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• **Supplementary File** •

Finite-Time Hybrid Cooperative Control of Nonlinear Time-Delay Multiagent Systems and Its Applications in Chua's Systems

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Appendix A Preliminaries

A weighted and undirected graph G defined as $G = (S, E, A)$ can be used to describe a MASs with n agents. $S = \{S_1, S_2, S_3, ..., S_n\}$ is a set of notes that a note represents an agent. $E \in S \times S$ is a edge set. If $(S_i, S_j) \in E$, it represents that there exists information transmission between the j-th agent and the i-th agent, where $i, j \in \{1, 2, 3, ..., n\}$. $A = [a_{ij}] \in R^{n\times n}$ is the weighted adjacency matrix. Especially, $[a_{ij}] = [a_{ji}]$ in the undirected graph and $[a_{ii}] = 0$. Let $\varkappa = diag\{\varkappa_1, \varkappa_2, ..., \varkappa_n\}$, If the i-th follower has access to the data provided by the leader, $b_i = 1$ if not $b_i = 0$, where b_i is the element of diagonal matrix B. The Laplacian matrix is $L = \varkappa - A$ and $H = L + B$.

The operational rules for sign ($\mathbb{A}$) and sat($\mathbb{B}$) are

$$\mathrm{sign}(\mathbb{A}) = \begin{cases} -1, & \mathbb{A} < 0 \\ 0, & \mathbb{A} = 0 \\ 1, & \mathbb{A} > 0 \end{cases},$$

$$\mathrm{sat}(\mathbb{B}) = \begin{cases} 1, & \mathbb{B} > 1 \\ \mathbb{B}, & \mathbb{B} \in [-1, 1] \\ -1, & \mathbb{B} < -1 \end{cases}.$$

Lemma 1. [1] If the undirected graph G is a connected graph, its Laplacian matrix L is positive semidefinite, and L has eigenvalues of $0, \lambda_2, \ldots, \lambda_N$, satisfying $0 < \lambda_2 \leqslant \cdots \leqslant \lambda_N$, what is more $\boldsymbol{I}^T x = 0$, where $x = [x_1, \ldots, x_N]^T$, then it can be concluded that

$$\lambda_2 x^T x \leqslant x^T L x,$$

and

$$x^T L x = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(x_i - x_j)^2.$$

Lemma 2. [2] Let two column vectors $g = (g_1, g_2, ..., g_n)^T$, $h = (h_1, h_2, ..., h_n)^T \in R^n$. There exists a set of diagonal matrixs W with diagonal elements that are exclusively either 0 or 1, and define W_i is the i-th element of W, $i = 1, 2, ..., 2^n$. Let $W_i^- = I - W_i$, where I is a identity matrix, $I \in R^{n\times n}$, evidently $W_i^- \in W$. If $|h_i| \leqslant 1$, then sat(g) $\in co\{W_i g + W_i^- h : i \in \{1, 2, ..., 2^n\}\}$.

Remark 1. Taking Lemma 2 as an illustration, in the case where n = 2, the matrix W has a dimension of 2, encompassing four matrix elements:

$$W_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, W_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, W_3 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, W_4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

When $n = 3$, W has a dimension of 3, consisting of eight matrix elements:

$$W_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, W_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, W_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, W_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

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$$W_5=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},W_6=\begin{bmatrix}1&0&0\\0&0&0\\0&0&1\end{bmatrix},W_7=\begin{bmatrix}0&0&0\\0&1&0\\0&0&1\end{bmatrix},W_8=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.$$

By extension, this pattern continues accordingly. What is more, one can obtain $\mathrm{sat(Mx)} \in \mathrm{co}\{\mathrm{W_iMx + W_i^- Sx : i \in \{1,2,...,2^n\}}\}$ with the condition of $\|Sx\|_\infty \leqslant 1$, where $x \in R^n$ and $M, S \in R^{n\times n}$. When the following conditions are satisfied,

$$\begin{cases}0 \leqslant f_i \leqslant 1\\ \sum_{i=1}^{2^n} f_i = 1\end{cases}$$

$\mathrm{sat(Mx)}$ can be further simplified as $\mathrm{sat(Mx)} = \sum_{\mathrm{i=1}}^{2^{\mathrm{n}}} \mathrm{f_i(W_iM + W_i^- S)x}$.

Lemma 3. [3] Provided that the continuous and non-negative function $V(\zeta)$ satisfy the following criteria for all solutions of a system

$$\dot{V}(x(\zeta)) \leqslant -cV^{\alpha}(x(\zeta))$$

where $c > 0, 0 < \alpha < 1$. The equilibrium point is globally asymptotically stable with a finite-time stability property, and the stability time T can be deduced that

$$T \leqslant \frac{V^{1-\alpha}(x_0)}{c(1-\alpha)}$$

where $V(x_0)$ is the initial value of $V(x(\zeta))$.

Lemma 4. [4] If $k >> 1$, then function $sign(x)$ is approximately equal to function $tanh(kx)$.

Lemma 5. [5] If m, n are two odd numbers satisfying $n > m > 0$, and $\mathbb{A}(a), \mathbb{B}(a) \in R$, then ones have

$$\left|\mathbb{A}(a)^{\frac{m}{n}} - \mathbb{B}(a)^{\frac{m}{n}}\right| \leqslant 2^{1-\frac{m}{n}} \left|\mathbb{A}(a) - \mathbb{B}(a)\right|^{\frac{m}{n}}.$$

Appendix B Proof of Theorem 1

It is assumed that the state of every agent is right-continuous, which implies $x_i(\zeta_k) = x_i(\zeta_k^-) = \lim_{\zeta\to\zeta_k^-} x_i(\zeta)$. The tracking error $e_{x_i}(\zeta)$ is designed as

$$e_{x_i}(\zeta) = x_i(\zeta) + (1-\delta(\zeta-\zeta_k))\int_{\zeta}^{\zeta+\nu} u_i(s-\nu)ds - x_0(\zeta) \tag{B1}$$

where $e_{x_i}(\zeta) \in R$. Thus, the error system is described by

$$\begin{cases}\dot{e}_{x_i}(\zeta) = -\varrho_1\Big(\sum_{j=1}^N a_{ij}\big(x_i(\zeta_{i^e}) - x_j(\zeta_{i^e})\big)\Big)^{\frac{m}{n}} - \varrho_2 \mathrm{sign}\Big(\sum_{\mathrm{j=1}}^{\mathrm{N}} \mathrm{a_{ij}}\big(\mathrm{x_i}(\zeta_{\mathrm{i^e}}) - \mathrm{x_j}(\zeta_{\mathrm{i^e}})\big)\Big) & \\ \qquad +\varphi_i(\zeta) - \varphi_0(\zeta), & \zeta \neq t_k\\ \Delta e_{x_i}(\zeta_k) = \mathrm{sat}\Big(\mathrm{K_i b_i e_{x_i}}(\zeta_k^- - \nu)\Big), & \zeta = \zeta_k.\end{cases} \tag{B2}$$

From lemma 2, (B2) is able to be further simplified as

$$\begin{cases}\dot{e}_{x_i}(\zeta) = -\varrho_1\Big(\sum_{j=1}^N a_{ij}\big(x_i(\zeta_{i^e}) - x_j(\zeta_{i^e})\big)\Big)^{\frac{m}{n}} - \varrho_2 \mathrm{sign}\Big(\sum_{\mathrm{j=1}}^{\mathrm{N}} \mathrm{a_{ij}}\big(\mathrm{x_i}(\zeta_{\mathrm{i^e}}) - \mathrm{x_j}(\zeta_{\mathrm{i^e}})\big)\Big) & \\ \qquad +\varphi_i(\zeta) - \varphi_0(\zeta), & \zeta \neq \zeta_k\\ \Delta e_{x_i}(\zeta_k) = P_i e_{x_i}(\zeta_k^- - \nu), & \zeta = \zeta_k\end{cases} \tag{B3}$$

where $P = \sum_{i=1}^{2^N} f_i(W_iKB + W_i^- S)$ and $P = \mathrm{diag[P_1, P_2, ..., P_n]}$.

Definition 1. Taking control protocol (2) into consideration, it can be reasonably surmised that the FTC of MASs (1) is achievable if there exists $T \in (0, +\infty)$ satisfying

$$\begin{cases}\lim_{\zeta\to T}(x_i(\zeta) - x_0(\zeta)) = 0, i = 1,2,\cdots,N.\\ x_i(\zeta) - x_0(\zeta) = 0, \zeta > T.\end{cases}$$

Construct the Lyapunov function$V(\zeta) = \frac{1}{2}e_x^T(\zeta)Le_x(\zeta)$.Denote $e_x(\zeta) = [e_{x_1}(\zeta), \ldots, e_{x_N}(\zeta)]^T$, $Q(\zeta) = [Q_1(\zeta), \ldots, Q_N(\zeta)]^T$, $T_i(\zeta_{i^e}) = \sum_{j=1}^N a_{ij}(x_i(\zeta_{i^e}) - x_j(\zeta_{i^e}))$, $T(\zeta) = [T_1(\zeta), \ldots, T_N(\zeta)]^T$, $\varphi(\zeta) = [\varphi_1(\zeta) - \varphi_0(\zeta), \ldots, \varphi_N(\zeta) - \varphi_0(\zeta)]^T$, when $\zeta \neq \zeta_k$

$$\begin{aligned}\dot{V}(\zeta) = e_x^T(\zeta)L\dot{e}_x(\zeta) &= e_x^T(\zeta)L)\Big(\varphi(\zeta) - \varrho_1 T^{\frac{m}{n}}(\zeta_{i^e}) - \varrho_2\mathrm{sign}\big(\mathrm{T}(\zeta_{\mathrm{i^e}})\big)\Big)\\ &= e_x^T(\zeta)L\Big(\varphi(\zeta) - Q(\zeta) - \varrho_1 T^{\frac{m}{n}}(\zeta) - \varrho_2\mathrm{sign}\big(\mathrm{T}(\zeta)\big)\Big)\\ &= \sum_{i=1}^N T_i(\zeta)\Big(-Q_i(\zeta) - \varrho_1 T_i^{\frac{m}{n}}(\zeta) - \varrho_2\mathrm{sign}\big(\mathrm{T_i}(\zeta)\big) + \varphi_\mathrm{i} - \varphi_0\Big).\end{aligned} \tag{B4}$$

According to Assumption 1, $\varphi_i - \varphi_0 \leqslant 2\gamma$ can be obtained. It then follow that

$$\dot{V}(\zeta) \leqslant -\sum_{i=1}^N T_i(\zeta)Q_i(\zeta) - \varrho_1\sum_{i=1}^N T_i^{\frac{m}{n}+1}(\zeta) + \sum_{i=1}^N |T_i(\zeta)|(-\varrho_2 + 2\gamma)$$

$$\leqslant \sum_{i=1}^{N}\Big(-T_i(\zeta)Q_i(\zeta)-\varrho_1\tau_1|T_i(\zeta)|T_{x_i}^{\frac{m}{n}}(\zeta)-\varrho_2\tau_2\big|T_i(\zeta)\big|\Big)-(1-\tau_1)\varrho_1\sum_{i=1}^{N}T_i^{\frac{m+n}{n}}(\zeta)$$
$$+\big(2\gamma-(1-\tau_2)\varrho_2\big)\sum_{i=1}^{N}\big|T_i(\zeta)\big|. \tag{B5}$$

When event is triggered, one has $|Q_i(\zeta)|-\tau_1\varrho_1\left|\sum_{j=1}^{N}a_{ij}\big(x_i(\zeta)-x_j(\zeta)\big)\right|^{\frac{m}{n}}-\tau_2\varrho_2>0$. The simplification of (B5) gives $\dot V(\zeta)\leqslant (2\gamma-(1-\tau_2)\varrho_2)\sum_{i=1}^{N}|T_i(\zeta)|-(1-\tau_1)\varrho_1\sum_{i=1}^{N}T_i^2(\zeta)^{\frac{m+n}{2n}}$. It follow from (8) that

$$\dot V(\zeta)\leqslant -(1-\tau_1)\varrho_1\sum_{i=1}^{N}T_i^2(\zeta)^{\frac{m+n}{2n}}. \tag{B6}$$

Combined with Assumption 2, a conclusion can be easily drawn that a positive definite matrix G is present, satisfying $L=G^TG$. Subsequently it can be concluded from Lemma 1 that

$$\sum_{i=1}^{N}T_i^2(\zeta)=e_x^T(\zeta)L^TLe_x(\zeta)=e_x^T(\zeta)G^TLGe_x(\zeta)\geqslant\lambda_{\min}e_x^T(\zeta)G^TGe_x(\zeta)=2\lambda_{\min}V(\zeta)$$

where λ_{min} is the minimum eigenvalue of L. To proceed, from (B6), ones have

$$\dot V(\zeta)\leqslant -(1-\tau_1)\varrho_1(2\lambda_{\min})^{\frac{m+n}{2n}}V^{\frac{m+n}{2n}}(\zeta)\leqslant\theta V(\zeta) \tag{B7}$$

where $\theta>1$. When $\zeta=\zeta_k$,

$$\begin{aligned}V(\zeta_k^+) &= \frac{1}{2}e^T(\zeta_k^+)Le(\zeta_k^+)\\ &= \frac{1}{4}\sum_{i,j=1}^{N}a_{ij}\big(e_i(\zeta_k^+)-e_j(\zeta_k^+)\big)^2\\ &= \frac{1}{4}\sum_{i,j=1}^{N}a_{ij}\Big((1+P_i)\big(e_i(\zeta_k^--\nu)-e_j(\zeta_k^--\nu)\big)\Big)^2\leqslant\frac{1}{2}(1+P_{\max})^2V(\zeta_k^--\nu)\end{aligned} \tag{B8}$$

where $P_{\max}=\max\{P_1,\ldots,P_n\}$, which means $P_{\max}$ is equivalent to taking the maximum value of $\{P_1\ldots P_n\}$. From (B7), obviously $\dot V(\zeta)\leqslant\theta V(\zeta),\zeta\neq\zeta_k$. Therefore, when $\zeta\in[\zeta_0,\zeta_1]$, clearly $V(\zeta)\leqslant e^{\theta(\zeta-\zeta_0)}V(\zeta_0)$. To proceed, when $\zeta\in(\zeta_k,\zeta_{k+1}]$, one has

$$\begin{aligned}V(\zeta)&\leqslant e^{\theta(\zeta-\zeta_k)}V(\zeta_k^+)\\ &\leqslant\frac{(1+P_{\max})^2}{2}e^{\theta(\zeta-\zeta_k)}V(\zeta_k^--\nu)\\ &\leqslant\frac{(1+P_{\max})^2}{2}e^{\theta(\zeta-\zeta_k)}e^{\theta(\zeta_k-\nu-\zeta_{k-1})}V(\zeta_{k-1})\\ &\leqslant\frac{(1+P_{\max})^{2k}}{2^k}e^{\theta(\zeta-\zeta_0-k\nu)}V(\zeta_0)\\ &\leqslant e^{k\left(ln\frac{(1+P_{\max})^2}{2}-\theta\nu\right)+\theta(\zeta-\zeta_0)}V(\zeta_0)\\ &\leqslant e^{\frac{\left(ln\frac{(1+P_{\max})^2}{2}-\theta\nu\right)(\zeta-\zeta_0)+\theta(\zeta-\zeta_0)}{\epsilon}}e^{\left(\theta\nu-ln\frac{(1+P_{\max})^2}{2}\right)}V(\zeta_0)\\ &\leqslant e^{\left(\frac{ln\frac{(1+P_{\max})^2}{2}-\theta\nu}{\epsilon}+\theta\right)(\zeta-\zeta_0)}\frac{2e^{\theta\nu}}{(1+P_{\max})^2}V(\zeta_0)\end{aligned}$$

which then ends the proofs.

Remark 2. In the aforementioned proof and lemma 3, it has been demonstrated that the control action triggered by events can enable the system to achieve FTC. Moreover, the control action governing the follower agents during pulse instants is also effective. As long as all agents have not yet reached consensus, the event-triggered control during non-pulse instances remains in effect. Consequently, under the influence of the entire control (2), the MASs (1) can achieve FTC.

Appendix C Proof of Theorem 2

From (5), one has

$$\begin{aligned}|Q_i(\zeta)|\ \leqslant\ &\varrho_1\left|\left(\sum_{j=1}^{N}a_{ij}\big(x_i(\zeta_{i^e})-x_j(\zeta_{i^e})\big)\right)^{\frac{m}{n}}-\left(\sum_{j=1}^{N}a_{ij}\big(x_i(\zeta)-x_j(\zeta)\big)\right)^{\frac{m}{n}}\right|\\ &+\varrho_2\left|\text{sign}\left(\sum_{j=1}^{N}a_{ij}\big(x_i(\zeta_{i^e})-x_j(\zeta_{i^e})\big)\right)-\text{sign}\left(\sum_{j=1}^{N}a_{ij}\big(x_i(\zeta)-x_j(\zeta)\big)\right)\right|.\end{aligned} \tag{C1}$$

Being combined with Lemma 4 and Lemma 5, the above inequality (C1) about $Q_i(t)$ yields

$$\begin{aligned}|Q_i(\zeta)| \leqslant\ & 2^{\frac{n-m}{n}}\varrho_1\left|\int_{\zeta_{i^e}}^{\zeta}\sum_{j=1}^{N}a_{ij}\left(\dot{x_i}(\eta)-\dot{x_j}(\eta)\right)d\eta\right|^{\frac{m}{n}}+\varrho_2\left|\tanh\left(k\sum_{j=1}^{N}a_{ij}\left(x_i(\zeta_{i^e})-x_j(\zeta_{i^e})\right)\right)\right.\\ & \left.-\tanh\left(k\sum_{j=1}^{N}a_{ij}\left(x_i(\zeta)-x_j(\zeta)\right)\right)\right|\\ =\ & 2^{\frac{n-m}{n}}\varrho_1\left|\int_{\zeta_{i^e}}^{\zeta}\sum_{j=1}^{N}a_{ij}\left(\dot{x_i}(\eta)-\dot{x_j}(\eta)\right)d\eta\right|^{\frac{m}{n}}+\varrho_2\left|\int_{\zeta_{i^e}}^{\zeta}k\sum_{j=1}^{N}a_{ij}\left(\dot{x_i}(\eta)-\dot{x_j}(\eta)\right)*\right.\\ & \left.\left(1-\tanh^2\left(k\sum_{j=1}^{N}a_{ij}\left(x_i(\eta)-x_j(\eta)\right)\right)\right)d\eta\right|.\end{aligned} \tag{C2}$$

According to the expression of $T_i(\zeta)$, one can give

$$|\dot{T}_i(\zeta)| \leqslant \left|\sum_{j=1}^{N}a_{ij}\left(u_i(\zeta-\nu)-u_j(\zeta-\nu)+\varphi_i-\varphi_j\right)\right| \leqslant \left|\sum_{j=1}^{N}L_{ij}u_j(\zeta-\nu)\right|+2c_i\gamma$$

where c_i is the number of the i-th agent's neighbors. It is concluded from the above that the MASs (1) can achieve FTC. Consequently, there is a positive numbers ξ_1 being the upper bound of $|\dot{T}_i(\zeta)|$, then (C2) gives

$$|Q_i(\zeta)| \leqslant \frac{n-m}{n}\varrho_1\left(\int_{\zeta_{i^e}}^{\zeta}\xi_1 d\eta\right)^{\frac{m}{n}}+k\varrho_2\int_{\zeta_{i^e}}^{\zeta}\xi_1 d\eta = 2^{\frac{n-m}{n}}\varrho_1\xi_1^{\frac{m}{n}}(\zeta-\zeta_{i^e})^{\frac{m}{n}}+k\varrho_2\xi_1(\zeta-\zeta_{i^e}).$$

Due to $g_i(\zeta) > 0$ when the event is triggered, there is $|Q_i(\zeta)| > \tau_2\varrho_2$, then the following inequality can be obtained at the next trigger time.

$$2^{\frac{n-m}{n}}\varrho_1\xi_1^{\frac{m}{n}}(\zeta-\zeta_{i^e})^{\frac{m}{n}}+k\varrho_2\xi_1(\zeta-\zeta_{i^e}) > \tau_2\varrho_2. \tag{C3}$$

Denote $\Delta_1 = 2^{\frac{n-m}{n}}\varrho_1\xi_1^{\frac{m}{n}}$, $\Delta_2 = k\varrho_2\xi_1$, $F(x) = \Delta_1 x^{\frac{m}{n}} + \Delta_2 x - \tau_2\varrho_2$, then the derivation of $F(x)$ can be deduced as

$$\dot{F}(x) = \frac{\Delta_1 m}{n}x^{\frac{m-n}{n}} + \Delta_2 > 0.$$

If (C3) hold, then $F(x) > 0$. It can be found that $F(0) < 0$. Therefore, there exists a unique zero point $x_0 > 0$. In other words,

$$\zeta_{i^{e+1}} - \zeta_{i^e} > x_0 > 0. \tag{C4}$$

(C4) indicates that the duration between successive event triggers is a positive value, which means that there will be no "Zeno phenomenon".

Appendix D Illustrative examples

The following section will present a numerical illustration, which is an empirical demonstration of the efficacy of the designed consensus control protocol. The communication topology of MAS (1) is is depicted in Fig. D1. It comprises one leader agent and four follower agents, with their initial state values are setted as $x_0(0) = 8, x(0) = [5, 3, -10, 14]^T$ respectively, and $\varphi_i = 0.12sin(x_i(\zeta)) + 0.01sin(\zeta), i = 1, 2, 3, 4,\ \varphi_0 = 0.12sin(x_0(\zeta)) + 0.01sin(\zeta), \varrho_1 = 5.8, \varrho_2 = 0.35, \tau_1 = 0.78, \tau_2 = 0.45,\ K = diag[-0.8, 0, -0.6, -0.8]$, $B = diag[1, 0, 1, 1]$ and $m = 21, n = 23, \nu = 0.03, \epsilon = 0.06$.

Fig. D2 and Fig. D3 illustrate the state and error of the leader and four followers, while Fig. D4 portrays the variation of the control protocol u. Additionally, Fig. D5 depicts the event-triggered instances for the four follower. Evidently, as inferred from the aforementioned numerical simulations. Thus a conclusion is drawn that the MASs (1), under the influence of the design consensus control protocol (2), is capable of achieving FTC.

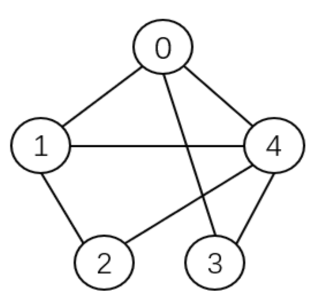

Figure D1 Communication topology

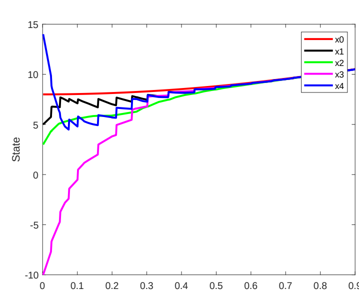

Figure D2 State of leader and followers

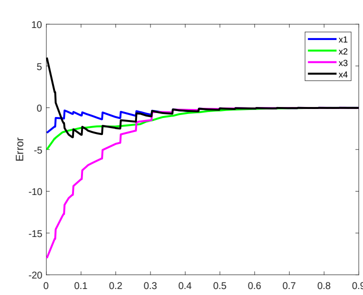

Figure D3 Error between the followers and the leader

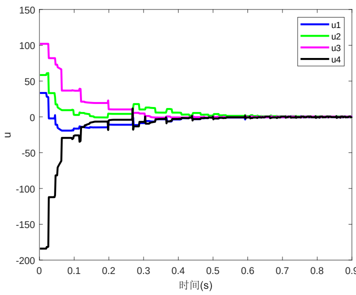

Figure D4 The variations curve of consensus control protocol

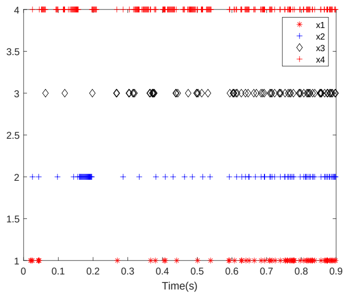

Figure D5 Event-triggered instances of the four followers

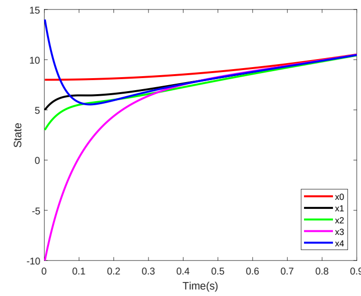

Figure D6 State of agents by consensus control protocol without event-triggered strategy

Table D1 Number of triggers of each follower agent

Method	Agent1	Agent2	Agent3	Agent4
Without impulsive	38	27	55	227
This paper	93	118	104	266

Table D2 The mean interval between each trigger event

Method	Agent1	Agent2	Agent3	Agent4
Without impulsive	0.0231	0.0325	0.0160	0.0039
This paper	0.0110	0.0086	0.0099	0.0038

Remark 3. Table D1 and Table D2 compare the proposed control protocol and the event-triggered control protocol without impulsive control. In accordance with the data presented in the tables, the proposed control protocol has more event triggers and shorter average trigger time, which also indicates that the control protocol (2) can reduce resource consumption more effectively and faster response speed.

Remark 4. Fig. D6 and Fig. D7 portray the state plot and the errors of agents under the influence of the traditional control protocol respectively, without considering time delays and input saturation. When compared to Fig. D2, it becomes evident that the proposed consensus control protocol exhibits stability and rapid response even when considering time delays and input saturation. Simultaneously, it significantly reduces the number of communications between followers and the leader, as precisely demonstrated in Fig. D8. The bar chart on the left side of Fig. D8 illustrates the communication frequency between followers and the leader for control protocol (2), while the bar chart on the right side depicts the communication frequency for the traditional control protocol.

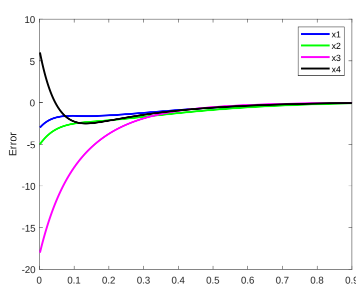

Figure D7 Error of agents by consensus control protocol without event-triggered strategy

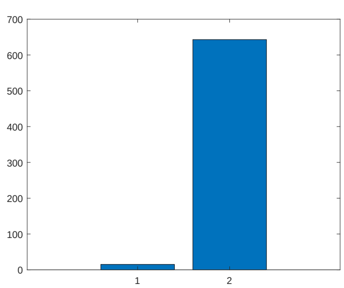

Figure D8 The number of communications between followers and the leader

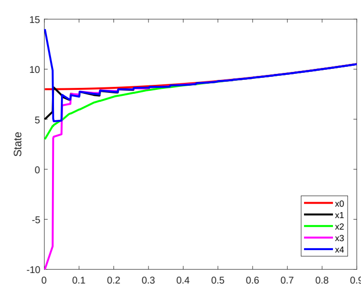

Figure D9 State of leader and followers without input saturation

Remark 5. To further illustrate the performance of our proposed method in reducing communication frequency and energy consumption, we conducted a comparative analysis of simulation experiments under different communication environments. With the initial values and controller parameters held constant, Fig. D9, Fig. D10 and Fig. D11 present the experimental results without input saturation constraints. As can be observed from these figures, the convergence rate of the system is significantly faster than that in the case with input saturation.

Assuming the circuit structure of every agent in system (1) is a classical Chua's system, as shown in Fig. D12. Its dynamical

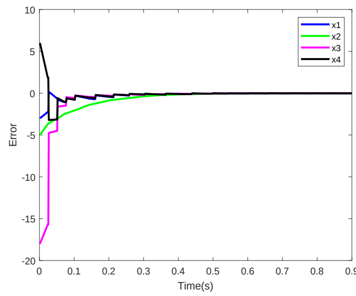

Figure D10 Error between the followers and leader without input saturation

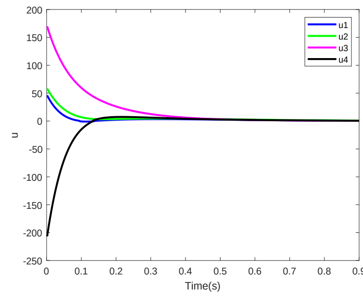

Figure D11 The variations curve of consensus control protocol without input saturation

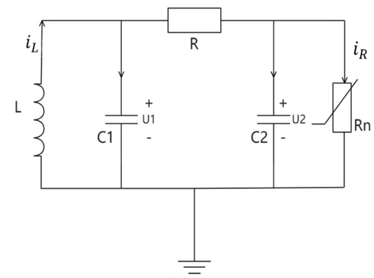

Figure D12 The circuit of the Chua's agents

equations are as follows,

$$\begin{cases} \frac{dU_1}{dt} = \frac{U_2}{RC_1} - \frac{U_1}{RC_1} + \frac{i_L}{C_1} \\ \frac{dU_2}{dt} = \frac{U_1}{RC_2} - \frac{U_2}{RC_2} - \frac{f(U_2)}{C_2} \\ \frac{di_L}{dt} = -\frac{U_1}{L} \end{cases}$$

where U_1, U_2, and i_L are the voltage across the energy storage component capacitor terminals C_1, C_2, and the current through the inductor terminal L respectively. R_n is a nonlinear resistor, and its current $i_R = f(U_2)$ depending on the voltage U_2. $R = 1\Omega$, $L = 13.45mH$, $C_1 = 20mF$, $C_2 = 200mF$, and $f(U_2) = -0.68U_2 - 0.259(|U_2 + 1| - |U_2 - 1|)$. When no control is applied, the system is in a chaotic state, as shown in Fig. D13. After applying the proposed consensus control protocol, the states of U_1, U_2, and i_L successfully follow the leader agent within finite-time, as shown in Fig. D14, and the error relative to the leader agent is depicted in Fig. D15.

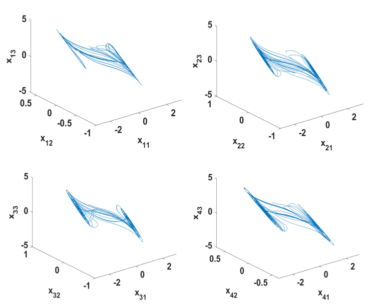

Figure D13 The circuit trace of the Chua's agents

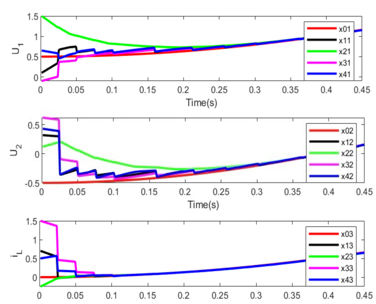

Figure D14 States of the Chua's agents

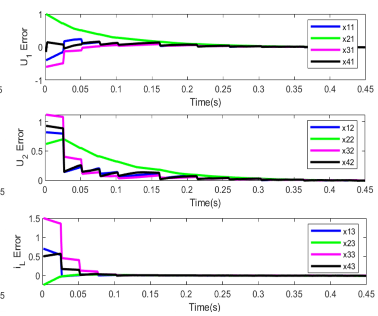

Figure D15 Errors of the Chua's agents

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