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p-moment stabilization of inhomogeneous Markov jump linear systems with time-varying transition probabilities

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Abstract This paper investigates the p-moment stability and stabilization problems of an inhomogeneous Markov jump linear system (MJLS) with time-varying and unknown transition probabilities. First, several technical lemmas are established to solve the p-moment stability/stabilization of MJLS. Then, p-moment stability is explored for even integer p and any real number $p > 0$, and mode-dependent state-feedback control laws are designed to ensure the p-moment stability. Finally, two illustrative examples have been provided to verify the analytical results obtained.

Keywords inhomogeneous Markov jump, linear system, moment stability, stabilization, state feedback control

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1 Introduction

Systems subject to stochastic switching are widely used to model phenomena involving abrupt changes, such as component failures, sudden parameter variations, or structural reconfigurations. Among these, Markov jump linear systems (MJLSs) form an important class, characterized by multiple linear subsystems (referred to as modes) and a switching signal governed by a Markov chain. This switching signal is defined by a transition rate matrix, which encodes information about both the stochastic residence time in each mode and the transition probabilities between modes. When the transition matrix is fully known, MJLSs have been extensively studied, and fundamental properties such as moment stability are well understood. For comprehensive treatments, see [1–8] and references therein.

In practical engineering applications, however, obtaining a complete and accurate transition matrix is often difficult due to the high cost or infeasibility of data acquisition. For instance, in high-reliability systems such as aircraft engines, certain severe failure modes occur with such low probability that it is nearly impossible to collect enough data to estimate their transition rates accurately. Similarly, cascading failures in power grids are rare events, resulting in sparse data for modeling. As a result, research attention has shifted from fully known transition matrices to partially unknown cases (see [9–13]). Yet, many studies still assume that the unknown transition probabilities remain constant, an assumption that may not hold in practice. For example, the transition probabilities among flight modes (e.g., “cruise”, “hover”, and “return”) for an environmental monitoring drone can vary over time as battery power depletes. This has motivated increasing interest in the stability and performance of MJLSs with time-varying transition probabilities (see [14, 15]).

This paper focuses on the limiting case where no information is available about the transition probabilities between modes. Instead, only the mode-dependent distribution of the sojourn time until the next switch is known, while the transition matrix itself varies over time. Such systems, composed of linear subsystems with Markov jumps and time-varying transition probabilities, have also been referred to as Poisson jump linear systems (PJLSs) in several studies. Preliminary stability analyses of PJLSs can be found in [16–18]. The admissibility of stochastic singular PJLSs was addressed in [19], and a subclass of PJLSs with positive dynamics was studied in [20].

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Another central challenge in Markov switching systems is the design of easily implementable stabilization controllers under highly uncertain switching rules. For example, Fragoso and Costa [9] successfully designed a second-moment stabilizing controller for MJLSs with partially unknown transition parameters using a linear matrix inequality (LMI) approach. However, their work did not address the stabilization of higher-order moments. Fang and Loparo [21] reduced the problem of p-moment stabilization to an optimal control framework, but solving such problems remains challenging due to their inherent complexity. To the best of our knowledge, no existing results formulate state-feedback controllers for high-order moment stabilization in an LMI framework, even for Markov chains with fully known and constant transition probabilities. This gap motivates the present work.

In this paper, we aim to establish computable criteria for moment stability of MJLSs and to develop a state-feedback control strategy for moment stabilization. As observed in [16], second-moment stability of MJLSs can be shown equivalent to the stability of a higher-dimensional deterministic switched system under arbitrary switching. Here, we extend this equivalence from second-moment to p-moment stability, and our main contributions are as follows: (1) inspired by the studies of Costa [2] and Molchanov [22], we derive sufficient conditions for p-moment stability of MJLSs using Kronecker products for even integers p; (2) by introducing a new technical lemma, we translate the proposed p-moment stability criteria into tractable LMI conditions for state-feedback control design, thereby enabling the stabilization of higher-order moments; (3) in addition, we extend the discussion of p-moment stability and stabilization to arbitrary real values $p > 0$.

Notations. Throughout this paper, $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant 0}, \mathbb{P})$ is a complete probability space with a right continuous family $\{\mathcal{F}_t\}_{t\geqslant 0}$ of sub σ-fields of $\mathcal{F}$ each containing all $\mathbb{P}$-null sets. $\mathbf{E}(\cdot)$ represents expectation. The indicator function of an event $\mathcal{A}$ is represented by $\mathbf{1}_{\mathcal{A}}$. The sets of natural numbers and positive integers are denoted by $\mathbb{N}$, $\mathbb{N}^+$, respectively. The symbol $|\cdot|$ refers to the Euclidean norm in $\mathbf{R}^n$ and the trace norm in $\mathbf{R}^{n\times m}$. $\mathbf{R}_+^{n\times m}$ is the space of real $n\times m$-matrices with all entries non-negative. Given a collection of matrices $\{M_i\}$, the block diagonal matrix formed by arranging M_i along the diagonal is expressed as $\mathrm{diag}\{M_i\}$. For any matrix A, $\mathrm{vec}\{A\}$ refers to the column vector obtained by stacking all columns of M in sequence. Similarly, given a set of vectors $\{v_i\}$, the column vector formed by arranging them sequentially is written as $\mathrm{col}\{v_i\}$. The Kronecker product is denoted by $\otimes$, whereas the Kronecker sum is indicated by $\oplus$. For any $n \in \mathbb{N}^+$, the notation $A^{[n]}$ is recursively defined as $A^{[n]} = A^{[n-1]} \otimes A$, with $A^{[0]} = 1$. The symbol $e_{k,n}$ stands for the k-th column of the n-dimensional identity matrix I_n. Let $C([0,+\infty), \mathbf{R}^{n\times n})$ be the space of all continuous $\mathbf{R}^{n\times n}$-valued differentiable functions on $[0,+\infty)$.

2 Problem formulation

Consider the following inhomogeneous MJLS:

$$\dot{x}(t) = A_{\theta(t)}x(t) + B_{\theta(t)}u(t), \tag{1}$$

where x represents the n-dimensional state vector, u denotes the m-dimensional control input, and $\theta(t)$ is a right-continuous inhomogeneous Markov switching signal that takes values from the finite set $\Delta = \{1, 2, \ldots, M\}$. $A_i \in \mathbf{R}^{n\times n}$, $B_i \in \mathbf{R}^{n\times m}$, $i \in \Delta$, all are given. When $\theta(t) = i$, we refer to the system as being in mode i at time t. Each mode corresponds to a linear time-invariant subsystem characterized by the matrices (A_i, B_i).

In this paper, we let a sequence of switching time instants, denoted as $t_k, k \in \mathbb{N}$ and $t_0 = 0$. The duration between two successive switching is given by $T_k = t_{k+1} - t_k$, with expected value $\mathbf{E}[T_k] = \pi_{\theta(t_k)}^{-1}$. Consequently, the Markov jump process $\theta(t)$ subject to a transition rate matrix $\Pi(t)$ that contains uncertainties:

$$\Pi(t) = \begin{bmatrix} -\pi_1 & \pi_{12}(t) & \ldots & \pi_{1M}(t) \\ \pi_{21}(t) & -\pi_2 & \ldots & \pi_{2M}(t) \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{M1}(t) & \pi_{M2}(t) & \ldots & -\pi_M \end{bmatrix}, \tag{2}$$

where all off-diagonal elements $\pi_{ij}(t), i \neq j$, remain unknown, and $\sum_{j=1,j\neq i}^{M} \pi_{ij}(t) = \pi_i$. Unlike the conventional MJLS framework, where both the transition probabilities and dwell times are typically specified, this formulation considers a scenario in which only the dwell-time between successive jumps is explicitly modeled. Consequently, the sequence of modes remains entirely indeterminate, as no transition probability information is provided.

For $M > 2$, the summation constraint $\sum_{j=1,j\neq i}^{M} \pi_{ij}(t) = \pi_i$ ensures that the uncertain matrix $\Pi(t)$ at any given t belongs to $\mathcal{H}$, which represents the convex hull spanned by the set of vertix matrices $\bar{\Pi}_\tau$, $\tau \in \Upsilon =$

$\{1, 2, \ldots, (M-1)^M\}$ [16]. More precisely, each vertex matrix $\bar{\Pi}_\tau$ is structured such that the i-th row follows the expression

$$\pi_i(e^\top_{j,(M-1)^M} - e^\top_{i,(M-1)^M}), \quad j \neq i.$$

The vertex matrices are then determined by systematically considering all possible row combinations. Namely, the transition rate matrix satisfies the convex decomposition:

$$\Pi(t) = \sum_{\tau=1}^{(M-1)^M} a_\tau(t)\bar{\Pi}_\tau, \tag{3}$$

where $a_\tau(t) \geqslant 0$, $\sum_{\tau=1}^{(M-1)^M} a_\tau(t) = 1$.

Remark 1. Consider the case with $M = 3$, then vertix matrices $\bar{\Pi}_\tau$, $\tau \in \Upsilon = \{1, 2, \ldots, 8\}$ are defined as follows:

$$\bar{\Pi}_1 = \begin{bmatrix} -\pi_1 & \pi_1 & 0 \\ \pi_2 & -\pi_2 & 0 \\ \pi_3 & 0 & -\pi_3 \end{bmatrix}, \quad \bar{\Pi}_2 = \begin{bmatrix} -\pi_1 & \pi_1 & 0 \\ \pi_2 & -\pi_2 & 0 \\ 0 & \pi_3 & -\pi_3 \end{bmatrix}, \quad \bar{\Pi}_3 = \begin{bmatrix} -\pi_1 & \pi_1 & 0 \\ 0 & -\pi_2 & \pi_2 \\ \pi_3 & 0 & -\pi_3 \end{bmatrix}, \quad \bar{\Pi}_4 = \begin{bmatrix} -\pi_1 & 0 & \pi_1 \\ \pi_2 & -\pi_2 & 0 \\ 0 & \pi_3 & -\pi_3 \end{bmatrix},$$

$$\bar{\Pi}_5 = \begin{bmatrix} -\pi_1 & 0 & \pi_1 \\ \pi_2 & -\pi_2 & 0 \\ \pi_3 & 0 & -\pi_3 \end{bmatrix}, \quad \bar{\Pi}_6 = \begin{bmatrix} -\pi_1 & 0 & \pi_1 \\ \pi_2 & -\pi_2 & 0 \\ 0 & \pi_3 & -\pi_3 \end{bmatrix}, \quad \bar{\Pi}_7 = \begin{bmatrix} -\pi_1 & 0 & \pi_1 \\ 0 & -\pi_2 & \pi_2 \\ \pi_3 & 0 & -\pi_3 \end{bmatrix}, \quad \bar{\Pi}_8 = \begin{bmatrix} -\pi_1 & 0 & \pi_1 \\ 0 & -\pi_2 & \pi_2 \\ 0 & \pi_3 & -\pi_3 \end{bmatrix}.$$

For any matrix

$$\Pi = \begin{bmatrix} -\pi_1 & \pi_{12} & \pi_{13} \\ \pi_{21} & -\pi_2 & \pi_{23} \\ \pi_{31} & \pi_{32} & -\pi_3 \end{bmatrix} \in \mathcal{H} = \mathrm{conv}\{\bar{\Pi}_1, \bar{\Pi}_2, \ldots, \bar{\Pi}_8\},$$

we can choose the coefficients as $a_1 = \frac{\pi_{31}}{\pi_3}$, $a_2 = a_5 = a_6 = a_7 = 0$, $a_3 = \frac{\pi_{32}}{\pi_3} - \frac{\pi_{23}}{\pi_2}$, $a_4 = \frac{\pi_{23}}{\pi_2} - \frac{\pi_{13}}{\pi_1}$ such that $\Pi = \sum_{\tau=1}^{8} a_\tau \bar{\Pi}_\tau$, and $\sum_{\tau=1}^{8} a_\tau = 1$.

Remark 2. $\theta(t)$ is an inhomogeneous Markov process with completely unknown time-varying transition probability for each pair of modes, leading to matrix $\Pi(t)$, whose off-diagonal terms are completely unknown, with the only constraint that the sum of the off-diagonal terms along row i equals π_i. Note that the formulation $\Pi(t)$ is not exactly equivalent to what is considered in [10], where the off-diagonal terms are assumed unknown but constant. As is recognized in the literature on asymptotic stability, the difference between time-varying and constant parameters is significant [23, 24]. Hence, the issue tackled in this paper differs from the one addressed in [10].

Definition 1. The system (1) with $u(t) = 0$ is said to be p-moment stable if, for any initial state $x(0)$ and any initial mode $\theta(0)$, $\lim_{t\to\infty} \mathbf{E}|x(t)|^p = 0$.

The p-moment stability of MJLSs is a crucial concept, as it also implies almost sure stability, i.e., almost all state trajectories tend to zero asymptotically.

3 Some technical lemmas

This paper seeks to work out the problem of p-moment stability and stabilization for MJLS (1). For this, we give several technical lemmas. First, a similar proof (based on Theorem 1 of [7]) applies for MJLS (1), we immediately have the following result.

Lemma 1. Let even $p \in \mathbb{N}^+$, then MJLS (1) with $u(t) = 0$ is p-moment stable if and only if the following linear time-varying system

$$\dot{y}(t) = (\mathrm{diag}\{M_{j,p}\} + \Pi(t)^\top \otimes I_{n^p})y(t) \tag{4}$$

is asymptotic stable, where $M_{j,p} = \sum_{l=1}^{p} I_{n^{l-1}} \otimes A_j \otimes I_{n^{p-l}}$.

The detailed proof of this lemma can be found in Appendix A.

Remark 3. Unlike the conclusion in Theorem 1 of [7], due to the transition probability matrix $\Pi(t)$ of Markov signal $\theta(t)$ is time-varying, the stability of system (4) cannot be directly determined by the Hurwitz stability of the coefficient matrix $\mathrm{diag}\{M_{j,p}\} + \Pi(t)^\top \otimes I_{n^p}$.

In addition, in Lemma 1, p is assumed to be even because if p were odd, Eq. (A2) may not hold (see Example 1). However, if system (1) is positive (i.e., matrices A_i are Mezler, and the initial value $x_0 \in \mathbf{R}^n_+$), then in Lemma 1, p can be any positive integer. Specifically, in Theorem 1 of [25], since the studied system is not positive, the domain of the parameter p should be adjusted from the set of positive integers to only include strictly positive even numbers.

Example 1. Let X be a scalar value random variable, and the probability distribution function of X be

$$P(X=1)=0.5,\ P(X=-1)=0.5.$$

For any $t \geqslant 0$, let $x(t)=X$. Then $\lim_{t\to+\infty}\mathbf{E}|x(t)|^{2k+1}=1$, and $\lim_{t\to+\infty} y(t)=\mathbf{E}X^{[2k+1]}=0$, where $k\in\mathbb{N}$. This implies, for odd $p\in\mathbb{N}^+$,

$$\lim_{t\to+\infty}\mathbf{E}|x(t)|^p=0 \not\Leftrightarrow \lim_{t\to+\infty} y(t)=0.$$

Substituting Eq. (3) into system (4), the following Lemma 2 can be directly derived from Lemma 1.

Lemma 2. Let even $p\in\mathbb{N}^+$, then the linear time-varying system (4) is asymptotic stable if and only if the following deterministic polytopic system:

$$\dot{y}(t)=\left(\sum_{\tau=1}^{(M-1)^M} a_\tau(t)\Phi_{\tau,p}\right)y(t) \tag{5}$$

is asymptotic stable, where $\Phi_{\tau,p}=\operatorname{diag}\{M_{i,p}\}+\bar{\Pi}_\tau^\top\otimes I_{n^p}$.

Remark 4. Further, for any $t,t_0\in[0,+\infty)$, $t\geqslant t_0$, denote the state transition matrix of system (5) (or (4)) by $\Theta(t,t_0)$. Assume that for all $\tau\in\Upsilon$, $a_\tau(t)$ are piecewise continuous functions defined on $[0,+\infty)$, then from Lemma 1 and Theorem 1 of Zhou [23], we have that the asymptotic stability of the deterministic polytopic system (5) is equivalent to either Proposition (i) or Proposition (ii), where Proposition (i): for any $t,t_0\in[0,+\infty)$, $t\geqslant t_0$, there exists a positive constant $M(t_0)$ such that the state transition matrix $|\Theta(t,t_0)|<M(t_0)$, and $\lim_{t\to+\infty}|\Theta(t,t_0)|=0$; Proposition (ii): there exist an asymptotic stable function $\upsilon(t)$ [23], a matrix $Q(t)=Q^\top(t)\in C([0,+\infty),\mathbf{R}^{Nn^p\times Nn^p})$, and a constant $b>0$ such that for any $t\geqslant 0$, the following conditions hold: $Q(t)>bI_{Nn^p}$, and

$$\dot{Q}(t)+\sum_{\tau=1}^{(M-1)^M} a_\tau(t)\Phi_{\tau,p}^\top Q(t)+\sum_{\tau=1}^{(M-1)^M} a_\tau(t)Q(t)\Phi_{\tau,p}\leqslant 2\upsilon(t)Q(t).$$

Based on a well-established stability result for polytopic systems [22], we immediately have the following equivalent proposition about the stability of a deterministic polytopic system (5).

Lemma 3. Let even $p\in\mathbb{N}^+$, then system (5) is asymptotic stable if and only if the deterministic system under arbitrary switching

$$\dot{y}(t)=\Phi_{\vartheta(t),p}y(t) \tag{6}$$

is asymptotic stable, where $\vartheta(t)$ is a switching signal that take values in set Υ.

Lemma 4. For any $p,m\in\mathbb{N}$ with $m\leqslant p$, $\varpi\in\mathbf{R}^n$, then

$$(I_{n^m}\otimes U_{n,n^{p-m-1}})\varpi^{[p]}=\varpi^{[p]},$$

where $U_{n,n^{p-m-1}}=\sum_{k=1}^n\sum_{l=1}^{n^{p-m-1}}(e_{k,n}e_{l,n^{p-l}}^\top)\otimes(e_{l,n^{p-l}}e_{k,n}^\top)$.

The detailed proof of this lemma can be found in Appendix B.

4 Main results

4.1 *p*-moment stability

Based on Lemmas 1–3, we shall give p-moment stability result of system (1) in terms of differential Lyapunov inequalities.

Theorem 1. Let even $p\in\mathbb{N}^+$, and there exist positive definite matrix $Q\in\mathbf{R}^{Nn^2\times Nn^2}$ satisfying, for all $\tau\in\Upsilon$,

$$\Phi_{\tau,p}Q+Q\Phi_{\tau,p}^\top<0, \tag{7}$$

then system (1) with $u(t)=0$ is p-moment stable.

Proof. For system (6), we take a common quadratic Lyapunov function $V(y) = y^{\top}Q^{-1}y$. Then, according to [26], system (6) is stable under any arbitrary switching. Hence, by Lemmas 1–3, the result of Theorem 1 can be derived.

Remark 5. Clearly, if system (6) is stable under arbitrary switching, then the vertex matrices $\Phi_{\tau,p}$ must be Hurwitz stable. That is, a prerequisite for p-moment stability of system (5) is that all matrices $\Phi_{\tau,p}$, $\tau \in \Upsilon$, defined in (6) are Hurwitz.

Remark 6. The above Theorem 1 gives the sufficient conditions for p-moment stability of system (1) in terms of LMIs. In fact, when uncertain transition rate matrix $\Pi(t)$ reduces to constant matrix, and $p = 2$, the conditions (7) are the sufficient and necessary criteria for the mean-square stability of MJLS (1) (see Theorem 3.3 of [21]).

Next, let p be any given positive constant, and by using the Lyapunov function method, obtain the stability theorem for p-moment.

Theorem 2. Let $p > 0$, the system (1) with $u(t) = 0$ is p-moment stable if there exist constants $a_{ij} > 0$, $i, j \in \Delta, j \neq i$, and positive definite matrices $P_i \in \mathbf{R}^{n\times n}$, $i \in \Delta$, such that $\forall i, j \in \Delta, j \neq i$,

$$\frac{p}{2}(A_i^{\top}P_i + P_iA_i) + \pi_i(a_{ij}^{\frac{p}{2}} - 1)P_i < 0, \tag{8}$$

$$P_j - a_{ij}P_i < 0. \tag{9}$$

Proof. When initial state $x(0) = 0$, it is evident that $x(t) \equiv 0$. Therefore, system (1) achieves p-moment stability under the condition $x(0) = 0$. For any $x(0) \neq 0$, we introduce stochastic Lyapunov function $V(x, \theta) = (x^{\top}P_{\theta}x)^{\frac{p}{2}}$. According to Lemma 2.1 in [27], we know that for any initial condition $x(0) \neq 0$, $x(t)$ will never reach zero with probability one. Consequently, based on condition (8), we can compute the infinitesimal generator $\mathcal{L}V(x, i)$ at time t with the position $\theta(t) = j$, $x(t) = x$ satisfies the following:

$$\begin{aligned}
&\mathcal{L}V(x, j)\\
&= (x^{\top}P_jx)^{\frac{p}{2}-1}x^{\top}\left[\frac{p}{2}(A_j^{\top}P_j + P_jA_j) - \pi_jP_j\right]x + \sum_{i=1,i\neq j}^{M}\pi_{ji}(t)(x^{\top}P_ix)^{\frac{p}{2}}\\
&= \sum_{i=1,i\neq j}^{M}\frac{\pi_{ji}(t)}{\pi_j}\left\{x^{\top}\left[\frac{p}{2}(A_j^{\top}P_j + P_jA_j) - \pi_jP_j\right]x(x^{\top}P_jx)^{\frac{p}{2}-1} + \pi_j(x^{\top}P_ix)^{\frac{p}{2}}\right\}\\
&< \sum_{i=1,i\neq j}^{M}\frac{\pi_{ji}(t)}{\pi_j}\left[-\pi_ja_{ji}^{\frac{p}{2}}(x^{\top}P_jx)(x^{\top}P_jfx)^{\frac{p}{2}-1} + \pi_j(x^{\top}P_ix)^{\frac{p}{2}}\right]\\
&= \sum_{i=1,i\neq j}^{M}\pi_{ji}(t)\left[(x^{\top}P_ix)^{\frac{p}{2}} - a_{ji}^{\frac{p}{2}}(x^{\top}P_jx)^{\frac{p}{2}}\right].
\end{aligned} \tag{10}$$

We assert that a positive definite matrix $Q \in \mathbf{R}^{n\times n}$ exists, satisfying that for any $j \in \Delta$, the inequality

$$\mathcal{L}V(x, j) < -x^{\top}Qx \tag{11}$$

holds. Utilizing Dynkin's formula, it follows immediately from this assertion that system (1) is p-moment stable [28].

To prove this, we set, for each $i, j \in \Delta$,

$$\omega_{ij}(x) = (x^{\top}a_{ij}P_ix)^{\frac{p}{2}} - (x^{\top}P_jx\pi)^{\frac{p}{2}}, \quad m_{ij} = \min_{x\in\mathcal{B}}\omega_{ij}(x),$$

where $\mathcal{B} = \{x : |x| = 1\}$. Then by condition (9), for any $x \in \mathcal{B}$, we have $x^{\top}P_jx < a_{ij}x^{\top}P_ix$, and hence

$$\omega_{ij}(x) > 0, \tag{12}$$

then this implies that $m_{ij} > 0$.

We choose $\delta_{ij} \in (0, m_{ij}^{\frac{2}{p}})$ and set $Q_{ij} = \delta_{ij}I_n$, where I_n is identity matrix. It is easy to see that for any $x \in \mathbf{R}^n$,

$$(x^{\top}Q_{ij}x)^{\frac{p}{2}} < m_{ij}(x^{\top}x)^{\frac{p}{2}} \leqslant \omega_{ij}(x). \tag{13}$$

Let $\delta = \min_{i,j\in\Delta}\{\delta_{ij}\}$, $\pi = \min_{j\in\Delta}\{\pi_j\}$, then the above inequality (13) gives that

$$\begin{aligned}\sum_{j=1,j\neq i}^{M} \pi_{ij}(t)\omega_{ij}(x) &\geqslant \sum_{j=1,j\neq i}^{M} \pi_{ij}(t)\delta(x^\top x)^{\frac{p}{2}}\\ &= \pi_i\delta(x^\top x)^{\frac{p}{2}}\\ &\geqslant \pi\delta(x^\top x)^{\frac{p}{2}}\\ &:= (x^\top Qx)^{\frac{p}{2}},\end{aligned}$$

where $Q = (\pi\delta)^{\frac{2}{p}} I_n$. Combining this with (10) we obtain the inequality (11). This proves the theorem.

Remark 7. Of course the infinitesimal generator

$$\mathcal{L}V(x,i) < 0$$

can be obtained by combining inequalities (10) and (12). But one should note that, to prove the p-moment stability of system (1), only the operator $\mathcal{L}V(x,i) < 0$ of (10) is not sufficient. Because the infinitesimal generator $\mathcal{L}V(x,i)$ contains time-varying transition rate $\pi_{ij}(t)$, it is necessary to verify that the $\mathcal{L}V(x,i)$ operator has a negative upper bound, i.e., condition (11) holds.

4.2 p-moment stabilization

Based on the results obtained above, this subsection will design a mode-dependent state-feedback law to achieve p-moment stability for the MJLS (1). In this paper, we consider a memoryless mode-dependent controller structured as

$$u(t) = K_{\theta(t)}x(t).$$

Both the stochastic process $\theta(t)$ and the state variable $x(t)$ are assumed to be measurable. The problem of p-moment stabilization consists in finding a set of gains K_i, $i \in \Delta$, such that

$$\dot{x}(t) = \left(A_{\theta(t)} + B_{\theta(t)}K_{\theta(t)}\right) x(t) \tag{14}$$

is p-moment stable.

Based on Theorem 1, p-moment stabilization problem of the closed-loop MJLS (14) can be transformed to a stabilization problem for deterministic linear system (6) under arbitrary switching. Let $A_i + B_iK_i$ replace A_i of the differential Lyapunov inequalities (7), the closed-loop deterministic linear system under arbitrary switching is stable, i.e., the closed-loop MJLS (14) is p-moment stable. Note that, in this situation, there will exist the coupling terms diag $\{\sum_{l=1}^{p} I_{n^{l-1}} \otimes (B_jK_j) \otimes I_{n^{p-l}}\}\, Q$, which is failed to obtain linear matrix inequalities for the stabilization of the closed-loop MJLS (14).

We now give the equivalent closed-loop deterministic linear system with arbitrary switching of the closed-loop MJLS (14). First, we replace A_i by $A_i + B_iK_i$ in (A1), we have

$$\begin{aligned}\dot{y}_j(t) &= \left[\sum_{l=1}^{p} I_{n^{l-1}} \otimes (A_j + B_jK_j) \otimes I_{n^{p-l}}\right] y_j(t) + \sum_{i\in\Delta}\pi_{ij}(t)y_i(t)\\ &= \left[\sum_{l=1}^{p} I_{n^{l-1}} \otimes A_j \otimes I_{n^{p-l}} + \sum_{l=1}^{p} I_{n^{l-1}} \otimes (B_jK_j) \otimes I_{n^{p-l}}\right] y_j(t) + \sum_{i\in\Delta}\pi_{ij}(t)y_i(t).\end{aligned} \tag{15}$$

By applying Lemma 4 above and Lemma 1 of [29], for any $l \in \mathbb{N}$, $l \leqslant p$, $j \in \Delta$,

$$\begin{aligned}&[I_{n^{l-1}} \otimes (B_jK_j) \otimes I_{n^{p-l}}]\, y_j(t)\\ &= [I_{n^{l-1}} \otimes ((B_j \otimes I_{n^{p-l}})(K_j \otimes I_{n^{p-l}}))]y_j(t)\\ &= (I_{n^{l-1}} \otimes B_j \otimes I_{n^{p-l}})\,[I_{n^{l-1}} \otimes (K_j \otimes I_{n^{p-l}})]\, y_j(t)\\ &= (I_{n^{l-1}} \otimes B_j \otimes I_{n^{p-l}})[I_{n^{l-1}} \otimes (U_{m,n^{p-l}}(I_{n^{p-l}} \otimes K_j)U_{n,n^{p-l}})]y_j(t)\\ &= (I_{n^{l-1}} \otimes B_j \otimes I_{n^{p-l}})[(\mathbb{I}_{n^{l-1}} \otimes U_{m,n^{p-l}})(I_{n^{l-1}} \otimes I_{n^{p-l}} \otimes K_j)(I_{n^{l-1}} \otimes U_{n,n^{p-l}})]y_j(t)\\ &= (I_{n^{l-1}} \otimes B_j \otimes I_{n^{p-l}})(I_{n^{l-1}} \otimes U_{m,n^{p-l}})(I_{n^{p-1}} \otimes K_j)(I_{n^{l-1}} \otimes U_{n,n^{p-l}})\mathbf{E}\left\{x^{[p]}\mathbf{1}_{\theta(t)=j}\right\}\end{aligned}$$

$$
\begin{aligned}
&= (I_{n^{l-1}} \otimes B_j \otimes I_{n^{p-l}})(I_{n^{l-1}} \otimes U_{m,n^{p-l}})(I_{n^{p-1}} \otimes K_j)\mathbf{E}\left\{(I_{n^{l-1}} \otimes U_{n,n^{p-l}})x^{[p]}\mathbf{1}_{\theta(t)=j}\right\} \\
&= (I_{n^{l-1}} \otimes B_j \otimes I_{n^{p-l}})(I_{n^{l-1}} \otimes U_{m,n^{p-l}})(I_{n^{p-1}} \otimes K_j)\mathbf{E}\left\{x^{[p]}\mathbf{1}_{\theta(t)=j}\right\} \\
&= (I_{n^{l-1}} \otimes B_j \otimes I_{n^{p-l}})(I_{n^{l-1}} \otimes U_{m,n^{p-l}})(I_{n^{p-1}} \otimes K_j)y_j(t),
\end{aligned}
$$

where $U_{s,n^{p-l}} = \sum_{k=1}^{s}\sum_{l=1}^{n^{p-l}}(e_{k,s}e_{l,n^{p-l}}^{\top}) \otimes (e_{l,n^{p-l}}e_{k,s}^{\top})$, $s = m, n$.

Substituting the above equality into (15) yields

$$
\begin{aligned}
\dot{y}_j(t) =\ & \left[\sum_{l=1}^{p}\Big(I_{n^{l-1}} \otimes A_j \otimes I_{n^{p-l}} + (I_{n^{l-1}} \otimes B_j \otimes I_{n^{p-l}})(I_{n^{l-1}} \otimes U_{m,n^{p-l}})(I_{n^{p-1}} \otimes K_j)\Big)\right] y_j(t) \\
&+ \sum_{i\in\Delta}\pi_{ij}(t)y_i(t),
\end{aligned}
$$

and consequently,

$$
\begin{aligned}
\dot{y}(t) &= \left(\operatorname{diag}\left\{\sum_{l=1}^{p}(I_{n^{l-1}} \otimes A_j \otimes I_{n^{p-l}} + (I_{n^{l-1}} \otimes B_j \otimes I_{n^{p-l}})(I_{n^{l-1}} \otimes U_{m,n^{p-l}})(I_{n^{p-1}} \otimes K_j))\right\} + \Pi(t)^{\top} \otimes I_{n^p}\right) y(t) \\
&= \sum_{\tau=1}^{(M-1)^M} a_\tau(t)\Xi_{\tau,p}y(t),
\end{aligned}
\tag{16}
$$

where

$$
\begin{aligned}
\Xi_{\tau,p} =\ & \operatorname{diag}\left\{\sum_{l=1}^{p} I_{n^{l-1}} \otimes A_i \otimes I_{n^{p-l}}\right\} + \bar{\Pi}_\tau^{\top} \otimes I_{n^p} \\
&+ \operatorname{diag}\left\{\sum_{l=1}^{p}(I_{n^{l-1}} \otimes B_i \otimes I_{n^{p-l}})(I_{n^{l-1}} \otimes U_{m,n^{p-l}})\right\}\operatorname{diag}\{I_{n^{p-1}} \otimes K_i\}.
\end{aligned}
$$

Hence, the closed-loop deterministic linear system under arbitrary switching is constructed as follows:

$$
\dot{y}(t) = \Xi_{\vartheta(t),p}y(t), \tag{17}
$$

where $\vartheta(t)$ is the same as in (6).

Combining this with Theorem 1, the following result can be directly derived.

Theorem 3. Let $p \in \mathbb{N}^+$ be even, and there exist positive definite matrices $P_j \in \mathbf{R}^{n\times n}$ and matrices $H_j \in \mathbf{R}^{m\times n}$, $j \in \Delta$, such that, for all $\tau \in \Upsilon$, the inequalities

$$
\operatorname{diag}\{\hat{A}_i\hat{P}_i + \hat{P}_i\hat{A}_i^{\top} + \hat{B}_i\hat{H}_i + \hat{H}_i^{\top}\hat{B}_i^{\top}\} + (\bar{\Pi}_\tau^{\top} \otimes I_{n^p})\operatorname{diag}\{\hat{P}_i\} + \operatorname{diag}\{\hat{P}_i\}(\bar{\Pi}_\tau \otimes I_{n^p}) < 0 \tag{18}
$$

are satisfied, where $\hat{P}_i = I_{n^{p-1}} \otimes P_i$, $\hat{A}_i = \sum_{l=1}^{p} I_{n^{l-1}} \otimes A_i \otimes I_{n^{p-l}}$, $\hat{B}_i = \sum_{l=1}^{p}(I_{n^{l-1}} \otimes B_i \otimes I_{n^{p-l}})(I_{n^{l-1}} \otimes U_{m,n^{p-l}})$, $\hat{H}_i = I_{n^{p-1}} \otimes H_i$. Then, the closed-loop system (14) is p-moment stabilized by the set of state-feedback gains $K_j = H_jP_j^{-1}$.

Proof. By letting $P = \operatorname{diag}\{\hat{P}_i\}$, and $H_i = K_iP_i$, the condition (18) can be expressed as

$$
\begin{aligned}
&[\operatorname{diag}\{\hat{A}_i\} + \operatorname{diag}\{\hat{B}_i\}\operatorname{diag}\{I_{n^{p-1}} \otimes K_i\} + (\bar{\Pi}_\tau^{\top} \otimes I_{n^p})]P \\
&+ P[\operatorname{diag}\{\hat{A}_i\} + \operatorname{diag}\{\hat{B}_i\}\operatorname{diag}\{I_{n^{p-1}} \otimes K_i\} + (\bar{\Pi}_\tau^{\top} \otimes I_{n^p})]^{\top} < 0,
\end{aligned}
\tag{19}
$$

which implies

$$
\Xi_{\tau,p}P + \Xi_{\tau,p}^{\top}P < 0.
$$

Then, by applying Theorem 1, the proof is completed.

Remark 8. It is worth mentioning that, in the existing literature, Theorem 3 is the first to propose LMI-based conditions for p-moment stabilization of closed-loop MJLS, which is made possible by the introduction of Lemma 4. It is evident that matrix Y_i in the matrix inequality (13) of Theorem 2 in [25] includes matrices K and $\hat{P}$; therefore, the stabilization result in Theorem 2 of [25] is not formulated in terms of LMIs. In fact, Lemma 4 can be applied to Theorem 2 of [25] to derive LMI-based conditions for p-moment asymptotic stabilization of a stochastic system.

Next, from the above Theorems 2, an immediate design strategy can be present in the following result.

Theorem 4. Let $p > 0$, if there exist constants $a_{ij} > 0$, $i, j \in \Delta, j \neq i$, and positive definite matrices $Q_i \in \mathbf{R}^{n\times n}$ and matrices $W_i \in \mathbf{R}^{m\times n}$, $i \in \Delta$, such that the following inequalities:

$$\frac{p}{2}\left(Q_i A_i^\top + A_i Q_i + W_i^\top B_i^\top + B_i W_i\right) + \pi_i(a_{ij}^{\frac{p}{2}} - 1)Q_i < 0, \tag{20}$$

$$\begin{bmatrix} -a_{ij}Q_i & Q_i \\ Q_i & -Q_j \end{bmatrix} < 0 \tag{21}$$

are satisfied for all $i, j \in \Delta$, $j \neq i$, then the closed-loop system (14) is p-moment stabilized by the set of state-feedback gains $K_i = W_i Q_i^{-1}$.

Proof. Let $Q_i = P_i^{-1}$, $W_i = K_i Q_i$. By applying the congruent transformation Q_i to condition (8) of Theorem 2, condition (20) is immediately obtained. Performing the congruent transformation Q_i to condition (9) of Theorem 2, condition (21) is immediately obtained, and the proof is complete.

5 Numerical examples

In order to better highlight the theoretical aspects treated in the paper, a couple of numerical examples are discussed. The first example deals with 2-moment stability analysis and shows how an unstable MJLS can be stabilized through a state-feedback mode-dependent controller based on the results from Theorem 3. The second example designs the 3-moment stabilization scheme based on Theorem 4.

Example 2. Consider the MJLS (1) with $u(t) = 0, M = 3$, and

$$A_1 = \begin{bmatrix} -5 & 0 & 0 \\ -2 & 0.5 & 2 \\ 0.8 & 0 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.8 & 2 & 2 \\ 0.5 & -3 & 1 \\ 1 & 0.2 & -6 \end{bmatrix}, \quad A_3 = \begin{bmatrix} -2 & 0 & 2 \\ -1 & -2 & -2 \\ 1 & 0 & -3 \end{bmatrix}.$$

It is important to note that mode 3 is stable, while the first two modes are unstable. The parameters π_1 and π_2 are both set to 4, while $\pi_3 > 0$ is treated as a free parameter. Observe that as π_3 tends to zero, the average residence-time in the stable mode grows substantially. It is expected that increasing π_3 may reduce stability. The objective here is to determine the maximum range of $\pi_3 > 0$ for which 2-moment stability of the MJLS is guaranteed.

As emphasized in Remark 5, to ensure 2-moment stability of the MJLS (1), it is necessary that all vertex matrices Φ_τ remain stable. Using a line-search approach, it is found that this property holds for values of π_3 up to 1.776. Again applying the line-search method, differential Lyapunov inequalities satisfying the requirements of Theorem 1 are obtained for π_3 up to 1.698, which is very near the upper bound $\pi_3 = 1.776$ derived from the necessary conditions.

For simulation purposes, the initial state is set to $x(0) = [2, 5, 3]^\top$, and the parameter π_3 is set to $1 < 1.698$ and $8 > 1.776$, respectively. When $\pi_3 = 1 < 1.698$, the conditions of Theorem 1 are satisfied, and thus the MJLS (1) is 2-moment stable (see Figure 1 for an illustration). When $\pi_3 = 8 > 1.776$, the necessary conditions highlighted in Remark 5 are not met, and the system exhibits unstable behavior (see Figure 2).

Next, for the unstable case with $\pi_3 = 8$, we apply Theorem 3 to design the control strategy for the closed-loop system. Consider the corresponding closed-loop MJLS (14) with

$$B_1 = [1.5, 1, 0.2]^\top, \; B_2 = [2, 3, 5.5]^\top, \; B_3 = [4, 0.9, 3]^\top.$$

The controller design is achieved by solving the LMI's presented in Theorem 3. A set of feasible gains is given by

$$K_1 = [-2.7021, -3.3411, -1.1719], K_2 = [-1.1752, -1.5528, -1.8115], K_3 = [-0.5199, -0.0259, -0.7590].$$

The corresponding closed-loop system (14) is 2-moment stable (see Figure 3 for an illustration).

Example 3. Consider the closed-loop system (14) as given in Example 2 and $\pi_3 = 8$. Note that without control, system (14) is unstable in the 3-moment sense (see Figure 4 for an illustration). Next, by utilizing Theorem 4, we achieve the 3-moment stabilization criteria of closed-loop system (14).

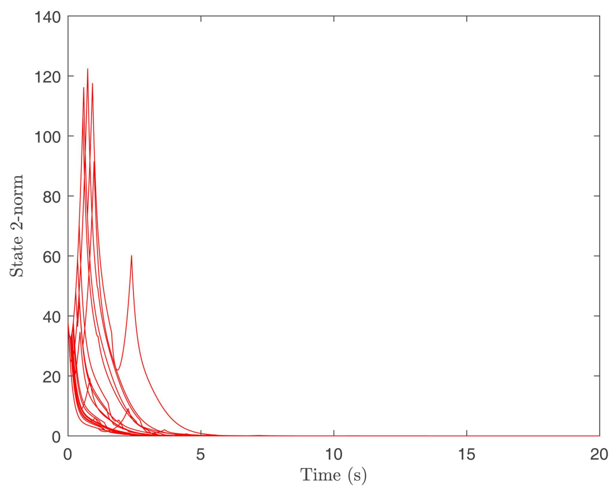

Figure 1 (Color online) For $\pi_3 = 1$, 20 trajectories of the 2-norm of state $x(t)$ for the MJLS (1) without feedback control.

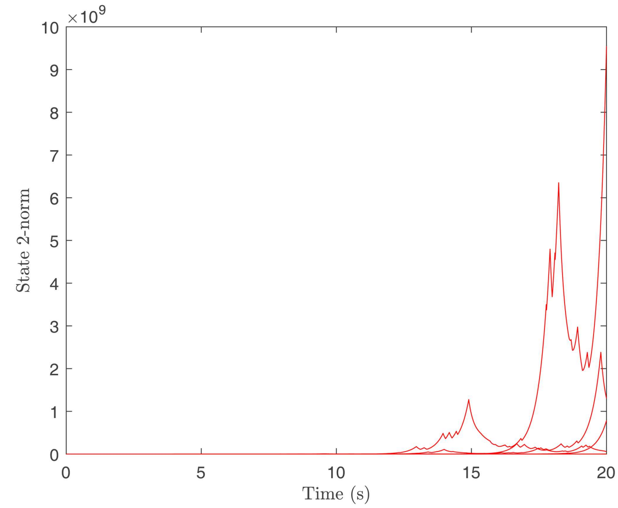

Figure 2 (Color online) For $\pi_3 = 8$, 20 trajectories of the 2-norm of state $x(t)$ for the MJLS (1) without feedback control.

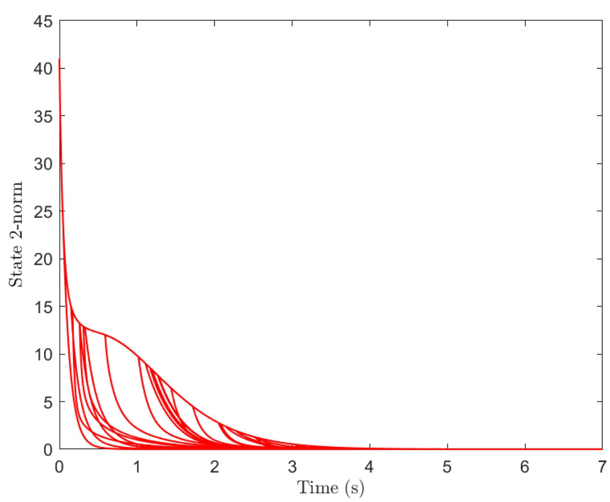

Figure 3 (Color online) For $\pi_3 = 8$, 20 trajectories of the 2-norm of state $x(t)$ for the MJLS (1) with feedback control designed via Theorem 3.

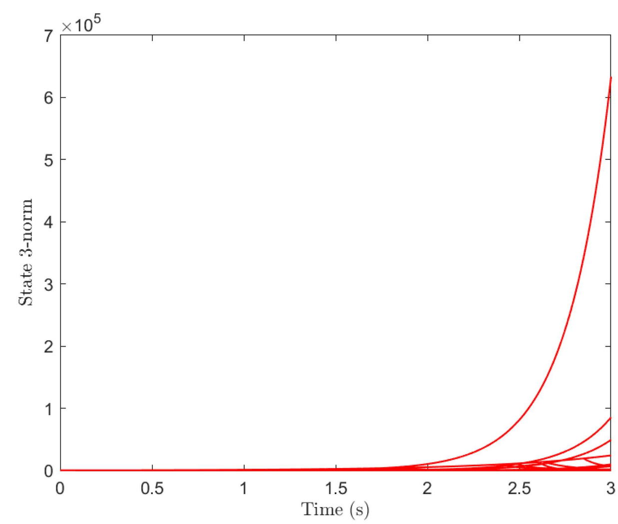

Figure 4 (Color online) 20 trajectories of the 3-norm of state $x(t)$ for the MJLS (14) without feedback control.

Set the constants as $a_{12} = 1,\ a_{13} = 3/2,\ a_{21} = 1.01,\ a_{23} = 5/4,\ a_{31} = 2/3 + 0.01,\ a_{32} = 4/5 + 0.01$. Using the LMI's of Theorem 4, the controller design yields the following set of feasible gains:

$$K_1 = [2.5686, -2.3720, -2.0210], \quad K_2 = [-0.8647, -0.5032, 0.8641], \quad K_3 = [0.1941, 0.6432, 0.1292].$$

The closed-loop system (14) achieves 3-moment stability, as illustrated in Figure 5.

6 Concluding remark

In this paper, the p-moment stability and stabilization problem of MJLSs with time-varying transition probabilities has been addressed. Specifically, when $p \in \mathbb{N}^+$ is even, the sufficient conditions ensuring p-moment stability of MJLSs are presented. Using a technical lemma, LMI-based conditions for p-moment stabilization of closed-loop

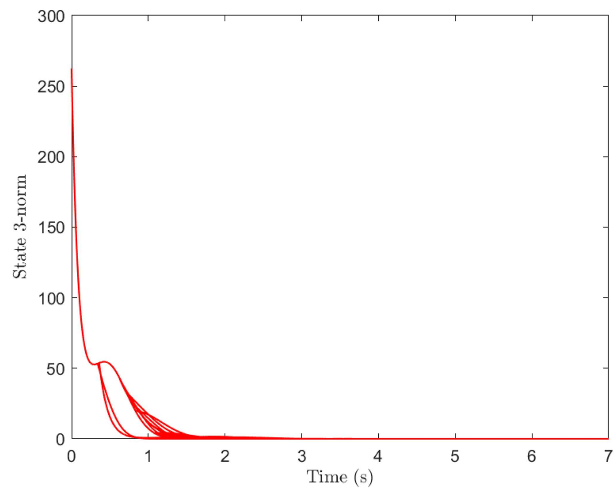

Figure 5 (Color online) 20 trajectories of the 3-norm of state $x(t)$ for the MJLS (14) with feedback control via Theorem 4.

MJLSs have been rigorously established. By the Lyapunov function approach, sufficient conditions for $p(p > 0)$-moment stability and stabilization of MJLSs have been built.

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Appendix A Proof of Lemma 1

Denote

$$y_j(t) = \mathbf{E}(x^{[p]}\mathbf{1}_{\theta(t)=j}),$$

$$y(t) = \mathrm{col}\{y_i\}.$$

Then, for any given $\Pi(t)$, by applying Itô rule and Lemma 3.6 of [2], for any $j \in \Delta$,

$$\begin{aligned} dy_j(t) &= \mathbf{E}(dx^{[p]}\mathbf{1}_{\theta(t)=j}) + \mathbf{E}(x^{[p]}d\mathbf{1}_{\theta(t)=j}) \\ &= \sum_{l=1}^{p}\mathbf{E}(x^{[l-1]} \otimes dx \otimes x^{[p-l]}\mathbf{1}_{\theta(t)=j}) \\ &\quad +\mathbf{E}(x^{[p]}d\mathbf{1}_{\theta(t)=j}) \\ &= \left[M_{j,p}y_j(t) + \sum_{i\in\Delta}\pi_{ij}(t)y_i(t)\right]dt. \end{aligned} \tag{A1}$$

For each even $p \in \mathbb{N}^+$, it is straightforward to confirm that

$$\lim_{t\to+\infty}\mathbf{E}|x(t)|^p = 0 \Longleftrightarrow \lim_{t\to+\infty} y(t) = 0. \tag{A2}$$

This result implies that the p-moment stability of the MJLS (1) is equivalent to asymptotic convergence of $y(t)$ to zero, which corresponds to asymptotic stability of system (4).

Appendix B Proof of Lemma 4

Note that

$$\begin{aligned} &(I_{n^m} \otimes U_{n,n^{p-m-1}})\varpi^{[p]} \\ &= (\mathbb{I}_{n^m} \otimes U_{n,n^{p-m-1}})(\varpi^{[m]} \otimes \varpi^{[p-m]}) \\ &= \varpi^{[m]} \otimes \Big[(\textstyle\sum_{k=1}^{n}\sum_{l=1}^{n^{p-m-1}}(e_{k,n}e_{l,n^{p-m-1}}^\top) \otimes (e_{l,n^{p-m-1}}e_{k,n}^\top))\varpi^{[p-m]} \Big] \\ &= \varpi^{[m]} \otimes \Big[(\textstyle\sum_{k=1}^{n}\sum_{l=1}^{n^{p-m-1}}(e_{k,n}e_{l,n^{p-m-1}}^\top) \otimes (e_{l,n^{p-m-1}}e_{k,n}^\top))(\varpi^{[p-m-1]} \otimes \varpi) \Big] \\ &= \varpi^{[m]} \otimes \Big(\textstyle\sum_{k=1}^{n}\sum_{l=1}^{n^{p-m-1}} \Big(e_{k,n}e_{l,n^{p-m-1}}^\top\varpi^{[p-m-1]}\Big) \otimes \Big(e_{l,n^{p-m-1}}e_{k,n}^\top\varpi\Big) \Big) \\ &= \varpi^{[m]} \otimes \Big(\textstyle\sum_{k=1}^{n}\sum_{l=1}^{n^{p-m-1}} \Big(e_{k,n}^\top\varpi e_{k,n}\Big) \otimes \Big(e_{l,n^{p-m-1}}^\top\varpi^{[p-m-1]}\mathbf{e}_{l,n^{p-m-1}}\Big) \Big) \\ &= \varpi^{[m]} \otimes \left(\sum_{k=1}^{n} e_{k,n}^\top\varpi e_{k,n}\right) \otimes \left(\sum_{l=1}^{n^{p-m-1}} e_{l,n^{p-m-1}}^\top\varpi^{[p-m-1]}e_{l,n^{p-m-1}}\right) \\ &= \varpi^{[p]}. \end{aligned}$$

This completes the proof.