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Pinning control of simplicial complexes with noise

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This study addresses the problem of pinning control in simplicial complexes influenced by two types of noise: dynamic noise and communication noise. It establishes the intricate relationships among synchronization behavior, node dynamics, higher-order structures, coupling strengths, noise characteristics, and control strategies. The findings reveal that both dynamic and communication noise can exert either beneficial or detrimental effects on synchronization. Specifically, the impact of dynamic noise is highly dependent on the intrinsic node dynamics, the inner-coupling functions, and the noise diffusion pattern. In contrast, communication noise demonstrates a dual role in synchronizability—either promoting or impairing synchronization—depending on factors such as network topology, the choice of pinned nodes, coupling strengths, and feedback gains (see Appendix C).

Problem formulation. Consider a D-simplicial complex composed of N identical nodes. The dynamics of the ith node is described by

$$\begin{aligned} dx_i = &\left[F(x_i) + \sum_{d=1}^{D}\sigma_d \sum_{j_1,\cdots,j_d=1}^{N} a^{(d)}_{ij_1\cdots j_d} H^{(d)}(x_i, x_{j_1},\cdots,x_{j_d})\right]dt \\ &+ \sigma_v g(x_i) dW(t) \\ &+ \sigma_c \left[\sum_{d=1}^{D}\sigma_d \sum_{j_1,\cdots,j_d=1}^{N} a^{(d)}_{ij_1\cdots j_d} H^{(d)}(x_i, x_{j_1},\cdots,x_{j_d})\right] db(t), \\ & i = 1,2,\cdots,N, \end{aligned} \tag{1}$$

where $x_i = [x_{i1}, x_{i2}, \cdots, x_{im}]^{\mathrm{T}} \in \mathbb{R}^m$ is the state vector of node i, $F(\cdot): \mathbb{R}^m \to \mathbb{R}^m$ is a function that describes the intrinsic dynamics of an individual node, and $H^{(d)}: \mathbb{R}^{(d+1)m} \to \mathbb{R}^m$ $(d = 1,2,\cdots,D)$ are inner-coupling functions, satisfying $H^{(d)}(x,x,\cdots,x) \equiv 0$, $\forall d$. The parameters $\sigma_d \in \mathbb{R}^+$ $(d = 1,2,\cdots,D)$ denote the coupling strengths, and $a^{(d)}_{ij_1\cdots j_d}$ are the elements of the adjacency tensors $A^{(d)}$, $d = 1,2,\cdots,D$. The term $\sigma_v g(x_i) dW(t)$ denotes the uncertainty in the node dynamics, where σ_v is the intensity of the dynamic noise, $g(\cdot): \mathbb{R}^m \to \mathbb{R}^m$ is the noise diffusion function, and $W(t)$ is 1-dimensional Brownian motion, representing the dynamic noise. The final term represents the uncertainty that exists in communication, where σ_c is the intensity of the communication noise, and $b(t)$ is a one-dimensional Wiener process independent of $W(t)$, used to represent the communication noise.

In this study, we employ pinning control on the simplicial complex, enabling the noise-perturbed system described by (1) to synchronize to a desired target state x_s, which satisfies

$$dx_s = F(x_s)dt + \sigma_v g(x_s) dW(t). \tag{2}$$

Therefore, the controlled D-simplicial complex can be described as

$$\begin{aligned} dx_i = &\left[F(x_i) + \sum_{d=1}^{D}\sigma_d \sum_{j_1,\cdots,j_d=1}^{N} a^{(d)}_{ij_1\cdots j_d} H^{(d)}(x_i, x_{j_1},\cdots,x_{j_d}) \right. \\ &\left. + \xi_i u_i \right] dt + \sigma_v g(x_i) dW(t) \\ &+ \sigma_c \left[\sum_{d=1}^{D}\sigma_d \sum_{j_1,\cdots,j_d=1}^{N} a^{(d)}_{ij_1\cdots j_d} H^{(d)}(x_i, x_{j_1},\cdots,x_{j_d}) \right. \\ &\left. + \xi_i u_i \right] db(t), \quad i = 1,2,\cdots,N \end{aligned} \tag{3}$$

with

$$u_i = b_i \sigma_1 H^{(1)}(x_i, x_s), \quad i = 1,2,\cdots,N, \tag{4}$$

where ξ_i indicates whether the ith node is controlled, and b_i denotes the corresponding feedback gain. Specifically, if node i is controlled, then $\xi_i = 1$ and $b_i > 0$; otherwise, $\xi_i = 0$ and $b_i = 0$. Let $\sum_{i=1}^{N}\xi_i = l$, where l represents the number of controlled nodes, satisfying $1 \leqslant l < N$. It is important to emphasize that pinning control is applied only to a limited subset of nodes in the simplicial complex.

Assumption 1. Assume that the functions $F(\cdot)$, $g(\cdot)$, and $H^{(d)}$ are C^1 continuously differentiable (i.e., first-order derivatives exist and are continuous) and Lipschitz continuous on the neighborhood Ω of the synchronous state x_s.

Definition 1. The controlled simplicial complex (3) is said to reach complete stochastic synchronization onto x_s if

$$\mathbb{P}\left(\lim_{t\to\infty} \|x_i - x_s\|_2 = 0\right) = 1, \quad \text{a.s.}, \quad i = 1,2,\cdots,N,$$

where $\mathbb{P}(w)$ denotes the probability of an event w.

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Next, we analyze the stability of the controlled noise-affected simplicial complex. Due to the complexity of (3), we first analyze the case $D = 2$ for simplicity, without loss of generality. It is worth noting that the analysis for $D = 2$ can be readily extended to higher-order cases with $D > 2$.

When $D = 2$, Eq. (3) can be rewritten as

$$\begin{aligned} dx_i = &\Bigg[F(x_i) + \sigma_1 \sum_{j_1=1}^{N} a_{ij_1}^{(1)} H^{(1)}(x_i, x_{j_1}) \\ &+ \sigma_2 \sum_{j_1=1}^{N} \sum_{j_2=1}^{N} a_{ij_1j_2}^{(2)} H^{(2)}(x_i, x_{j_1}, x_{j_2}) + \xi_i u_i\Bigg] dt \\ &+ \sigma_v g(x_i) dW(t) \\ &+ \sigma_c \Bigg[\sigma_1 \sum_{j_1=1}^{N} a_{ij_1}^{(1)} H^{(1)}(x_i, x_{j_1}) \\ &+ \sigma_2 \sum_{j_1=1}^{N} \sum_{j_2=1}^{N} a_{ij_1j_2}^{(2)} H^{(2)}(x_i, x_{j_1}, x_{j_2}) + \xi_i u_i\Bigg] db(t), \\ &i = 1, 2, \cdots, N, \end{aligned} \tag{5}$$

where σ_1 and σ_2 denote the coupling strengths for two-body and three-body interactions, respectively.

Assumption 2. Assume that

$$\begin{aligned} H^{(1)}(x_i, x_j) &= \omega^{(1)}(x_j) - \omega^{(1)}(x_i), \\ H^{(2)}(x_i, x_j, x_k) &= \omega^{(2)}(x_j, x_k) - \omega^{(2)}(x_i, x_i), \end{aligned} \tag{6}$$

where $\omega^{(d)} : \mathbb{R}^{dm} \to \mathbb{R}^m$ $(d = 1, 2)$ satisfy the following condition:

$$\omega^{(2)}(x, x) = \omega^{(1)}(x). \tag{7}$$

The local stability condition for network synchronization can be derived.

Theorem 1. Suppose Assumptions 1 and 2 hold. The pinned simplicial complex (5) achieves complete stochastic synchronization if the largest Lyapunov exponent $\Lambda < 0$, where Λ is related to σ_v and σ_c.

The proof of Theorem 1 is provided in Appendix D. The synchronized region of the system is defined as the set of parameters for which $\Lambda < 0$ holds. Unlike conventional network models, the extended master stability equation incorporates noise terms and is not a differential equation with respect to a single parameter (see Appendix D). Moreover, in the case of simplicial complexes, the extended master stability equation no longer depends solely on the eigenvalues of a single Laplacian matrix—as is typical in traditional networks—but instead involves the eigenvalues of multiple generalized Laplacian matrices, together with the sum of the products of the corresponding coupling strengths.

Remark 1. It is worth emphasizing that this study differs from existing studies. On one hand, in [1], pinning control was studied in lower-order networks subject to both dynamic and communication noise. By contrast, we investigate the pinning control of higher-order networks modeled by simplicial complexes, which provide a more suitable framework for representing real-world systems. On the other hand, in [2], a virtual node called a pinner is introduced, whose dynamics are not affected by those of the controlled nodes. In this setting, control actions are modeled as directed hyperedges from the pinner to a subset of nodes, allowing multiple nodes to share the same control action. In our work, however, pinning control is pairwise: the control actions are applied only to a small fraction of nodes, with each action targeting a single node. This scheme reflects a more localized and resource-efficient control strategy, where the influence of each controller is confined to an individual node rather than being simultaneously distributed across multiple nodes. Such a design not only reduces the complexity of implementation in practical systems but also facilitates a clearer analysis of how local control propagates through higher-order interactions in simplicial complexes.

Conclusion. This work investigates the pinning control of simplicial complexes under two types of noise. Within the master stability function (MSF) framework, we derive an extended master stability equation and corresponding synchronization stability criteria. By applying different nonlinear systems as intrinsic node dynamics, we analyze how noise affects synchronization, focusing on the roles of diffusion and coupling functions. The results show that noise can modulate synchronization patterns, inducing phase transitions between stable and unstable regimes. For dynamic noise, the impact depends on both intrinsic dynamics and diffusion patterns, and when applied to all state variables, it often enhances synchronization in a certain range. Communication noise, however, can either promote or hinder synchronization, depending on network topology, pinned nodes, coupling strengths, and feedback gains. These findings extend traditional synchronization and control paradigms, offering a more realistic perspective for designing control strategies in higher-order networks subject to noise.

Future research should further address synchronization in higher-order networks with time delays [3] and heterogeneous dynamics, as well as compare noise effects between lower- and higher-order networks. Exploring the structural differences between hypergraphs and simplicial complexes, optimizing network topology [4], conducting robustness analysis under varying noise intensities, and integrating with machine learning algorithms [5] will advance interdisciplinary frameworks and open new directions for network science and its applications to real-world systems.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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