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Output and estimated state event-based output feedback control of high-order fully actuated systems

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Abstract This paper focuses on the output feedback event-triggered control problem for high-order fully actuated systems (HOFASs). Unlike existing results, two event-triggered mechanisms are employed for the output and estimated states, respectively, whose trigger thresholds are time-varying functions. First, only based on the triggered output, a novel state observer is constructed to estimate unmeasurable states. Second, for the estimated states, an event-triggered mechanism is proposed to reduce their update frequency. Then, an event-triggered controller is established with the help of the fully actuated system method, which only utilizes the triggered estimated states, and avoids the introduction of virtual controllers. By the use of Lyapunov stability theory, it is strictly proven that all signals of closed-loop systems converge to adjustable boundaries. Finally, the simulation on an RLC circuit serves as a practical illustration to verify the reliability of the proposed control algorithm.

Keywords high-order fully actuated systems, output feedback control, event-triggered control

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1 Introduction

Over the last few decades, since nearly all real-world systems possess an inherent nonlinearity [1–3], the control problem of nonlinear systems has garnered significant interest, which can generally be categorized into two types: state feedback control [4] and output feedback control [5], based on whether the system states are completely measurable. In state feedback control, all state variables are measurable and can be used for controller design. For example, Ref. [6] considered the prescribed-time control problem of nonlinear systems with uncertainties. For output feedback control, only partial state information, such as the output, can be measured and used for controller design. In this case, the state observer [7] and K-filter [8] are usually employed to estimate the unmeasurable state variables. Furthermore, Ref. [9] considered the distributed leader-following consensus control problem for a class of nonlinear multiagent systems, which was contingent on stochastic output sensing noises under a fixed directed topology. However, all of the aforementioned studies face a common problem: how to reduce resource utilization (such as computing resources and communication bandwidth) while ensuring system performance.

In actual systems, there are often network resources and bandwidth limitations [10]. In this case, continuous sampling control that requires real-time collection and transmission of system information seems powerless. Towards this problem, event-triggered control (ETC) has emerged, which only samples and transmits information when the system information meets the pregiven events, effectively reducing the number of variable updates while ensuring system performance [11,12]. Ref. [13] established a state-feedback ETC algorithm for nonlinear systems by using the assumption of input-to-state (ISS) stability and the small gain theorem. To eliminate the ISS condition, Ref. [14] co-designed a controller and an event-triggered mechanism (ETM). Similarly, Ref. [15] proposed another approach to develop an ETC algorithm. Along this line, Ref. [16] further considered the ETMs on the controller and parameter estimator. It is important to note that although these ETMs [14–16] significantly reduce the frequency of controller transmissions, they rely on real-time state sampling, which exerts pressure on limited network bandwidth resources. To break through this technical difficulty, the ETM proposed in [17] was based on system states, which introduced an innovative theoretical approach to event-driven control, and showed significant advantages in reducing network communication load. Based on the triggered output signal, Ref. [18] established a state observer and designed

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a corresponding output feedback controller. It is worth noting that the implementation of [17, 18] is based on backstepping method, which requires the derivative of the virtual controller to be constant and the trigger threshold to also be constant. The conditions are a bit strict, and limit the universality of engineering application scenarios.

As we all know, the numerous physical laws determine the operational patterns of nature, such as Newton's Law, and the physical models derived from them are a series of second-order or higher-order systems [19]. Although many excellent studies [1–5, 7, 8, 10–18] have emerged on first-order systems, it should be admitted that the first-order system approach primarily lies in states and is better suited for solving and estimating states [20]. Towards this problem, the fully actuated system (FAS) approach was established by [21], which directly focuses on high-order systems and is more suitable for designing controllers. Following this work, researchers have already made a series of attempts towards related problems. For example, Ref. [22] focused on applying the FAS method to strict-feedback systems, which handled the control problem under subsystems with increased dimensions. Ref. [23] established a FAS control algorithm for a class of flexible servo systems by means of the singular perturbation method. Different from the aforementioned deterministic FAS methods, Ref. [24] constructed a stochastic high-order fully actuated system (HOFAS) model and investigated the sensor fault problem. Ref. [25] delved into the HOFASs under an unknown control gain function. Ref. [26] separately addressed the sliding mode control problems of strict-feedback systems under the framework of the FAS method, as well as the problem of an unknown control direction. Meanwhile, Refs. [25, 26] discussed the ETM for the controller of systems, reducing only the number of information transfers from the controller to the actuator.

Encouraged by the above observations, this paper aims to investigate the output feedback ETC problem of HOFASs, specifically constructing a state observer based on the triggered output (TO), and designing a controller based on the triggered estimated states (TESs). The main contributions of this paper can be outlined as follows.

(i) For HOFASs, this paper establishes a new state observer based on discontinuous output signals to estimate unmeasurable states, achieving maximum reduction of unnecessary resource consumption while ensuring estimation accuracy.

(ii) The ETM is taken into account for the output and the estimated states, which effectively reduces the number of sampling and updating of output and estimated states.

(iii) Based on the TESs, the event-triggered controller is constructed under the guidance of the FAS method, which avoids the limitation in traditional controller design that the partial derivatives of the virtual controllers must be constants.

Notation: For the purpose of brevity and without causing ambiguity, we will refrain from elaborating on the specifications of variables within this paper. For example, y^e expresses the triggered output $y^e(t)$. I_m represents the m-dimensional identity matrix; 0_m denotes the m-dimensional null matrix; $\Re^m$ expresses an m-dimensional vector space; $\Re^{m\times n}$ denotes an $m\times n$ matrix space; $i\in\mathbb{N}_{1:n}$ expresses $i=1,\ldots,n$; $\inf(\cdot)$ is the infimum of a variable; $\lambda_{\min}(\cdot)$ represents the minimum eigenvalue of a matrix; $\|\cdot\|$ denotes 2-norm for a vector; $\xi^{(n)}$ represents the n-th derivative of the variable ξ; for a variable $\xi\in\Re^r$, $\xi^{(0\sim n-1)}=[(\xi)^{\mathrm{T}},(\dot{\xi})^{\mathrm{T}},\ldots,(\xi^{(n-1)})^{\mathrm{T}}]^{\mathrm{T}}\in\Re^{nr}$; for a positive constant matrix $k_i\in\Re^{r\times r}$, $i\in\mathbb{N}_{0:n-1}$, $k_{0\sim n-1}=[k_0,\ldots,k_{n-1}]\in\Re^{r\times nr}$; for a matrix $M_{kj}\in\Re^{nr\times r}$, $k\in\mathbb{N}_{1:2}$, $j\in\mathbb{N}_{1:n}$, $M_k=[M_{k1},M_{k2},\ldots,M_{kn}]^{\mathrm{T}}\in\Re^{nr\times nr}$;

$$\Phi(0_{0\sim n-1})=\begin{bmatrix}0_r & I_r & \cdots & 0_r\\ \vdots & \vdots & \ddots & \vdots\\ 0_r & 0_r & \cdots & I_r\\ 0_r & 0_r & \cdots & 0_r\end{bmatrix}\in\Re^{nr\times nr};\ \Phi(k_{0\sim n-1})=\begin{bmatrix}0_r & I_r & \cdots & 0_r\\ \vdots & \vdots & \ddots & \vdots\\ 0_r & 0_r & \cdots & I_r\\ -k_0 & -k_1 & \cdots & -k_{n-1}\end{bmatrix}\in\Re^{nr\times nr}.$$

2 Problem formulation

In this paper, motivated by [20], we consider the following HOFAS:

$$\begin{aligned}\xi^{(n)}(t)&=f(\xi^{(0\sim n-1)}(t))+u(t),\\ y(t)&=C\xi^{(0\sim n-1)}(t),\end{aligned}\tag{1}$$

where $\xi(t),\dot{\xi}(t),\ldots,\xi^{(n)}(t)$ are states, which are r-dimensional; $u(t)\in\Re^r$ is a controller; $y(t)\in\Re^m$ is a real output; $C=[C_1,\ldots,C_n]\in\Re^{m\times nr}$ is a constant matrix with $C_j\in\Re^{m\times r}$, $j\in\mathbb{N}_{1:n}$; $f(\cdot)\in\Re^r$ is a nonlinear function.

Then, we can get its closed-loop systems as follows:

$$\dot{\xi}^{(0\sim n-1)}=\Phi(0_{0\sim n-1})\xi^{(0\sim n-1)}+Bf(\xi^{(0\sim n-1)})+Bu,\tag{2}$$

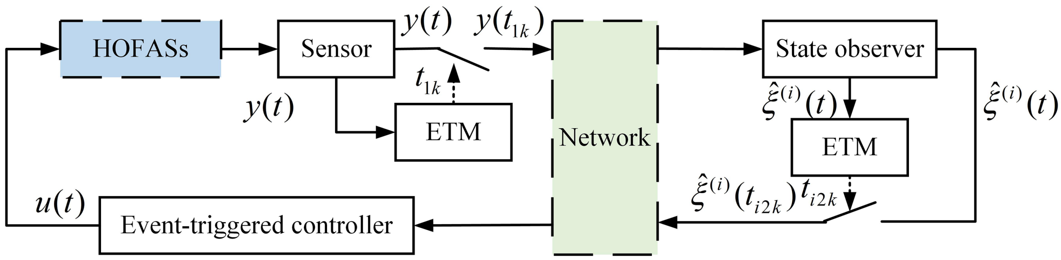

Figure 1 (Color online) The structure diagram of the proposed algorithm.

where $B = [0_r, \ldots, 0_r, I_r]^{\mathrm{T}} \in \Re^{nr\times r}$.

Remark 1. Due to the fact that almost the entire physical world is governed by physical laws, the resulting system models are FASs. Most existing studies of FASs [20,21,23–26] only considered state feedback control problems, which is difficult to achieve under certain conditions, such as the measurement of the states, pressure and temperature inside the system. In addition, Refs. [25, 26] investigated the ETC problem for FASs, which are still necessary to collect real-time state information from actual systems. Different from their studies, this paper focuses on the output feedback ETC problem for HOFAS (1) with the TO and TESs, which is of great theoretical and practical significance. The control structure diagram is given in Figure 1.

Assumption 1. There exists a positive constant ϑ satisfying $f(p) - f(q) \leqslant \vartheta\|p-q\|$ for variables $p, q \in \Re^{nr}$, and nonlinear functions $f(\cdot) \in \Re^{r}$, where p can be $\xi^{(0\sim n-1)}$ or $\hat{\xi}^{(0\sim n-1)}$, q can be $\hat{\xi}^{(0\sim n-1)}$ or $\hat{\xi}_e^{(0\sim n-1)}$ in this paper.

Objective: The control objective of this article is to propose an output feedback event-triggered controller based on the TO and TESs, which ensures that all signals of the closed-loop systems are bounded.

3 State observer design

To avoid continuous transmission of output signals, we introduce the following ETM:

$$\textbf{Event-O:} \begin{cases} y^e(t) = y(t_{1k}), \forall t \in [t_{1k}, t_{1,k+1}), \\ t_{1,k+1} = \inf\{t > t_{1k} | \|y(t) - y^e(t)\| \geqslant v_1(t)\}, \end{cases} \tag{3}$$

where t_{1k} is the output transmission time; $v_1(t) = v_{10}e^{-b_1 t} + v_{11}$ with v_{10}, v_{11} and b_1 being positive constants.

To estimate the states of system (2), based on TO y^e, we construct the following state observer:

$$\dot{\hat{\xi}}^{(0\sim n-1)} = \Phi_0\hat{\xi}^{(0\sim n-1)} + Bf(\hat{\xi}^{(0\sim n-1)}) + \sigma(y^e - C\hat{\xi}^{(0\sim n-1)}) + Bu, \tag{4}$$

where $\hat{\xi}^{(0\sim n-1)} \in \Re^{nr}$ is an estimate of state $\xi^{(0\sim n-1)} \in \Re^{nr}$; $\Phi_0 = \Phi(0_{0\sim n-1})$, $\sigma = [\sigma_1^{\mathrm{T}}, \ldots, \sigma_n^{\mathrm{T}}]^{\mathrm{T}} \in \Re^{nr\times m}$ is a constant matrix with $\sigma_i \in \Re^{r\times m}$, $i \in \mathbb{N}_{1:n}$ being a positive constant matrix.

Define the estimation error $e^{(0\sim n-1)}(t) = \xi^{(0\sim n-1)}(t) - \hat{\xi}^{(0\sim n-1)}(t) \in \Re^{nr}$. It can be calculated that

$$\dot{e}^{(0\sim n-1)} = \Phi_0 e^{(0\sim n-1)} - \sigma(Ce^{(0\sim n-1)}) + \sigma(y - y^e) + BF, \tag{5}$$

where $F = f(\xi^{(0\sim n-1)}) - f(\hat{\xi}^{(0\sim n-1)}) \in \Re^{r}$.

The Lyapunov function for the estimation error $e^{(0\sim n-1)}$ is chosen as

$$V_1 = (e^{(0\sim n-1)})^{\mathrm{T}} M_1 e^{(0\sim n-1)}, \tag{6}$$

where $M_1 \in \Re^{nr\times nr}$ is a positive definite matrix and satisfies

$$M_1(\Phi_0 - \sigma C) + (\Phi_0 - \sigma C)^{\mathrm{T}} M_1 \leqslant -\mu I_{nr}, \tag{7}$$

in which μ is a positive constant.

Based on (5), the time derivative of V_1 can be obtained as

$$\dot{V}_1 = 2(e^{(0\sim n-1)})^{\mathrm{T}} M_1(\Phi_0 e^{(0\sim n-1)} - \sigma(Ce^{(0\sim n-1)}) + \sigma(y - y^e) + BF). \tag{8}$$

Based on Assumption 1 and **Event-O** (3), we can get

$$2(e^{(0\sim n-1)})^{\mathrm{T}}M_1BF \leqslant 2\vartheta\|e^{(0\sim n-1)}\|^2\|M_1\|, \tag{9a}$$

$$2(e^{(0\sim n-1)})^{\mathrm{T}}M_1\sigma(y-y^e) \leqslant \frac{v_1^2}{4r_0}+r_0\|e^{(0\sim n-1)}\|^2\|M_1\|^2\|\sigma\|^2, \tag{9b}$$

where r_0 is a positive constant.

With (7), substituting (9) into (8), we can get

$$\begin{aligned}\dot{V}_1 \leqslant & -\mu\|e^{(0\sim n-1)}\|^2+2\vartheta\|M_1\|\|e^{(0\sim n-1)}\|^2+\frac{v_1^2}{4r_0}\\ & +r_0\|e^{(0\sim n-1)}\|^2\|M_1\|^2\|\sigma\|^2.\end{aligned} \tag{10}$$

Let $\mu_0=\mu-2\vartheta\|M_1\|-r_0\|M_1\|^2\|\sigma\|^2-\frac{1}{4}$, if the parameters μ, r_0, and matrices M_1, σ are chosen properly, the parameter μ_0 is a positive constant. Then, Eq. (10) can be rewritten as

$$\dot{V}_1 \leqslant -\mu_0\|e^{(0\sim n-1)}\|^2+\frac{v_1^2}{4r_0}-\frac{1}{4}\|e^{(0\sim n-1)}\|^2. \tag{11}$$

Remark 2. It should be acknowledged that currently, little consideration has been given to the output feedback control problem for HOFASs. However, the state observer in the output feedback control algorithm is an important tool to estimate the unmeasurable states, and is the foundation for designing a controller based on the FAS method in subsequent work. Although many researchers [7,8] have conducted research on the output feedback control problem for nonlinear systems, their results are based on continuous output signals, which require real-time acquisition of output from system. In this paper, by means of the **Event-O** (3), the output is only sampled with the given event being satisfied, which avoids real-time transmission of system output information to the observer.

4 Event-triggered controller design

In this section, to avoid continuous transmission of estimated states to the controller, the following ETM is taken into account for the estimated state:

$$\textbf{Event-S:}\begin{cases}\hat{\xi}_e^{(i)}=\hat{\xi}^{(i)}(t_{i2k}), \forall t\in[t_{i2k},t_{i2,k+1}),\\ t_{i2,k+1}=\inf\{t>t_{i2k}\,|\,\|\hat{\xi}^{(i)}(t)-\hat{\xi}_e^{(i)}(t)\|\geqslant v_{i2}(t)\}, i\in\mathbb{N}_{0:n-1},\end{cases} \tag{12}$$

where t_{i2k} is the estimated state transmission time; $\hat{\xi}_e^{(i)}$ is the TES; $v_{i2}(t)=v_{i20}e^{-b_{i2}t}+v_{i22}$ with v_{i20}, v_{i22} and b_{i2} being positive constants; $t_{2k}=\min\{t_{02k},t_{22k},\ldots,t_{n-1,2k}\}$; $v_2(t)=\max\{v_{02}(t),\ldots,v_{n-1,2}(t)\}$.

In the following, we will give the event-triggered controller design process and conduct the stability analysis on the closed-loop systems, based on the FAS theory.

First, choose the Lyapunov function for the estimated state $\hat{\xi}^{(0\sim n-1)}$ as

$$V_2=\frac{1}{2}(\hat{\xi}^{(0\sim n-1)})^{\mathrm{T}}M_2\hat{\xi}^{(0\sim n-1)}, \tag{13}$$

where $M_2=[M_{21},M_{22},\ldots,M_{2n}]^{\mathrm{T}}\in\Re^{nr\times nr}$ is a positive definite matrix with $M_{2j}\in\Re^{nr\times r}$, $j\in\mathbb{N}_{1:n}$. M_2 satisfies

$$\Phi_k^{\mathrm{T}}M_2+M_2\Phi_k\leqslant -2bI_{nr}, \tag{14}$$

where $\Phi_k=\Phi(k_{0\sim n-1})$, b is a positive constant.

The time derivative of V_2 is

$$\dot{V}_2=(\hat{\xi}^{(0\sim n-1)})^{\mathrm{T}}M_2(\Phi_0\hat{\xi}^{(0\sim n-1)}+\sigma(y-C\hat{\xi}^{(0\sim n-1)}))+(\hat{\xi}^{(0\sim n-1)})^{\mathrm{T}}M_{2n}(f(\hat{\xi}^{(0\sim n-1)})+u). \tag{15}$$

From Young's inequality, it can be calculated that

$$(\hat{\xi}^{(0\sim n-1)})^{\mathrm{T}}M_2\sigma(y-C\hat{\xi}^{(0\sim n-1)})\leqslant\|\hat{\xi}^{(0\sim n-1)}\|^2\|M_2\|^2\|\sigma\|^2\|C\|^2+\frac{1}{4}\|e^{(0\sim n-1)}\|^2. \tag{16}$$

By substituting (16) into (15), we can obtain

$$\begin{aligned}\dot{V}_2 \leqslant &(\hat{\xi}^{(0\sim n-1)})^{\mathrm{T}} M_2 \Phi_0 \hat{\xi}^{(0\sim n-1)} + \frac{1}{4}\|e^{(0\sim n-1)}\|^2 + (\hat{\xi}^{(0\sim n-1)})^T M_{2n}(f(\hat{\xi}^{(0\sim n-1)}) + u) \\ &+ \|\hat{\xi}^{(0\sim n-1)}\|^2 \|M_2\|^2 \|\sigma\|^2 \|C\|^2.\end{aligned} \tag{17}$$

Design the controller u as

$$u = -\left[k_{0\sim n-1}\hat{\xi}_e^{(0\sim n-1)} + f(\hat{\xi}_e^{(0\sim n-1)})\right]. \tag{18}$$

Taking (18) into (17), it occurs that

$$\begin{aligned}\dot{V}_2 \leqslant &(\hat{\xi}^{(0\sim n-1)})^{\mathrm{T}} M_2 \Phi_k \hat{\xi}^{(0\sim n-1)} + (\hat{\xi}^{(0\sim n-1)})^{\mathrm{T}} M_{2n}(u - \bar{u}) + \frac{1}{4}\|e^{(0\sim n-1)}\|^2 \\ &+ \|\hat{\xi}^{(0\sim n-1)}\|^2 \|M_2\|^2 \|\sigma\|^2 \|C\|^2,\end{aligned} \tag{19}$$

where $\bar{u} = -[k_{0\sim n-1}\hat{\xi}^{(0\sim n-1)} + f(\hat{\xi}^{(0\sim n-1)})]$.

Based on **Event-S** (12), it can be calculated that

$$\|\hat{\xi}^{(0\sim n-1)} - \hat{\xi}_e^{(0\sim n-1)}\| \leqslant \sqrt{n} v_2. \tag{20}$$

Considering Assumption 1 and Young's inequality, we can acquire

$$\begin{aligned}&(\hat{\xi}^{(0\sim n-1)})^{\mathrm{T}} M_{2n}(u - \bar{u}) \\ &\leqslant \|\hat{\xi}^{(0\sim n-1)}\| \|M_{2n}\| \|k_{0\sim n-1}\| \|\hat{\xi}^{(0\sim n-1)} - \hat{\xi}_e^{(0\sim n-1)}\| + \|\hat{\xi}^{(0\sim n-1)}\| \|M_{2n}\| \|f(\hat{\xi}^{(0\sim n-1)}) - f(\hat{\xi}_e^{(0\sim n-1)})\| \\ &\leqslant \|\hat{\xi}^{(0\sim n-1)}\| \|M_{2n}\| (\|k_{0\sim n-1}\|\sqrt{n} v_2 + \vartheta\sqrt{n} v_2) \\ &\leqslant r_1 \|\hat{\xi}^{(0\sim n-1)}\|^2 \|M_{2n}\|^2 \|k_{0\sim n-1}\|^2 n + \frac{1}{4r_1} v_2^2 + r_2 \|\hat{\xi}^{(0\sim n-1)}\|^2 \|M_{2n}\|^2 n\vartheta^2 + \frac{1}{4r_2} v_2^2,\end{aligned} \tag{21}$$

where r_1 and r_2 are positive constants.

Substituting (21) into (20), one can obtain

$$\begin{aligned}\dot{V}_2 \leqslant &(\hat{\xi}^{(0\sim n-1)})^{\mathrm{T}} M_2 \Phi_k \hat{\xi}^{(0\sim n-1)} + \frac{1}{4}\|e^{(0\sim n-1)}\|^2 + \|\hat{\xi}^{(0\sim n-1)}\|^2 \|M_2\|^2 \|\sigma\|^2 \|C\|^2 + \frac{1}{4r_2} v_2^2 \\ &+ r_1 \|\hat{\xi}^{(0\sim n-1)}\|^2 \|M_{2n}\|^2 \|k_{0\sim n-1}\|^2 n + r_2 \|\hat{\xi}^{(0\sim n-1)}\|^2 \|M_{2n}\|^2 n\vartheta^2 + \frac{1}{4r_1} v_2^2.\end{aligned} \tag{22}$$

Recalling (14), Eq. (22) can be modified into

$$\begin{aligned}\dot{V}_2 \leqslant &-b\|\hat{\xi}^{(0\sim n-1)}\|^2 + \|\hat{\xi}^{(0\sim n-1)}\|^2 (\|M_2\|^2 \|\sigma\|^2 \|C\|^2 + \|M_{2n}\|^2 (r_1 \|k_{0\sim n-1}\|^2 n + r_2 n\vartheta^2)) \\ &+ \frac{1}{4r_1} v_2^2 + \frac{1}{4r_2} v_2^2 + \frac{1}{4}\|e^{(0\sim n-1)}\|^2 \\ \leqslant &- b_0 \|\hat{\xi}^{(0\sim n-1)}\|^2 + \frac{1}{4}\|e^{(0\sim n-1)}\|^2 + \frac{v_2^2}{4r_1} + \frac{v_2^2}{4r_2},\end{aligned} \tag{23}$$

where b_0 satisfies

$$b_0 = b - (\|M_2\|^2 \|\sigma\|^2 \|C\|^2 + r_1 \|M_{2n}\|^2 \|k_{0\sim n-1}\|^2 n + r_2 \|M_{2n}\|^2 n\vartheta^2),$$

which can be a positive constant by selecting appropriate parameters r_1, r_2, and matrices M_2, $k_{0\sim n-1}$.

The Lyapunov function of the closed-loop systems is defined as

$$V = V_1 + V_2. \tag{24}$$

Combining (11) with (23), the time derivative of V is

$$\dot{V} \leqslant -\mu_0 \|e^{(0\sim n-1)}\|^2 - b_0 \|\hat{\xi}^{(0\sim n-1)}\|^2 + \frac{1}{4r_1} v_2^2 + \frac{1}{4r_2} v_2^2 + \frac{v_1^2}{4r_0}. \tag{25}$$

Based on the **Event-O** (3) and **Event-S** (12), we can get $v_1 \leqslant \bar{v}_1$, $v_2 \leqslant \bar{v}_2$, where $\bar{v}_1$ is a positive constant satisfying $\bar{v}_1 = v_{10} + v_{11}$, $\bar{v}_2$ is a positive constant satisfying $\bar{v}_2 = \max\{v_{020}, \ldots, v_{n-1,20}\} + \max\{v_{02}, \ldots, v_{n-1,2}\}$.

Then, Eq. (25) can be rewritten as

$$\dot{V} \leqslant -RV + S, \tag{26}$$

where $R = \min\{\frac{\mu_0}{\lambda_{\max}(M_1)}, \frac{b_0}{\lambda_{\max}(M_2)}\}$, $S = \frac{1}{4r_1}\bar{v}_2^2 + \frac{1}{4r_2}\bar{v}_2^2 + \frac{\bar{v}_1^2}{4r_0}$.

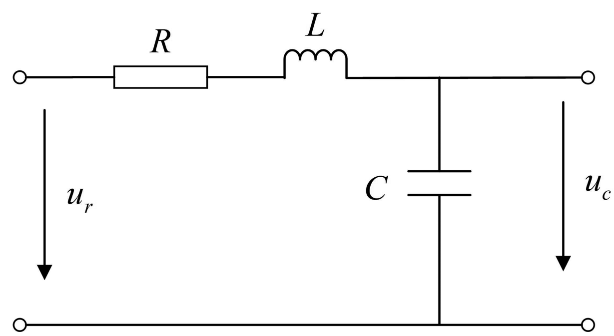

Figure 2 RLC circuit.

Theorem 1. For HOFASs (1) satisfying Assumption 1, under the **Event-O** (3) and **Event-S** (12), with the observer (4), the output feedback event-triggered controller (18) can ensure that all signals of the closed-loop system are bounded.

Proof. By integrating (26), when t tends to infinity, we can obtain that the Lyapunov function V satisfies $V \leqslant \frac{S}{R}$. Considering the definitions of Lyapunov functions V, V_1 and V_2, it can be calculated that $\|e^{(0\sim n-1)}\| \leqslant \sqrt{\frac{R}{\lambda_{\min}(M_1)S}}$, and $\|\xi^{(0\sim n-1)}\| \leqslant \sqrt{\frac{2R}{\lambda_{\min}(M_2)S}}$. Based on the definitions of the parameters R and S, we can get the conclusion that all signals of the closed-loop systems converge to adjustable boundaries.

In the following, we will prove that there is no Zeno phenomenon in the proposed **Event-O** (3) and **Event-S** (12). It can be obtained that the measurement error $e_y = y - y^e$ and $\|e_y\| \leqslant v_1(t) < \bar{v}_1$ for $\forall t \in [t_{1k}, t_{1,k+1})$. Then, it is easy to verify that $\frac{d}{dt}\|e_y\| \leqslant |\frac{d}{dt}\|e_y\|| = \|\dot{e}_y\| \leqslant \|\dot{y}\|$. Based on Theorem 1, there must exist a constant τ_y satisfying $\|\dot{y}\| \leqslant \tau_y$, which implies $\frac{d}{dt}\|e_y\| \leqslant \tau_y$. Because of $e_y(t_{1k}) = 0$ and $e_y(t_{1,k+1}) = v_{10}e^{-b_1 t_{1,k+1}} + v_{11} \geqslant v_{11}$, the minimum execution time $t_1^* = t_{1,k+1} - t_{1k}$ satisfies $t_1^* \geqslant \frac{v_{11}}{\tau_y}$. In a similar way, it can be obtained that the minimum execution time $t_{i2}^* = t_{i2,k+1} - t_{i2k}$ for **Event-S** (12) is $t_{i2}^* \geqslant \frac{v_{i22}}{\tau_{xi}}$, where τ_{xi} is the constant such that $\|\dot{\xi}^{(i)}\| \leqslant \tau_{xi}$. Therefore, the designed **Event-O** and **Event-S** will not exhibit the Zeno phenomenon.

Remark 3. Compared with the existing ETM on states [17,18], this paper proposes a trigger threshold driven by a time-varying decay function $v_i(t) = v_{i0}e^{-b_i t} + v_{i1}$. Then, the trigger condition can be adjusted adaptively. In detail, the initial trigger threshold $v_i(0) = v_{i0} + v_{i1}$ is jointly determined by parameters v_{i0} and v_{i1}, while parameter b_i governs the exponential decay rate. A larger v_{i0} increases the initial threshold magnitude to minimize unnecessary triggers at system startup, while a larger v_{i1} raises the steady-state threshold level to balance responsiveness and energy efficiency. The decay parameter b_i accelerates threshold convergence towards v_{i1}, which allows for flexible adjustment from the initial trigger threshold to the final trigger threshold. Thus, the proposed ETM can adjust the triggering threshold based on time, effectively balancing the relationship between communication frequency and system performance.

Remark 4. There have been some studies on ETC problems, their trigger mechanisms are designed for inputs, such as [11, 12, 14, 15], which greatly avoids the problem caused by the non-differentiability of triggered signals. Refs. [17, 18] established a constant ETM for states and output, and designed the state feedback event-triggered controller and output feedback event-triggered controller, respectively. However, to address the challenges posed by virtual controller differentiation, the conditions in their studies are strict, such as the partial derivative of the virtual controller needs to be a constant. To deal with the problem, we not only introduce a time-varying function in the trigger condition that allows for the adjustment of the trigger threshold based on time, but also effectively avoid the introduction of virtual controllers by means of the FAS method, which expands the application scope of the resulting ETC algorithm.

5 Simulation

In this section, the RLC circuit shown in Figure 2 is taken into account to demonstrate the efficiency of the proposed output feedback ETC algorithm.

With the help of the Kirchhoff's Laws of Voltage and Current, we can establish a mathematical model for the RLC circuit as follows:

$$LC\ddot{\xi} + \Delta(\xi, \dot{\xi}) + RC\dot{\xi} = u,$$

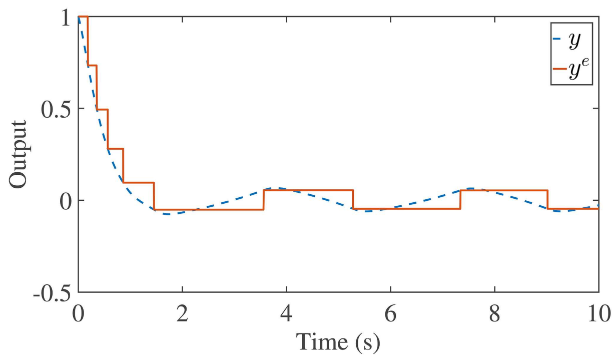

Figure 3 (Color online) Responses of y, y^e.

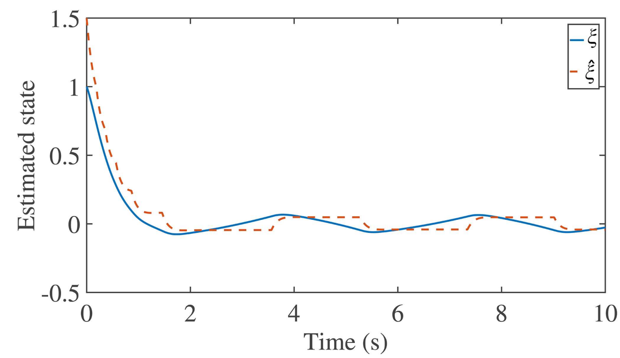

Figure 4 (Color online) Responses of ξ, $\hat{\xi}$.

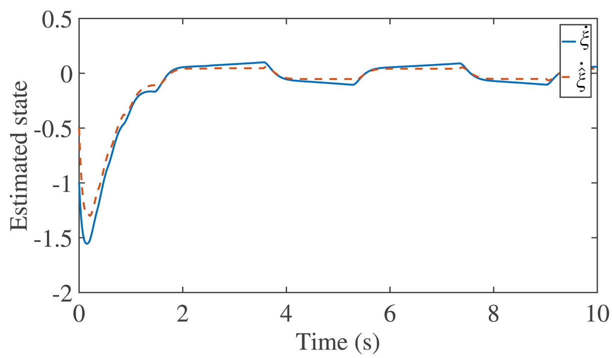

Figure 5 (Color online) Responses of $\dot{\xi}$, $\hat{\dot{\xi}}$.

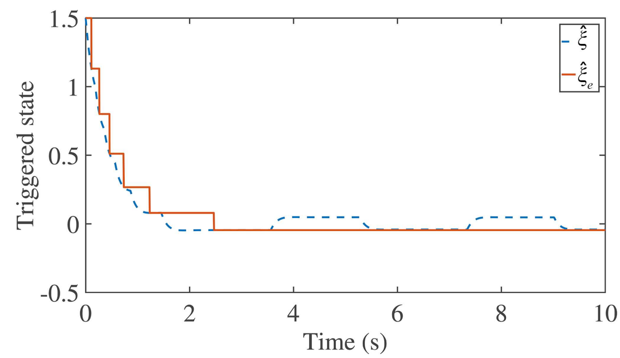

Figure 6 (Color online) Responses of $\hat{\xi}$, $\hat{\xi}_e$.

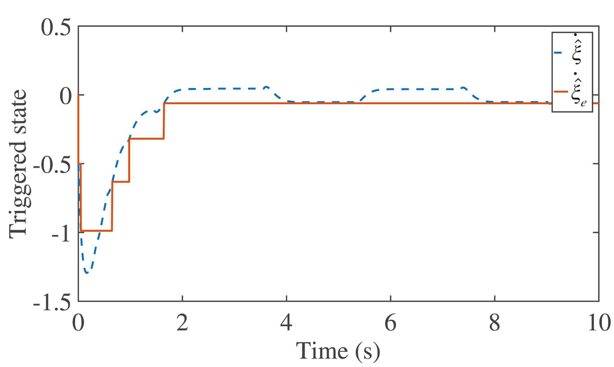

Figure 7 (Color online) Responses of $\hat{\dot{\xi}}$, $\hat{\dot{\xi}}_e$.

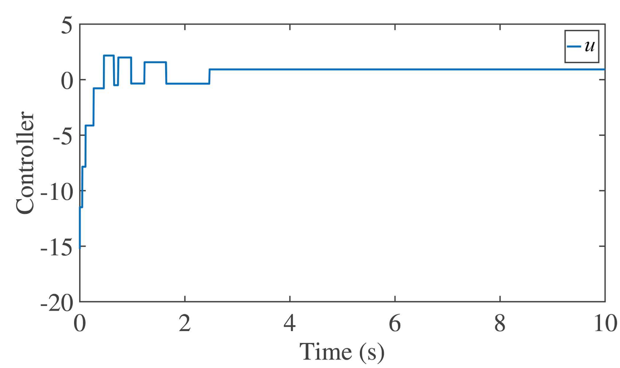

Figure 8 (Color online) Response of u.

where L is the inductance of inductor coil; C is the capacitance of capacitor; R is the resistance; $\Delta(\xi,\dot{\xi}) = -0.25\sin(\xi)$ is the uncertainty; $\xi = u_c(V)$, $\dot{\xi} = \frac{du_c}{dt}(V/t)$, $\ddot{\xi} = \frac{d^2u_c}{dt^2}(V/t^2)$, $u = u_r(V)$; $L = 1(H)$, $R = 0.5(\Omega)$, $C = 1(F)$.

The related parameters are selected as $\sigma_1 = 8$, $\sigma_2 = 6$, $k_0 = 10$, $k_1 = 8$, $v_{10} = 0.2$, $v_{11} = 0.1$, $b_1 = 1$, $v_{120} = 0.3$, $v_{122} = 0.1$, $b_{12} = 1$, $v_{220} = 0.3$, $v_{222} = 0.1$, $b_{22} = 1$. The simulation results are shown in Figures 3–9.

From Figures 3–9, it can be seen that all signals in the closed-loop systems converge to adjustable boundaries, and there is no Zeno phenomenon in the proposed ETMs. To be specific, the responses of the output y and its triggered signal y^e are given in Figure 3. Based on the TO, the responses of states ξ, $\dot{\xi}$ and their estimated states $\hat{\xi}$ and $\hat{\dot{\xi}}$ are depicted in Figures 4 and 5, which show that the proposed algorithm can effectively estimate states while reducing the number of output updates. The TESs $\hat{\xi}_e$ and $\hat{\dot{\xi}}_e$ are drawn in Figures 6 and 7, and the corresponding controller designed by them is plotted in Figure 8. Finally, the trigger times t_{1k}, t_{12k} and t_{22k} for output y^e, estimated states $\hat{\xi}_e$ and $\hat{\dot{\xi}}_e$ are provided in Figure 9, which imply that there is no Zeno phenomenon in the proposed ETMs.

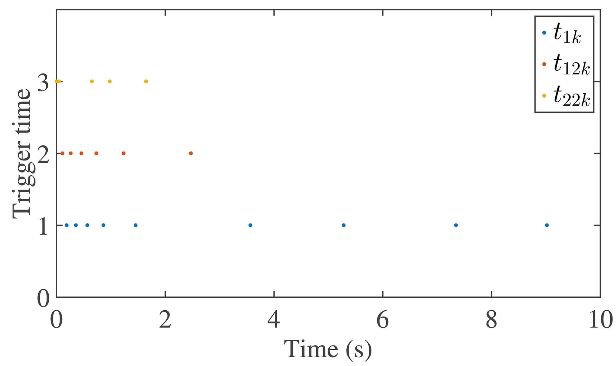

Figure 9 (Color online) Responses of t_{1k}, t_{12k}, and t_{22k}.

6 Conclusion

This paper focused on the output feedback ETC problem for HOFASs, where the output and the estimated states are triggered by the designed ETMs. The state observer that only uses the TO has been designed to achieve effective state estimation with less output information. The estimated states generated by the observer were subject to ETMs specifically designed to limit the number of estimated state updates. Then, the output feedback event-triggered controller was constructed only using the TO and TESs. However, it should be acknowledged that the considered system does not involve unknown parameters, while many practical systems have unknown uncertainties. In future, we will further investigate the adaptive output feedback ETC problem of uncertain systems.

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