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Special Topic: Logical System Control

A Boolean algebra approach for the disturbance decoupling problem of large-scale Boolean control networks

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Boolean networks (BNs) are discrete dynamical systems composed of logical operators and Boolean variables. Boolean control networks (BCNs) extend BNs through the incorporation of external control inputs and outputs. Both BNs and BCNs are widely used to model gene regulatory networks (GRNs), where gene activity is abstracted into ON/OFF states. In GRNs, disturbances are inevitable, such as transcription/translation noise, gene mutations, and external environmental stimuli. These disturbances can drive the network toward undesirable cellular states, including disease states like cancer. Therefore, addressing the disturbance decoupling problem (DDP) in BCNs becomes crucial. The objective of disturbance decoupling is to identify a set of intervention targets (e.g., specific genes or proteins) that block the adverse effects of disturbances, thereby ensuring the network converges to a desired state, such as a healthy cellular state, even under persistent disturbance. In recent years, Cheng and his team developed an algebraic state-space approach based on the semi-tensor product (STP) of matrices [1], which has led to substantial progress in solving the DDP for BCNs [1–4]. However, these methods generally suffer from exponential computational complexity, hindering their application to large-scale BCNs [5]. Therefore, developing new approaches to tackle DDP in large-scale BCNs remains an important and challenging research problem.

This study addresses the DDP for large-scale BCNs by leveraging Boolean algebra to overcome the challenge of high computational complexity. We first establish a necessary and sufficient solvability condition for the DDP, which is verifiable in polynomial time using restriction expressions and the properties of Boolean algebra. Furthermore, a simplified Karnaugh map (K-map) is introduced, leading to a constructive algorithm for designing state feedback controllers. The algorithm achieves a time complexity of $O(2^{n_1})$, where n_1 is typically much smaller than the total number of state variables n, significantly reducing the computational burden compared to existing methods. The effectiveness of our approach is demonstrated through a numerical example with 53 nodes.

Preliminaries. Let $\mathcal{B}$ and $\mathcal{B}^n$ denote $\{0,1\}$ and $\underbrace{\mathcal{B}\times\mathcal{B}\times\cdots\times\mathcal{B}}_{n}$, respectively, where $\times$ is the Cartesian product. $[m,n]$ denotes the set of integers i satisfying $m \leqslant i \leqslant n$, where $m < n$ are two integers. The symbols $\vee$, $\wedge$, and $\neg$ represent the logical operators OR, AND, and NOT, respectively.

A BCN with n state variables, m control inputs, q disturbances, and p outputs can be described as

$$\begin{cases} x_i(t+1) = f_i(u_1(t),\ldots,u_m(t),x_1(t),\ldots,x_n(t), \\ \qquad \xi_1(t),\ldots,\xi_q(t)), i=1,2,\ldots,n, \\ y_j(t) = h_j(x_1(t),\ldots,x_n(t)), j=1,2,\ldots,p, \end{cases} \tag{1}$$

where $x=(x_1,x_2,\cdots,x_n)$, $u=(u_1,u_2,\cdots,u_m)$, $\xi=(\xi_1,\xi_2,\cdots,\xi_q)$, and $y=(y_1,y_2,\cdots,y_p)$ represent the state, control, disturbance, and output of the system, respectively. $f_i: \mathcal{B}^{m+n+q}\rightarrow\mathcal{B}, i=1,2,\ldots,n$ and $h_j:\mathcal{B}^n\rightarrow\mathcal{B}, j=1,2,\ldots,p$ are Boolean functions.

Let f be a Boolean expression in $\mathcal{B}^n$. For a variable x_i, the restricted expressions $f|_{x_i=1}$ and $f|_{x_i=0}$ in $\mathcal{B}^{n-1}$ are defined by fixing x_i to 1 and 0, respectively, as given in (2a) and (2b).

$$f|_{x_i=1} = f(x_1,\ldots,x_{i-1},1,x_{i+1},\ldots,x_n), \tag{2a}$$
$$f|_{x_i=0} = f(x_1,\ldots,x_{i-1},0,x_{i+1},\ldots,x_n). \tag{2b}$$

The original expression can then be reconstructed as

$$f = (x_i \wedge f|_{x_i=1}) \vee (\bar{x}_i \wedge f|_{x_i=0}). \tag{3}$$

A variable x_i is called redundant if $f|_{x_i=1} = f|_{x_i=0}$, meaning it can be eliminated in a simplified form of the expression. Therefore, the influence of a logical variable on the Boolean expression can be determined directly by examining its restricted expressions. In this study, we consider the following disturbance decoupling definition introduced in [3].

Definition 1. The disturbance decoupling of BCN (1) is said to be implemented if there exists a feedback control $u(t)=\phi(x(t))$ such that, for every initial state $x(0)$, the output $y(t)$ is independent of disturbance $\xi(t-1)$ for all $t \geqslant 0$.

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Consider BCN (1). For each $j \in [1, p]$, the output y_j can be rewritten as

$$y_j(t) = \bar{h}_j(x_1(t-1), \ldots, x_n(t-1), u_1(t-1), \ldots, u_m(t-1), \xi_1(t-1), \ldots, \xi_q(t-1)). \tag{4}$$

For a given y_j in (4), if $y_j(t)|_{\xi_i=1} = y_j(t)|_{\xi_i=0}$, where $i \in [1, q]$, then ξ_i is a redundant variable for y_j and can be removed from $\bar{h}_j$. On the other hand, if $y_j(t)|_{\xi_i=1} \neq y_j(t)|_{\xi_i=0}$ for some i, then y_j depends on ξ_i, and a controller must be designed to decouple this disturbance.

This study solves the DDP for BCN (1) in two stages: first, determining the solvability of the DDP via restricted expressions and Boolean algebra; and second, if solvable, constructing the controller using a simplified Karnaugh map (K-map).

Theorem 1. BCN (1) is disturbance decoupled if and only if, for any $k \in [1, q]$, every $y_j(t), j \in [1, p]$, satisfies one of the following conditions.

(1) The restricted expressions $y_j(t)|_{\xi_k(t-1)=1}$ and $y_j(t)|_{\xi_k(t-1)=0}$ share the same expression.

(2) Let

$$y_j(t)|_{\xi_k(t-1)=1} = y_j(t)|_{\xi_k(t-1)=0}. \tag{5}$$

Eq. (5) can be rewritten in one of the following forms: $E \vee u = E, E \wedge u = E, E \vee u = 1, E \wedge u = 0, E \wedge u = u, E \vee u = u$, where E is a Boolean expression not dependent on u.

The proof of Theorem 1 can be seen in Appendix A. Based on Theorem 1, an algorithm (See Algorithm 1 in Appendix B) for verifying the solvability of DDP is provided. Upon completion of Algorithm 1, a list of constraint expression pairs required for controller design is obtained. If any output fails to satisfy the conditions of Theorem 1, the DDP is unsolvable, and the list (if generated) is discarded.

In the following, we focus on controller design using a simplified K-map. A K-map is a graphical tool used for the simplification of Boolean algebra expressions. In a conventional K-map, there are 2^n cells, where n is the number of Boolean variables. Each cell corresponds to a minterm, and the symbol X denotes a "don't care" value, which can be either 0 or 1 in $\mathcal{B}$. In this study, to reduce computational complexity, we introduce a simplified K-map that utilizes only the output-related state variables, rather than all state variables. The construction of the simplified K-map proceeds as follows.

Step 1: Based on Algorithm 1, select a pair of restricted expressions from the list required for controller design, denoted as $y_j(t)|_{\xi_k(t-1)=1}$ and $y_j(t)|_{\xi_k(t-1)=0}$.

Step 2: Let $y_j(t)|_{\xi_k(t-1)=1} = y_j(t)|_{\xi_k(t-1)=0}$ and transform the resulting equation into one of the following forms: $E \vee u = E$, $E \wedge u = E$, $E \vee u = 1$, $E \wedge u = 0$, $E \wedge u = u$, $E \vee u = u$, where E is a Boolean expression.

Step 3: The simplified K-map is constructed based on the state variables appearing in the Boolean expression E. In the case where E comprises only one state variable, the K-map is formed by introducing one additional variable not present in E, thereby enabling a meaningful tabular representation. When E contains multiple variables, the addition of a further variable is permitted but not required.

When a BCN is solvable, Algorithm 1 yields a list of restricted expression pairs required for controller design. To guide the controller design, we provide the design rules summarized in Table 1, which are derived from the identity laws in Boolean algebra. For any Boolean expression E with identity elements $\{0, 1\}$ and operators $\{+, \cdot\}$, the identity and complement laws are expressed as $E + 0 = E, M + 1 = 1, E \cdot 1 = E, E \cdot 0 = 0, E + \bar{E} = 1, E \cdot \bar{E} = 0$. Here, operations '$\cdot$' and '$+$' in Boolean algebra are equivalent to logical AND and logical OR, respectively. As stated in [4], for any restricted expression pair obtained from Algorithm 1 that does not conform to a structure listed in Table 1, it can be transformed into a compatible structure by applying the duality principle or simplifying the expression using Boolean algebraic laws. Based on the above analysis, we propose the following algorithm for designing feedback controllers to solve the DDP (See Algorithm 2 in Appendix C).

Table 1 Controller design based on the simplified K-map.

Rule no.	Pair structure	Control design rules
1	$E \vee u = E$	Retain the 0 entries in the simplified K-map of E and set the remaining cells as "don't care".
2	$E \wedge u = E$	Retain the 1 entries in the simplified K-map of E and set the remaining cells as "don't care".
3	$E \vee u = 1$	Retain the 1 entries in the simplified K-map of $\bar{E}$ and set the remaining cells as "don't care".
4	$E \wedge u = 0$	Retain the 0 entries in the simplified K-map of $\bar{E}$ and set the remaining cells as "don't care".
5	$E \wedge u = u$	Retain the 0 entries in the simplified K-map of E and set the remaining cells as "don't care".
6	$E \vee u = u$	Retain the 1 entries in the simplified K-map of E and set the remaining cells as "don't care".

For comparisons with existing results and illustrative examples, see Appendixes D and E.

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Supporting information Appendixes A–E. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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