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Special Topic: Logical System Control

Robust stabilization of random impulsive logical dynamical systems via a hybrid-index model

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Impulsive phenomena are commonly encountered in various biological systems, especially processes of gene expression. Random impulsive logical dynamical systems (RILDSs) with a hybrid-index model are able to capture such natural phenomena in biological systems [1]. Recently, with the theoretical development of the semi-tensor product of matrices [2, 3], there are numerous novel results on RILDSs. In particular, various stabilization problems of RILDSs were solved [4, 5]. It is worth noting that disturbances are prevalent and unavoidable phenomena in biological systems which arise from experimental noise, biological uncertainties, and inherent modeling errors. As was pointed out in [6], the theoretical models neglecting disturbance effects may not be as effective as expected in real applications. However, there are few results for RILDSs with disturbances.

This study focuses on the robust finite-time feedback stabilization with probability one (FFSPO) of RILDSs subject to disturbances. We present a criterion to verify the robust FFSPO of RILDSs with disturbances. Furthermore, we provide an algorithm to construct the state-feedback controllers. Based on the algorithm, we can determine all time-optimal state-feedback controllers for the robust FFSPO of RILDSs with disturbances. Compared with [5] which focuses on the stabilization of RILDSs, we propose a general framework for the case of RILDSs with disturbance inputs by constructing the disturbance reachable set.

Preliminaries. In algebraic form, the hybrid-index model of RILDS is described as follows:

$$\begin{cases} x(t+1,j) = L\xi(t)u(t)x(t,j),\ \theta(t,j)=0, \\ x(t,j+1) = Gx(t,j),\ \theta(t,j)=1, \end{cases} \tag{1}$$

where $t, j \in \mathbb{N}$, $x(t,j) \in \Delta_{k^n}$, $u(t) \in \Delta_{k^m}$, and $\xi(t) \in \Delta_{k^l}$ denote the state, input, and disturbance variables, respectively. $L \in \mathcal{L}_{k^n \times k^{l+m+n}}$ and $G \in \mathcal{L}_{k^n \times k^n}$. The index $(t,j) \in \mathbb{N}\times\mathbb{N}$ is called the hybrid index, where t represents the time index and j represents the impulse index. $\theta(t,j) \in \mathbb{N}_0^1$ is a random Boolean variable, and the value of $\theta(t,j)$ obeys the following probability distribution: $\mathbb{P}\{\theta(t,j)=0 \mid x(t,j)=\delta_{k^n}^i\} := p_i \in \mathbb{R}_0^1$ and $\mathbb{P}\{\theta(t,j)=1 \mid x(t,j)=\delta_{k^n}^i\} := q_i \in \mathbb{R}_0^1$, $i \in \mathbb{N}_1^{k^n}$. Then, we construct the stepping and jumping probability vectors as $\mathbf{p} := [p_1\ \cdots\ p_{k^n}]$ and $\mathbf{q} := [q_1\ \cdots\ q_{k^n}]$, respectively. Obviously, $\mathbf{p}+\mathbf{q} = \mathbf{1}_{k^n}^\top$ holds.

A stochastic sequence $\{x(t_s,j_s)\}_{s\in\mathbb{N}}$ is called a solution to RILDS (1), if for any $s \in \mathbb{N}$, one of the following two conditions is satisfied: $\theta(t_s,j_s)=0$, $x(t_{s+1},j_{s+1}) = L\xi(t_s)u(t_s)x(t_s,j_s)$, $t_{s+1}=t_s+1$ and $j_{s+1}=j_s$; $\theta(t_s,j_s)=1$, $x(t_{s+1},j_{s+1}) = Gx(t_s,j_s)$, $t_{s+1}=t_s$ and $j_{s+1}=j_s+1$. Let $t_0=j_0=0$. Then $t_s+j_s=s$, $\forall\ s\in\mathbb{N}$.

When RILDS (1) is controlled by a state-feedback controller

$$u(t) = Hx(t, j_t^+), \tag{2}$$

where $H := \delta_{k^m}[\mu_1\ \cdots\ \mu_{k^n}] \in \mathcal{L}_{k^m\times k^n}$, and j_t^+ represents the value of index j after the final jump at time instant t. Let $\overline{\mathbf{u}}$ be the control sequence generated by state-feedback controller (2).

The forward completeness of impulsive systems is important for effectively avoiding the Zeno phenomenon.

Definition 1 ([5]). RILDS (1) is said to be forward complete, if for any initial state $x_0 \in \Delta_{k^n}$, any state-feedback controller (2), any disturbance sequence $\boldsymbol{\xi} := \{\xi(\vartheta)\}_{\vartheta\in\mathbb{N}} \subseteq \Delta_{k^l}$, and any positive integer T, there exists an integer d such that $\mathbb{P}\{t_s \geqslant T | x(0,0)=x_0, \overline{\mathbf{u}}, \boldsymbol{\xi}\} = 1$, $\forall\ s \geqslant d$.

Definition 2 ([5]). RILDS (1) is said to be robust finite-time feedback x_e-stabilizable with probability one (x_e-FFSPO), if it is forward complete and there exists an integer d and a state-feedback controller (2) such that

$$\mathbb{P}\{x(t_s,j_s)=x_e | x(0,0)=x_0, \overline{\mathbf{u}}, \boldsymbol{\xi}\} = 1 \tag{3}$$

holds for any $x_0 \in \Delta_{k^n}$, any $\boldsymbol{\xi} \subseteq \Delta_{k^l}$, and any positive integer $s \geqslant d$.

Forward completeness analysis. Construct the jumping transition probability matrix of RILDS (1) as

$$\boldsymbol{\Phi}_J := G * \mathbf{q} \in \mathbb{R}_{k^n\times k^n}. \tag{4}$$

Based on the construction of $\boldsymbol{\Phi}_J$, we have $(\boldsymbol{\Phi}_J)_{\beta,\alpha} = \mathbb{P}\{x(t,j+1)=\delta_{k^n}^\beta, \theta(t,j)=1 | x(t,j)=\delta_{k^n}^\alpha\}$.

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Proposition 1. Given $\alpha, \beta \in \mathbb{N}_1^{k^n}$ and $d \in \mathbb{Z}_+$, it holds that $((\mathbf{\Phi}_J)^d)_{\beta,\alpha} = \mathbb{P}\Big\{x(t, j+d) = \delta_{k^n}^{\beta}, \prod_{s=0}^{d-1}\theta(t, j+s) = 1 | x(t, j) = \delta_{k^n}^{\alpha}\Big\}$.

See Appendix B.1 for the proof.

According to Proposition 1, one derives

$$\sum_{\beta=1}^{k^n}((\mathbf{\Phi}_J)^d)_{\beta,\alpha} = \mathbb{P}\Big\{\prod_{s=0}^{d-1}\theta(t, j+s) = 1 | x(t, j) = \delta_{k^n}^{\alpha}\Big\}$$
$$= \mathbb{P}\{t_{s+d} = t_s | x(t_s, j_s) = \delta_{k^n}^{\alpha}, \overline{\mathbf{u}}, \boldsymbol{\xi}\} \quad (5)$$

holds for any $\overline{\mathbf{u}} \subseteq \Delta_{k^m}$ and any $\boldsymbol{\xi} \subseteq \Delta_{k^l}$.

Proposition 2. For $\alpha \in \mathbb{N}_1^{k^n}$, if $Col_{\alpha}((\mathbf{\Phi}_J)^{k^n}) \neq \mathbf{0}_{k^n}$, then $Col_{\alpha}((\mathbf{\Phi}_J)^t) \neq \mathbf{0}_{k^n}$, $\forall\ t \in \mathbb{N}$.

See Appendix B.2 for the proof.

Theorem 1. RILDS (1) is forward complete if and only if $(\mathbf{\Phi}_J)^{k^n} = \mathbf{0}_{k^n \times k^n}$.

See Appendix B.3 for the proof.

Robust stabilization of RILDSs with disturbances. We define the following s-domain system of RILDS (1):

$$z(s+1) = \begin{cases} L\xi_{\phi}(s)u_{\phi}(s)z(s),\ \theta(s) = 0, \\ Gz(s),\ \theta(s) = 1, \end{cases} \quad (6)$$

where $z(s) := x(t_s, j_s)$, $\theta(s) := \theta(t_s, j_s)$, $\forall\ s \in \mathbb{N}$ and $z(0) := x(0, 0)$. When $s = t_s + j_{t_s}^+$, we have $u_{\phi}(s) := u(t_s)$ and $\xi_{\phi}(s) := \xi(t_s)$. The s-domain system (6) evolves regardless of the stepping process or jumping process at each instant and merely evolves with s, where $s = t_s + j_s$.

Note that variables $u_{\phi}(s)$ and $\xi_{\phi}(s)$ are not defined for $s \neq t_s + j_{t_s}^+$. Letting $u_{\phi}(s)$ and $\xi_{\phi}(s)$, $s \neq t_s + j_{t_s}^+$, be arbitrarily chosen from Δ_{k^m} and Δ_{k^l}, respectively, one can extend $u_{\phi}(s)$ and $\xi_{\phi}(s)$ to the sequences defined on $\mathbb{N}$ without influencing the original transition. Letting $\hat{G} := \mathbf{1}_{k^{m+l}}^{\top} \otimes G$, the s-domain system (6) can be equivalently expressed as

$$z(s+1) = \begin{cases} L\xi_{\phi}(s)u_{\phi}(s)z(s),\ \theta(s) = 0, \\ \hat{G}\xi_{\phi}(s)u_{\phi}(s)z(s),\ \theta(s) = 1. \end{cases} \quad (7)$$

Remark 1. According to (1) and (7), one can derive $\mathbb{P}\{z(s+1) = \delta_{k^n}^{\beta} | z(s) = \delta_{k^n}^{\alpha}, u_{\phi}(s) = \delta_{k^m}^{\nu}, \xi_{\phi}(s) = \delta_{k^l}^{\varepsilon}\} = \mathbb{P}\{x(t_{s+1}, j_{s+1}) = \delta_{k^n}^{\beta} | x(t_s, j_s) = \delta_{k^n}^{\alpha}, u(t_s) = \delta_{k^m}^{\nu}, \xi(t_s) = \delta_{k^l}^{\varepsilon}\}$, $\forall\ s \in \mathbb{N}$.

See Appendix B.4 for the proof.

We split L into k^{l+m} equal blocks. Let $L := [L_1^1\ \cdots\ L_{k^m}^1\ \cdots\ L_1^{k^l}\ \cdots\ L_{k^m}^{k^l}]$. We construct the transition probability matrix of s-domain system (7) as $\mathbf{\Phi} := [\mathbf{\Phi}_1^1\ \cdots\ \mathbf{\Phi}_{k^m}^1\ \cdots\ \mathbf{\Phi}_1^{k^l}\ \cdots\ \mathbf{\Phi}_{k^m}^{k^l}]$, where

$$\mathbf{\Phi}_{\nu}^{\varepsilon} := L_{\nu}^{\varepsilon} * \mathbf{p} + \mathbf{\Phi}_J \in \Upsilon_{k^n \times k^n},\ \forall\ \nu \in \mathbb{N}_1^{k^m},\ \forall\ \varepsilon \in \mathbb{N}_1^{k^l}.$$

Proposition 3. For any states $\delta_{k^n}^{\alpha}, \delta_{k^n}^{\beta} \in \Delta_{k^n}$, any control $\delta_{k^m}^{\nu} \in \Delta_{k^m}$ and any disturbance $\delta_{k^l}^{\varepsilon} \in \Delta_{k^l}$, it holds that $\mathbb{P}\{z(s+1) = \delta_{k^n}^{\beta} | z(s) = \delta_{k^n}^{\alpha}, u_{\phi}(s) = \delta_{k^m}^{\nu}, \xi_{\phi}(s) = \delta_{k^l}^{\varepsilon}\} = (\mathbf{\Phi}_{\nu}^{\varepsilon})_{\beta,\alpha}$.

See Appendix B.5 for the proof.

Let $x_e = \delta_{k^n}^{\eta}$ and $R_0(x_e) := \Big\{\eta$: there exists $\nu \in \mathbb{N}_1^{k^m}$ such that $\sum_{\varepsilon=1}^{k^l}(\mathbf{\Phi}_{\nu}^{\varepsilon})_{\eta,\eta} = k^l\Big\}$. We construct the following disturbance reachable sets:

$$R_d(x_e) := \Big\{\alpha \in \mathbb{N}_1^{k^n} : \text{there exists } \nu \in \mathbb{N}_1^{k^m} \text{such that}$$
$$\sum_{\varepsilon=1}^{k^l}\sum_{\beta \in R_{d-1}(x_e)}(\mathbf{\Phi}_{\nu}^{\varepsilon})_{\beta,\alpha} = k^l\Big\},\ d \in \mathbb{Z}_+. \quad (8)$$

Based on (8), one concludes that $R_d(x_e) = R_{d'}(x_e)$, $\forall\ d \geqslant d' \in \mathbb{N}$, if $R_{d'+1}(x_e) = R_{d'}(x_e)$. Furthermore, if $R_0(x_e) = \{\eta\}$, then $R_d(x_e) \subseteq R_{d+1}(x_e)$, $\forall\ d \in \mathbb{N}$.

Theorem 2. RILDS (1) is robust x_e-FFSPO, if and only if the following two items hold:

(i) $(\mathbf{\Phi}_J)^{k^n} = \mathbf{0}_{k^n \times k^n}$,

(ii) there exists $d \in \mathbb{N}_1^{k^n}$ such that $R_d(x_e) = \mathbb{N}_1^{k^n}$.

See Appendix B.6 for the proof.

Design of state-feedback controllers. Given initial state x_0 and state-feedback controller (2), let $T^*(x_0, \overline{\mathbf{u}})$ denote the minimum sum of hybrid indices t_s and j_s such that Eq. (3) holds for any disturbance sequence $\boldsymbol{\xi}$. Let $\mathbf{u}^*$ be the set of state-feedback controllers such that RILDS (1) is robust x_e-FFSPO. Moreover, let $T^*(x_0) := \min_{\overline{\mathbf{u}} \in \mathbf{u}^*}\{T^*(x_0, \overline{\mathbf{u}})\}$.

Definition 3. State-feedback controller $\overline{\mathbf{u}} \in \mathbf{u}^*$ is called the time-optimal state-feedback controller, if $T^*(x_0, \overline{\mathbf{u}}) := T^*(x_0)$, $\forall\ x_0 \in \Delta_{k^n}$.

Theorem 3. Let $d^* := \min\{d \in \mathbb{N}_1^{k^n} : R_d(x_e) = \mathbb{N}_1^{k^n}\}$. The controllers designed by Algorithm 1 are all time-optimal state-feedback controllers of system (1).

See Appendix B.7 for the proof.

Algorithm 1 The design of the feedback gain matrix H.

Require: Matrices $\mathbf{\Phi}_J$, $\mathbf{p}$ and L;
Ensure: The feedback gain matrix H;
$\Lambda_{\eta} = \Big\{\nu \in \mathbb{N}_1^{k^m} : \sum_{\varepsilon=1}^{k^l}(\mathbf{\Phi}_{\nu}^{\varepsilon})_{\eta,\eta} = k^l\Big\}$;
for $d = 1, \cdots, d^*$ **do**
  for all $\alpha \in R_d(x_e) \backslash R_{d-1}(x_e)$ **do**
    $\Lambda_{\alpha} = \Big\{\nu \in \mathbb{N}_1^{k^m} : \sum_{\varepsilon=1}^{k^l}\sum_{\beta \in R_{d-1}(x_e)}(\mathbf{\Phi}_{\nu}^{\varepsilon})_{\beta,\alpha} = k^l\Big\}$;
  end for
end for
return H where $\mathrm{Col}_{\alpha}(H) \in \Lambda_{\alpha}$, $\forall\ \alpha \in \mathbb{N}_1^{k^n}$.

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Supporting information Appendixes A–C. The supporting information is available online at **info.scichina.com** and **link.springer.com**. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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