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Predefined-time-synchronized control for Euler-Lagrange systems

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Settling time is one of the most concerning topics in control technology [1]. Despite the settling time of the overall system, the individual settling time of each state element is likewise critical to the overall system performance. In some applications, multiple state elements of a system (or multiple states of cooperative systems) are demanded to work together in the perspective of time and reach some desired value simultaneously. Furthermore, special tasks may have a narrow time window, such that the settling time of both the overall system and each state element should converge in a small time period.

To this end, we proposed predefined-time-synchronized control, in which both the predefined convergence [2] of the system and the time-synchronized arrival [3] of each state element are considered. The definition of predefined-time-synchronized stability is proposed with three categories according to specific convergence requirements, and the corresponding Lyapunov conditions and controller designs are presented. True-predefined-time-synchronized stability is achieved, where every state element converges precisely at the given time. Moreover, the disturbance rejection problem is considered; thus the robustness of the system can be guaranteed.

Problem formulation. Consider an Euler-Lagrange system in the disturbed form [4],

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau(t) + d(t), \tag{1}$$

where $q \in \mathbb{R}^n$ is the state vector, $\tau(t)$ is the control input, and $d(t)$ is the time-varying low-frequency disturbance. System matrices $M(q), C(q,\dot{q})$, and $g(q)$ represent the inertia matrix, the centrifugal and Coriolis forces, and the gravitational force, respectively. The system matrices are continuous in terms of both state x and time t.

Definition 1. Given a predefined time constant T_c, system $\dot{x} = f(x,t)$ is said to be the following:

- predefined-time-synchronized stable (PTSS) if the settling time T satisfies $T \leqslant T_c$ and every non-zero element of the state x reaches the equilibrium simultaneously at T;
- true-predefined-time-synchronized stable (true-PTSS) if every non-zero element of the state x reaches the origin simultaneously at $T = T_c$;
- ε-predefined-time-synchronized stable (ε-PTSS) if the initially non-zero state element $x_i, i = 1, \cdots n$ satisfies $x_i > 0$ for $t < T_c$ and $\|x\| \leqslant \varepsilon$ for $t \geqslant T_c$, where ε is a small positive constant.

Assumption 1. The initial value and the first-order derivative of the disturbance $d(t)$ are both bounded. Mathematically, $\|d(0)\| \leqslant \sigma$ and $\|\dot{d}(t)\| \leqslant \delta$.

PTSS control. The PTSS controller for system (1) is designed with $d(t) = 0$. Define a reference sliding-mode surface s,

$$s = \dot{q} + \zeta_1 \left(p_1 \mathrm{sig}_{\mathrm{n}}^{\alpha_1}(q) + g_1 \mathrm{sig}_{\mathrm{n}}^{\beta_1}(q) \right), \tag{2}$$

where ζ_1 and $\mathrm{sig}_{\mathrm{n}}(x)$ have the following formulations:

$$\zeta_1 = \lambda\left(p_1, g_1, 1, (\alpha_1+1)/2, (\beta_1+1)/2, 2T_c\right), \tag{3}$$

$$\lambda(p,g,k,\alpha,\beta,T_c) = \frac{p^k(1-k\alpha) + g^k(k\beta-1)}{p^k g^k T_c (1-k\alpha)(k\beta-1)}, \tag{4}$$

$$\mathrm{sign}_{\mathrm{n}}(x) \triangleq \frac{x}{\|x\|}, \ \mathrm{sig}_{\mathrm{n}}^{\alpha}(x) \triangleq \|x\|^{\alpha} \mathrm{sign}_{\mathrm{n}}(x). \tag{5}$$

The control input τ is designed to drive s to zero,

$$\begin{aligned}\tau(t) = & -M(q)\left[\zeta_2\left(p_2 \mathrm{sig}_{\mathrm{n}}^{\alpha_2}(s) + g_2 \mathrm{sig}_{\mathrm{n}}^{\beta_2}(s)\right) + \zeta_1 \theta(q)\dot{q}\right] \\ & + C(q,\dot{q})\dot{q} + g(q),\end{aligned} \tag{6}$$

where s is presented in (2). ζ_2 and $\theta(q): \mathbb{R}^n \to \mathbb{R}^n \times \mathbb{R}^n$ are formulated as

$$\zeta_2 = \lambda\left(p_2, g_2, 1, (\alpha_2+1)/2, (\beta_2+1)/2, 2\bar{T}_c\right), \tag{7}$$

$$\begin{aligned}\theta(q) = & p_1(\alpha_1-1)\|q\|^{\alpha_1-3} q_1 q_1^{\mathrm{T}} + p_1\|q\|^{\alpha_1-1} I_n \\ & + g_1\|q\|^{\beta_1-1} I_n + g_1(\beta_1-1)\|q\|^{\beta_1-3} q_1 q_1^{\mathrm{T}},\end{aligned} \tag{8}$$

where $\bar{T}_c < T_c$ is a time constant and I_n is the identity matrix. The stability analysis can be found in Appendix A.

True-PTSS control. The Lyapunov condition for true-PTSS control is presented. Then the true-PTSS controller for system (1) is designed with $d(t) = 0$.

Theorem 1. System $\dot{x} = f(x,t)$ is true-PTSS with predefined settling time T_c if the following satisfies.

(1) There exist a continuous function $\mu(t) : [0, T_c) \to \mathbb{R}$ and a class K-function $W(t, \|x\|) : [0, T_c) \times \mathbb{R}_+ \to \mathbb{R}_+$ with respect to

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$\|x\|$, such that $\forall x \in \mathbb{R}^n$ and $\forall t \in [0, T_c)$,

$$\dot{W}(t, \|x\|) = \mu(t) \left(e^{W(t,\|x\|)} - 1\right) / e^{W(t,\|x\|)}, \tag{9}$$

and the function $\rho(t) = -\int_0^t \mu(s)\mathrm{d}s$ satisfies

$$\lim_{t\to T_c^-} \rho(t) = +\infty \quad \text{and} \quad \rho(t)|_{t<T_c} < +\infty. \tag{10}$$

(2) x is ratio persistent with $x/\|x\| = \zeta f(x,t)/\|f(x,t)\|$ for $x \neq 0$ and $\zeta \in \{1, -1\}$.

Let the sliding-mode surface be

$$s = \begin{cases} \dot{q} + \frac{p_1\left(1-e^{-\|q\|^{\alpha_1}}\right)}{T_c - t}\mathrm{sig}_{\mathrm{n}}^{1-\alpha_1}(q), & \text{if } 0 \leqslant t < T_c, \\ 0, & \text{otherwise}, \end{cases} \tag{11}$$

where $p_1 > 0$ and $0 < \alpha_1 < 1$.

The controller is designed as

$$\tau(t) = \begin{cases} M(q)p_2\left[1 - e^{-\|s\|^{\alpha_2}}\right]\mathrm{sig}_{\mathrm{n}}^{1-\alpha_2}(s)/(\bar{T}_c - t) + g(q), \\ -M(q)\mu_1(t)[\theta_1(q) + \theta_2(q)\dot{q}] + C(q,\dot{q})\dot{q}, \ \text{if } t \in [0, \bar{T}_c), \\ C(q,\dot{q})\dot{q} + g(q), \ \text{otherwise}, \end{cases}$$

where $0 < \alpha_2 < 1, p_2 > 0, \bar{T}_c < T_c, \mu_1(t) = -\alpha_1 p_1/(T_c - t), \theta_1(q)$ and $\theta_2(q)$ are formulated as

$$\begin{aligned} \theta_1(q) =& \eta(q)\mathrm{sig}_{\mathrm{n}}^{1-\alpha_1}(q)/(T_c - t), \\ \theta_2(q) =& \eta(q)\left(\|q\|^{-\alpha_1}I_n - \alpha_1\|q\|^{-\alpha_1-2}qq^{\mathrm{T}}\right) + e^{-\|q\|^{\alpha_1}}qq^{\mathrm{T}}, \\ \eta(q) =& 1 - e^{-\|q\|^{\alpha_1}}. \end{aligned}$$

The proof and stability analysis can be found in Appendix B.

ε-PTSS control. We introduce the theorem for ε-PTSS.

Theorem 2. System $\dot{x} = f(x,t)$ is ε-PTSS within T_c if the state x varies in the following manner:

$$\dot{x} = \begin{cases} -\frac{p\left(1-e^{-\|x\|^{\alpha}}\right)}{T_c + \Delta T - t}\mathrm{sig}_{\mathrm{n}}^{1-\alpha}(x), & \text{if } t < T_c, \\ -\frac{p\left(1-e^{-\|x\|^{\alpha}}\right)}{\Delta T}\mathrm{sig}_{\mathrm{n}}^{1-\alpha}(x), & \text{otherwise}, \end{cases} \tag{12}$$

where $p > 0$ and $0 < \alpha < 1$. Let $x(0)$ be the initial state at $t = 0$. ΔT is calculated as

$$\Delta T = \left(\frac{e^{\varepsilon^{\alpha}} - 1}{e^{\|x(0)\|^{\alpha}} - 1}\right)^{-\frac{1}{\alpha p}}. \tag{13}$$

When $d(t)$ in (1) exists, the disturbance should be estimated before the predefined time T_c. A disturbance observer is designed, which converges to the true value before T_c.

$$\begin{aligned} \dot{z}_0 =& -k_{z1}\mathrm{sig}_{\mathrm{c}}^{\frac{1}{2}}(\tilde{z}_0) - k_{z2}\mathrm{sig}_{\mathrm{c}}^{\nu}(\tilde{z}_0) + z_1 \\ &+ M^{-1}(q)\left(\tau(t) - C(q,\dot{q})\dot{q} - g(q)\right), \end{aligned} \tag{14}$$

$$\dot{z}_1 = -k_{z3}\mathrm{sign}_{\mathrm{c}}(\tilde{z}_0), \tag{15}$$

where $\nu > 1$ is a constant parameter, $k_{zi} > 0$ with $i = 1, 2, 3$ and z_0 and z_1 are the estimated values of $\dot{q}$ and $M^{-1}(q)d(t)$.

The ε-PTSS controller is designed on the basis of Theorem 2. A sliding-mode controller is constructed,

$$s = \begin{cases} \dot{q} + \frac{p_1\left(1-e^{-\|q\|^{\alpha_1}}\right)}{T_c + \Delta T - t}\mathrm{sig}_{\mathrm{n}}^{1-\alpha_1}(q), & \text{if } t < T_c, \\ \dot{q} + \frac{p_1\left(1-e^{-\|q\|^{\alpha_1}}\right)}{\Delta T}\mathrm{sig}_{\mathrm{n}}^{1-\alpha_1}(q), & \text{otherwise}, \end{cases} \tag{16}$$

where $p_1 > 0$ and $0 < \alpha_1 < 1$. And ΔT can be calculated as in (13).

The ε-PTSS controller is designed as

$$\begin{aligned} \tau(t) =& -M(q)\left[\theta_3(q) + \theta_4(q)\dot{q}\right] + C(q,\dot{q})\dot{q} + g(q) - M(q)z_1 \\ &- \lambda_{\varepsilon}M(q)\left(p_2\mathrm{sig}_{\mathrm{n}}^{\alpha_2}(s) + g_2\mathrm{sig}_{\mathrm{n}}^{\beta_2}(s)\right), \end{aligned} \tag{17}$$

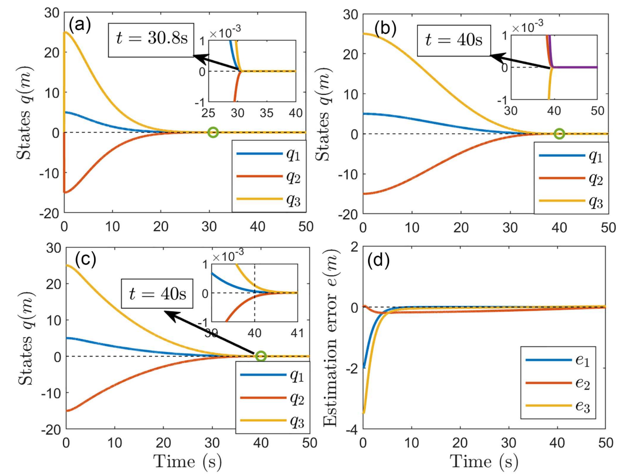

Figure 1 (Color online) Simulation results of PTSS (a), true-PTSS (b), ε-PTSS (c), and the estimation error of ε-PTSS (d).

where $\lambda_{\varepsilon} = \lambda\left(p_2, g_2, 1, (\alpha_2+1)/2, (\beta_2+1)/2, 2T_c\right)$, $\alpha_2 > 1$, $\beta_2 < 1$, p_2, and g_2 are positive parameters, and $\theta_3(q)$ and $\theta_4(q)$ are formulated as

$$\begin{aligned} \theta_3(q) =& \eta_1\mathrm{sig}_{\mathrm{n}}^{1-\alpha_1}(q), \\ \theta_4(q) =& \eta_2\left(\|q\|^{-\alpha_1}I_n - \alpha_1\|q\|^{-\alpha_1-2}qq^{\mathrm{T}}\right) + \eta_2\alpha_1 e^{-\|q\|^{\alpha_1}}qq^{\mathrm{T}}, \end{aligned}$$

$$\eta_1 = \begin{cases} -\alpha_1 p_1\left(1 - e^{-\|q\|^{\alpha_1}}\right)/(T_c - \Delta T - t)^2, \ \text{if } t < T_c, \\ -\alpha_1 p_1\left(1 - e^{-\|q\|^{\alpha_1}}\right)/\Delta T^2, \ \text{otherwise}, \end{cases}$$

$$\eta_2 = \begin{cases} \alpha_1 p_1\left(1 - e^{-\|q\|^{\alpha_1}}\right)/(T_c - \Delta T - t), \ \text{if } t < T_c, \\ \alpha_1 p_1\left(1 - e^{-\|q\|^{\alpha_1}}\right)/\Delta T, \ \text{otherwise}. \end{cases}$$

The proof of Theorem 2, convergence analysis of the disturbance observer and the ε-PTSS controller can be found in Appendix C.

Numerical simulations. We use the satellite in the local-vertical-local-horizontal rotating frame for controller verification [5]. Figure 1 illustrates the simulation results, where $T_c = 40$ s is set for true-PTSS and ε-PTSS control. The predefined-time-synchronized property can be clearly found in these plots. Please refer to Appendix D for more details.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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