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• Supplementary File •

A novel Polar-coded multipath-cluster-alignment receiver for high-data-rate long-range deep-sea acoustic communications

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Appendix A Multipath-cluster-alignment equalization and analysis

Consider a single-input multiple-output (SIMO) deep-sea acoustic communication system which consists of Mreceiving elements, where the channel impulse response (CIR) of the m-th element exhibits a clustered-sparse multipath, modeled as

$$h_m(n) = \sum_{k=1}^{K}\sum_{l=0}^{L_k-1} a_{m,k,l}\delta(n-\tau_{m,k,l}). \tag{A1}$$

where K is the number of MCs, $a_{m,k,l}$ denotes the complex amplitude of the l-th path in the k-th MC at the m-th receiving element, and $\tau_{m,k,l}$ is the corresponding time delay. The baseband signal received at the m-th element can be expressed as

$$y_m(n) = \sum_{k=1}^{K}\sum_{l=0}^{L_k-1} a_{m,k,l}a_{m,k,l}s(n-\tau_{m,k,l}) + v_m(n). \tag{A2}$$

$s(n)$ is the transmitted symbol, and $v_m(n)$ is the additive Gaussian white noise (AWGN) with a mean of zero and a variance of σ_v^2.

The CIR of k-th MC can be expressed as

$$h_{m,k}(n) = \sum_{l=0}^{L_k-1} a_{m,k,l}\delta(n-\tau_{m,k,l}). \tag{A3}$$

For the large spread channel $h_m(n)$ as a whole, a high-order, high-complexity equalizer would be generally required. Since the delay spread inside each MC is significantly smaller, it would be advantageous to decompose the high-order equalizer into K sub-equalizers, each of which addresses an individual MC in parallel by aligning its input to the corresponding MC (see in Figure C1).

Specifically, MCs in the channel $h_m(n)$ are extracted by identifying the most prominent K peaks. The delay $\tau_{m,k}$ for each MC is calculated by cross-correlating the received signal and the known preamble. Aligning the k-th MC with $\tau_{m,k}$, the input to the k-th sub-equalizer is calculated from the contribution of the aligned k-th MC and the other non-aligned MCs, through the following equation

$$\begin{aligned} y^{\mathrm{al}}_{m,k}(n) &= y_m(n) * \delta(n+\tau_{m,k}) \\ &= \{s(n) * h_m(n) + v_m(n)\} * \delta(n+\tau_{m,k}) \\ &= s(n) * h^{\mathrm{al}}_{m,k}(n) + \sum_{i=1,i\neq k}^{K} s(n+\tau_{m,k}) * h_{m,i}(n) \\ &\quad + v_m(n+\tau_{m,k}) \\ &= s(n) * h^{\mathrm{al}}_{m,k}(n) + v'_{m,k}(n). \end{aligned} \tag{A4}$$

where $h^{\mathrm{al}}_{m,k}(n)$ denotes the aligned CIR of the k-th MC, and $v'_{m,k}(n)$ includes ISI from other MCs and shifted AWGN, which can be assumed to be independent additive noise. This is because the delay between adjacent MCs is larger than the order of sub-equalizer itself, with variance $\sigma^2_{v'} \approx \sigma_v^2 + \sum_{i=1,i\neq k}^{K}\sum_{l=0}^{L_i-1}|a_{m,i,l}|^2$. Thus, the D-order weight coefficients $\mathbf{w}^H_{m,k}$ of each sub equalizer could be iteratively optimized by using mainstream adaptive algorithm, such as the recursive least squares (RLS) algorithm, by minimizing the corresponding error function between the known training sequence $s_{TD}(n)$ and the combined output of all sub-equalizers along m and k, expressed as

$$e(n) = s_{TD}(n) - \sum_{m=1}^{M}\sum_{k=1}^{K}\hat{s}_{m,k}(n). \tag{A5}$$

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where

$$\hat{s}_{m,k}(n) = \mathbf{w}_{m,k}^H \mathbf{y}_{m,k}^{al}. \tag{A6}$$

Compared to the mainstream equalization manner (Direct adaptive RLS) that generally needs a filter order $D \geqslant \Delta_\tau$, corresponding to a complexity of $\mathcal{O}(MD^2)$, to directly address the whole multipath channel $h_m(n)$ with a CIR spanning $\Delta_\tau = (\tau_{m,K,L_k-1} - \tau_{m,1,0})$, the proposed MC-alignment equalizer is equivalent to decomposing the DA-RLS equalizer into multiple compact sub-equalizers that need a smaller $D' \geqslant \Delta_{\tau'}$, corresponding to a much lower complexity of $\mathcal{O}(MKD'^2)$, to address a simple MC response spanning a duration of $\Delta_{\tau'} = (\tau_{m,k,L_k-1} - \tau_{m,k,0})$.

According to the steady-state excess mean square error (EMSE) of the RLS algorithm [1], the output SNR (OSNR) of the mainstream multi-channel DA-RLS equalizer is given by

$$OSNR_{CM} = \frac{G_{opt}}{\xi_{min}(1 + M_{global})} + \frac{\sigma_v^2}{G_{div}}. \tag{A7}$$

where G_{div} is spatial diversity gain given by Brennan's formula, G_{opt} is the optimal spatial matching gain, and M_{global} is the global misalignment of RLS algorithm respectively, expressed as

$$\begin{aligned} G_{div} &= M \cdot \frac{1-\rho}{1+(M-1)\rho} \\ G_{opt} &= \sigma_s^2 \cdot |\mathbf{h}_m^H \mathbf{R}_{ss} \mathbf{h}_m|^2 \\ M_{global} &= \frac{1-\lambda}{1+\lambda} D + \frac{\sigma_v^2 \, \mathrm{tr}(\mathbf{R}_{\mathrm{ss}}^{\text{-}1})}{D \cdot \xi_{min}} + \frac{\kappa(\mathbf{H})}{M} \cdot \frac{Var(\mathbf{w}_m)}{\|\mathbf{w}_{opt}\|^2}. \end{aligned} \tag{A8}$$

where λ represents the forgetting factor, $\mathbf{R}_{ss}$ is the autocorrelation matrix of the input signals, ξ_{min} is its minimum eigenvalue.ρ represents the average channel correlation coefficient, $\kappa(\mathbf{H})$ is the condition number of the channel matrix $\mathbf{H}$, and $\mathbf{w}_{opt}$ is the optimal weight of equalizer. Obviously, under the mainstream equalizer structure that addressing the multipath as a whole, the presence of multipath, equivalent to correlated noise, greatly degrades G_{div}, G_{opt} and M_{global}. Moreover, a large D required to match the large spread deep-sea channel degrade the M_{global}. All of these factors reduce the OSNR.

The OSNR of the MC-alignment equalizer can be expressed as

$$OSNR_{MA} = \frac{G_{opt}^{'} \cdot G_{MA}}{\xi_{min}(1 + M_{global}^{'})} + \frac{\sigma_{v'}^2}{G_{div}^{'} \cdot G_{MA}}. \tag{A9}$$

where $G_{opt}^{'} = \sigma_s^2 \cdot |\mathbf{h}_{m,k}^H \mathbf{R}_{ss} \mathbf{h}_{m,k}|^2$,$G_{div}^{'} = MK \cdot \frac{1-\rho}{1+(MK-1)\rho}$, and G_{MA} is the MC diversity gain, expressed as

$$G_{MA} = \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{l=0}^{L_k-1} |a_{m,k,l}|^2. \tag{A10}$$

the corresponding M_{global} could be written as

$$M_{global}^{'} = \frac{1-\lambda}{1+\lambda} D' + \frac{\sigma_{v'}^2 \, \mathrm{tr}(\mathbf{R}_{\mathrm{ss}}^{\text{-}1})}{D' \cdot \xi_{min}} + \frac{\kappa(\mathbf{H})}{M} \cdot \frac{Var(\mathbf{w}_{m,k})}{\|\mathbf{w}_{opt}\|^2}. \tag{A11}$$

Different from the mainstream equalizer structure, as each sub-equalizer of the proposed MC-alignment equalizer is designed to address an individual MC, only a much smaller D' is needed. Meanwhile, interference caused by other MCs could be formulated as uncorrected noise, as for deep-sea channel the delay between different MCs is much larger than the equivalent duration of the small D', thus equivalent to decorrelating the input signals. As a result, a higher $G_{opt}^{'}$ is obtained. Meanwhile, a much smaller D' also contribute to improve $M_{global}^{'}$ as given in Eq.$(A11)$.

Appendix B Polar encoding scheme

To incorporate with the MC-alignment equalizer to achieve high-data-rate long-range deep-sea acoustic communication, a low complexity Polar coding scheme is designed to accommodate low input SNR caused by long-range, as well as multipath uncertainty caused by limited element number. Specifically, with a list of L and a codelength of N, the Successive Cancellation List (SCL) decoding algorithm that has a complexity of $\mathcal{O}(LN \log N)$ is adopted [2]. Different from mainstream research that relied on a large L for the SCL algorithm to handle lower SNR at the price of an increasing decoding complexity, processing gain yielded with the proposed MC-alignment equalizer would enable the adoption of a low complexity SCL decoding with a small L.

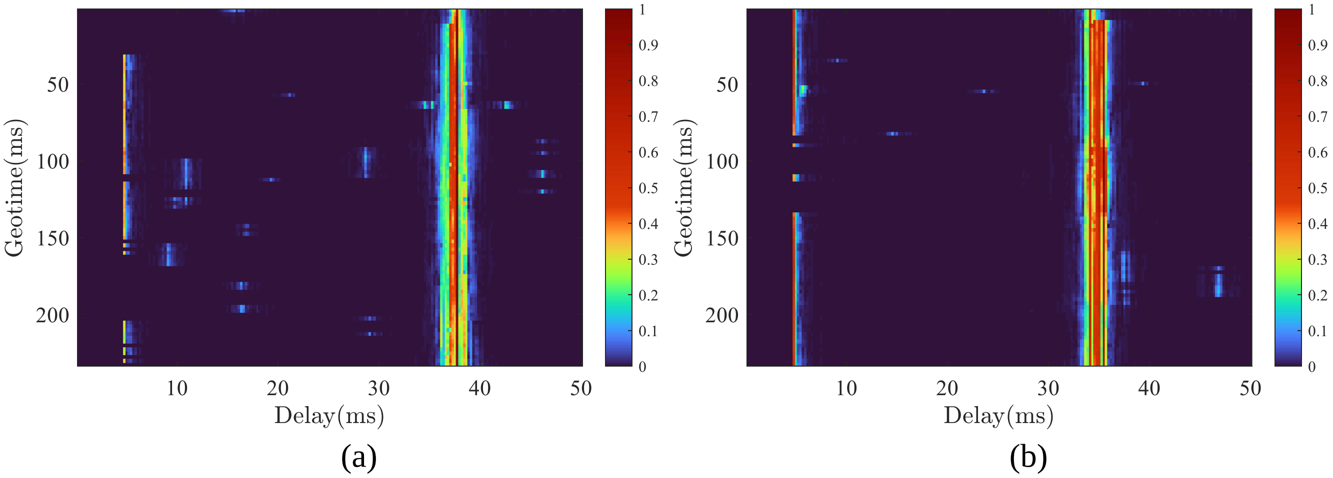
Appendix C Deep-sea field test

The specific parameters of the deep-sea acoustic communication system are provided in Table C1. Each data packet consists of a 100-symbol preamble followed by a 1024-symbol data payload, with no interval inserted between them, i.e., corresponding to a training overhead of 8.9%. The preamble simultaneously acts as synchronization, training sequence, MC detection and Doppler estimation.

Table C1 Deep-sea acoustic communication system parameters.

Parameters	Value
Sampling rate	48kHz
Carrier frequency	6kHz
Bandwidth	4kHz
Modulation scheme	QPSK
Number of training symbols	100 (25ms)
Number of data symbols	1024
Coding type	Polar code
Coderate	1/2
Coding length	2048
Decoding algorithm	SCL

The proposed dual-channel MC-alignment equalizer, along with mainstream dual-channel RLS-driven equalizers (including small/large order, different sparsity exploitation, and linear/decision-feedback structures), are adopted to process the received signals based on a 12-frame of each receiving element transmission comprising total 26, 976 QPSK symbols.Afterward, the equalizer output is fed into the subsequent SCL Polar decoder to obtain the final receiver output. With Doppler effects addressed using an established approach, the communication results are presented in Table C2, using two frames from receiving elements 1 and 2, as shown in Figure C1 C1(a) and (b), respectively, to illustrate the specific parameters selection in second column of Table C2 . The results indicate that the proposed equalizer achieves error-free performance after Polar decoding, while all reference mainstream equalizers fail to achieve effective communication, even with Polar decoding. The reason is that the received SNR is obviously lower than that generally needed for the mainstream equalizer to obtain enough processing gain to accommodate the Polar decoding.



(a) (b)

Figure C1 The measured CIRs. (a) receiving element1. (b) receiving element2

Table C2 The dual-channel RLS-driven equalizers

Equalizers	Parameters&Values	OSNR(dB)	Orginal BER	Polar decoded BER
RLS [1]	D = 5; $\lambda = 0.9999$; $\delta = 0.01$	-0.041	3.5×10^{-1}	F1)
	D = 100; $\lambda = 0.999$;$\delta = 1$	-2.18	4.8×10^{-1}	F
l_0-constrained-RLS [3]	D = 5; $\gamma = 1$;$\lambda = 0.9999$;$\delta = 0.01$	-0.094	3.5×10^{-1}	F
	D = 100; $\gamma = 1$;λ= 0.999;$\delta = 1$	-2.35	4.8×10^{-1}	F
Proportionate-RLS [3]	D = 5; $\alpha = 0$; $\mu = 10$; $\lambda = 0.9999$;$\delta = 0.1$	0.001	3.3×10^{-1}	F
	D = 100; $\alpha = 0$; $\mu = 10$; $\lambda = 0.9999$;$\delta = 0.1$	-0.38	4.8×10^{-1}	F
RLS-DFE [4]	ff_D2)=5; fb_D=2; λ=0.999;$\delta = 0.1$	-0.99	4.1×10^{-1}	F
	ff_D=100; fb_D=30; λ=0.9999;$\delta = 10$	-2.68	4.9×10^{-1}	F
MCs_JPE [5]	D = 5; $\lambda = 0.9999$;$\rho = 0.01$;$\sigma = 0.35$;$mu = 100$;$\alpha = 0$;	2.93	1.5×10^{-1}	F
MC-alginment	D = 5; $\lambda = 0.9999$;$\delta = 0.1$	4.54	8.3×10^{-2}	0

1) F: Polar Decoded BER lg 1×10^{-1} 2) ff_D: Feedforward filter length; fb_D: Feedback filter length 3) The parameters for all reference equalizers were determined via a systematic grid search along maximizing the Output SNR (OSNR).

In accordance with the parameters of all equalizers presented in Table C2, it is evident that a smaller filter order enhances both OSNR and original BER performance. Consequently, a smaller filter order (e.g., D=5) was selected, and the parameters of each equalizer were tuned accordingly. The average original BER and OSNR were computed based on a 12-frame transmission consisting of 26,976 QPSK symbols, which ensured robust statistical support and reproducibility of the experimental results. As shown in Table C3, all reference mainstream equalizers failed to achieve effective communication, even when Polar decoding was employed. In contrast, by leveraging large-delay MC diversity, the proposed receiver successfully achieved error-free communication across all 12 frames. Under low SNR conditions, where the average SNR for receiving elements 1 and 2 was measured at 3.08 dB and 1.71 dB, respectively, the proposed receiver outperformed the nonlinear optimized equalizer, which was only capable of achieving error-free communication over 6 frames.

Table C3 The dual-channel RLS-driven equalizers for 12 frames

	RLS	l_0-constrained-RLS	PRLS	RLS-DFE	MCs_JPE	MC-alginment
Average of OSNR (dB)	-1.18	-1.22	-1.65	-1.74	3.41	4.31
Average of original BER	3.4×10^{-1}	3.5×10^{-1}	3.4×10^{-1}	3.6×10^{-1}	1.3×10^{-1}	9.2×10^{-2}
Number of error-free frames	0	0	0	0	6	12

References

1 Sayed A H. Adaptive filters[M]. John Wiley & Sons, 2011.

2 Tal I, Vardy A. List decoding of polar codes[J]. IEEE transactions on information theory, 2015, 61(5): 2213-2226.

3 Zhu Z, Tong F, Zhou Y. Enhanced sparsity-aware rls algorithm for efficient estimation of time-varying sparse system[J]. IEEE Transactions on Circuits and Systems II: Express Briefs, 2023, 70(12): 4589-4593.

4 Stojanovic, Milica, Josko Catipovic, and John G. Proakis. "Adaptive multichannel combining and equalization for underwater acoustic communications." The Journal of the Acoustical Society of America 94.3 (1993): 1621-1631.

5 Zhao Y, Jiang F T W, Zhou Y, et al. SpatialTemporal Multi-Path Clusters Joint Equalization for DeepSea Acoustic Communication in Large Delay Spread Channels[J]. IEEE Internet of Things Journal, 2024.