

• Supplementary File •

# Adaptive small-family population-guided swarm intelligence optimization algorithm

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## Appendix A The standard test function details

**Table A1** Description of unimodal benchmark functions

Function	Dimension	Range	$F_{\min}$
$F_1(x) = \sum_{i=1}^n x_i^2$	50	$[-100, 100]^D$	0.0000
$F_2(x) = \sum_{i=1}^n  x_i  + \prod_{i=1}^n  x_i $	50	$[-10, 10]^D$	0.0000
$F_3(x) = \max_i \{ x_i , 1 \leq i \leq n\}$	50	$[-100, 100]^D$	0.0000
$F_4(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	50	$[-30, 30]^D$	0.0000
$F_5(x) = \sum_{i=1}^n ( x_i + 0.5 )^2$	50	$[-100, 100]^D$	0.0000
$F_6(x) = \sum_{i=1}^n i x_i^4 + \text{random}[0, 1)$	50	$[-1.28, 1.28]^D$	0.0000

**Table A2** Description of multimodal benchmark functions

Function	Dimension	Range	$F_{\min}$
$F_7(x) = \sum_{i=1}^n -x_i \sin(\sqrt{ x_i })$	50	$[-500, 500]^D$	-2094.9145
$F_8(x) = \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i) + 10]$	50	$[-5.12, 5.12]^D$	0.0000
$F_9(x) = -20 \exp(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}) - \exp(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i)) + 20 + e$	50	$[-32, 32]^D$	0.0000
$F_{10}(x) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos(\frac{x_i}{\sqrt{i}}) + 1$	50	$[-600, 600]^D$	0.0000

## Appendix B Experimental results

### Appendix C Semi-Real problem detailed definition

#### Appendix C.1 Planetary gear train design optimization problem

Minimize [2],

$$F_{31}(x) = \max |i_k - i_{0k}|, k = \{1, 2, \dots, R\}$$

subject to,

$$I_1 = \frac{N_6}{N_4}, I_{01} = 3.11, I_2 = \frac{N_6(N_1 N_3 + N_2 N_4)}{N_1 N_3(N_6 - N_4)}, I_{0R} = -3.11,$$

$$I_R = -\frac{N_2 N_6}{N_1 N_3}, \bar{x} = (p, N_6, N_5, N_4, N_3, N_2, N_1, m_2, m_1),$$

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**Table A3** Description of fixed-dimension multimodal benchmark functions

Function	Dimension	Range	$F_{\min}$
$F_{11}(x) = \left( \frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^2 (x_i - a_{ij})^6} \right)^{-1}$	2	$[-65.536, 65.536]^D$	1.0000
$F_{12}(x) = \sum_{i=1}^{11} \left[ a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4} \right]^2$	4	$[-5, 5]^D$	0.0003
$F_{13}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	2	$[-5, 5]^D$	-1.0316
$F_{14}(x) = \left( x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6 \right)^2 + 10 \left( 1 - \frac{1}{8\pi} \right) \cos x_1 + 10$	2	Low= $[-5, 0]$ ; Up= $[10, 15]$	0.3980
$F_{15}(x) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2 \times (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	2	$[-2, 2]^D$	3.0000
$F_{16}(x) = -\sum_{i=1}^4 c_i \exp \left( -\sum_{j=1}^3 a_{ij}(x_j - p_{ij})^2 \right)$	3	$[0, 1]^D$	-3.8600
$F_{17}(x) = -\sum_{i=1}^4 c_i \exp \left( -\sum_{j=1}^6 a_{ij}(x_j - p_{ij})^2 \right)$	6	$[0, 1]^D$	-3.3200
$F_{18}(x) = -\sum_{i=1}^5 [(X - a_i)(X - a_i)^T + c_i]^{-1}$	4	$[0, 10]^D$	-10.1532
$F_{19}(x) = -\sum_{i=1}^7 [(X - a_i)(X - a_i)^T + c_i]^{-1}$	4	$[0, 10]^D$	-10.4028
$F_{20}(x) = -\sum_{i=1}^{10} [(X - a_i)(X - a_i)^T + c_i]^{-1}$	4	$[0, 10]^D$	-10.5363

$$\begin{aligned}
 g_1(\bar{x}) &= m_3(N_6 + 2.5) - D_{max} \leq 0, \\
 g_2(\bar{x}) &= m_1(N_1 + N_2) + m_1(N_2 + 2) - D_{max} \leq 0, \\
 g_3(\bar{x}) &= m_3(N_4 + N_5) + m_3(N_5 + 2) - D_{max} \leq 0, \\
 g_4(\bar{x}) &= |m_1(N_1 + N_2) - m_3(N_6 - N_3)| - m_1 - m_3 \leq 0, \\
 h_1(\bar{x}) &= \frac{N_6 - N_4}{p} = \text{integer}.
 \end{aligned}$$

where,

$$\delta_{22} = \delta_{33} = \delta_{55} = \delta_{35} = \delta_{56} = 0.5.$$

with bounds,

$$\begin{aligned}
 p &= 3, 4, 5, \\
 m_1 &= 1.75, 2.0, 2.25, 2.5, 2.75, 3.0, \\
 m_3 &= 1.75, 2.0, 2.25, 2.5, 2.75, 3.0.
 \end{aligned}$$

## Appendix C.2 Rolling element bearing

Minimize [3],

$$F_{32}(\bar{x}) = \begin{cases} f_c Z^{\frac{2}{3}} D_b^{1.8} & , \text{if } D_b \leq 25.4 \text{mm} \\ 3.647 f_c Z^{\frac{2}{3}} D_b^{1.4} & , \text{otherwise} \end{cases}$$

subject to,

$$\begin{aligned}
 g_1(\bar{x}) &= Z - \frac{\phi_0}{2 \sin^{-1} \left( \frac{D_b}{D_m} \right)} - 1 \leq 0, \\
 g_2(\bar{x}) &= K_{Dmin}(D - d) - 2D_b \leq 0, \\
 g_3(\bar{x}) &= 2D_b - K_{Dmax}(D - d) \leq 0, \\
 g_4(\bar{x}) &= D_b - \zeta B_w \leq 0, \\
 g_5(\bar{x}) &= 0.5(D + d) - D_m \leq 0, \\
 g_6(\bar{x}) &= D_m - (0.5 + e)(D + d) \leq 0, \\
 g_7(\bar{x}) &= \varepsilon D_b - 0.5(D - D_m - D_b) \leq 0, \\
 g_8(\bar{x}) &= 0.515 - f_i \leq 0, \\
 g_9(\bar{x}) &= 0.515 - f_0 \leq 0.
 \end{aligned}$$

**Table A4** Description of composite benchmark functions [1] (Part 1)

Function	Dimension	Range	$F_{\min}$
<p>F21: Hybrid Composition Function</p> <p><math>f_{1-2}(x)</math> : Rastrigin's Function</p> <p><math>f_{3-4}(x)</math> : Weierstrass Function</p> <p><math>f_{5-6}(x)</math> : Griewank's Function</p> <p><math>f_{7-8}(x)</math> : Ackley's Function</p> <p><math>f_{9-10}(x)</math> : Sphere Function</p> <p><math>\sigma_i = 1</math> for <math>i = 1, 2, \dots, D</math></p> <p><math>\lambda = [1, 1, 10, 10, 5/60, 5/60, 5/32, 5/32, 5/100, 5/100]</math></p> <p><math>M_i</math> : identity matrices</p>	50	$[-5, 5]^D$	$F(x^*) = f_{\text{bias}} = 120$
<p>F22: Rotated Hybrid Composition Function</p> <p>Except <math>M_i</math> are different linear transformation matrices with condition number of 2, all other settings are the same as F21.</p>	50	$[-5, 5]^D$	$F(x^*) = f_{\text{bias}} = 120$
<p>F23: Rotated Hybrid Composition Function with Noise</p> <p>F22 with Noise in Fitness</p> <p>Let <math>(F22 - f_{\text{bias}22})</math> be <math>G(x)</math>, then</p> <p><math>F23(x) = G(x) * (1 + 0.2N(0, 1)) + f_{\text{bias}23}</math></p> <p>All settings are the same as F22.</p>	50	$[-5, 5]^D$	$F(x^*) = f_{\text{bias}} = 120$
<p>F24: Rotated Hybrid Composition Function</p> <p><math>f_{1-2}(x)</math> : Ackley's Function</p> <p><math>f_{3-4}(x)</math> : Rastrigin's Function</p> <p><math>f_{5-6}(x)</math> : Sphere Function</p> <p><math>f_{7-8}(x)</math> : Weierstrass Function</p> <p><math>f_{9-10}(x)</math> : Griewank's Function</p> <p><math>\sigma = [1, 2, 1.5, 1.5, 1, 1, 1.5, 1.5, 2, 2]</math></p> <p><math>\lambda = [2 * 5/32; 5/32; 2 * 1; 1; 2 * 5/100; 5/100; 2 * 10; 10; 2 * 5/60; 5/60]</math></p> <p><math>M_i</math> : rotation matrices (condition numbers: [2 3 2 3 2 3 20 30 200 300])</p> <p><math>\sigma_{10} = [0, 0, \dots, 0]</math></p>	50	$[-5, 5]^D$	$F(x^*) = f_{\text{bias}} = 10$
<p>F25: Rotated Hybrid Composition Function with a Narrow Basin for the Global Optimum</p> <p>All settings are the same as F18 except</p> <p><math>\sigma = [0.1, 2, 1.5, 1.5, 1, 1, 1.5, 1.5, 2, 2]</math></p> <p><math>\lambda = [0.1 * 5/32; 5/32; 2 * 1; 1; 2 * 5/100; 5/100; 2 * 10; 10; 2 * 5/60; 5/60]</math></p>	50	$[-5, 5]^D$	$F(x^*) = f_{\text{bias}} = 10$

**Table A5** Description of composite benchmark functions [1] (Part 2)

Function	Dimension	Range	$F_{\min}$
<p>F26: Rotated Hybrid Composition Function with the Global Optimum on the Bounds                      All settings are the same as F24 except after load the data file, set <math>o_{1(2j)} = 5</math> for <math>j = 1, 2, \dots, \lfloor \frac{D}{2} \rfloor</math></p>	50	$[-5, 5]^D$	$F(x^*) = f_{\text{bias}} = 10$
<p>F27: Rotated Hybrid Composition Function  <math>f_{1-2}(x)</math> : Rotated Expanded Scaffer's Function  <math>f_{3-4}(x)</math> : Rastrigin's Function  <math>f_{5-6}(x)</math> : F8F2 Function                      F2: Shifted Schwefel's Problem 1.2                      F8: Shifted Rotated Ackley's Function with Global Optimum on Bounds  <math>f_{7-8}(x)</math> : Weierstrass Function  <math>f_{9-10}(x)</math> : Griewank's Function  <math>\sigma = [1, 1, 1, 1, 1, 2, 2, 2, 2, 2]</math>  <math>\lambda = [5 * 5/100; 5/100; 5 * 1; 1; 5 * 1; 1; 5 * 10; 10; 5 * 5/200; 5/200]</math>  <math>M_i</math> : orthogonal matrices</p>	50	$[-5, 5]^D$	$F(x^*) = f_{\text{bias}} = 360$
<p>F28: Rotated Hybrid Composition Function with High Condition Number Matrix                      All settings are the same as F27 except <math>M_i</math>'s condition numbers are [10 20 50 100 200 1000 2000 3000 4000 5000]</p>	50	$[-5, 5]^D$	$F(x^*) = f_{\text{bias}} = 360$
<p>F29: Rotated Hybrid Composition Function  <math>f_1(x)</math> : Weierstrass Function  <math>f_2(x)</math> : Rotated Expanded Scaffer's Function  <math>f_3(x)</math> : F8F2 Function  <math>f_4(x)</math> : Ackley's Function  <math>f_5(x)</math> : Rastrigin's Function  <math>f_6(x)</math> : Griewank's Function  <math>f_7(x)</math> : Non-Continuous Expanded Scaffer's Function  <math>f_8(x)</math> : Non-Continuous Rastrigin's Function  <math>f_9(x)</math> : High Conditioned Elliptic Function  <math>f_{10}(x)</math> : Sphere Function with Noise in Fitness  <math>\sigma_i = 2</math> for <math>i = 1, \dots, D</math>  <math>\lambda = [10; 5/20; 1; 5/32; 1; 5/100; 5/50; 1; 5/100; 5/100]</math>  <math>M_i</math> : rotation matrices (condition numbers: [100 50 30 10 5 5 4 3 2 2])</p>	50	$[-5, 5]^D$	$F(x^*) = f_{\text{bias}} = 260$
<p>F30: Rotated Hybrid Composition Function without Bounds                      All settings are the same as F29 except no exact search range set for this test function.</p>	50	Initialize population in $[2, 5]^D$	$F(x^*) = f_{\text{bias}} = 260$

**Table B1** Validation of the effectiveness of the elite reverse learning strategy

Functions Index	ASIOA		ASPSIOA	
	mean	std	mean	std
F4	47.90	0.23	46.07	0.18
F6	0.00	0.00	0.01	0.00
F7	-5827.21	536.91	-5496.98	327.27
F11	1.00	0.00	0.99	0.00
F14	0.40	0.00	0.39	0.00
F17	-3.14	0.09	-3.32	0.00
F18	-5.23	0.99	-7.07	0.35
F19	-4.98	0.04	-7.73	1.34
F20	-5.17	0.51	-9.92	1.69
F21	822.41	69.69	651.48	62.68
F22	542.37	31.54	546.46	41.53
F23	571.87	33.90	550.83	29.95
F24	1119.56	54.19	910.00	0.00
F25	1126.10	49.49	910.00	0.00
F26	1116.91	75.51	910.00	0.00
F27	1695.21	57.05	1593.89	81.52
F28	1525.11	38.80	1459.21	42.83
F29	1563.95	20.66	1564.41	21.35
F30	1566.48	20.93	1556.18	16.09

**Table B2** Comparison of test function optimization results for different npop values

Functions Index	npop=4		npop=6		npop=10		npop=12		npop=15	
	mean	std								
F4	47.39	0.20	46.07	0.18	47.81	0.16	47.89	0.13	48.03	0.10
F6	0.01	0.00	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.00
F7	-5809.65	454.60	-5496.98	327.27	-5865.67	439.48	-5745.23	437.51	-5968.31	455.73
F9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.35	4.36
F10	0.00	0.00	0.00	0.00	0.02	0.09	0.00	0.00	0.01	0.08
F11	1.00	0.00	0.99	0.00	1.00	0.00	1.00	0.00	1.00	0.00
F14	0.40	0.00	0.39	0.00	0.40	0.00	0.40	0.00	0.40	0.00
F17	-3.10	0.08	-3.32	0.00	-3.14	0.07	-3.13	0.06	-3.10	0.06
F18	-4.98	0.04	-7.07	0.35	-5.59	1.24	-5.23	0.75	-5.45	1.01
F19	-5.46	1.11	-7.73	1.34	-5.75	1.22	-5.68	1.18	-5.74	1.18
F20	-5.53	1.10	-9.92	1.69	-5.83	1.22	-6.05	1.30	-6.50	1.50
F21	677.92	66.18	651.48	62.68	694.56	89.88	1056.33	33.75	690.57	75.05
F22	555.91	46.54	546.46	41.53	526.55	34.14	738.36	61.21	553.27	43.53
F23	590.94	52.88	550.83	29.95	579.39	54.64	862.46	87.77	588.26	39.03
F27	1614.15	49.37	1593.89	81.52	1639.12	37.25	1398.52	4.92	1653.22	21.94
F28	1465.03	28.51	1459.21	42.83	1484.51	36.30	1388.41	72.75	1531.01	31.04
F29	1571.53	16.33	1564.41	21.35	1574.14	12.86	1398.82	160.49	1574.08	10.37
F30	1569.75	20.12	1556.18	16.09	1573.05	17.12	1387.82	157.20	1577.22	10.64

where,

$$f_\epsilon = 37.91 \left\{ 1 + \left\{ 1.04 \left( \frac{1-\gamma}{1-\gamma} \right)^{1.72} \left( \frac{f_i(2f_0-1)}{f_0(2f_i-1)} \right)^{0.41} \right\}^{\frac{10}{3}} \right\}^{-0.3},$$

$$\gamma = \frac{D_b \cos(\alpha)}{D_m}, f_i = \frac{r_i}{D_b}, f_0 = \frac{r_0}{D_b},$$

$$\phi_0 = 2\pi - 2 \bullet \cos^{-1}([(D-d)/2 - 3 \left(\frac{T}{4}\right)^2 +$$

$$D/2 - \left(\frac{T}{4}\right)^2 - D_b^2 - d/2 + \left(\frac{T}{4}\right)^2] / 2D - d2 - 3T4D2 - T4 - Db)$$

$$T = D - d - 2D_b, D = 160, d = 90, B_w = 30.$$

with bounds,

$$0.5(D+d) \leq D_m \leq 0.6(D+d),$$

$$0.15(D-d) \leq D_b \leq 0.45(D-d).$$

### Appendix C.3 Topology optimization

Minimize [4],

$$F_{33}(\bar{x}) = \mathbf{L}^T \mathbf{G} \mathbf{L} = \sum_{e=1}^N (x_e)^{p_l} l_e^T h_0 l_0$$

subject to,

$$h_1(\bar{x}) = \frac{V(\bar{x})}{V_0} - f \leq 0,$$

**Table B3** Comparison of test function optimization results under different exploration factor coefficient combinations

Functions	(-3.5,1,40)		(-3.5,1,60)		(-3.5,2,50)		(-3.5,3,40)		(-3.5,3,60)	
Index	mean	std								
F4	47.82	0.17	47.77	0.16	46.07	0.18	47.82	0.15	47.88	0.14
F6	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00
F7	-5803.03	391.88	-5925.59	366.67	-5496.98	327.27	-5954.10	406.83	-5848.29	482.62
F8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F11	1.00	0.00	2.56	4.04	0.99	0.00	1.00	0.00	1.00	0.00
F14	0.40	0.00	0.40	0.00	0.39	0.00	0.40	0.00	0.40	0.00
F17	-3.15	0.08	-3.16	0.08	-3.32	0.00	-3.17	0.07	-3.18	0.08
F18	-5.21	0.83	-5.08	0.71	-7.07	0.35	-5.02	0.36	-5.16	0.51
F19	-5.25	0.86	-5.13	0.50	-7.73	1.34	-5.06	0.37	-5.53	1.04
F20	-5.49	1.03	-5.16	0.77	-9.92	1.69	-5.37	0.81	-6.10	1.56
F21	821.45	21.26	831.93	34.94	651.48	62.68	816.26	34.53	814.64	41.10
F22	550.12	39.98	560.69	46.82	546.46	41.53	562.93	49.29	542.63	36.66
F23	580.56	34.42	599.40	53.20	550.83	29.95	598.63	44.98	594.08	45.44
F24	1120.39	64.88	1092.02	89.31	910.00	0.00	1081.42	91.67	1104.73	82.73
F25	1107.47	72.20	1071.30	95.39	910.00	0.00	1095.70	90.16	1117.72	49.17
F26	1089.06	88.13	1106.63	82.30	910.00	0.00	1082.09	96.22	1124.94	57.33
F27	1561.19	78.42	1619.43	35.62	1593.89	81.52	1626.34	37.73	1494.87	61.78
F28	1524.74	41.12	1525.58	44.68	1459.21	42.83	1527.94	37.46	1523.59	30.74
F29	1573.31	13.34	1579.62	10.84	1564.41	21.35	1572.95	12.38	1572.53	13.23
F30	1572.89	21.65	1572.56	23.68	1556.18	16.09	1571.89	16.08	1571.61	13.85

**Table B4** Comparison of test function optimization results for different  $w_3$  values

Functions Index	$w_3 = 0.5$		$w_3 = 0.8$		$w_3 = 2$		$w_3 = 3$		$w_3 = 3.5$	
	mean	std	mean	std	mean	std	mean	std	mean	std
F1	0.00	0.00	0.00	0.00	0.00	0.00	/	/	/	/
F2	0.00	0.00	0.00	0.00	0.00	0.00	15.79	1.62	34.86	5.69
F3	0.00	0.00	0.00	0.00	0.00	0.00	18.99	2.19	19.60	1.68
F4	48.01	0.15	47.80	0.23	46.07	0.18	/	/	/	/
F5	0.00	0.00	0.00	0.00	0.00	0.00	/	/	/	/
F6	0.00	0.00	0.00	0.00	0.01	0.00	3.55	1.26	7.97	2.50
F7	-5795.90	362.21	-5765.32	458.95	-5496.98	327.27	-6115.01	519.28	-5987.03	481.83
F8	0.00	0.00	0.00	0.00	0.00	0.00	/	/	/	/
F9	15.50	5.22	7.79	1.47	0.00	0.00	17.96	1.61	18.75	0.83
F10	0.00	0.00	0.00	0.00	0.00	0.00	31.84	7.81	65.58	14.68
F11	1.00	0.01	1.07	0.36	0.99	0.00	1.00	0.00	1.00	0.00
F14	0.40	0.00	0.40	0.00	0.39	0.00	0.40	0.00	0.40	0.00
F15	3.01	0.00	3.00	0.00	3.00	0.00	3.00	0.00	3.00	0.00
F17	-3.02	0.08	-3.07	0.08	-3.32	0.00	-3.12	0.07	-3.11	0.05
F18	-4.80	0.94	-4.75	0.56	-7.07	0.35	-5.41	1.15	-5.15	0.58
F19	-4.88	0.68	-5.29	0.86	-7.73	1.34	-5.56	1.05	-5.53	1.16
F20	-5.16	0.73	-5.10	0.78	-9.92	1.69	-6.10	1.38	-5.78	1.45
F21	885.12	84.25	808.88	76.03	651.48	62.68	695.99	91.63	681.81	110.00
F22	666.34	67.75	620.56	50.65	546.46	41.53	538.85	35.83	550.62	31.30
F23	664.98	50.40	647.27	42.26	550.83	29.95	565.73	33.73	593.32	53.97
F27	1716.97	18.73	1703.21	18.98	1593.89	81.52	1624.73	19.91	1597.68	65.04
F28	1680.06	25.67	1638.18	26.86	1459.21	42.83	1471.32	33.72	1463.88	44.74
F29	1625.69	12.26	1608.28	10.13	1564.41	21.35	1565.14	14.58	1575.44	23.83
F30	1624.03	11.45	1609.05	11.59	1556.18	16.09	1574.13	19.03	1572.60	23.38

**Table B5** Comparison of test function optimization results for different  $w_4$  values

Functions Index	$w_4 = 0.5$		$w_4 = 0.8$		$w_4 = 2$		$w_4 = 3$		$w_4 = 3.5$	
	mean	std	mean	std	mean	std	mean	std	mean	std
F1	0.00	0.00	0.00	0.00	0.00	0.00	/	/	/	/
F2	0.00	0.00	0.00	0.00	0.00	0.00	/	/	/	/
F3	0.00	0.00	0.00	0.00	0.00	0.00	/	/	/	/
F4	48.70	0.32	48.61	0.42	46.07	0.18	/	/	/	/
F5	0.00	0.00	0.00	0.00	0.00	0.00	/	/	/	/
F6	0.00	0.00	0.00	0.00	0.01	0.00	314.41	58.57	365.45	58.70
F7	-5472.65	489.92	-5658.55	474.25	-5496.98	327.27	-6044.55	574.36	-6192.92	551.64
F8	0.00	0.00	0.00	0.00	0.00	0.00	/	/	/	/
F9	0.00	0.00	0.00	0.00	0.00	0.00	19.96	0.00	19.96	0.00
F10	0.00	0.00	0.00	0.00	0.00	0.00	/	/	/	/
F11	1.20	0.61	1.00	0.00	0.99	0.00	1.02	0.04	1.01	0.02
F14	0.40	0.00	0.40	0.00	0.39	0.00	0.40	0.00	0.40	0.00
F15	3.00	0.00	3.00	0.00	3.00	0.00	3.05	0.05	3.07	0.10
F17	-3.085	0.05	-3.078	0.05	-3.32	0.00	-3.071	0.06	-3.034	0.06
F18	-5.194	1.07	-5.719	1.51	-7.07	0.35	-5.098	1.22	-4.776	0.71
F19	-5.200	0.84	-5.522	1.39	-7.73	1.34	-4.703	0.49	-4.551	0.70
F20	-5.391	1.15	-6.036	1.83	-9.92	1.69	-4.182	0.91	-3.887	1.35
F21	861.10	108.64	842.86	101.89	651.48	62.68	956.81	91.47	974.11	86.33
F22	656.70	63.92	630.80	70.10	546.46	41.53	704.24	77.33	696.28	67.91
F23	643.22	69.48	624.52	48.43	550.83	29.95	679.12	69.53	695.82	60.52
F24	910.00	0.00	910.00	0.00	910.00	0.00	992.21	15.56	994.86	20.02
F25	910.00	0.00	910.00	0.00	910.00	0.00	991.80	13.29	994.88	14.63
F26	910.00	0.00	910.00	0.00	910.00	0.00	996.53	15.86	997.04	15.26
F27	1737.33	22.82	1722.46	23.68	1593.89	81.52	1718.22	48.82	1739.32	22.31
F28	1669.88	41.35	1650.55	42.54	1459.21	42.83	1639.79	133.10	1622.00	104.81
F29	1624.07	16.18	1617.94	16.50	1564.41	21.35	1645.32	14.33	1638.00	15.05
F30	1624.60	18.76	1621.46	17.16	1556.18	16.09	1641.79	17.99	1640.60	15.87

**Table B6** Comparison of test function optimization results for different  $w_2$  values

Functions	$w_2 = 0.5$		$w_2 = 0.7$		$w_2 = 1.2$		$w_2 = 2$		$w_2 = 3$	
Index	mean	std	mean	std	mean	std	mean	std	mean	std
F1	/	/	0.00	0.00	/	/	/	/	/	/
F2	/	/	0.00	0.00	/	/	/	/	/	/
F3	21.73	1.87	0.00	0.00	48.63	4.43	81.66	6.19	92.29	2.16
F4	/	/	47.95	0.18	/	/	/	/	/	/
F5	/	/	0.00	0.00	/	/	/	/	/	/
F6	0.79	0.26	0.01	0.00	50.34	15.38	328.75	55.47	358.29	66.41
F7	-5774.56	401.48	-5784.05	498.94	-7672.33	847.54	-8211.44	686.03	-8979.75	304.05
F8	365.51	19.62	0.00	0.00	563.59	33.52	718.79	32.68	719.71	25.79
F9	5.12	1.02	0.00	0.00	19.96	0.00	19.96	0.00	19.96	0.00
F10	4.22	1.60	0.00	0.00	382.32	38.27	835.90	110.29	825.74	82.40
F11	1.00	0.00	1.00	0.00	1.00	0.00	1.01	0.03	1.39	0.63
F12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F13	-1.03	0.00	-1.03	0.00	-1.03	0.00	-1.03	0.00	-1.03	0.00
F14	0.40	0.00	0.40	0.00	0.40	0.00	0.40	0.00	0.40	0.01
F15	3.00	0.00	3.00	0.00	3.00	0.00	3.09	0.09	3.43	0.38
F16	-3.86	0.00	-3.86	0.00	-3.86	0.00	-3.85	0.01	-3.85	0.01
F17	-3.08	0.07	-3.10	0.05	-3.03	0.11	-2.96	0.09	-2.90	0.12
F18	-4.91	0.11	-5.05	0.38	-5.77	1.36	-4.60	0.95	-4.50	1.14
F19	-5.16	0.85	-5.34	0.77	-5.88	1.46	-4.10	1.31	-4.20	1.83
F20	-5.01	0.54	-5.80	1.13	-4.76	1.53	-3.73	1.40	-3.34	1.62
F21	908.95	77.64	886.91	68.41	896.46	110.17	933.54	101.42	962.04	112.90
F22	708.46	86.17	684.34	74.05	618.48	66.43	669.39	74.75	701.08	70.87
F23	680.36	67.45	685.85	59.46	614.63	59.12	690.24	60.11	689.47	68.49
F24	910.37	0.26	910.00	0.00	1003.14	20.19	1011.41	22.20	1009.50	18.50
F25	910.43	0.31	910.00	0.00	999.03	21.05	1012.15	15.60	1009.87	24.53
F26	910.44	0.34	910.00	0.00	1008.87	20.20	1012.11	18.52	1008.18	22.07
F27	1741.63	19.75	1731.25	15.99	1593.47	110.40	1593.91	80.20	1702.17	54.10
F28	1682.95	39.90	1655.72	45.78	1463.73	54.45	1586.61	75.46	1698.16	55.00
F29	1639.33	15.10	1630.83	13.81	1557.68	94.73	1557.02	89.13	1634.75	33.93
F30	1645.66	18.86	1639.88	17.44	1508.23	113.67	1588.53	69.96	1639.86	19.16

$$h_2(\bar{x}) = \mathbf{GL} - \mathbf{F} \leq 0.$$

with bounds,

$$0 < \bar{x}_{min} \leq x \leq 1.$$

### Appendix D Ethylene cracking furnace model

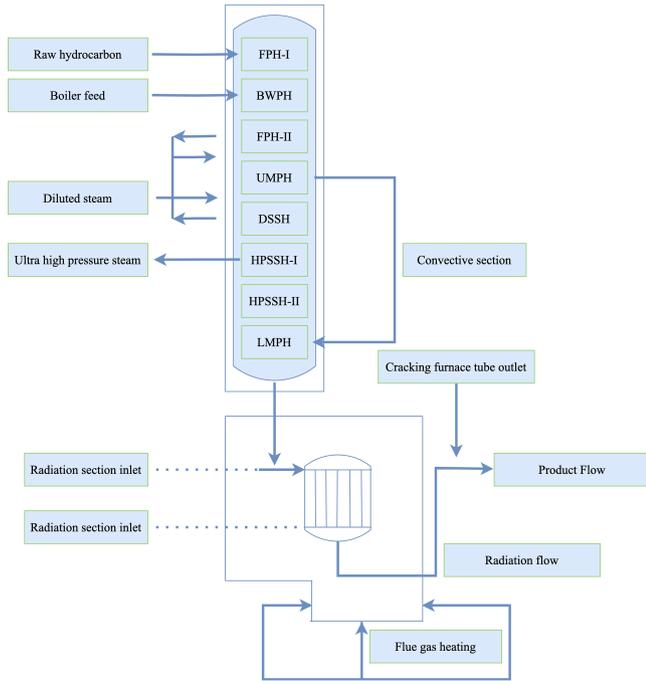


Fig. D1 The structure of the cracking furnace

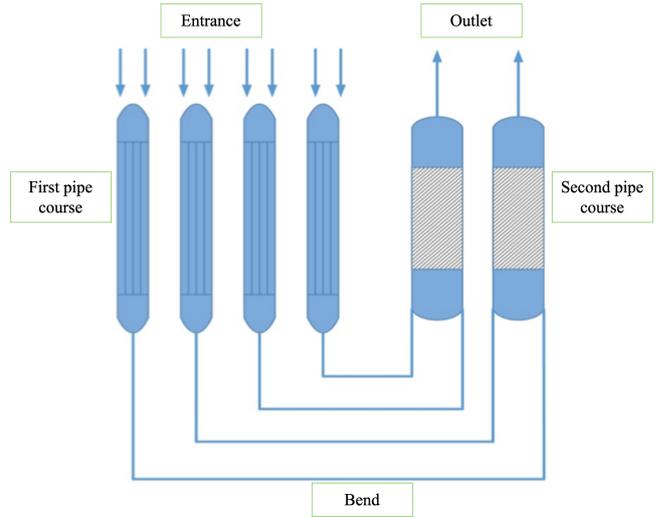


Fig. D2 The structure of the reaction tube

Table D1 Initial parameters of ethylene cracking furnace [5]

Parameters	Values
Inlet temperature of orifice (K)	875
Gas to hydrocarbon ratio	0.6
Naphtha flow rate (kg/h)	890.625
Outlet pressure (kPa)	178
Outlet temperature of orifice (K)	1122
Coking thickness (cm)	0

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