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Adaptive anti-disturbance controller for discrete-time multi-input multi-output systems with input delays

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Abstract This paper considers the anti-disturbance problem for the discrete-time multi-input multi-output (MIMO) system with input delays, where both the disturbance and state information are unavailable. A new adaptive output feedback controller is proposed to reject the unknown disturbances of discrete-time linear MIMO systems with input delays. First, an equivalent input delay-free system of discrete-time input delay system is established to facilitate the controller design. Second, based on the Youla parameterization method, a state observer is established and a multichannel decoupling law is designed, which helps to facilitate the design of an adaptive regulator. Next, the unknown disturbance complete attenuation condition is given based on the frequency domain characteristics of the disturbance. In addition, the direct adaptive parameter algorithm with a projection operator is devised to attenuate the unknown disturbance. Finally, the effectiveness of the proposed method is verified by numerical simulation examples.

Keywords anti-disturbance, discrete-time MIMO systems, input delays, adaptive output feedback, Youla parameterization

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1 Introduction

Many systems suffer from input delay due to signal transmission [1], sensor and actuator dynamics [2], such as hot rolling processes [3], adaptive optical systems [4], and space-teleoperated manipulators [5]. If the input delay is ignored, the performance will be degraded and even the stability of the system will be jeopardized. Then, the stability of input delay systems has received much attention. The Smith predictor (SP) is a pioneering work based on the frequency domain to compensate for input delays [6]. Next, many time-domain-based predictors have been reported, which include finite spectrum assignment (FSA) [7] and model reduction methods [8]. In [9], the model reduction approach was used to stabilize the discrete-time system with input delay. Wu et al. [10] ensured the stability of discrete linear time-varying systems with input delay by a model reduction approach. Duan et al. [11, 12] established a unified framework for nonlinear input delay systems using the actuated system theory.

Apart from input delay, anti-disturbance is also a concern in control systems, especially periodic disturbances commonly encountered in practice, for example, permanent magnet synchronous motors [13], adaptive optics systems [14], and microvibration isolators [15]. In [16], an adaptive fuzzy controller was reported to suppress unknown periodic disturbances in servo systems. Other similar reports can be found in [17, 18]. To simultaneously compensate for input delay and periodic disturbances, some predictor-based approaches [19–22] have been designed to enhance disturbance rejection performance by improving the prediction accuracy of the system state. In [20], a new predictor-based method was designed to suppress the unknown disturbance in continuous-time linear input delay systems. Sanz et al. [21] enhanced the prediction accuracy of the state by incorporating disturbance information into the prediction scheme and used the enhanced predictor to suppress the disturbance of the DC systems [22]. However, these predictor-based controllers require state information. Therefore, some output feedback methods have been reported [23]. Since state prediction errors cannot be eliminated, predictor-based control methods fail to satisfy the output regulation equation and thus cannot fully compensate for periodic disturbances. The equivalent input disturbance (EID) method is an effective method for attenuating disturbances and has been validated on state delay systems [24]. In [25], a composite controller combining SP and EID was used to compensate for unknown

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disturbances in input delay systems. Du et al. [26] further improved the disturbance rejection performance of the EID method in input delay systems. In addition, the disturbance-observer-based approach can also compensate for the unknown periodic disturbance. In [27], an adaptive output regulator based on the internal model principle was proposed, but it can only compensate for single-frequency periodic disturbances due to the limitation of the frequency estimator. In [14, 28], some disturbance-observer-based repetitive controllers (RCs) were designed to attenuate the multi-frequency periodic disturbances in the tip-tilt mirror system. But these frequency-domain-based design methods are only applicable to single-input single-output (SISO) systems. However, RCs can only compensate for the disturbance frequency components that must be integer multiples of the fundamental frequency. And RCs are only applicable to SISO systems.

Discrete-time input delay systems are widely present in practice, including computer network-based control systems [29] and sampling control systems [30]. Unfortunately, there are fewer results on anti-disturbance control for discrete-time input delay systems, especially for multiple-input multiple-output (MIMO) systems. The main challenges of existing controllers for continuous-time input delay systems that cannot be directly applied to discrete-time input delay systems include the following. (1) Using numerical calculations to approximate the integral term in continuous-time predictors generates computational errors and leads to complex stability problems. (2) Due to the multichannel cross-coupling in discrete-time MIMO systems, the disturbance observer for continuous-time input delay systems will fail. Liu et al. [30] designed a predictor based on extended state observers to improve the disturbance rejection of input delay sampled control systems. However, their method is limited by the filtered Smith predictor and is only applicable to SISO systems. In their subsequent work [31], a state & disturbance observer-predictor (SDOP) was designed for MIMO systems. In [32], an improved SDOP controller was reported. However, both the SDOP controller and its variants would need to know the system equations for the disturbances, which is very difficult in practice. To overcome this problem, Wu et al. [33] designed a parameterized predictor to improve the system's disturbance resistance by using higher-order state predictors. However, their approach is a state-feedback strategy, which limits potential applications. Apart from state feedback strategies, some output feedback methods have been reported. Li et al. [34] designed an output feedback controller to realize accurate scanning in atomic force microscopy. In [14], Feng et al. further reported a fractional-order RC to improve the attenuation of periodic disturbances. In addition, some observer-based output feedback methods have been presented [4, 35]. However, the above output feedback methods require inverse modeling, which means that they are only applicable to minimum-phase systems. Furthermore, they are not applicable to MIMO input delay systems with multichannel coupling. Overall, the anti-disturbance performance of discrete-time MIMO input delay systems needs to be further improved. Specifically, the performance that needs to be improved includes disturbance compensation accuracy and decay rate. The main reasons are as follows. (1) Due to the presence of state prediction errors, the existing methods are unable to completely attenuate the unknown disturbances [23, 33]. (2) The existing controllers are all centralized controllers, which do not decouple the multichannel coupling of the MIMO system [31, 32]. Then, the convergence is slow or even divergent. Selecting controller parameters is tedious, which is very inconvenient in practice. Therefore, it is a challenge to design output feedback controllers with higher anti-disturbance accuracy and faster anti-disturbance speed in discrete linear MIMO systems with input delays.

Motivated by the above challenges, this paper proposes an adaptive output feedback control strategy for discrete-time MIMO input delay systems based on Youla parameterization. It is well known that the Youla parameterized controller is a direct adaptive output feedback controller based on the internal model principle, which can completely attenuate periodic disturbances. Youla parameterized controllers are widely used in vibration control and noise control [36–38]. However, existing transfer function-based Youla parameterized controllers are only applicable to SISO systems and cannot simultaneously achieve closed-loop system stability and multichannel decoupling. Then, existing Youla parameterized controllers cannot be applied to discrete-time systems with input delays, especially MIMO systems with input delays. Compared to existing methods, the main contributions of this paper are as follows.

(1) A new adaptive output feedback anti-disturbance controller is proposed. Compared with the existing output feedback controllers, the proposed controller not only completely attenuates the disturbances but also does not require system matrix information of the disturbances [31, 32].

(2) All families of stabilizing controllers for discrete-time MIMO systems with input delays are derived using the Youla parameterization. Closed-loop system stabilization and multichannel decoupling are achieved simultaneously, which greatly reduces the design difficulty and computational complexity of adaptive controllers. Then, the proposed method has faster disturbance decay compared to the existing centralized controllers [26, 27, 33].

(3) For the first time, the adaptive Youla parameterization is used to compensate for unknown disturbances in discrete-time MIMO systems with input delays. Compared to existing adaptive Youla parameterized controllers [15, 37, 38], the limitations of adaptive Youla parameterized controllers for discrete-time input delay systems are

overcome by constructing equivalent systems of input delay systems.

The paper is organized as follows. Section 2 gives the problem statement. Section 3 designs an adaptive output feedback anti-disturbance controller. Section 4 verifies the effectiveness of the proposed method by numerical simulation. The conclusion is given in Section 5.

2 Problem statement

Discrete-time MIMO systems subject to input delays and unknown disturbances can generally be described as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + B_1 w(k) + B_2 u(k-h), \\ e(k) &= C_1 x(k) + D_1 w(k), \\ y(k) &= C_2 x(k) + D_2 w(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^{m\times 1}$ represent the system state, and $u(k-h) \in \mathbb{R}^{n\times 1}$ and $w(k) \in \mathbb{R}^{n\times 1}$ denote the input signal and unknown disturbance, respectively. h is a known delay constant. $A \in \mathbb{R}^{m\times m}, B_{1,2} \in \mathbb{R}^{m\times n}, C_{1,2} \in \mathbb{R}^{n\times m}, D_{1,2} \in \mathbb{R}^{n\times n}$ are related system matrices. $y(k) \in \mathbb{R}^{n\times 1}$ and $e(k) \in \mathbb{R}^{n\times 1}$ are the output and performance variables of the system, respectively.

It is well known that most disturbances can be decomposed into periodic signals in practice. Therefore, in this paper, the unknown disturbances are considered periodic disturbances with multiple frequencies, which can be written as follows:

$$\begin{aligned} w(k) &= [w_1(k), \cdots, w_n(k)]^{\mathrm{T}}, \\ w_i(k) &= \sum_{j=1}^{\bar{n}_i} a_{ij}(k) \sin[b_{ij}(k)k + \psi_{ij}(k)], i = 1, \cdots, n, \end{aligned} \tag{2}$$

where $a_{ij}(k)$, $b_{ij}(k)$ and $\psi_{ij}(k)$ are the unknown amplitude, frequency, and phase of periodic disturbances, respectively. $\bar{n}_i$ denotes the frequency number of the ith periodic disturbances.

The main goal of this paper is to design a control input $u(k)$, which measures only the system output signal $y(k)$ and ensures that the system performance variable $e(k)$ will converge to zero. Moreover, the proposed controller requires decoupling the cross-coupling between multiple channels to improve the convergence rate. To achieve the goal, the following Youla parameterized adaptive controller is designed.

3 Adaptive Youla parameterized controller

This section presents a novel adaptive controller based on Youla parameterization for attenuating the unknown disturbance in discrete-time linear MIMO systems with input delays.

3.1 Equivalent transformations of input delay systems

The transform matrix of (1) is defined as $\mathfrak{P}$. Then, Eq. (1) can be rewritten as follows:

$$\begin{bmatrix} e(k) \\ y(k) \end{bmatrix} = \begin{bmatrix} \mathfrak{P}_{11} & \mathfrak{P}_{12} \\ \mathfrak{P}_{21} & \mathfrak{P}_{22} \end{bmatrix} \begin{bmatrix} w(k) \\ u(k-h) \end{bmatrix}, \tag{3}$$

where $\mathfrak{P}_{11} = \left[\begin{array}{c|c} A & B_1 \\ \hline C_1 & D_1 \end{array}\right], \mathfrak{P}_{12} = \left[\begin{array}{c|c} A & B_2 \\ \hline C_1 & 0 \end{array}\right], \mathfrak{P}_{21} = \left[\begin{array}{c|c} A & B_1 \\ \hline C_2 & D_2 \end{array}\right], \mathfrak{P}_{22} = \left[\begin{array}{c|c} A & B_2 \\ \hline C_2 & 0 \end{array}\right]$.

Based on (3), the control block diagram for (1) can be found as shown in Figure 1(a), where F denotes the controller to be designed. To reduce the difficulty of the controller design, we first equivalently convert the input delay system to an input delay-free system. For this purpose, two auxiliary systems Ξ_1 and Ξ_2 are defined as follows:

$$\Xi_1 = \mathfrak{P}_{11} - z^{-h}\bar{\mathfrak{P}}_{11}, \Xi_2 = \bar{\mathfrak{P}}_{22} - z^{-h}\mathfrak{P}_{22}, \tag{4}$$

where $\bar{\mathfrak{P}}_{11} = \left[\begin{array}{c|c} A & A^h B_1 \\ \hline C_1 & 0 \end{array}\right], \bar{\mathfrak{P}}_{22} = \left[\begin{array}{c|c} A & B_2 \\ \hline C_2 A^{-h} & 0 \end{array}\right]$.

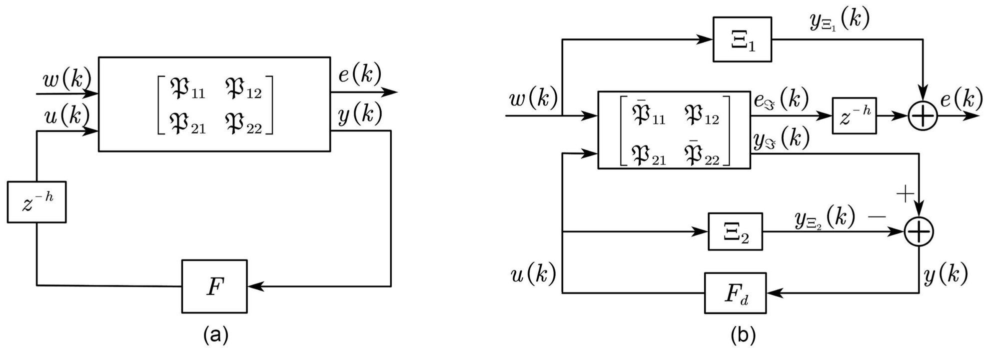

Figure 1 General structure with input delays. (a) Input delay system; (b) equivalent structure of the system in (a).

Lemma 1. Based on $\mathfrak{P}_{11}$, $\bar{\mathfrak{P}}_{11}$, $\mathfrak{P}_{22}$, and $\bar{\mathfrak{P}}_{22}$, it can be concluded that Ξ_1 and Ξ_2 are stable systems, regardless of $\mathfrak{P}$ stability. Moreover, the input delay system can be equivalently expressed as an input delay-free system, as shown in Figure 1(b).

Proof. First, we prove that Ξ_1 and Ξ_2 are stable. Combined with (3), $\bar{\mathfrak{P}}_{22}$, and (4), Ξ_2 can be written as follows:

$$\begin{aligned}\Xi_2 &= \bar{\mathfrak{P}}_{22} - z^{-h}\mathfrak{P}_{22}\\ &= C_2A^{-h}\left(zI-A\right)^{-1}B_2 - C_2z^{-h}\left(zI-A\right)^{-1}B_2\\ &= C_2A^{-h}\left(I-z^{-h}A^h\right)\left(zI-A\right)^{-1}B_2.\end{aligned} \tag{5}$$

Then, the impulse response of Ξ_2 can be expressed as follows:

$$y_{\Xi_2}(k) = C_2A^{-h}\sum_{v=0}^{k-1}A^{k-1-v}B_2\sigma(v) - C_2\sum_{v=0}^{k-1}A^{k-1-v}B_2\sigma(v-h), \tag{6}$$

where $\sigma(v)$ denotes the impulse signal.

By simply operating, Eq. (6) can be simplified as follows:

$$y_{\Xi_2}(k) = C_2A^{-h}\sum_{\alpha=k-h}^{k-1}A^{k-1-\sigma}B_2\sigma(\alpha). \tag{7}$$

From (7), it is concluded that Ξ_2 is a finite impulse response system. Therefore, Ξ_2 is stable regardless of $\mathfrak{P}$ stability. Ξ_1 can be proven similarly. Next, we will prove that the input delay system shown in Figure 1(a) is equivalent to the input delay-free system shown in Figure 1(b). From Figure 1(b), $e(k)$ and $y(k)$ can be written as follows, respectively:

$$\begin{aligned}e(k) &= z^{-h}e_{\Im}(k) + \Xi_1 w(k)\\ &= z^{-h}\bar{\mathfrak{P}}_{11}w(k) + z^{-h}\mathfrak{P}_{12}u(k) + \Xi_1 w(k)\\ &= \left(z^{-h}\bar{\mathfrak{P}}_{11} + \Xi_1\right)w(k) + z^{-h}\mathfrak{P}_{12}u(k),\end{aligned} \tag{8}$$

$$\begin{aligned}y(k) &= y_{\Im}(k) + \Xi_2 u(k)\\ &= \mathfrak{P}_{21}w(k) + \left(\bar{\mathfrak{P}}_{22} - \Xi_2\right)u(k).\end{aligned} \tag{9}$$

Substituting $\bar{\mathfrak{P}}_{11} = C_1\left(zI-A\right)^{-1}A^hB_1$ and $\bar{\mathfrak{P}}_{22} = C_2A^{-h}\left(zI-A\right)^{-1}B_2$ into (8) and (9), one has

$$\begin{aligned}z^{-h}\bar{\mathfrak{P}}_{11} + \Xi_1 &= z^{-h}C_1\left(zI-A\right)^{-1}A^hB_1 + C_1\left(zI-A\right)^{-1}\left(I-z^{-h}A^h\right)B_1\\ &= C_1\left(zI-A\right)^{-1}B_1 = \mathfrak{P}_{11},\end{aligned} \tag{10}$$

$$\begin{aligned}\bar{\mathfrak{P}}_{22} - \Xi_2 &= C_2A^{-h}(zI-A)^{-1}B_2 - C_2A^{-h}\left(I-z^{-h}A^h\right)(zI-A)^{-1}B_2\\ &= z^{-h}C_2(zI-A)^{-1}B_2 = z^{-h}\mathfrak{P}_{11}.\end{aligned} \tag{11}$$

From (10) and (11), it is clear that $e(k)$ and $y(k)$ in Figure 1(a) are equal to $e(k)$ and $y(k)$ in Figure 1(b). Then, the input delay system can be equated to the input delay-free system.

The proof of Lemma 1 is complete.

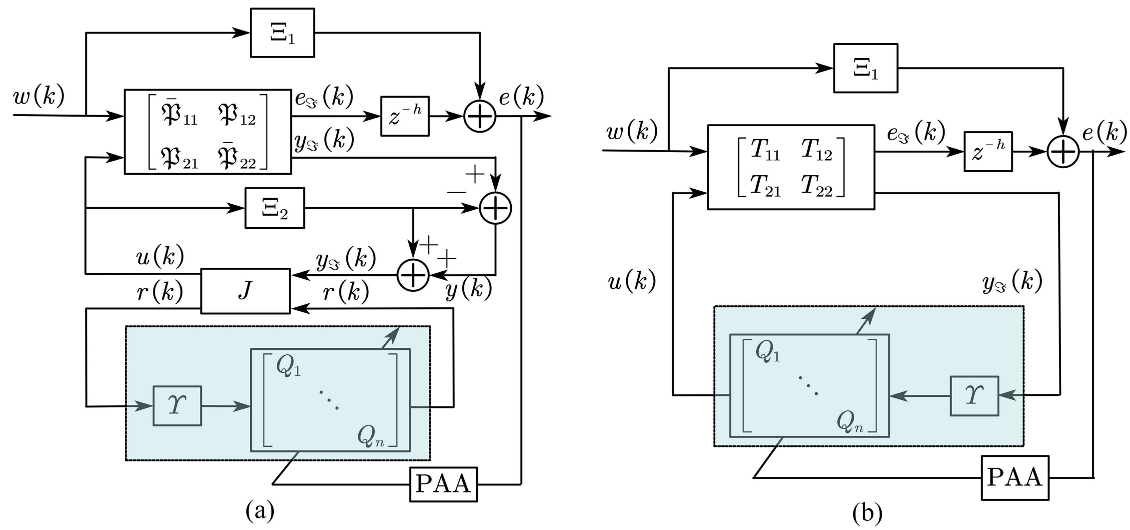

Figure 2 (Color online) Structure of the closed-loop system. (a) Block diagram of the closed-loop system with J and Q; (b) block diagram of the closed-loop system with lumped $\bar{\mathfrak{P}}$ and J.

Remark 1. By constructing the auxiliary system, the time delay in the input channel is shifted to the output channel. As a result, the difficulty of controller design is greatly reduced. Furthermore, this means that, in the future, many methods used to compensate for disturbances in input delay-free systems may be applied to input delay systems.

3.2 Youla parameterized controller design with decoupled law

Based on $\mathfrak{P}_{11}$, $\bar{\mathfrak{P}}_{11}$, $\mathfrak{P}_{22}$, and $\bar{\mathfrak{P}}_{22}$, the state space realization of the input delay-free system $\bar{\mathfrak{P}}$ can be formulated as follows:

$$\begin{aligned} \Im(k+1) &= A\Im(k) + A^h B_1 w(k) + B_2 u(k), \\ e_{\Im}(k) &= C_1 \Im(k), \\ y_{\Im}(k) &= C_2 A^{-h} \Im(k) + D_2 w(k), \end{aligned} \tag{12}$$

where $\Im(k)$ is state variable of input delay-free system. $y_{\Im}(k)$ and $e_{\Im}(k)$ are the output and residual error of an input delay-free system.

All stabilizing controllers of the input delay-free system (12) can be denoted by the Youla parameterization controller, which includes a Q part and a base controller J. The control block diagram of the closed-loop system is shown in Figure 2(a), where J can be given as follows:

$$J: \begin{cases} \hat{\Im}(k+1) = \left(A + B_2 K + L C_2 A^{-h}\right) \Im(k) \\ \qquad\qquad\quad - L y_{\Im}(k) + B_2 y_Q(k), \\ u(k) = K\hat{\Im}(k) + y_Q(k), \\ r(k) = y_{\Im}(k) - C_2 A^{-h} \hat{\Im}(k), \end{cases} \tag{13}$$

where $\hat{\Im}(k)$ is the estimated value of $\Im(k)$. $K \in \mathbb{R}^{n\times m}$ and $L \in \mathbb{R}^{m\times n}$ are the feedback and observer matrices, respectively. $y_Q(k)$ is the output of Q.

Based on the linear fractional transformation (LFT), the feedback controller of the input delay system can be formulated as follows:

$$F_d = \mathcal{F}_l(J,Q) = J_{11} + J_{12} Q \left(I - J_{22} Q\right)^{-1} J_{21}, \tag{14}$$

$$F = \left(I + F_d \Xi_2\right)^{-1} F_d, \tag{15}$$

where $J_{i,j}\ \ (i,j=1,2)$ can be given as follows:

$$J_{11}:\left[\begin{array}{c|c} A+B_2K+LC_2A^{-h} & -L \\ \hline K & 0 \end{array}\right], J_{12}:\left[\begin{array}{c|c} A+B_2K+LC_2A^{-h} & -L \\ \hline K & I \end{array}\right],$$
$$J_{21}:\left[\begin{array}{c|c} A+B_2K+LC_2A^{-h} & B_2 \\ \hline -C_2A^{-h} & I \end{array}\right], J_{22}:\left[\begin{array}{c|c} A+B_2K+LC_2A^{-h} & B_2 \\ \hline -CA^{-h} & 0 \end{array}\right]. \tag{16}$$

$\bar{\mathfrak{P}}$ and J can be merged into T block, as shown in Figure 2(b). Based on Figure 2(b), the transform function from $w\left(k\right)$ to $e\left(k\right)$ can be written as follows:

$$E\left(z\right)=\left[\Xi_1\left(z\right)+z^{-h}\left(T_{11}\left(z\right)+T_{12}\left(z\right)Q\left(z\right)T_{21}\left(z\right)\right)\right]W\left(z\right), \tag{17}$$

where $E\left(z\right)$ and $W\left(z\right)$ be the z-transformations of $e\left(k\right)$ and $w\left(k\right)$, respectively. T_{11}, T_{21}, and T_{21} can be written as follows:

$$T_{11}=\left[\begin{array}{cc|c} A+B_2K & -B_2K & A^hB_1 \\ 0 & A+LC_2A^{-h} & A^hB_1+LD_2 \\ \hline C_1 & 0 & D_1 \end{array}\right],$$
$$T_{12}=\left[\begin{array}{c|c} A+B_2K & B_2 \\ \hline C_1 & 0 \end{array}\right], T_{21}=\left[\begin{array}{c|c} A+LC_2A^{-h} & A^hB_1+LD_2 \\ \hline C_2A^{-h} & D_2 \end{array}\right]. \tag{18}$$

To improve the attenuation rate of disturbances and facilitate the design of adaptive algorithms, multichannel decoupling is necessary. Then, in this paper, a method is designed to make $T_{12}\left(z\right)Q\left(z\right)$ become a diagonal matrix by designing $T_{12}\left(z\right)$ and $Q\left(z\right)$.

Let the state space equation of the subsystem Σ_{12} be as follows:

$$\Sigma_{12}:\begin{cases} \varkappa\left(k+1\right)=A\varkappa\left(k\right)+B_2\bar{u}\left(k\right), \\ \bar{y}\left(k\right)=C_1\varkappa\left(k\right), \end{cases} \tag{19}$$

where $\varkappa\left(k\right)\in\mathbb{R}^n$ is state vector of Σ_{12}. $\bar{u}\left(k\right)\in\mathbb{R}^n$ and $\bar{y}\left(k\right)\in\mathbb{R}^n$ denote the input and output of Σ_{12}, respectively.

Designing the following state feedback control law: $\bar{u}\left(k\right)=K\varkappa\left(k\right)+\varUpsilon\rho\left(k\right)$, where $\rho\left(k\right)$ is the external signal. $\varUpsilon$ is the matrix to be designed. Substituting $\bar{u}\left(k\right)$ into (19), the following system can be obtained as follows:

$$\Sigma_{12}^{cl}:\begin{cases} \varkappa(k+1)=\left(A+B_2K\right)\varkappa\left(k\right)+B_2\varUpsilon\rho\left(k\right), \\ \bar{y}\left(k\right)=C_1\varkappa\left(k\right). \end{cases} \tag{20}$$

Therefore, the transfer matrix of Σ_{12}^{cl} is $C_1\left(zI-\left(A+B_2K\right)^{-1}\right)B_2\varUpsilon$. The desired goal is to make the transfer matrix of Σ_{12}^{cl} a diagonal matrix by designing K and L. To achieve this objective, let $C_1=\begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}^{\mathrm{T}}$, and define the following two matrices:

$$B^*=\begin{bmatrix} c_1^{\mathrm{T}}A^{\delta_1-1}B_2 \\ c_2^{\mathrm{T}}A^{\delta_2-1}B_2 \\ \vdots \\ c_n^{\mathrm{T}}A^{\delta_n-1}B_2 \end{bmatrix}, C^*=\begin{bmatrix} c_1^{*\mathrm{T}} \\ c_2^{*\mathrm{T}} \\ \vdots \\ c_n^{*\mathrm{T}} \end{bmatrix}=\begin{bmatrix} c_1^{\mathrm{T}}\eta_1\left(A\right) \\ c_2^{\mathrm{T}}\eta_2\left(A\right) \\ \vdots \\ c_n^{\mathrm{T}}\eta_n\left(A\right) \end{bmatrix}, \tag{21}$$

where $\eta_i\left(A\right)=A^{\delta_i}+\gamma_{i1}A^{\delta_i-1}+\gamma_{i2}A^{\delta_i-2}+\cdots+\gamma_{i\delta_i}I$, which is a polynomial of A and the integers δ_i can be calculated by the following:

$$\delta_i=\begin{cases} \min\left(j\mid c_i^{\mathrm{T}}A^{j-1}B_2\neq 0^{\mathrm{T}}, j=1,2,\cdots,n-1\right), \\ n-1;\ \text{if}\ c_i^{\mathrm{T}}A^{j-1}B_2=0^{\mathrm{T}}, j=1,2,\cdots,n. \end{cases} \tag{22}$$

In addition, to realize multichannel decoupling, the Q part is defined as follows:

$$\begin{aligned} Q(z) &= \Upsilon \times \operatorname{diag}\left[Q_1(z), Q_2(z), \cdots, Q_n(z)\right], \\ Q_i(z) &= \left(\sum_{\kappa=1}^{n_q} \theta_{i\kappa} z^{1-\kappa}\right) \xi_i(z), i = 1, 2, \cdots, n, \end{aligned} \tag{23}$$

where $\xi_i(z) = \frac{b_{i,1}z^{m-1}+\cdots+b_{i,m}}{z^m+a_{i,1}z^{m-1}+\cdots+a_{i,m}}$ is a stable function, which implies that Q_i is an FIR system. Υ is the matrix to be designed for decoupling.

Based on (17)–(23), the following Theorem 1 is proposed to ensure that $T_{12}(z) Q(z)$ is a diagonal matrix.

Theorem 1. If B^* is nonsingular, and let $\Upsilon = (B^*)^{-1}, K = -(B^*)^{-1} C^*$. Therefore, the transfer matrix $C_1 \left(zI - (A + B_2K)^{-1}\right) B_2\Upsilon$ will become a diagonal matrix, and $T_{12}(z) Q(z)$ can be simplified as follows:

$$T_{12}(z) Q(z) = \operatorname{diag}\left[\tfrac{Q_1(z)}{\eta_1(z)} \ \tfrac{Q_2(z)}{\eta_2(z)} \ \cdots \ \tfrac{Q_n(z)}{\eta_n(z)}\right]. \tag{24}$$

Proof. Define $\mathcal{G}(z) = C_1(zI - A)^{-1}B_2$. Let $g_i(z)$ be the ith row of $\mathcal{G}(z)$. Based on (21) and (22), we can immediately obtain as follows:

$$\begin{aligned} g_i^{\mathrm{T}}(z) &= c_i^{\mathrm{T}}(zI - A)^{-1}B_2 = c_i^{\mathrm{T}}B_2z^{-1} + c_i^{\mathrm{T}}AB_2z^{-2} + \cdots \\ &= c_i^{\mathrm{T}}A^{\delta_i-1}B_2z^{-\delta_i} + c_i^{\mathrm{T}}A^{\delta_i}B_2z^{-(\delta_i+1)} + \cdots \\ &= z^{-\delta_i}\left[c_i^{\mathrm{T}}A^{\delta_i-1}B_2 + c_i^{\mathrm{T}}A^{\delta_i}(zI - A)^{-1}B_2\right]. \end{aligned} \tag{25}$$

Multiplying both sides by $\left(z^{\delta_i} + \gamma_{i1}z^{\delta_i-1} + \cdots + \gamma_{i\delta_i}\right)$ of (25), one immediately obtains as follows:

$$\begin{aligned} &\left(z^{\delta_i} + \gamma_{i1}z^{\delta_i-1} + \cdots + \gamma_{i\delta_i}\right) g_i^{\mathrm{T}}(z) \\ &\quad = c_i^{\mathrm{T}}A^{\delta_i-1}B_2 + c_i^{*\mathrm{T}}B_2z^{-1} + c_i^{*\mathrm{T}}AB_2z^{-2} + \cdots \\ &\quad = c_i^{\mathrm{T}}A^{\delta_i-1}B_2 + c_i^{*\mathrm{T}}\left(Iz^{-1} + Az^{-2} + A^2z^{-3} + \cdots\right)B_2 \\ &\quad = c_i^{\mathrm{T}}A^{\delta_i-1}B_2 + c_i^{*\mathrm{T}}(zI - A)^{-1}B_2. \end{aligned} \tag{26}$$

From (26), it is known that $\mathcal{G}(z)$ can be expressed as follows:

$$\mathcal{G}(z) = \operatorname{diag}\left[\varsigma_1, \cdots, \varsigma_{\mathrm{m}}\right]\left[B^* + C^*(zI - A)^{-1}B_2\right], \tag{27}$$

where $\aleph_i = \left(z^{\sigma_i} + \gamma_{i1}z^{\sigma_i-1} + \cdots + \gamma_{i\sigma_i}\right)^{-1}, i = 1, 2, \cdots, n$.

Combined with (27) and $C_1 \left(zI - (A + B_2K)^{-1}\right) B_2\Upsilon$, $T_{12}(z) Q(z)$ can be given as follows:

$$\begin{aligned} T_{12}(z) Q(z) &= C_1\left[zI - (A + B_2K)\right]^{-1}B_2Q(z) \\ &= \mathcal{G}(z)\left[\Upsilon^{-1} + \Upsilon^{-1}K(zI - A)^{-1}B_2\right]^{-1} \\ &= \operatorname{diag}\left[\frac{Q_1(z)}{\eta_1(z)}, \frac{Q_2(z)}{\eta_2(z)}, \cdots, \frac{Q_n(z)}{\eta_n(z)}\right]. \end{aligned} \tag{28}$$

The proof of Theorem 1 is completed.

Remark 2. The core of Theorem 1 is that B^* is nonsingular, which is common in many systems, for example, active magnetic bearing systems [39, 40] and adaptive optical systems [41]. Therefore, the assumptions in Theorem 1 are reasonable.

Besides multichannel decoupling, the stability of the closed-loop system is also very important. Next, this paper presents Theorem 2 that guarantees the stability of the closed-loop system.

Theorem 2. Assume the input delay systems (1) is controllable and observable. If Theorem 1 holds, L and ξ_i are designed such that $A + LC_2A^{-h}$ and A_Q are Hurwitz, the input delay system can be stabilized by the output-feedback controller F_d.

Proof. Let $\chi(k)=\begin{bmatrix}\Im(k) & x_Q(k) & \tilde{\Im}(k)\end{bmatrix}^{\mathrm{T}}$ be the state variable of the closed-loop system, and $\tilde{\Im}(k)$ denotes the state observation error. Then, based on (12), (13), and (23), the closed-loop system can be written as follows:

$$\Sigma_{cl}:\begin{cases}\chi(k+1)=A_{cl}\chi(k)+B_{cl}w(k),\\ e_{cl}(k)=C_{cl}\chi(k)+D_{cl}w(k),\end{cases}\tag{29}$$

where

$$\begin{aligned}A_{cl}&=\begin{bmatrix}A+B_2K & B_2C_Q & B_2K\\ 0 & A_Q & -B_QC_2A^{-h}\\ 0 & 0 & A+LC_2A^{-h}\end{bmatrix},B_{cl}=\begin{bmatrix}A^hB_1\\ B_QD_2\\ -\left(LD_2+A^hB_1\right)\end{bmatrix},\\ C_{cl}&=\begin{bmatrix}C_1 & 0 & 0\end{bmatrix},D_{cl}=1.\end{aligned}\tag{30}$$

From (30), it is clear that if $A+B_2K, A_Q$, and $A+LC_2A^{-h}$ are stable, the closed-loop system is stable. It can be known from (30) that the stability of $A+B_2K, A_Q$, and $A+LC_2A^{-h}$ is independent of each other. Therefore, the closed-loop system is stable if $A+B_2K, A_Q$, and $A+LC_2A^{-h}$ are Hurwitz.

Based on (23), the state space equation of $Q_i(z)$ can be written as follows:

$$Q_i:\begin{cases}x_{Q_i}(k+1)=A_{Q_i}x_{Q_i}(k)+B_{Q_i}\left(y_{\Im}^i(k)-\hat{y}_{\Im}^i(k)\right),\\ y_{Q_i}(k)=C_{Q_i}x_{Q_i}(k),\end{cases}\tag{31}$$

where $x_{Q_i}(k)$ is the state vector of $Q_i(z)$. $y_{\Im}^i(k)$ and $\hat{y}_{\Im}^i(k)$ are the ith variable of $y_{\Im}(k)$ and $\hat{y}_{\Im}(k)$, respectively.

Define $\theta_i^{\mathrm{T}}=\left[\theta_{i1},\theta_{i2},\cdots,\theta_{in_q}\right]$. Then, in terms of (23), A_{Q_i}, B_{Q_i}, and C_{Q_i} can be given as follows:

$$\begin{aligned}A_{Q_i}&=\begin{bmatrix}0&\cdots&0&0&-a_{i,m}&0&0&0\\ 1&\cdots&0&0&-a_{i,m-1}&0&0&0\\ \vdots&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\\ 0&\cdots&1&0&-a_{i,2}&0&0&0\\ 0&\cdots&0&1&-a_{i,1}&0&0&0\\ 0&\cdots&0&0&1&0&0&0\\ 0&\cdots&0&0&0&\ddots&0&0\\ 0&\cdots&0&0&0&0&1&0\end{bmatrix}_{n_{Q_i}\times n_{Q_i}},B_{Q_i}=\begin{bmatrix}b_{i,m}\\ b_{i,m-1}\\ \vdots\\ b_{i,2}\\ b_{i,1}\\ 0\\ \vdots\\ 0\end{bmatrix}_{n_{Q_i}\times 1},\\ C_{Q_i}&=\left[0_{1\times(m-1)}\theta_i^{\mathrm{T}}\right].\end{aligned}\tag{32}$$

Based on (23) and (32), the state space realization of $Q(z)$ can be written as $Q=\left[\begin{array}{c|c}A_Q & B_Q\\ \hline C_Q & 0\end{array}\right]$, where $A_Q=\Upsilon\times\mathrm{diag}\left[A_{Q_1},\cdots,A_{Q_{\bar{n}}}\right], B_Q=\Upsilon\times\mathrm{diag}\left[B_{Q_1},\cdots,B_{Q_{\bar{n}}}\right]$, and $C_Q=\Upsilon\times\mathrm{diag}\left[C_{Q_1},\cdots,C_{Q_{\bar{n}}}\right]$. It can be known that Υ is a constant value matrix, which does not change the stability of A_{Q_i}. Therefore, A_Q is stable, if ξ_i is an FIR system. In addition, it can be concluded that $A+B_2K$ is Hurwitz if Theorem 1 holds. If L is designed so that $A+LC_2A^{-h}$ is Hurwitz, the closed-loop system is stable.

The proof of Theorem 2 is completed.

Remark 3. From Theorems 1 and 2, it can be concluded that closed-loop system stabilization and multichannel decoupling can be achieved simultaneously by designing matrices K, L, and A_Q. In addition, in this paper, the system is assumed to be controllable and observable, which is a common assumption in input delay systems. In addition, many real systems satisfy this assumption, including permanent magnet synchronous motors [13], magnetic fluid deformable mirror [41], and magnetic bearing systems [39, 40].

Substituting (24) into (17), it immediately obtains as follows:

$$\begin{aligned}
\begin{bmatrix} E_1(z) \\ \vdots \\ E_n(z) \end{bmatrix} &= \left[\begin{bmatrix} z^{-h}T_{12}^{11} & \cdots & z^{-h}T_{12}^{1n} \\ \vdots & \ddots & \vdots \\ z^{-h}T_{12}^{n1} & \cdots & z^{-h}T_{12}^{nn} \end{bmatrix} \varUpsilon \begin{bmatrix} Q_1 & & 0 \\ & \ddots & \\ 0 & & Q_n \end{bmatrix} \begin{bmatrix} T_{21}^{11} & \cdots & T_{21}^{1n} \\ \vdots & \ddots & \vdots \\ T_{21}^{n1} & \cdots & T_{21}^{nn} \end{bmatrix} \right] \begin{bmatrix} W_1(z) \\ \vdots \\ W_n(z) \end{bmatrix} \\
&\quad + \left[\begin{bmatrix} z^{-h}T_{11}^{11} & \cdots & z^{-h}T_{11}^{1n} \\ \vdots & \ddots & \vdots \\ z^{-h}T_{11}^{n1} & \cdots & z^{-h}T_{11}^{nn} \end{bmatrix} + \begin{bmatrix} \Xi_1^{11} & \cdots & \Xi_1^{1n} \\ \vdots & \ddots & \vdots \\ \Xi_1^{n1} & \cdots & \Xi_1^{nn} \end{bmatrix} \right] \begin{bmatrix} W_1(z) \\ \vdots \\ W_n(z) \end{bmatrix} \\
&= \left[\begin{bmatrix} \frac{z^{-h}Q_1(z)}{\eta_1(z)} & & 0 \\ & \ddots & \\ 0 & & \frac{z^{-h}Q_n(z)}{\eta_n(z)} \end{bmatrix} \begin{bmatrix} T_{21}^{11} & \cdots & T_{21}^{1n} \\ \vdots & \ddots & \vdots \\ T_{21}^{n1} & \cdots & T_{21}^{nn} \end{bmatrix} \right] \begin{bmatrix} W_1(z) \\ \vdots \\ W_n(z) \end{bmatrix} \\
&\quad + \begin{bmatrix} z^{-h}T_{11}^{11} & \cdots & z^{-h}T_{11}^{1n} \\ \vdots & \ddots & \vdots \\ z^{-h}T_{11}^{n1} & \cdots & z^{-h}T_{11}^{nn} \end{bmatrix} + \begin{bmatrix} \Xi_1^{11} & \cdots & \Xi_1^{1n} \\ \vdots & \ddots & \vdots \\ \Xi_1^{n1} & \cdots & \Xi_1^{nn} \end{bmatrix} \begin{bmatrix} W_1(z) \\ \vdots \\ W_n(z) \end{bmatrix} \\
&= \begin{bmatrix} \sum_{j=1}^{n} \left[z^{-h}T_{11}^{1j}(z) + \frac{z^{-h}Q_1(z)}{\eta_1(z)} T_{21}^{1j}(z) + \Xi_1^{1j}(z) \right] W_j(z) \\ \vdots \\ \sum_{j=1}^{n} \left[z^{-h}T_{11}^{nj}(z) + \frac{z^{-h}Q_n(z)}{\eta_n(z)} T_{21}^{nj}(z) + \Xi_1^{nj}(z) \right] W_j(z) \end{bmatrix}.
\end{aligned} \tag{33}$$

From (33), it can be seen that the performance variable $E_i(z)$ is only affected by $Q_i(z)$, which means that disturbances can be attenuated by choosing $Q_i(z)$ alone. This not only improves the convergence speed, but also facilitates the design of the parameter adaptive algorithm. In this section, we will further analyze the characteristics of the periodic disturbances to give further parameter selection guidelines. First, we give conditions for the complete attenuation of periodic disturbances in Lemma 2.

Lemma 2. Consider the closed-loop system performance variable $E(z)$. Assuming Theorems 1 and 2 hold, the interpolation condition for the complete attenuation of the periodic disturbance can be given as follows:

$$\theta_i \in \mathbb{S} : \{\Pi_{\theta_i}\theta_i + \mho_{\theta_i} = 0\}, \tag{34}$$

where $\theta_i = \left[\theta_{i1}, \theta_{i2}, \cdots, \theta_{in_q}\right]^{\mathrm{T}}, i = 1, \cdots, n, \Pi_{\theta_i} \in \mathbb{R}^{2\bar{n}_i \times n_q}, \mho_{\theta_i} \in \mathbb{R}^{2\bar{n}_i \times 1}$.

Proof. The z-transform of periodic disturbance responses can be written as follows:

$$\begin{bmatrix} E_1(s) \\ \vdots \\ E_n(s) \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n} \left[\left(\Xi_1^{1j} + z^{-h}T_{11}^{1j}(z) + z^{-h}\frac{Q_1(z)}{\eta_1(z)} T_{21}^{1j}(z) \right) \frac{N_j(z)}{D_j(z)} \right] \\ \vdots \\ \sum_{j=1}^{n} \left[\left(\Xi_1^{nj} + z^{-h}T_{11}^{nj}(z) + z^{-h}\frac{Q_n(z)}{\eta_n(z)} T_{21}^{nj}(z) \right) \frac{N_j(z)}{D_j(z)} \right] \end{bmatrix}, \tag{35}$$

where $N_j(z)$ and $D_j(z)$ denote the numerator and denominator polynomials of the z-transform of the periodic disturbance, respectively.

Based on (35), the time domain of periodic disturbance responses can be given as follows:

$$\begin{bmatrix} e_1(t) \\ \vdots \\ e_n(t) \end{bmatrix} = \begin{bmatrix} \mathcal{Z}^{-1} \left[\sum_{\varepsilon=1}^{n_p} \frac{1}{z-p_\varepsilon} \sum_{j \in S_\varepsilon} R_j(p_\varepsilon) \varGamma_1(p_\varepsilon) \right] + e_{10}(t) \\ \vdots \\ \mathcal{Z}^{-1} \left[\sum_{\varepsilon=1}^{n_p} \frac{1}{z-p_\varepsilon} \sum_{j \in S_\varepsilon} R_j(p_\varepsilon) \varGamma_n(p_\varepsilon) \right] + e_{n0}(t) \end{bmatrix}, \tag{36}$$

where $\mathcal{Z}^{-1}$ denotes the inverse z-transform and p_ε is a pair of conjugate poles of $W(z)$ on the unit circle. The total response $e_{i0}(k)$ can be considered the sum of the system's output due to nonzero initial conditions and partial fractions with $\mathit{\Gamma}_i(p_\varepsilon)$ poles. Since the closed-loop system is stable, $e_{i0}(k)$ will decay to zero, and

$$\mathit{\Gamma}_i=\sum_{j=1}^{n}\left(\Xi_1^{ij}+z^{-h}T_{11}^{ij}(z)+z^{-h}\frac{Q_i(z)}{\eta_i(z)}T_{21}^{ij}(z)\right)|_{z=P_\varepsilon}. \tag{37}$$

Based on the internal model principle (IMP), the response of the closed-loop system to periodic disturbances can converge to zero if $\theta_i=\left[\theta_{i1},\theta_{i2},\cdots,\theta_{in_q}\right]^{\mathrm{T}},i=1,\cdots,n$ satisfies the following:

$$\sum_{j\in S_\varepsilon}R_j(z)\sum_{j=1}^{n}\left(\Xi_1^{ij}(z)+z^{-h}T_{11}^{ij}(z)+z^{-h}\frac{Q_i(z)}{\eta_i(z)}T_{21}^{ij}(z)\right)N_j(z)\,|_{z=p_\varepsilon}=0. \tag{38}$$

Without loss of generality, the conjugate poles of periodic disturbance can be denoted as p_ε^{+} and p_ε^{-}, which denote the poles whose imaginary parts are positive and negative, respectively. Since the absolute value of the conjugate pole's imaginary part is equal, it follows that Eq. (38) has the same real part and equal absolute values of the imaginary parts at p_ε^{+} and p_ε^{-}. Then, substituting p_ε^{+} into (38), one has

$$\begin{bmatrix}\sum_{j\in S_\varepsilon}R_j^{re}\sum_{j=1}^{n}\left(\Xi_1^{ij,re}+z^{-h}T_{11}^{ij,re}+z^{-h}\frac{Q_i^{re}}{\eta_i^{re}}T_{21}^{ij,re}\right)N_j^{re}\\ +\mathrm{j}\sum_{j\in S_\varepsilon}R_j^{im}\sum_{j=1}^{n}\left(\Xi_1^{ij,im}+z^{-h}T_{11}^{ij,im}+z^{-h}\frac{Q_i^{im}}{\eta_i^{im}}T_{21}^{ij,im}\right)N_j^{im}\end{bmatrix}|_{z=p_\varepsilon^{+}}=0, \tag{39}$$

where $f^{re}(\cdot)$ and $f^{im}(\cdot)$ denote the real and imaginary parts of $f(\cdot)$, respectively. To simplify (39), define the following two expressions:

$$V_i(z)=\sum_{j\in S_\varepsilon}R_i(z)\sum_{j=1}^{n}\left[\Xi_1^{ij}(z)+z^{-h}T_{11}^{ij}(z)\right]N_j(z), \tag{40}$$

$$\Lambda_i(z)=\sum_{j\in S_\varepsilon}R_i(z)\sum_{j=1}^{n}\frac{z^{-h}T_{21}^{ij}(z)N_j(z)}{\eta_i(z)\xi_i(z)}\sum_{\kappa=1}^{n_q}z^{1-\kappa}. \tag{41}$$

Based on (40) and (41), Eq. (39) can be written as follows:

$$\left[V_i^{re}\left(p_\varepsilon^{+}\right)+\sum_{\kappa=1}^{n_q}\theta_{i\kappa}\Lambda_i^{re}\left(p_\varepsilon^{+}\right)\right]+\mathrm{j}\left[V_i^{im}\left(p_\varepsilon^{+}\right)+\sum_{\kappa=1}^{n_q}\theta_{i\kappa}\Lambda_i^{im}\left(p_\varepsilon^{+}\right)\right]=0. \tag{42}$$

Then, the equivalent form of (42) can be expressed as follows:

$$V_i^{re}\left(p_\varepsilon^{+}\right)+\sum_{\kappa=1}^{n_q}\theta_{i\kappa}\Lambda_i^{re}\left(p_\varepsilon^{+}\right)=0,V_i^{im}\left(p_\varepsilon^{+}\right)+\sum_{\kappa=1}^{n_q}\theta_{i\kappa}\Lambda_i^{im}\left(p_\varepsilon^{+}\right)=0. \tag{43}$$

Reorganizing (43), the conditions for the periodic disturbances to be completely attenuated are given as follows:

$$\Pi_{\theta_i}\theta_i+\mho_{\theta_i}=0, \tag{44}$$

where $\theta_i=\left[\theta_{i1},\cdots,\theta_{in_q}\right]^{\mathrm{T}},\Pi_{\theta_i}\in\mathbb{R}^{2\bar{n}_i\times n_q},\mho_{\theta_i}\in\mathbb{R}^{2\bar{n}_i\times 1}$, and

$$\Pi_{\theta_i}=\begin{bmatrix}V_1^{re}\left(p_1^{+}\right)&\cdots&V_{n_q}^{re}\left(p_1^{+}\right)\\V_1^{im}\left(p_1^{+}\right)&\cdots&V_{n_q}^{im}\left(p_1^{+}\right)\\\vdots&\cdots&\vdots\\V_1^{re}\left(p_{\bar{n}}^{+}\right)&\cdots&V_{n_q}^{re}\left(p_{\bar{n}}^{+}\right)\\V_1^{im}\left(p_{\bar{n}_i}^{+}\right)&\cdots&V_{n_q}^{im}\left(p_{\bar{n}_i}^{+}\right)\end{bmatrix}_{2\bar{n}_i\times n_{iq}},\mho_{\theta_i}=\begin{bmatrix}\Lambda^{re}\left(p_1^{+}\right)\\\Lambda^{im}\left(p_1^{+}\right)\\\vdots\\\Lambda^{re}\left(p_{\bar{n}}^{+}\right)\\\Lambda^{im}\left(p_{\bar{n}_i}^{+}\right)\end{bmatrix}_{2\bar{n}_i\times 1}. \tag{45}$$

The proof of Lemma 2 is completed.

Based on Lemma 2, there is a unique vector θ_i that completely attenuates periodic disturbances if $n_q=2\bar{n}_i$. However, the unique vector θ_i may not achieve satisfying results in practice due to the effect of measurement noise. Motivated by this, the over-parameterized method is used to address this problem. In addition, to overcome the parameter drift caused by over-parameterization, a robust recursive least-squares adaptive algorithm with a projection operator is designed later to update the Q-parameters.

Remark 4. The proposed method is a direct adaptive controller that compensates for periodic disturbances without identifying frequency and does not redesign the base controller J. Therefore, the computational burden is reduced. In addition, as an output feedback controller, the number of sensors is minimized, significantly reducing costs.

3.3 Parameter adaptive algorithm design

In the previous section, we have shown that there exists an optimal set of parameters $\theta^0=\left[\theta_1^0,\theta_2^0,\cdots,\theta_n^0\right]^{\mathrm{T}}$ that can completely attenuate the unknown disturbances. However, the values of $\theta^0=\left[\theta_1^0,\theta_2^0,\cdots,\theta_n^0\right]^{\mathrm{T}}$ are unknown. In this section, the parameter adaptive algorithm (PAA) will be proposed for online updating $\theta=\left[\theta_1,\theta_2,\cdots,\theta_n\right]^{\mathrm{T}}$ so that it can converge to $\theta^0=\left[\theta_1^0,\theta_2^0,\cdots,\theta_n^0\right]^{\mathrm{T}}$.

From (35), $e\left(k\right)$ can be expressed as follows:

$$e\left(k\right)=\left[\Xi_{11}\left(q^{-1}\right)+z^{-h}\left(T_{11}\left(q^{-1}\right)+T_{12}\left(q^{-1}\right)Q\left(q^{-1}\right)T_{21}\left(q^{-1}\right)\right)\right]w\left(k\right), \tag{46}$$

where q^{-1} denotes the time operation with one step delay.

Due to the non-commutativity of time-varying operators $Q\left(q^{-1}\right)$, $e\left(k\right)$ cannot be written as a linear parametric model. Then, the desired error $e^0\left(k\right)$ is used to construct a linear parameter model of the adaptive parameter. Since $\theta^0=\left[\theta_1^0,\theta_2^0,\cdots,\theta_n^0\right]^{\mathrm{T}}$ is the optimal solution satisfying Lemma 2, the related optimal performance variable $e^0=\left[e_1^0,e_2^0,\cdots,e_n^0\right]^{\mathrm{T}}$ can be expressed as follows:

$$e^0\left(k\right)=\left[\Xi_{11}\left(q^{-1}\right)+z^{-h}\left(T_{11}\left(q^{-1}\right)+T_{12}\left(q^{-1}\right)Q^0\left(q^{-1}\right)T_{21}\left(q^{-1}\right)\right)\right]w\left(k\right). \tag{47}$$

Substituting (47) into (46), we immediately obtain as follows:

$$\begin{aligned} e\left(k\right)&=z^{-h}T_{12}\left(q^{-1}\right)Q\left(q^{-1}\right)r\left(k\right)-z^{-h}T_{12}\left(q^{-1}\right)Q^0\left(q^{-1}\right)r\left(k\right)+e^0\left(k\right)\\ &=z^{-h}T_{12}\left(q^{-1}\right)\left(Q\left(q^{-1}\right)-Q^0\left(q^{-1}\right)\right)r\left(k\right)+e^0\left(k\right), \end{aligned} \tag{48}$$

where $r\left(k\right)=\left[r_1\left(k\right),\cdots,r_n\left(k\right)\right]^{\mathrm{T}}=T_{21}\left(q^{-1}\right)w\left(k\right)$.

Based on Theorem 1, $T_{12}\left(q^{-1}\right)Q\left(q^{-1}\right)$ is diagonal. Therefore, Eq. (48) can be simplified as follows:

$$\begin{bmatrix} e_1\left(k\right)\\ \vdots\\ e_n\left(k\right)\end{bmatrix}=\begin{bmatrix} \frac{q^{-h}\left(Q_1\left(q^{-1}\right)-Q_1^0\left(q^{-1}\right)\right)r_1(k)}{\eta_1\left(q^{-1}\right)}+e_1^0\left(k\right)\\ \vdots\\ \frac{q^{-h}\left(Q_n\left(q^{-1}\right)-Q_n^0\left(q^{-1}\right)\right)r_n(k)}{\eta_n\left(q^{-1}\right)}+e_n^0\left(k\right)\end{bmatrix}. \tag{49}$$

Let $\tilde{e}=\left[\tilde{e}_1,\tilde{e}_2,\cdots,\tilde{e}_n\right]^{\mathrm{T}}$ be the modified error, which is expressed as follows:

$$\tilde{e}_i\left(k\right)=e_i\left(k\right)-\left[q^{-h}Q_i\left(q^{-1}\right)-Q_i\left(q^{-1}\right)q^{-h}\right]r_i\left(k\right). \tag{50}$$

Substituting (23) into (50), $\tilde{e}_i\left(k\right)$ can be written as the following linear regression model:

$$\begin{aligned} \tilde{e}_i\left(k\right)=&\,q^{-h}Q_i\left(q^{-1}\right)r_i\left(k\right)-Q_i^0\left(q^{-1}\right)q^{-h}r_i\left(k\right)+e_i^0\left(k\right)\\ &-q^{-h}Q_i\left(q^{-1}\right)r_i\left(k\right)+Q_i\left(q^{-1}\right)q^{-h}r_i\left(k\right). \end{aligned} \tag{51}$$

In addition, based on Theorem 2, θ_i^0 is a constant vector, which implies that $Q_i^0\left(k\right)$ is not a time-varying operator. Then, one has the following:

$$Q_i^0q^{-h}r_i\left(k\right)=q^{-h}Q_i^0r_i\left(k\right). \tag{52}$$

Combined with (52), the modified performance variable $\tilde{e}_i\left(k\right)$ can be written as the following linear regression model:

$$\begin{aligned} \tilde{e}_i\left(k\right)&=q^{-h}Q_i\left(q^{-1}\right)r_i\left(k\right)-Q_i^0\left(q^{-1}\right)q^{-h}r_i\left(k\right)+e_i^0\left(k\right)\\ &\quad-q^{-h}Q_i\left(q^{-1}\right)r_i\left(k\right)+Q_i\left(q^{-1}\right)q^{-h}r_i\left(k\right)\\ &=\left[Q_i\left(q^{-1}\right)-Q_i^0\left(q^{-1}\right)\right]q^{-h}r_i\left(k\right)+e_i^0\left(k\right)\\ &=\left[\sum_{\kappa=1}^{n_q}\theta_{i\kappa}q^{1-\kappa}-\sum_{\kappa=1}^{n_q}\theta_{i\kappa}^0q^{1-\kappa}\right]\xi_i\left(q^{-1}\right)q^{-h}r_i\left(k\right)+e_i^0\left(k\right)\\ &=\phi_i^{\mathrm{T}}\left(k\right)\tilde{\theta}_i\left(k\right)+e_i^0\left(k\right), \end{aligned} \tag{53}$$

where $\phi_i(k)$ is the regression variable. $\tilde{\theta}_i(k)$ is the parameter estimation error, and

$$\begin{aligned}\phi_i(k) &= \left[-\xi_1\left(q^{-1}\right)q^{-h}r_i(k)\ \cdots\ -\xi_{n_q}\left(q^{-1}\right)q^{-h}r_i(k)\right]^{\mathrm{T}},\\ \tilde{\theta}_i(k) &= \left[\left(\theta_{i1}^0-\theta_{i1}\right),\cdots,\left(\theta_{in_q}^0-\theta_{in_q}\right)\right]^{\mathrm{T}}, i=1,\cdots,n.\end{aligned}\tag{54}$$

To attenuate the unknown disturbances, a decentralized recursive least squares (RLS) algorithm is designed to tune online unknown parameters $\theta(k)=[\theta_1(k),\cdots,\theta_n(k)]^{\mathrm{T}}$, and the parameters adaptive law with a projection operator are given as follows:

$$\hat{\theta}_i(k+1)=\mathrm{Proj}\left\{\hat{\theta}_i(k)+\frac{P_i(k)\phi_i(k+1)\tilde{e}_i(k+1)}{1+\phi_i^{\mathrm{T}}(k+1)P_i(k)\phi_i(k+1)}\right\},\tag{55}$$

$$P_i(k)=\frac{1}{\lambda_i(k)}\left[P_i(k-1)-\frac{P_i(k-1)\phi_i(k)\phi_i^{\mathrm{T}}(k)P_i(k-1)}{1+\phi_i^{\mathrm{T}}(k)P_i(k-1)\phi_i(k)}\right],\tag{56}$$

where $0<\lambda_i(k)\leqslant 1$ is the forgetting factor. $\mathrm{Proj}(\cdot)$ denotes the projection operator, which can be written as follows:

$$\mathrm{Proj}\left(\hat{\theta}_i(k)\right)=\begin{cases}\hat{\theta}_i(k), & \text{if}\quad \hat{\theta}_i(k)\in\mathbb{S}_{\theta_i},\\ P_i^{\frac{1}{2}}(k)\hat{\vartheta}_i^*(k), & \text{if}\quad \hat{\theta}_i(k)\notin\mathbb{S}_{\theta_i},\end{cases}\tag{57}$$

where $\hat{\vartheta}_i^*(k)$ denotes the orthogonal projection of $\hat{\vartheta}_i(k)=P^{-\frac{1}{2}}(k)\hat{\theta}_i(k)$ on $\mathscr{D}_{\vartheta_i}$. $\mathbb{S}_{\theta_i}:\left\|\hat{\theta}_i(k)\right\|_2^2<\mathfrak{h}<\infty$. The projection domain $\mathscr{D}_{\vartheta_i}$ can be given as follows:

$$\hat{\theta}_i(k)\in\mathbb{S}_{\theta_i},\hat{\vartheta}_i(k)=P_i^{-\frac{1}{2}}(k)\hat{\theta}_i(k)\in\mathscr{D}_{\vartheta_i}.\tag{58}$$

Remark 5. The performance of the adaptation is governed principally by the forgetting factor $\lambda(k)$, which is primarily related to the characteristics of periodic disturbances. For example, if the periodic disturbance is stationary or changes gradually, the forgetting factor should be chosen $\lambda(k)\in(0.95,1)$. Otherwise, $\lambda(k)$ must be chosen in each iteration to keep the trace of the gain matrix $P(k)$ invariant. In addition, it is clear from (48) that the calculation of $e(k)$ requires $e^0(k)$, which is the desired error corresponding to the nominal parameter $\theta^0(k)$. This implies that $e^0(k)$ can be considered zero. Therefore, when using $e(k)$ to update the adaptive parameter $\theta(k)$, $e^0(k)$ is not actually required.

Theorem 3. For the closed-loop system (29) consisting of the input delay system (1) and the adaptive output feedback controller (15). Using the parameter adaptive algorithm in (55) and (56), the performance variable $e_i(k)$ can achieve $\lim_{k\to\infty}e_i(k)=0$, and the interpolation condition (34) can be satisfied.

Proof. Consider the following Lyapunov function:

$$V_i(k)=\tilde{\theta}_i^{\mathrm{T}}(k)P_i^{-1}(k)\tilde{\theta}_i(k).\tag{59}$$

If $\hat{\theta}_i(k)\notin\mathbb{S}_{\theta_i}$, we have $\hat{\vartheta}_i(k)=P^{-\frac{1}{2}}(k)\hat{\theta}_i(k)$. Then, Eq. (59) can be rewritten as follows:

$$\begin{aligned}V_i(k)&=\tilde{\theta}_i^{\mathrm{T}}(k)P_i^{-1}(k)\tilde{\theta}_i(k)\\ &=\left[\theta_i^0-\hat{\theta}_i(k)\right]^{\mathrm{T}}P_i^{-1}(k)\left[\theta_i^0-\hat{\theta}_i(k)\right]\\ &=\left[P_i^{-\frac{1}{2}}(k)\left(\theta_i^0-\hat{\theta}_i(k)\right)\right]^{\mathrm{T}}\left[P_i^{-\frac{1}{2}}(k)\left(\theta_i^0-\hat{\theta}_i(k)\right)\right]\\ &=\left[\vartheta_i^0-\hat{\vartheta}_i(k)\right]^{\mathrm{T}}\left[\vartheta_i^0-\hat{\vartheta}_i(k)\right].\end{aligned}\tag{60}$$

$\hat{\vartheta}_i^*(k)$ is the orthogonal projection vector of $\hat{\vartheta}_i(k)$ onto $\mathscr{D}_{\vartheta_i}$ and $\vartheta_i(k)\in\mathscr{D}_{\vartheta_i}$. Then, $[\vartheta_i^0-\hat{\vartheta}_i^*(k)]^2\leqslant[\vartheta_i^0-\hat{\vartheta}_i(k)]^2$, which implies

$$\left[\theta_i^0-\mathrm{Proj}\left\{\hat{\theta}_i\right\}\right]^{\mathrm{T}}P_i^{-1}(k)\left[\theta_i^0-\mathrm{Proj}\left\{\hat{\theta}_i\right\}\right]\leqslant\left[\theta_i^0-\hat{\theta}_i\right]^{\mathrm{T}}P_i^{-1}(k)\left[\theta_i^0-\hat{\theta}_i\right].\tag{61}$$

It can be concluded from (61) that by choosing the Lyapunov function as in (59), the results of the convergence analysis of the algorithm without the projection operator are equally applicable to the results with the projection operator. Next, we will analyze the convergence of the parameter adaptive algorithm without projection operators.

The parameter adaptive algorithm without the projection operator can be written as follows:

$$\hat{\theta}_i(k+1) = \hat{\theta}_i(k) + \frac{P_i(k)\,\phi_i(k+1)\tilde{e}_i(k+1)}{1+\phi_i^{\mathrm{T}}(k+1)P_i(k)\phi_i(k+1)}, \tag{62}$$

$$P_i(k) = \frac{1}{\lambda_i(k)}\left[P_i(k-1) - \frac{P_i(k-1)\phi_i(k)\phi_i^{\mathrm{T}}(k)P_i(k-1)}{1+\phi_i^{\mathrm{T}}(k)P_i(k-1)\phi_i(k)}\right]. \tag{63}$$

By equivalent transformations, Eqs. (62) and (63) can also be rewritten as follows:

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \lambda(k+1)\,P(k+1)\,\phi(k+1)\,\tilde{e}_i(k+1), \tag{64}$$

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \lambda(k+1)\,P(k+1)\,\phi(k+1)\,\tilde{e}_i(k+1). \tag{65}$$

Based on (63), it is immediately as follows:

$$\begin{aligned}
&\lambda(k+1)\,P_i(k+1)\,\phi_i(k+1)\\
&\quad= P_i(k)\,\phi_i(k+1)\left(1-\frac{\phi_i^{\mathrm{T}}(k+1)\,P_i(k)\,\phi_i(k+1)}{1+\phi_i^{\mathrm{T}}(k+1)\,P_i(k)\,\phi_i(k+1)}\right)\\
&\quad= \frac{P_i(k)\,\phi_i(k+1)}{1+\phi_i^{\mathrm{T}}(k+1)\,P_i(k)\,\phi_i(k+1)}.
\end{aligned} \tag{66}$$

Based on (62)–(66), we have the following:

$$\begin{aligned}
V_i(k+1) &= \tilde{\theta}_i^{\mathrm{T}}(k+1)\,P_i^{-1}(k+1)\,\tilde{\theta}_i(k+1)\\
&= \left[\theta_i^0 - \hat{\theta}_i(k+1)\right]^{\mathrm{T}} P_i^{-1}(k+1)\left[\theta_i^0 - \hat{\theta}_i(k+1)\right]\\
&= \tilde{\theta}_i^{\mathrm{T}}(k)\,P_i^{-1}(k+1)\,\tilde{\theta}_i(k) - 2\lambda_i(k)\,\phi_i^{\mathrm{T}}(k)\,\tilde{\theta}_i(k)\,\tilde{e}_i(k)\\
&\quad + \lambda_i^2(k)\,\phi_i^{\mathrm{T}}(k)\,P_i(k+1)\,\phi_i(k)\,\tilde{e}_i^2(k).
\end{aligned} \tag{67}$$

Based on the Matrix inversion Lemma [42], Eq. (67) can be written as follows:

$$V_i(k+1) = \lambda_i(k)\left[V_i(k) + \left(\tilde{e}_i(k) - \tilde{\theta}_i^{\mathrm{T}}\phi_i(k)\right)^2 - \frac{\tilde{e}_i^2(k)}{1+\phi_i^{\mathrm{T}}(k)\,P_i(k)\,\phi_i(k)}\right]. \tag{68}$$

Since $0 < \lambda_i(k) \leqslant 1$, and $\tilde{e}_i(k) - \tilde{\theta}_i^{\mathrm{T}}\phi_i(k) = e_i^0(k)$, Eq. (68) can be written as follows:

$$V_i(k+1) - V_i(k) \leqslant \left(e_i^0(k)\right)^2 - \frac{\tilde{e}_i^2(k)}{1+\phi_i^{\mathrm{T}}(k)\,P_i(k)\,\phi_i(k)}, \tag{69}$$

which implies

$$\lim_{\mathcal{E}\to\infty} V_i(\mathcal{E}) - V_i(0) \leqslant \lim_{\mathcal{E}\to\infty}\left[\sum_{k=1}^{\mathcal{E}}\left[\left(e_i^0(k)\right)^2 - \frac{\tilde{e}_i^2(k)}{1+\phi_i^{\mathrm{T}}(k)\,P_i(k)\,\phi_i(k)}\right]\right]. \tag{70}$$

Since $e_i^0(k)$ is exponentially decaying, $\lim_{k\to\infty} e_i^0(k) = 0$, and $P_i(k) > 0$. We have the following:

$$\lim_{\mathcal{E}\to\infty}\sum_{k=1}^{\mathcal{E}}\frac{\tilde{e}_i^2(k)}{1+\phi_i^{\mathrm{T}}(k)\,P_i(k)\,\phi_i(k)} \leqslant \lim_{\mathcal{E}\to\infty}\left[\sum_{k=1}^{\mathcal{E}}\left(e_i^0(k)\right)^2 - V_i(\mathcal{E})\right] + V_i(0) \leqslant \infty, \tag{71}$$

which implies

$$\lim_{\mathcal{E}\to\infty}\frac{\tilde{e}_i^2(k)}{1+\phi_i^{\mathrm{T}}(k)\,P_i(k)\,\phi_i(k)} = 0. \tag{72}$$

Then,

$$\lim_{k\to\infty}\tilde{e}_i(k) = 0. \tag{73}$$

Based on (53) and (73), it immediately obtains as follows:

$$\lim_{k\to\infty}\left[\theta_i^0 - \hat{\theta}_i(k)\right] = 0 \Rightarrow \lim_{k\to\infty}\hat{\theta}_i(k) = \theta_i^0. \tag{74}$$

Based on (50) and (71), we have $z^{-h}Q_i\left(q^{-1}\right) - Q_i\left(q^{-1}\right)z^{-h} = 0$, then

$$\lim_{k\to\infty} e_i(k) = 0. \tag{75}$$

The proof of Theorem 3 is completed.

4 Numerical examples

In this section, the validity of the proposed method will be verified by comparing it with Wu et al. [33] and Du et al. [26] in an active magnetic bearing (AMB) system. It is well known that the AMB system is a typical MIMO system [39]. In addition, the AMB system is affected by input delay due to the demands of remote operation [40]. Because of the dynamic imbalance, the AMB system's rotor generates periodic disturbances with multiple frequencies at high rotational speeds, which limits the rotational speed of the rotor.

The dynamic model of the AMB systems can be written as follows [39]:

$$\mathcal{M}\ddot{S} + \mathcal{C}\dot{S} + \mathcal{K}S = \mathcal{B}\left(i + w\right), \tag{76}$$

where $S = \left[s_{ax}, s_{bx}, s_{ay}, s_{by}\right]^{\mathrm{T}}$ denotes the displacement in the x and y directions at the ends of the magnetic bearing, respectively. $i = \left[i_{ax}, i_{bx}, i_{ay}, i_{by}\right]^{\mathrm{T}}$ represents the control currents at the ends of the magnetic bearings in the x and y directions, respectively. w is the periodic disturbance due to rotor dynamic unbalance. $\mathcal{M} = I_{4\times 4}$, and

$$\begin{gathered}
\mathcal{C} = \begin{bmatrix} 0_{2\times 2} & \mathcal{C}_{12} \\ \mathcal{C}_{21} & 0_{2\times 2} \end{bmatrix}, \mathcal{K} = \begin{bmatrix} \mathcal{K}_{11} & 0_{2\times 2} \\ 0_{2\times 2} & \mathcal{K}_{22} \end{bmatrix}, \mathcal{B} = \begin{bmatrix} \mathcal{B}_{11} & 0_{2\times 2} \\ 0_{2\times 2} & \mathcal{B}_{22} \end{bmatrix}, \mathcal{C}_{12} = -\mathcal{C}_{21} = \begin{bmatrix} \frac{Hl_{as}}{J_{rr}l} & -\frac{Hl_{as}}{J_{rr}l} \\ -\frac{Hl_{bs}}{J_{rr}l} & \frac{Hl_{bs}}{J_{rr}l} \end{bmatrix}, \\
\mathcal{K}_{11} = \mathcal{K}_{22} = \begin{bmatrix} -\frac{k_x}{ml}a - \frac{k_x l_{\mathrm{as}}}{J_{rr}l}c & -\frac{k_x}{ml}b - \frac{k_x l_{\mathrm{as}}}{J_{rr}l}d \\ -\frac{k_x}{ml}a + \frac{k_x l_{\mathrm{bs}}}{J_{rr}l}c & -\frac{k_x}{ml}b + \frac{k_x l_{\mathrm{bs}}}{J_{rr}l}d \end{bmatrix}, \mathcal{B}_{11} = \mathcal{B}_{22} = \begin{bmatrix} \frac{k_i}{m} + \frac{k_i l_{\mathrm{as}} l_{\mathrm{am}}}{J_{rr}} & \frac{k_i}{m} - \frac{k_i l_{\mathrm{as}} l_{\mathrm{bm}}}{J_{rr}} \\ \frac{k_i}{m} - \frac{k_i l_{\mathrm{bs}} l_{\mathrm{am}}}{J_{rr}} & \frac{k_i}{m} + \frac{k_i l_{\mathrm{as}} l_{\mathrm{bm}}}{J_{rr}} \end{bmatrix},
\end{gathered} \tag{77}$$

where the values of the parameters in (77) can be found in [39].

Let $x = \left[S, \dot{S}\right]^{\mathrm{T}}$, the continuous-time state equation of the AMB system can be written as follows:

$$\begin{aligned}
\dot{x}\left(t\right) &= A_c x\left(t\right) + B_c\left(i\left(t\right) + w\left(t\right)\right), \\
e\left(t\right) &= y\left(t\right) = C_c x\left(t\right),
\end{aligned} \tag{78}$$

where $A_c = \begin{bmatrix} 0_{4\times 4} & I_{4\times 4} \\ -\mathcal{M}^{-1}\mathcal{K} & \mathcal{M}^{-1}\mathcal{C} \end{bmatrix}, B_c = \begin{bmatrix} 0_{4\times 4} \\ \mathcal{M}^{-1}\mathcal{B} \end{bmatrix}, C_c = \begin{bmatrix} I_{4\times 4} & 0_{4\times 4} \end{bmatrix}$.

Considering the input delay due to teleoperation in the AMB system is three sampling times [40], and using the zero-order holder, the discrete-time state equation of (78) can be written as follows:

$$\begin{aligned}
x\left(k+1\right) &= Ax\left(k\right) + B_1 w\left(k\right) + B_2 i\left(k-h\right), \\
e\left(k\right) &= y\left(k\right) = C_1 x\left(k\right),
\end{aligned} \tag{79}$$

where $A = \sum_{q=0}^{\infty} \frac{A_c^q T_s^q}{q!}, B_1 = B_2 = \sum_{q=0}^{\infty} \frac{A_c^q T_s^{q+1}}{(q+1)!} B_c, C_1 = C_c$. T_s is the sampling time, and $h = 3$.

In the simulation, the unknown disturbance is considered a periodic disturbance with three frequencies, 34, 55, and 67 Hz, respectively. Let $\gamma_{i1} = \gamma_{i2} = \gamma_{i3} = \gamma_{i4} = 0.1$. Therefore, by using Theorem 2, the state feedback matrix K can be given as follows:

$$K = \begin{bmatrix}
-1.23\times 10^6 & 1.16\times 10^5 & 3.77\times 10^4 & -3.77\times 10^4 & -1.11\times 10^3 & 113.86 & -17.02 & 17.03 \\
2.08\times 10^5 & -3.92\times 10^5 & -3.77\times 10^4 & 3.77\times 10^4 & 189.43 & -358.44 & 17.06 & -17.07 \\
-3.77\times 10^4 & 3.78\times 10^4 & -1.23\times 10^6 & 1.16\times 10^5 & 17.02 & -17.03 & -1.11\times 10^3 & 113.87 \\
3.77\times 10^4 & 3.77\times 10^4 & 2.08\times 10^5 & -3.92\times 10^5 & -17.06 & 17.03 & 189.43 & -358.44
\end{bmatrix}. \tag{80}$$

Let ξ_i be a bandpass filter with a cutoff frequency of 10–150 Hz. Let the desired pole of $A + LC_2A^{-h}$ be $[-0.9, 0.93, 0.95, -0.84, 0.91, -0.92, 0.976, -0.88]^{\mathrm{T}}$. Based on the pole configuration method, the state observation

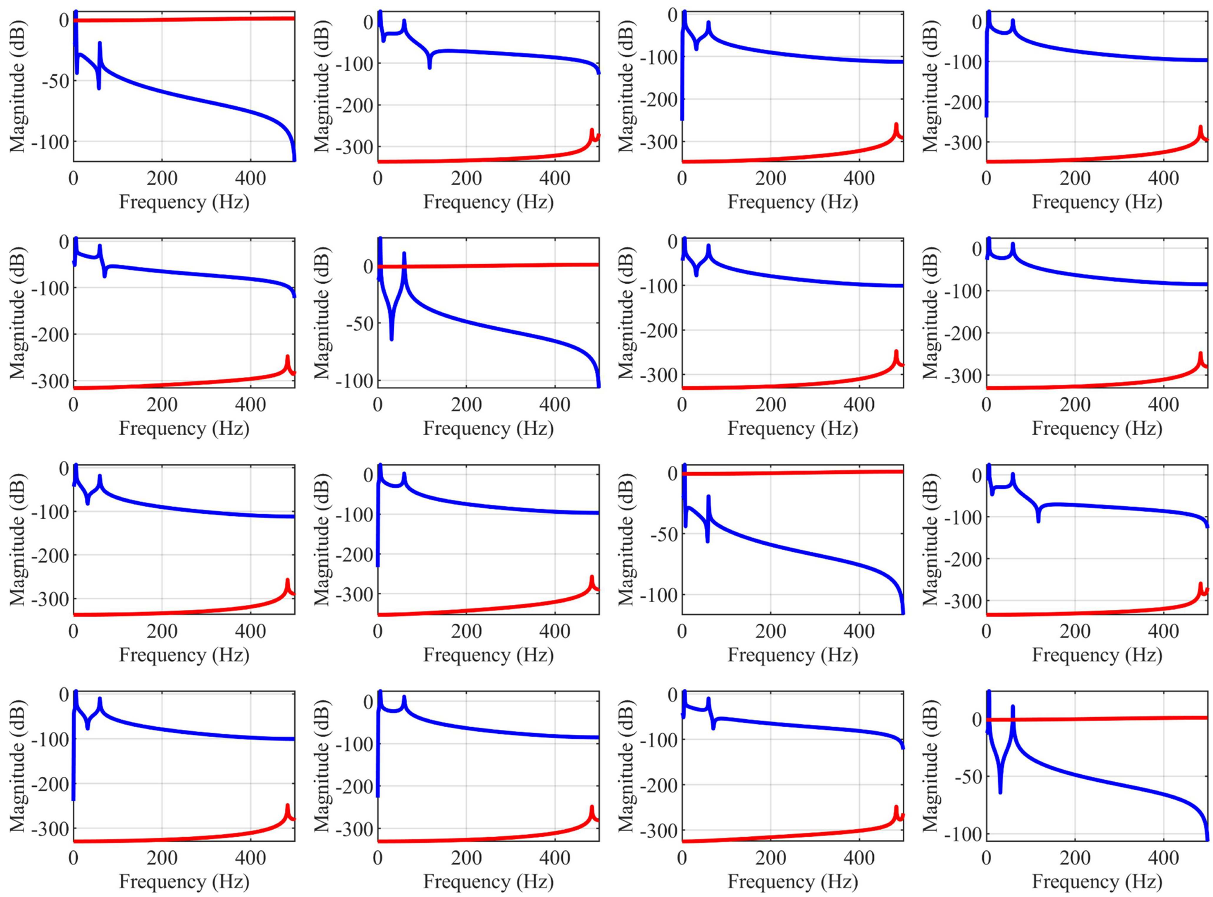

Figure 3 (Color online) Frequency response characteristics of the AMB system before (blue line) and after (red line) decoupling.

matrix L can be obtained as follows:

$$L = \begin{bmatrix} -2.21 & -0.53 & -0.29 & 0.29 \\ -0.80 & -1.49 & 1.35 & -1.32 \\ 0.30 & -0.30 & -2.29 & -0.63 \\ -1.33 & 1.34 & -0.97 & -1.42 \\ -129.49 & -280.44 & -21.49 & 1.53 \\ -447.44 & 307.84 & 270.14 & -165.12 \\ 26.96 & -3.01 & -171.64 & -300.39 \\ -269.30 & 194.40 & -491.19 & 315.14 \end{bmatrix}. \tag{81}$$

In addition, the forgetting factor is chosen as $\lambda_i(k) = 0.975$. The initial value of $\theta_i(0)$ and $P_i(0)$ is selected as $\theta_i(0) = 0_{12\times 1}$, $P_i(0) = I_{12\times 12}$, respectively. The simulation results are shown in Figures 3–5. The frequency characteristics of the AMB system before and after decoupling are shown in Figure 3, which shows the cross-coupling between the different channels. It can be known from Figure 3 that the AMB system is severely cross-coupled in the range of 0–100 Hz. By using the proposed method, the transfer function matrix of the system becomes the main diagonal matrix, and the influence of other transfer functions is almost zero. Therefore, the proposed method can effectively decouple the cross-coupling of the AMB system.

The simulation comparison results of the three methods are shown in Figure 4. According to the comparison results, it can be known that the proposed method can completely compensate for the disturbances. This is because the proposed method is an IMP-based controller. In addition, using the proposed method, the performance variable converges to zero within 0.7 s. The steady-state residual errors of the performance variables in the other two methods cannot be completely eliminated. Specifically, using the observer-predictor-based controller designed by Wu et al. in [33], the AMB system oscillates at ±9, ±13, ±8, and ±6 μm vibration displacements at the A- and

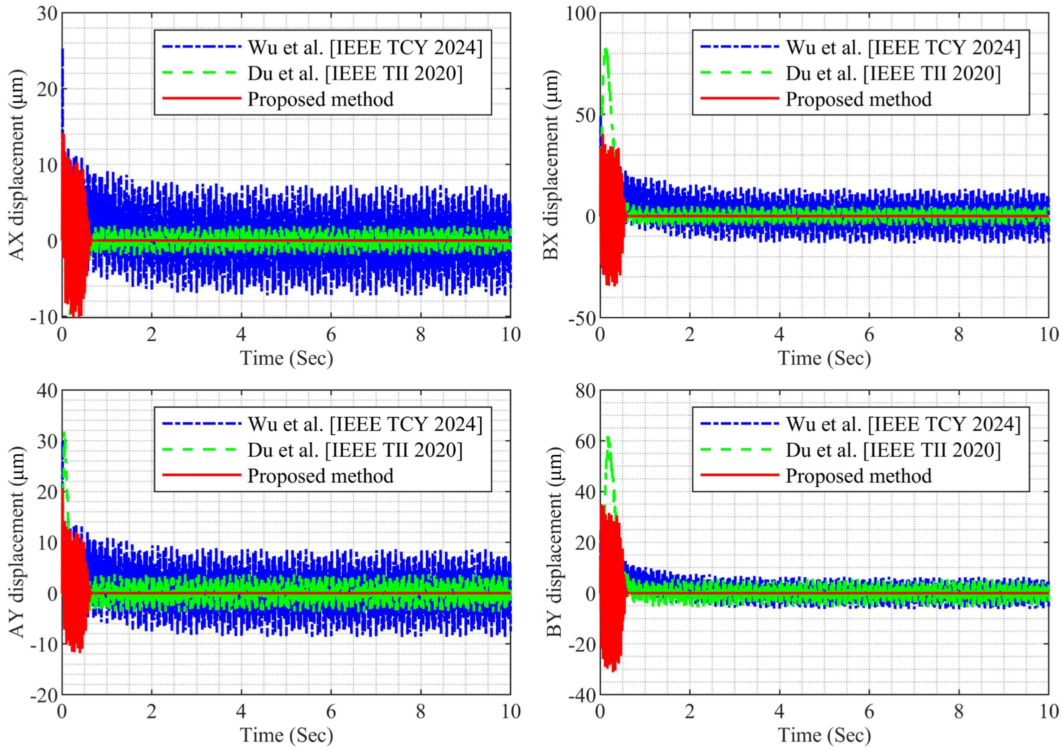

Figure 4 (Color online) Residual vibrations at the A and B ends of the AMB system.

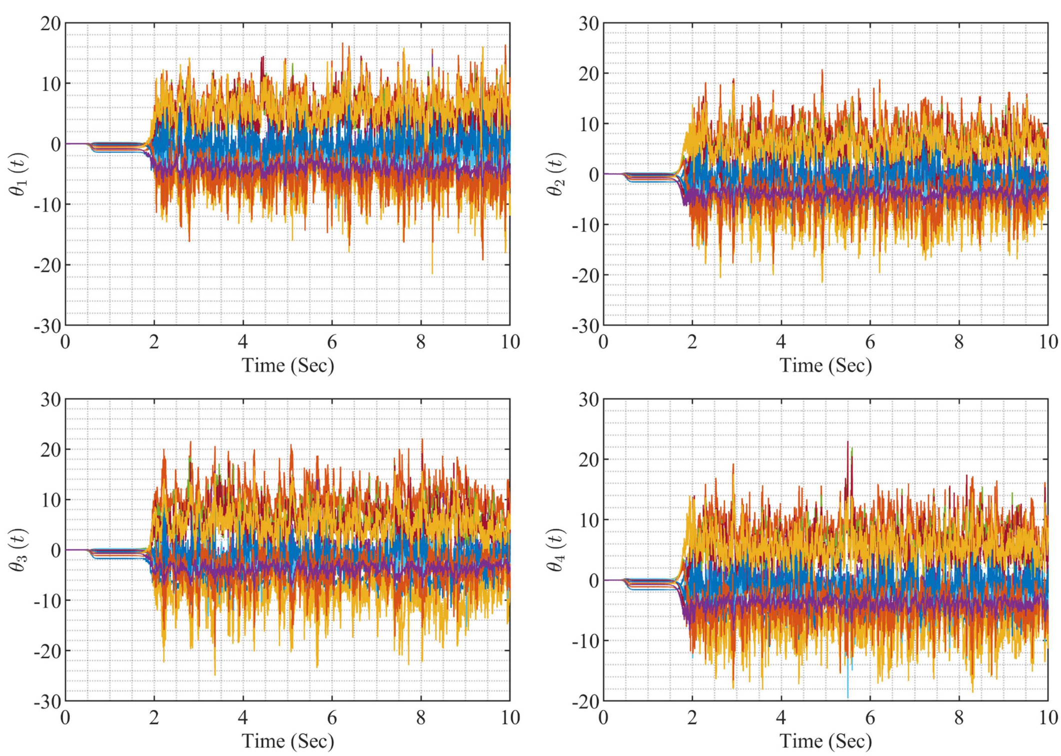

Figure 5 (Color online) Adaptive parameter update results on different channels.

B-ends, respectively. Similarly, the method in [33] was unable to completely attenuate the disturbance, and the AMB system oscillates at ± 2, ± 5, ± 4, and ± 5 μm vibration displacements at the A- and B-ends, respectively. This is because predictor-based control strategies cannot accurately predict the state information of the input delay system. Therefore, the anti-disturbance of the proposed method is better than the methods in [26,33]. Moreover, the convergence speed of the proposed method is much better than the other two methods, which is mainly attributed to the fact that the proposed method can decouple multiple channels. In addition, the proposed method is a direct adaptive method that does not need to identify the disturbance information. However, the methods proposed by Wu et al. and Du et al. are centralized controllers. Due to the strong coupling between multiple channels, the convergence speed is relatively slow. The evolutionary results of the adaptive parameters corresponding to the different channels are shown in Figure 5.

5 Conclusion

This paper proposes an output feedback adaptive controller with a multichannel decoupling design to compensate for unknown periodic disturbances in MIMO systems subject to input delay. By applying an equivalent transformation to the input delay system, it is converted into an input delay-free system. This approach addresses the limitation that Youla parameterization cannot be applied to input delay systems. To improve algorithm convergence speed, an inner-loop base controller J module with decoupling effects is designed. On this basis, the interpolation condition for complete attenuation of periodic disturbances in MIMO input delay systems is established. The parameter adaptive algorithm is designed to achieve adaptive compensation for unknown periodic disturbances in MIMO input delay systems. Simulation results for the AMB system demonstrate that the proposed method exhibits significantly superior disturbance attenuation performance compared to other approaches.

In future research, we will generalize the proposed method to variable-parameter systems subject to time-varying input delay. Furthermore, sensor and actuator failures will also be further considered.

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