3$) closest to the target to pursue the target robot in Figure C2. Moreover, as shown in Figure C4, a similar task is executed on the multiple E-puck2 robot experimental platform.

Conclusion. In this study, a distributed privacy-preserving k-WTA network named DPP-kWTA network has been constructed. Specifically, a dynamic privacy-preserving consensus filter suitable for k-WTA networks has been designed and used to deal with the risk of privacy leakage caused by direct information interaction without affecting the consensus among individuals in the network, which has been verified by theoretical analyses. Moreover, the proposed DPP-kWTA network has theoretically eliminated the lagging errors, which means that it has a fast reaction to dynamic information. Besides, we elaborated on the possible research directions for the future in Appendix D.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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