

Adaptive consensus control of heterogeneous multi-agent time-delay systems with motor backlash compensator and its application

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Received 23 April 2025/Revised 29 August 2025/Accepted 15 October 2025/Published online 15 January 2026

Citation Li H, Zhang X Y, Li Z, et al. Adaptive consensus control of heterogeneous multi-agent time-delay systems with motor backlash compensator and its application. *Sci China Inf Sci*, 2026, 69(4): 149202, <https://doi.org/10.1007/s11432-025-4730-y>

Heterogeneous multi-agent systems (MAS) are increasingly moving toward adaptive, multi-modal collaborative frameworks. However, maintaining closed-loop stability remains a significant challenge due to the complex interactions between heterogeneous dynamics, actuator nonlinearities, and stochastic communication delays. Current solutions have notable limitations. For example, many rely on assumptions of homogeneity, which makes them unsuitable for heterogeneous systems. Additionally, strict requirements on communication delays, especially their variability and stochastic nature, greatly limit the practical applicability and flexibility of these methods [1, 2]. Moreover, actuator nonlinearities such as backlash, combined with a heavy reliance on precise models or limited compensation strategies, result in a performance gap between theoretical models and real-world implementations. This underscores the pressing need for more robust, practical, and adaptable cooperative control approaches.

To address the above issues, a new consensus control method based on an observer is proposed for the backlash nonlinearity input control problem in heterogeneous time-delay multi-agent systems. The main contributions are summarized as follows. (1) A state observer is developed using the finite cover lemma and radial basis function neural networks (RBFNNs) to simultaneously estimate unknown time-delay nonlinear functions and unmeasurable states. This approach removes the strict assumptions on time-delay functions in traditional Lyapunov-Krasovskii methods in [3], reduces conservatism, and provides a unified state observation framework for heterogeneous time-delay systems. (2) Unlike [4], which develops a compensator based on linear parameterization, we propose a distributed control law incorporating a backlash inverse compensator. This compensator mitigates the effects of non-smooth nonlinear actuators without requiring an exact inverse model, thereby enabling collaborative consensus control in heterogeneous multi-agent systems. (3) A collaborative platform comprising unmanned ground vehicles (UGVs), a wheeled mobile robot, and quadrotor unmanned aerial vehicles (UAVs) is constructed. Comparative experiments on motor backlash elimina-

tion, conservatism reduction in time-delay handling, and dynamic surface control with command-filtered compensation verify the effectiveness of the proposed methods.

Problem formulation. Consider a class of multi-agent systems composed of N nonlinear time-delay systems with backlash inputs, and the dynamics of the j -th agent can be described by

$$\begin{cases} \dot{x}_{j,i} = x_{j,i+1} + f_{j,i}(\bar{x}_{j,i}, \bar{x}_{j,i\tau}) + d_{j,i}(t), \\ \dot{x}_{j,n} = b_{j,0}w(u_j) + f_{j,n}(\bar{x}_{j,n}, \bar{x}_{j,n\tau}) + d_{j,n}(t), \\ y_j = x_{j,1}, i = 1, \dots, n-1, j = 1, \dots, N \end{cases} \quad (1)$$

with $\bar{x}_{j,i} := [x_{j,1}, x_{j,2}, \dots, x_{j,i}]^T \in \mathbb{R}^i$ denoting the state vector, and $\bar{x}_{j,i\tau} := [x_{j,1}(t - \tau_{j,1}), x_{j,2}(t - \tau_{j,2}), \dots, x_{j,i}(t - \tau_{j,i})]^T$ representing the delayed state vector. The unknown smooth nonlinear functions $f_i(\bar{x}_i, \bar{x}_{i\tau})$, $i = 1, \dots, n$ contain unknown time-delay constants $\tau_{j,i}$. $d_{j,i}(t)$ denotes bounded disturbances. $w(u_j)$ describes the actuator output under backlash effects, driven by control input $u_j(t)$.

Preliminaries. The assumptions for the model are listed in Appendix B. Detailed descriptions of the algebraic graph theory used for communication modeling can be found in Appendix A. The Prandtl-Ishlinskii model and its approximated inverse compensator for backlash compensation are described in Appendix C. The time-delay compensating RBFNNs-based state observer design is elaborated in Appendix E.

Main results. The structure of the proposed control scheme is shown in Figure 1(b). Correspondingly, the control algorithms are designed in Figure 1(a), with the notations of design parameters being given in Appendix D.

Theorem 1. For the control system that is composed of the system (1) with time-delay functions and backlash input nonlinearity, the compensation signals (T1.9), (T1.13) and (T1.17), the adaptive laws (T1.7), (T1.8), (T1.12), (T1.15) and (T1.16), and the designed controller (T1.14), the following conclusion can be derived.

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$$\begin{aligned}
\nu_{j,1} &= \sum_{k=1}^N a_{jk} (\hat{y}_j - \hat{y}_k) + \mathcal{B}_j (\hat{y}_j - y_r) & (T1.1) \\
\nu_{j,i} &= \hat{x}_{j,i} - \varsigma_{j,i}, i = 2, \dots, n & (T1.2) \\
\eta_{j,1} &= \nu_{j,1} - \mathcal{H}_j q_{j,1} & (T1.3) \\
\eta_{j,i} &= \nu_{j,i} - q_{j,i}, i = 2, \dots, n & (T1.4) \\
\varsigma_{j,i+1} + \iota_{j,i} \varsigma_{j,i+1} &= \alpha_{j,i}, \quad \varsigma_{j,i+1}(0) = \alpha_{j,i}(0) & (T1.5)
\end{aligned}$$

Step 1 ($i = 1$)

$$\begin{aligned}
\alpha_{j,1} &= \frac{1}{\mathcal{H}_j} (\mathcal{B}_j \hat{y}_r - \hat{\vartheta}_{j,L}^T \psi_{j,L} (\bar{x}_{k,1}, \bar{x}_{k,1\tau})) - c_{j,1} \nu_{j,1} \\
&\quad - \hat{\vartheta}_{j,1}^T \psi_{j,1} (\bar{x}_{j,1}, \bar{x}_{j,1\tau}) - k_{j,1} (y_j - \hat{y}_j) & (T1.6)
\end{aligned}$$

$$\dot{\hat{\vartheta}}_{j,1} = \tau_{j,1} (\eta_{j,1} \psi_{j,1} (\bar{x}_{j,1}, \bar{x}_{j,1\tau}) - \gamma_{j,1} \hat{\vartheta}_{j,1}) \quad (T1.7)$$

$$\dot{\hat{\vartheta}}_{j,L} = \frac{\tau_{j,L}}{\mathcal{H}_j} (\eta_{j,L} \psi_{j,L} (\bar{x}_{i,1}, \bar{x}_{i,1\tau}) - \gamma_{j,L} \hat{\vartheta}_{j,L}) \quad (T1.8)$$

$$\dot{q}_{j,1} = -c_{j,1} q_{j,1} + q_{j,2} + \varsigma_{j,2} - \alpha_{j,1} \quad (T1.9)$$

Step i ($i = 2, \dots, n-1$)

$$\begin{aligned}
\alpha_{j,i} &= -c_{j,i} \nu_{j,i} - \hat{\vartheta}_{j,i}^T \psi_{j,i} (\bar{x}_{j,i}, \bar{x}_{j,i\tau}) \\
&\quad - k_{j,i} (y_j - \hat{y}_j) + \dot{\varsigma}_{j,i} - \nu_{j,i-1} & (T1.10)
\end{aligned}$$

$$\dot{\hat{\vartheta}}_{j,i} = \tau_{j,i} (\eta_{j,i} \psi_{j,i} (\bar{x}_{j,i}, \bar{x}_{j,i\tau}) - \gamma_{j,i} \hat{\vartheta}_{j,i}) \quad (T1.11)$$

$$\dot{q}_{j,i} = -c_{j,i} q_{j,i} + q_{j,i+1} - q_{j,i-1} + \varsigma_{j,i+1} - \alpha_{j,i} \quad (T1.12)$$

Step n ($i = n$)

$$u_{j,d} = \hat{q}_j u_{j,\varrho} \quad (T1.13)$$

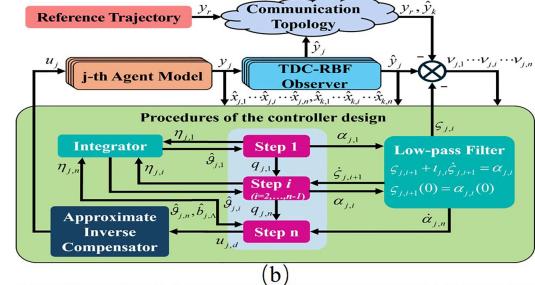
$$\begin{aligned}
u_{j,\varrho} &= -c_{j,n} \nu_{j,n} - \hat{\vartheta}_{j,n}^T \psi_{j,n} (\bar{x}_{j,n}, \bar{x}_{j,n\tau}) \\
&\quad - k_{j,n} (y_j - \hat{y}_j) + \dot{\varsigma}_{j,n} - \nu_{j,n-1} & (T1.14)
\end{aligned}$$

$$\dot{\hat{\vartheta}}_{j,n} = \tau_{j,n} (\eta_{j,n} \psi_{j,n} (\bar{x}_{j,n}, \bar{x}_{j,n\tau}) - \gamma_{j,n} \hat{\vartheta}_{j,n}) \quad (T1.15)$$

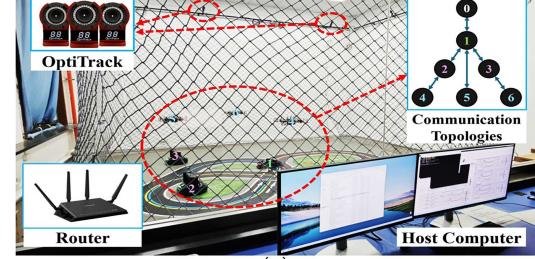
$$\dot{\hat{q}}_j = \tau_{j,\varrho} (-\eta_{j,n} u_{j,\varrho} - \gamma_{j,\varrho} \hat{q}_j) \quad (T1.16)$$

$$\dot{q}_{j,n} = -c_{j,n} q_{j,n} - q_{j,n-1} \quad (T1.17)$$

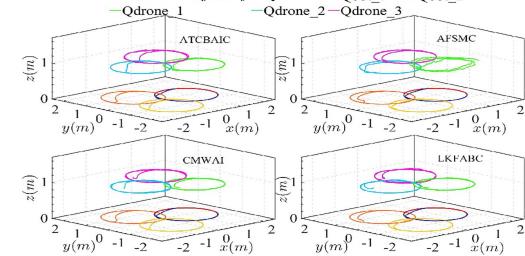
(a)



(b)



(c)



(d)

Figure 1 (Color online) (a) The controller structure; (b) the structure of the ATCBAIC scheme; (c) the experimental environment; (d) experimental results.

(1) For bounded initial conditions and a given positive constant G_j , if $V(0) \leq \sum_{j=1}^N G_j$ holds, then by properly selecting the design parameters $k_{j,i}, c_{j,i}, \tau_{j,i}, \iota_{j,i}, \gamma_{j,i}, i = 1, \dots, n, \tau_{j,L}, \tau_{j,\varrho}, \gamma_{j,\varrho}$ and $\gamma_{j,L}$, all signals in the closed-loop system, e.g., $e_{j,i}, \eta_{j,i}, \hat{\vartheta}_{j,i}, \alpha_{j,i}, i = 1, \dots, n, \hat{\vartheta}_{j,L}$ and \hat{q}_j , are bounded.

(2) The tracking error $\varepsilon_j = y_j - y_r, j = 1, \dots, N$ can be ensured to converge to a residual set that can be made arbitrarily small by adjusting the design parameters.

Proof. Please see Appendix F.

Experimental verification. A multi-agent experimental platform (Figure 1(c)) was established to validate the effectiveness of the proposed control scheme. The results of this validation are presented in Figure 1(d), with further details available in Appendix G.

Conclusion. This article investigates the consensus control problem for heterogeneous multi-agent systems with backlash nonlinearity and time delays. To address the challenges posed by time delays, we propose a heterogeneous-compatible state observer based on the finite cover lemma and RBFNNs, effectively estimating unmeasurable states and unknown time-delay functions. This method overcomes the conservatism of the Lyapunov-Krasovskii approach, resulting in improved control performance. A distributed control law with backlash compensation is also developed, which suppresses non-smooth input effects without requiring precise backlash inverse models. Experimental validation on QMAEP shows a 50% improvement in tracking accuracy and a 20% reduction in phase lag compared to benchmarks. Note that the approach

has limitations in scalability, as re-identification of backlash parameters is required for different systems. Future work will explore online reinforcement learning methods for real-time backlash estimation and compensation, enhancing adaptability to various systems. Another future research direction is to further investigate time-synchronized stability and control, which can provide synchronized convergence in heterogeneous multi-agent systems [5].

Acknowledgements This work was supported in part by National Natural Science Foundation of China (Grant No. 62373092) and Science and Technology Development Plan Project of Jilin Province (Grant No. 20250201086GX).

Supporting information Appendixes A–G. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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