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Finite-time hybrid cooperative control of nonlinear time-delay multiagent systems and its applications in Chua's systems

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As is commonly acknowledged, a central focus of multi-agent systems (MASs) research is achieving consensus control [1,2]. This entails individuals reaching consensus in specific states through information transmission and coordinated control. Expanding on previous research, scholars have started to address practical constraints related to limited resources and communication bandwidth [3]. In response to these challenges, event-triggered strategies have been introduced to optimize communication resources by minimizing unnecessary data sampling and transmission. Among the various control protocols, the event-triggered control strategy stands out as it updates the control function when specific event-triggered conditions are established. Such strategies are particularly critical in resource-constrained scenarios like unmanned aerial vehicle (UAV) swarms for search-and-rescue missions or industrial Internet of Things (IoT) device coordination, where timely convergence (i.e., finite-time performance) is often mission-critical. Our work aims to bridge this gap by enhancing both efficiency and rapidity in cooperative control.

Investigating the stability of nonlinear systems, particularly in systems with time delays, has been undertaken through a combination of event-triggered and impulsive control techniques. However, in many actual cases, we occasionally need the system to achieve convergence in a certain time, that is, to achieve finite-time convergence (FTC). The authors in [4] investigated the FTC problem under disturbances and uncertain parameters, utilizing an event-triggered control strategy. Consequently, a protocol by melting event-triggered with impulsive control is proposed to achieve FTC for first-order leader-following MASs with uncertain disturbance. The key contributions of the work can be summed up as follows.

(1) In the case that the followers communicate with the leader only at impulsive instants, this work introduces a hybrid impulsive control strategy that incorporates an event-triggered mechanism. This approach, grounded in practical considerations, significantly reduces communication costs, and it is shown that the proposed method can mitigate the "Zeno phenomenon" effectively.

(2) In view that communication delays inevitably take place in network congestion or interference in signal transmission between different individuals, in contrast to [5], this work takes communication delays into the design of the consensus control protocol.

(3) This work takes into account situations of control input saturation arising during communication between agents. Even where communication delays and input saturation exist, the control protocol proposed in this work enables first-order leader-following MASs to achieve FTC, and the simulation examples are provided for the purpose of clearly demonstrating the effectiveness of our proposed analysis.

Preliminaries and problem formulation. Preliminaries are provided in Appendix A.

Problem 1. Consider first-order leader-following MASs. The dynamics of each agent is

$$\begin{cases} \dot{x}_i(\zeta) = u_i(\zeta - \nu) + \varphi_i(\zeta), \\ \dot{x}_0(\zeta) = \varphi_0(\zeta), \end{cases} \tag{1}$$

where $x_i(\zeta) \in R$ stands for the state of i-th agent and $i \in \{1, 2, 3, \ldots, N\}$, $\zeta \in R$ stands for time, $u_i(\zeta) \in R$ is the input of control and ν is time delay of it, $\varphi_0(\zeta)$ and $\varphi_i(\zeta)$ are the disturbance of the system, and $x_0(\zeta) \in R$ is the leader's state.

Assumption 1. $\varphi_0(\zeta)$ and $\varphi_i(\zeta)$ satisfy $|\varphi_0(\zeta)|, |\varphi_i(\zeta)| \leqslant \gamma$, where constant $\gamma \geqslant 0$.

Assumption 2. The communication topology graph G between followers of the system (1) is connected. Moreover, at least one follower communicates with the leader.

For system (1), a consensus protocol $u_i(\zeta)$ with time delays is presented based on an event-triggered mechanism and an impulsive control strategy.

$$u_i(\zeta - \nu) = u_i^l(\zeta - \nu) + u_i^f(\zeta - \nu), \tag{2}$$

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$$u_i^l(\zeta-\nu) = \mathrm{sat}\left(\sum_{k=1}^{+\infty}\delta(\zeta-\zeta_k)K_i b_i\big(x_i(\zeta-\nu)-x_0(\zeta-\nu)\big)\right), \quad (3)$$

$$\begin{aligned} u_i^f(\zeta-\nu) = & -\varrho_1\left(\sum_{j=1}^{N}a_{ij}\left(x_i(\zeta_{i^e}-\nu)-x_j(\zeta_{i^e}-\nu)\right)\right)^{\frac{m}{n}} \\ & -\varrho_2\mathrm{sign}\left(\sum_{j=1}^{N}a_{ij}\big(x_i(\zeta_{i^e}-\nu)-x_j(\zeta_{i^e}-\nu)\big)\right), \end{aligned} \quad (4)$$

where adjustable parameters $\varrho_1, \varrho_2 > 0$. ζ_{i^e} and $K_i \in [-1, 0]$ being elements of diagonal matrix K are the e-th trigger time and adjustable impulsive intensity of agent i respectively. There are two odd-number m, n satisfying $n > m > 0$. The constant impulsive time series $\{\zeta_k\}$ satisfies $0 < \zeta_1 < \cdots < \zeta_k < \zeta_{k+1} < +\infty$, besides $\lim_{k\to+\infty}\zeta_k = +\infty$, $\zeta_{k+1}-\zeta_k \leqslant \epsilon$. In addition, $\delta(t)$ is denoted as a Dirac function.

Remark 1. In practical applications, communication between leaders and followers is often limited by environmental or communication conditions, making real-time continuous communication difficult to achieve. Compared with the impulsive control strategy in [6], the hybrid event-triggered control and impulsive control proposed in this work not only enable followers to communicate with the leader only at impulsive moments but also ensure that communication among followers in the control action occurs only at event-triggered moments. This has a stronger practical application value and further reduces communication energy consumption.

Denote the measurement error $Q_i(\zeta)$ as

$$\begin{aligned} Q_i(\zeta) = & \varrho_1\left(\sum_{j=1}^{N}a_{ij}\big(x_i(\zeta_{i^e})-x_j(\zeta_{i^e})\big)\right)^{\frac{m}{n}} \\ & +\varrho_2\mathrm{sign}\left(\sum_{j=1}^{N}a_{ij}\big(x_i(\zeta_{i^e})-x_j(\zeta_{i^e})\big)\right) \\ & -\varrho_1\left(\sum_{j=1}^{N}a_{ij}\big(x_i(\zeta)-x_j(\zeta)\big)\right)^{\frac{m}{n}} \\ & -\varrho_2\mathrm{sign}\left(\sum_{j=1}^{N}a_{ij}\big(x_i(\zeta)-x_j(\zeta)\big)\right). \end{aligned} \quad (5)$$

Combined with (8), it follows that

$$\begin{aligned} u_i^f(\zeta) = & -\left(Q_i(\zeta)+\varrho_1\left(\sum_{j=1}^{N}a_{ij}(x_i(\zeta)-x_j(\zeta))\right)^{\frac{m}{n}}\right. \\ & \left.+\varrho_2\mathrm{sign}\left(\sum_{j=1}^{N}a_{ij}(x_i(\zeta)-x_j(\zeta))\right)\right). \end{aligned} \quad (6)$$

From (8), event-triggered function $g_i(\zeta)$ is proposed,

$$g_i(\zeta) = |Q_i(\zeta)| - \tau_1\varrho_1\left|\sum_{j=1}^{N}a_{ij}\big(x_i(\zeta)-x_j(\zeta)\big)\right|^{\frac{m}{n}} - \tau_2\varrho_2, \quad (7)$$

where adjustable event-triggered parameters $\tau_1, \tau_2 \in (0, 1)$. When $g_i(\zeta) > 0$, $\zeta > \zeta_{i^e}$, the consensus protocol $u_i(\zeta)$ will be next updated.

Theorem 1. Under Assumptions 1 and 2, the MASs (1) can achieve FTC with the hybrid control protocol (2), if

$$\begin{aligned} &\text{(i)}\ \ 2\gamma-(1-\tau_2)\varrho_2<0, \\ &\text{(ii)}\ \ \frac{\ln\frac{(1+P_{\max})^2}{2}-\theta\nu}{\epsilon}+\theta<0. \end{aligned} \quad (8)$$

Theorem 2. Under the influence of control protocol (2), the MASs (1) ensures the absence of the "Zeno phenomenon" during the attainment of finite-time consensus. This specifically implies the presence of a positive time interval between successive event-triggering instances, thereby averting the occurrence of rapid successive triggering.

The proof of Theorems 1 and 2 can be found in Appendixes B and C, at last numerical examples are given in Appendix D.

Conclusion. The study addresses the FTC for a leader-following MASs under uncertain disturbances, communication delays and input saturation of first-order dynamics. A control protocol is proposed that integrates event-triggering and impulsive control techniques, effectively reducing communication between the leader and the following, which leads to a significant reduction in communication frequency and energy consumption, enhancing communication efficiency. This protocol enables the system to achieve FTC while eliminating the occurrence of the "Zeno phenomenon". In future research, the FTC control will be investigated under more intricate disturbances and in the context of variable stochastic communication delays.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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