

A survey of braking control techniques for railway vehicles

Jiuhe WANG¹, Zhiyong CHEN^{2*}, Zhiwen CHEN¹, Chunhua YANG¹ & Weihua GUI¹

¹*School of Automation, Central South University, Changsha 410083, China*

²*School of Engineering, The University of Newcastle, Callaghan NSW 2308, Australia*

Received 11 March 2025/Revised 17 April 2025/Accepted 5 August 2025/Published online 4 February 2026

Abstract Braking control techniques for railway vehicles have seen significant advancements over the past few decades, with notable developments in recent years. Due to the complexity of railway vehicle systems, these techniques span a wide range of tasks. This paper provides a comprehensive review of the latest braking technologies, organized into an integrated framework across three levels: vehicle, carriage, and wheel. The focus is placed on their mathematical formulations and practical relevance. At each of these levels, the key functionalities are braking trajectory optimization, braking force distribution, and braking torque distribution, respectively. The review covers both theoretical progress and real-world applications, with an emphasis on the common braking mode rather than emergency braking. Additionally, it highlights recent advancements in validation platforms and explores potential future directions for braking control techniques.

Keywords braking control techniques, railway vehicles, common braking mode, trajectory optimization, braking force distribution

Citation Wang J H, Chen Z Y, Chen Z W, et al. A survey of braking control techniques for railway vehicles. *Sci China Inf Sci*, 2026, 69(3): 131201, <https://doi.org/10.1007/s11432-025-4671-y>

1 Introduction

Railway transportation, which includes urban rail transit systems such as metros, subways, and underground networks, as well as high-speed trains, is an efficient mode of public transport reliant on vehicles operating on railway tracks. Over the past four decades, there has been a steadily growing focus, both theoretical and practical, on the development of advanced braking control technologies. This progress has been driven by advancements in sensing, computing, and distributed technologies, which have the potential to revolutionize rail transportation and deliver significant societal benefits.

According to incomplete statistics, over a hundred railway-related accidents occur annually. Nearly 80% of these accidents fall into three primary categories: collisions (28.2%), derailments (32.1%), and level crossing incidents (19.2%). Among these, 43.6% are attributed to technical failures, while 52.6% involve at least one instance of human error [1]. Braking actions serve as the last line of defense for railway vehicles, reducing the likelihood of collisions or derailments caused by technical failures or human error and playing a crucial role in enhancing train safety. The successful launch of the Japanese Shinkansen in 1964 marked the beginning of the high-speed transportation era and underscored the need for stable, high-performance braking systems [2]. Reliable braking operations are essential for the development of next-generation autonomous train technologies [3].

During the early Shinkansen era, rheostatic and pneumatic brakes were the primary braking methods used in high-speed trains (HSTs). However, these systems had notable limitations, as the kinetic energy from high-speed operations was dissipated as heat, leading to significant energy loss [4]. Heavy-haul trains emerged as early as the 1950s [5] and became a practical reality in the 1970s. Unlike HSTs, heavy-haul trains feature extended carriage configurations, which initially required the exclusive use of pneumatic braking systems [6]. To address the delay issues inherent in pneumatic brakes, the Association of American Railroads developed the electronically controlled pneumatic (ECP) braking system. This innovation utilizes electronic command signals to apply and release braking forces promptly, eliminating response delays and ensuring more consistent and efficient braking [7].

The 1970s marked a significant milestone in braking technology with the introduction of regenerative braking, designed to recycle electrical energy generated during braking for train use [8]. This advancement over traditional pneumatic braking led to the development of electric braking, commonly known as regenerative braking, which

* Corresponding author (email: zhiyong.chen@newcastle.edu.au)

was first implemented in subway vehicles [9], demonstrating the feasibility of energy conversion during the braking process.

Meanwhile, research on braking systems has accelerated in both academia and the railway industry. As engineers explored various braking modes, they recognized that electric braking often needed to work in conjunction with air braking to achieve optimal performance, particularly in high-speed railways [10]. This realization led to the development of an advanced electro-pneumatic mixed braking system (EPMBS) that integrates both electric braking (EB) and air braking (AB) modes. The EB mode efficiently converts kinetic energy into electrical energy, which can be reused to power train systems, while the AB mode ensures safety during emergency braking. As a result, EPMBSs have been widely adopted in HSTs and subways worldwide [11]. These systems retain the benefits of ECP braking while maximizing energy efficiency through EB mode, and they continue to be widely used today.

Despite significant advancements, the implementation of braking control techniques continues to face substantial challenges in real-world operational environments. For example, real-time estimation of wheel-rail adhesion remains infeasible under varying environmental conditions, complicating the deployment of control strategies. Additionally, the trade-off between braking performance and energy recovery requires careful management to ensure safety compliance. Centralized control architectures also introduce risks such as communication latency, cybersecurity threats, and data integrity issues, all of which must be explicitly addressed during system design. Ensuring robustness against environmental disturbances (e.g., precipitation, crosswinds) and component failures (e.g., sensor malfunctions, partial actuator saturation) is crucial for maintaining reliable braking performance. These operational constraints must be systematically incorporated into the development of advanced braking systems. Addressing these challenges is a key driver of technological innovation in this field.

The development of modern braking systems has been reviewed in various survey papers, including [12–18], each offering different perspectives. Compared to these studies, this paper presents several distinct contributions. It provides a comprehensive review of the latest braking technologies related to EPMBS, structured within an integrated framework across three levels: vehicle, carriage, and wheel. The focus is placed on their mathematical formulations and practical relevance. This approach offers clear control objectives for braking tasks at each level and systematically reviews the methods for achieving these objectives, fostering a deeper understanding of system-level braking principles.

The three-level framework reflects the hierarchical decision-making architecture commonly used in braking system design, encompassing braking trajectory planning, braking force distribution, and braking torque distribution, which primarily occur at these levels. Additionally, the paper highlights recent advancements in validation platforms and explores potential future directions for braking control techniques.

The remainder of the paper is organized as follows. Section 2 provides an overview of hierarchical decision-making processes and design methodologies. The review of mathematical formulations and methods at the vehicle, carriage, and wheel levels is presented in Sections 3–5, respectively. Section 6 examines the development of validation platforms, while Section 7 explores promising directions for future research. Finally, Section 8 concludes the paper.

2 Decision-making hierarchy in braking systems: an overview

Braking decisions in railway systems are classified into two types based on urgency: common braking and emergency braking. The common braking mode operates under normal conditions, where the braking force is adjusted in discrete levels (e.g., Level 1 to Level 7) to control deceleration. The driver manually operates the brake lever, assuming all braking units function correctly and provide adequate force. In contrast, emergency braking is activated in abnormal situations, applying maximum force instantly to minimize stopping distance. This survey focuses on the common braking mode, analyzing its decision-making architecture and the roles of its components.

A braking system is a semi-autonomous decision-making unit, integrated into railway vehicles as a large-scale power-actuated system. Its decision-making process depends on cooperation with the operation control system or human guidance in non-automatic operation mode. The operation control system continuously processes data from onboard sensors, such as global positioning system (GPS) and odometry sensors. By combining these observations with prior knowledge of the railway line, environment, vehicle dynamics, and sensor models, it automatically determines control variables for the braking trajectory. From a macro perspective, this enables an optimal braking trajectory, minimizing energy consumption while reducing collision risks.

The decision-making process in a typical braking system follows a hierarchical structure with three levels (cf. Figure 1): vehicle, carriage, and wheel. Each level employs specific algorithms: braking trajectory planning at the vehicle level, braking force distribution at the carriage level, and braking torque distribution at the wheel level. The roles and functions of these levels, as well as their integration, are discussed in detail below.

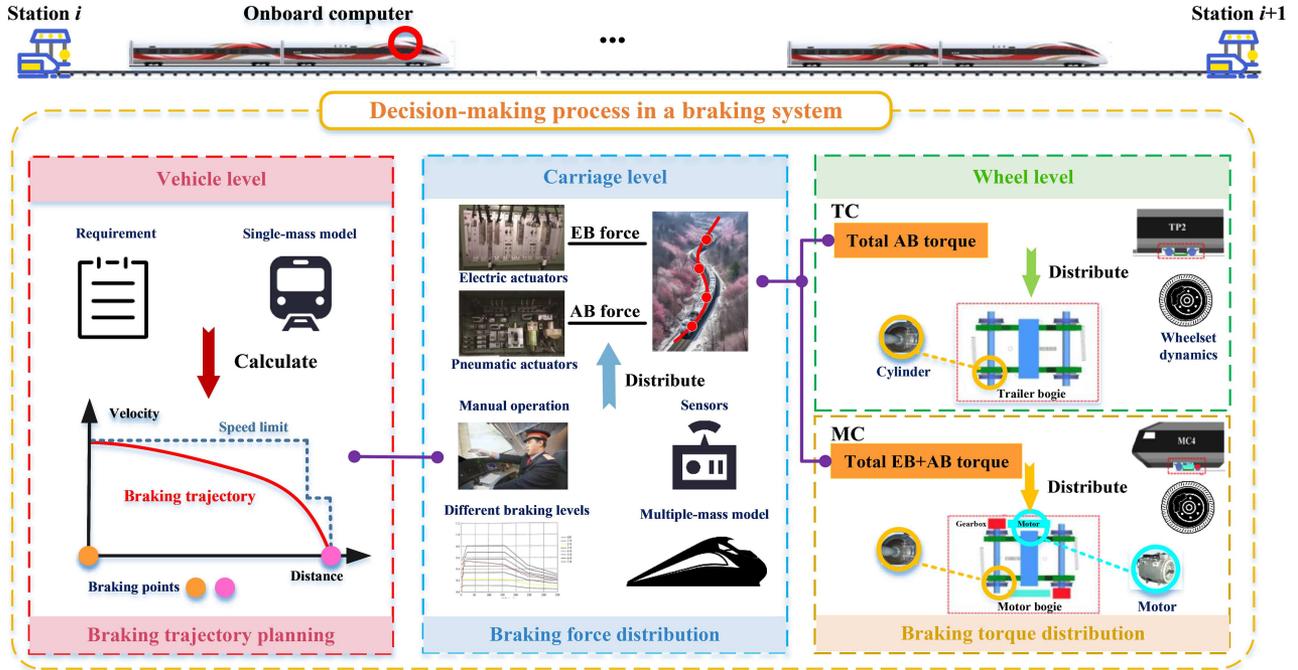


Figure 1 (Color online) Illustration of the hierarchical decision-making process in a braking system.

2.1 Vehicle level: braking trajectory planning

At the highest level, the planning layer of braking systems selects optimization indicators based on different scenarios to generate ideal braking trajectories that comply with operational requirements. Figure 1 illustrates a railway segment between two adjacent stations, where a vehicle, guided by a decision-making process, prepares to depart. Before departure, the onboard computer receives confirmation messages containing travel details, including the distance to the next station, line gradients, speed limits, and arrival times. Notably, arrival and dwell times, sourced from the timetable established by the operation control system [19], ensure punctuality and enhance railway reliability.

Based on these data, the onboard computer generates a recommended speed profile within speed limits. Multiple position-velocity trajectories can satisfy arrival time and parking constraints [3,20–22]. The operation control system optimizes the speed profile to minimize energy consumption and enhance passenger comfort. This profile, comprising acceleration, coasting, cruising, and braking phases, begins and ends at 0 km/h [23–25] and is typically computed offline before departure. This paper focuses on braking techniques without considering specialized methods for signal blocks, network junctions, or external disturbances like crosswinds in the speed profile.

The braking systems under investigation include both regenerative and air brakes. One principle of an optimal braking trajectory is maximizing regenerative braking energy (RBE) [26–28]. Some railway vehicles, such as heavy-haul trains, rely solely on air brakes, focusing on minimizing cyclic braking time on downhill slopes to reduce actuator wear [29]. This paper does not cover such vehicles; for further details, see [30–32].

The technical development of braking trajectory planning (BTP) at the vehicle level is discussed in Section 3.

2.2 Carriage level: braking force distribution

A vehicle interacts with its environment and activates the control loop to track the optimal braking trajectory from the BTP, maximizing total RBE. At the carriage level, a controller distributes braking force across all carriages, ensuring their combined RBE matches the predetermined total while maintaining reliable control of a moving vehicle.

The braking force distribution (BFD) algorithm embedded in the controller must adhere to operational conventions and braking regulations, specifically the force prioritization principle [33]. A typical priority order is: EB forces of motor carriages (MCs) are higher than AB forces of trailer carriages (TCs), which are higher than AB forces of MCs. The BFD at the carriage level selects an appropriate force distribution mechanism in real-time based on sampled feedback data, rail conditions, actuator signals, and threshold alarms from fault diagnosis systems.

The technical development of BFD at the carriage level is detailed in Section 4.

2.3 Wheel level: braking torque distribution

When the BFD control determines the desired braking force for each carriage, the feedback controller at the wheel level regulates the actual braking force to match this target. It adjusts actuator braking torques, such as those from the traction motor and brake cylinder, forming braking torque distribution. This directly influences wheelset states, modifying adhesive force to achieve the desired braking force [34].

Although adhesion coefficients and wheelset states may vary among wheels on a carriage, the wheel-level controller estimates the adhesive upper bounds of each wheelset and distributes braking torques accordingly. This ensures adhesive force remains close to the expected value while preventing wheelset or carriage slippage [35].

While regulating braking force to the desired value enables the planned braking action, errors may arise due to modeling inaccuracies and environmental uncertainties. Thus, ensuring the closed-loop system's robustness and stability is critical.

The technical development of braking torque distribution (BTD) at the wheel level is detailed in Section 5.

2.4 Hierarchical integration of three levels

The three-level framework is well-suited for analyzing large-scale systems, as the levels are inherently interconnected. Their integration will be systematically presented through detailed formulations in the subsequent sections. An overall view is provided below.

At the vehicle level, the generated braking trajectory establishes the global deceleration profile, which subsequently decomposes into specific braking force requirements at the carriage level. While not directly controlling individual wheelsets, the vehicle-level controller imposes hierarchical constraints that govern force distribution and execution throughout the subordinate carriage level. This top-down propagation forms a command-driven coupling across the braking hierarchy.

At the carriage level, BFD design must simultaneously satisfy the aggregate deceleration requirements propagated from the vehicle level while accounting for carriage-specific parameters including carriage mass, actuator operational status, and localized adhesion condition. The resultant braking force references are subsequently transmitted to the wheel-level controllers, where physical implementation through precise braking torque regulation occurs. Thus, the carriage level is an intermediary bridging layer that transforms global deceleration objectives into locally executable torque commands while maintaining consistency throughout the hierarchical control structure.

At the wheel level, braking torque commands received from the carriage level must be physically realized while accounting for local adhesion constraints and wheel-rail interaction dynamics. The wheel-level controller transforms these reference commands into precise actuator torque signals, ensuring adherence to physical limitations dictated by the instantaneous contact conditions. Thus, this wheel level completes the hierarchical execution process, maintaining operational consistency and disturbance rejection capabilities within the overall feedforward control architecture.

In subsequent sections, we detail the signal flow across the three layers during the complete braking process, with particular emphasis on the coupling relationships governed by explicit mathematical formulations. It is important to note that the empirical formulas employed for certain physical quantities, such as adhesion limitation and actuator saturation, are derived from extensive field testing conducted during the vehicle commissioning phases. These parameters exhibit robustness for vehicles operating on specific lines, ensuring that the nominal system modeling accurately captures real-world behaviors. As a result, the use of empirical models does not compromise the integrity of the problem definitions or the rigor of the conclusions. Instead, it highlights the engineering application focus of this study, aiming to bridge theoretical insights with practical feasibility without relying on modeling assumptions under uncertainty.

3 Braking trajectory planning

This section introduces the commonly used dynamic models for online BTP. The model approximates a railway vehicle by treating all carriages as a single mass point, responding to braking control under relevant operating conditions. Based on this, we present the mathematical formulation for braking trajectory optimization and review solution methodologies.

3.1 Mathematical formulation

In the most basic practical model, a railway vehicle is represented as a single-point control model, moving along a straight track and focusing solely on longitudinal motion and braking dynamics. This formulation aims to capture the dominant factors (e.g., resistance, EB and AB forces) that govern longitudinal motion and energy consumption, which are critical for trajectory planning. Other features such as lateral forces, carriage dynamics, turns, and external disturbances are omitted, as their impact on longitudinal deceleration profiles is negligible under normal conditions [36–38].

The braking trajectory can be approximated by a Newton's equation:

$$\frac{dt}{dx} = \frac{1}{v}, \quad mv \frac{dv}{dx} = u(x) - F^r(x) - F^g(x), \quad (1)$$

where t is the time, m is the mass of vehicle, $x \in \mathbb{R}$ is the position, $v \in \mathbb{R}$ is the velocity, and $u \in \mathbb{R}$ is the braking force. This formulation allows all trajectory parameters, $v(x)$, $u(x)$, $F^r(x)$, and $F^g(x)$, to be expressed as functions of position x . Choosing x over time t as the independent variable simplifies both analytical and numerical analysis [39].

The force $F^r(x)$ in (1) represents both mechanical resistance and aerodynamic drag. It is computed using an empirical model

$$F^r(x) = m(c_0 + c_1 v(x) + c_2 v(x)^2), \quad (2)$$

where c_0 , c_1 , and c_2 are Davis coefficients. The force $F^g(x)$ represents gradient resistance and is given by

$$F^g(x) = mg \sin(\alpha(x)), \quad (3)$$

where $\alpha(x)$ is the slope angle at position x , and g is the gravitational acceleration.

A railway vehicle applies both electric and air braking, expressed as

$$u(x) = u^{\text{elec}}(x) + u^{\text{air}}(x),$$

where u^{elec} is the EB force and u^{air} is the AB force at position x . The instantaneous RBE $E^{\text{elec}}(x)$ is given by

$$\dot{E}^{\text{elec}}(x) = \frac{dE^{\text{elec}}(x)}{dx} = u^{\text{elec}}(x), \quad (4)$$

indicating that the braking operation with $u^{\text{elec}}(x)$ increases RBE.

The total braking distance X is assumed to be a known parameter when a vehicle must stop at a designated point, such as the next station, within a defined time window. Braking is assumed to occur between position points x_0 and x_f on the track, where $x_f - x_0 = X$. Additionally, it is assumed that the speed limit $\sigma_v(x)$ and the total braking time T are known, with an initial speed of v_0 and a final speed of v_f , where $v_0 > 0$ and $v_0 > v_f \geq 0$. The boundary conditions are

$$\begin{aligned} E^{\text{elec}}(x_0) &= 0, \quad t(x_f) - t(x_0) = T, \\ v(x_0) &= v_0, \quad v(x_f) = v_f, \quad v(x) \leq \sigma_v(x). \end{aligned} \quad (5)$$

The constraints on the control variables (i.e., braking forces u^{elec} and u^{air}) depend on the vehicle's speed, denoted as $\sigma^{\text{elec}}(x)$ and $\sigma^{\text{air}}(x)$, respectively. The vehicle's braking forces must satisfy the following constraints:

$$\begin{aligned} 0 &\leq u^{\text{elec}}(x) \leq \sigma^{\text{elec}}(x), \\ 0 &\leq u^{\text{air}}(x) \leq \sigma^{\text{air}}(x). \end{aligned} \quad (6)$$

The upper limits have the following typical expressions:

$$\sigma^{\text{elec}}(x) = N_m h(v(x)), \quad \sigma^{\text{ad}}(x) = mg\epsilon(v(x)), \quad (7)$$

where N_m is the number of MCs and $\sigma^{\text{ad}}(x) = \sigma^{\text{elec}}(x) + \sigma^{\text{air}}(x)$.

The maximum EB force $\sigma^{\text{elec}}(x)$ depends on the velocity $v(x)$ and is represented by an empirical nonlinear piecewise function h , which corresponds to the maximum EB force for a single MC. The maximum adhesion limitation, $\sigma^{\text{ad}}(x)$, prevents wheel slippage and is calculated based on m , g , and the optimal adhesion coefficient

$\epsilon(v(x))$. The maximum AB force $\sigma^{\text{air}}(x)$ is limited by both the adhesion limitation $\sigma^{\text{ad}}(x)$ and the maximum EB force $\sigma^{\text{elec}}(x)$. The calculation of the optimal adhesion coefficient is detailed in [40], and is given by

$$\epsilon(v(x)) = \theta_1 + \frac{\theta_2}{v(x) + \theta_3},$$

where θ_1 , θ_2 , and θ_3 are physical parameters of the rail surface.

The problem of determining the braking operation that maximizes RBE $E^{\text{elec}}(x_f)$ while driving the railway vehicle from one point to another can be formulated as follows.

Problem 1 (Braking trajectory planning). Given the vehicle model (1) and the braking requirements x_0 , x_f , v_0 , v_f , and T , solve

$$\max_{t(x), v(x), u^{\text{elec}}(x), u^{\text{air}}(x)} \int_{x_0}^{x_f} u^{\text{elec}}(x) dx \quad \text{s.t. (5) and (6)}. \quad (8)$$

3.2 Methods

With rising energy demand and a growing emphasis on carbon reduction, energy conservation has become a key focus in rail transportation [41]. Researchers in academia and industry have developed various energy-efficient technologies to minimize consumption. Early studies, such as [42], explored efficient use of traction energy and the application of EB mode for kinetic energy recovery. Without major infrastructure changes, speed trajectory optimization remains an effective strategy for reducing rail energy consumption [43–45].

Various algorithms have been developed to determine the optimal speed trajectory for minimizing energy consumption or other associated costs. A common approach is coasting operations within a set time window, which reduces non-RBE by adjusting coasting duration. Since the coasting phase shuts down the motors, optimizing its duration balances operation time and energy consumption [46].

Regenerative braking is highlighted as a key feature for railway vehicles with EPMBs, as it converts kinetic energy into reusable electrical energy [47]. This recovered energy can be fed back into the power grid in alternating current (AC) networks, used by nearby trains via direct current (DC) networks, or stored in energy devices [48]. If unused, the energy is dissipated as heat through a resistance bank, leading to significant energy loss.

The studies in [49, 50] focus on maximizing RBE in speed trajectory planning, examining the application of regenerative braking in electrified railway systems. Some research optimizes train scheduling to increase the overlap between acceleration and braking phases, thereby enhancing RBE utilization [51–53]. Notably, Ref. [54] incorporated electric load flow calculations, emphasizing network modeling and the impact of network losses on RBE recovery. These studies generally treat regenerative braking or the energy it generates as a single component in optimization objectives or constraints.

While maximizing RBE does not always yield the highest overall travel energy efficiency, real-time BTP becomes crucial in emergencies when the vehicle cannot follow the offline optimal speed trajectory set by the operation control system [55]. In such cases, railway engineers or drivers are primarily concerned with a key question: how much RBE can be recovered during braking within a given distance and time frame? To address this, various methods based on different mathematical formulations have been proposed, as outlined below. These methods are also summarized in Table 1 [27, 56–81], which provides a comparison of their objectives along with their respective pros and cons.

(1) **Variational algorithms.** These algorithms represent the performance index as an objective functional parameterized by a finite-dimensional vector, enabling the calculation of the optimal braking trajectory through nonlinear continuous optimization techniques [82]. They converge rapidly to locally optimal solutions. However, they typically lack the ability to find globally optimal solutions unless a suitable initial guess is provided [83]. Two subclasses of variational algorithms, direct and indirect approaches, are discussed below.

In a direct approach, the continuous-time optimal control problem is converted into a nonlinear programming (NLP) problem [56], which is then solved using established techniques like sparse nonlinear optimizer (SNOPT) [57, 58] and interior point optimizer (IPOPT) [59, 60]. Direct methods are particularly effective for handling inequality constraints on states and inputs [61]. Since the 2010s, a specific class of direct methods, known as pseudospectral (PS), has been applied to BTP for railway vehicles. For instance, Ref. [62] explored the optimal control problem for vehicle operations with constraints and fixed arrival times using the PS approach. Building on this, Ref. [63] extended the problem to multiple vehicles under fixed block and moving block signaling systems. Several studies, such as [64–66], have also employed the PS approach to improve computational efficiency in BTP.

In an indirect approach, Pontryagin’s minimum principle (PMP) provides the necessary optimality conditions for solving Problem 1 [84, 85]. Indirect methods, as the name suggests, solve the problem by finding solutions that

Table 1 Comparison of different braking trajectory planning techniques.

Refs.	Objectives	Pros	Cons
(1) Variational algorithms			
[56–61]	<ul style="list-style-type: none"> ○ Energy/travel time minimization ○ RBE maximization 	<ul style="list-style-type: none"> ○ Fast convergence (with suitable init.) ○ Commercial solver compatibility 	<ul style="list-style-type: none"> ○ Global optimality not guaranteed ○ Limited interpretability
[62–66]	<ul style="list-style-type: none"> ○ RBE maximization ○ Physical constraint satisfaction ○ Multi-stage decision integration ○ Energy/travel time minimization ○ Switching point determination ○ Terminal constraint satisfaction 	<ul style="list-style-type: none"> ○ Fast convergence (with suitable init.) ○ High accuracy for simplified models ○ Commercial solver compatibility ○ High accuracy for simplified models ○ Interpretable control structure ○ Mode-switching problem suitability 	<ul style="list-style-type: none"> ○ Global optimality not guaranteed ○ Sensitive to init. and parameters ○ Instability in non-smooth problems ○ Complex boundary-value solving ○ Initialization sensitivity ○ Limited solver availability
(2) Mathematical programming algorithms			
[27]	<ul style="list-style-type: none"> ○ RBE maximization ○ Graph-based global planning 	<ul style="list-style-type: none"> ○ Multi-path structure compatibility ○ Efficient graph-based path evaluation 	<ul style="list-style-type: none"> ○ Limited to discrete modeling ○ Dependent on graph quality
[67–69]	<ul style="list-style-type: none"> ○ Energy/travel time minimization ○ Minimum-cost trajectory search ○ Global discrete-space optimization 	<ul style="list-style-type: none"> ○ Global optimality in the discrete domain ○ Flexible objective function design ○ Robustness to model uncertainty 	<ul style="list-style-type: none"> ○ Curse of dimensionality ○ Dependence on discretization ○ High cost for large spaces
[70–72]	<ul style="list-style-type: none"> ○ Energy/travel time minimization ○ Piecewise model approximation ○ Time-bounded near-optimality 	<ul style="list-style-type: none"> ○ Scalability to large-scale problems ○ Efficient near-optimal solutions ○ Commercial solver compatibility 	<ul style="list-style-type: none"> ○ Model linearization requirements ○ Sensitivity to model formulation ○ Limited nonlinear handling
(3) Heuristic algorithms			
[73]	<ul style="list-style-type: none"> ○ RBE maximization 	<ul style="list-style-type: none"> ○ No strict model requirements 	<ul style="list-style-type: none"> ○ Susceptible to local optima
[74]	<ul style="list-style-type: none"> ○ RBE maximization ○ Multi-objective trade-off 	<ul style="list-style-type: none"> ○ Fast convergence ○ Gradient-free optimization capability 	<ul style="list-style-type: none"> ○ Susceptible to local optima ○ Sensitive to initial distribution
[75]	<ul style="list-style-type: none"> ○ Non-convex energy optimization ○ Robustness to uncertainty 	<ul style="list-style-type: none"> ○ Ability to escape local minima ○ Suitable for non-convex problems 	<ul style="list-style-type: none"> ○ Slow convergence ○ Large-scale inefficiency
[76–79]	<ul style="list-style-type: none"> ○ RBE maximization ○ Multi-objective trade-off 	<ul style="list-style-type: none"> ○ Extensive solution space exploration ○ Hybrid method integration 	<ul style="list-style-type: none"> ○ Susceptible to local optima ○ Sensitive to initial population
[80, 81]	<ul style="list-style-type: none"> ○ RBE maximization ○ Multi-objective trade-off 	<ul style="list-style-type: none"> ○ Problem-type adaptability ○ Enhanced global-local optimization 	<ul style="list-style-type: none"> ○ Implementation complexity ○ Parameter coordination challenges

satisfy these conditions, typically formulated as an augmented system of ordinary differential equations (ODEs) governing the evolution of states and co-states. A key advantage of indirect methods is their ability to reduce the optimization problem’s dimensionality to that of the state space, making them well-suited for BTP.

Applying PMP reformulates Problem 1 as determining the optimal sequence of control regimes (e.g., maximum braking, maximum electric braking, coasting) [27] and the switching points between them under varying conditions and vehicle types. For instance, Ref. [86] used a continuous-time model with constant efficiency to identify optimal switching points. Simple cases, such as a single speed limit on a flat track, were studied in [87], while Ref. [88] extended the analysis to undulating tracks with steep grades. For a broader review of variational approaches, see [89–91].

(2) Mathematical programming algorithms. Variational methods are valuable in railway vehicle applications but are limited by their tendency to converge only to local minima. Indirect approaches using PMP struggle with complex train dynamics, variable speed limits, and gradients. To overcome these challenges, Ref. [92] reformulated train operation as a multi-stage decision problem, applying mathematical programming to perform a global search in a discretized optimization space and directly obtain the optimal control strategy. These methods impose fewer requirements on the objective function while balancing optimization performance and computational time [93]. However, they depend on numerical iterations and precise mathematical modeling. With proper linearization and problem modeling, they can be both robust and adaptive, ensuring global optimality in convex cases [94].

To enhance computational efficiency in real-world applications, efforts focus on balancing performance and processing time. A dynamic programming (DP) approach has been used to address this trade-off [69]. In [67], DP with a distance-time model was applied to obtain the optimal trajectory within a discrete search space for a single train. An improved DP method in [68] introduced event-based decomposition, significantly reducing the search space and computational time. The Bellman-Ford algorithm, which integrates DP and graph theory, has also been shown to achieve near-optimal solutions [27]. To further reduce computation time, the BTP problem is reformulated as a mixed-integer linear program, approximated with piecewise affine functions, and solved using commercial software [70–72].

Mathematical programming algorithms often require approximations, typically through discretization. However, this leads to the “curse of dimensionality”, where computational cost grows exponentially with the state space

dimension. Therefore, selecting an appropriate model for trajectory planning is critical when applying mathematical programming techniques.

(3) **Heuristic algorithms.** In addition to variational and mathematical programming approaches, heuristic algorithms have also been applied to the BTP problem. When practical constraints are considered, the problem may become “nondeterministic polynomial time (NP)-complete” [95]. As approximate optimization methods, heuristic algorithms provide near-optimal solutions with reduced computational time due to their minimal modeling requirements.

However, most heuristic algorithms, such as genetic algorithms [96], ant colony optimization [73], particle swarm optimization [74], and simulated annealing [75], lack guarantees of optimality and convergence. Among them, genetic algorithms are widely used in BTP [78]. For instance, Ref. [76] utilized a genetic algorithm to balance energy consumption and journey time, while Ref. [77] applied it to optimize coast-brake points by jointly considering energy efficiency and punctuality. Integrating train operation and scheduling, Ref. [79] employed a genetic algorithm to minimize total net energy, leveraging regenerative braking in multi-train operations. A key advantage of heuristic methods is their flexibility, allowing different algorithms to be applied to the same problem for comparative studies [80, 81].

The selection of appropriate algorithms for specific application scenarios fundamentally depends on operational requirements, available computational resources, infrastructure characteristics, and control objectives. For instance, high-speed railways demand precise braking under strict safety, energy efficiency, and comfort requirements, making variational and mathematical programming methods, especially direct approaches like NLP and PS, well-suited due to their ability to generate smooth, constraint-satisfying deceleration profiles in centralized control settings. Urban transit systems, characterized by frequent acceleration and short interstation distances, benefit from heuristic algorithms (e.g., genetic algorithm (GA), simulated annealing (SA), and hybrids) for their simplicity, flexibility, and robustness, though they may yield suboptimal solutions; in contrast, DP and mixed-integer linear programming (MILP) offer superior results for timetable alignment and station-level energy optimization. In scenarios with limited sensing or computational resources, low-complexity methods such as PMP or graph-based techniques like Bellman-Ford algorithm (BF) provide efficient, interpretable solutions with minimal implementation demands.

4 Braking force distribution

The section first formulates the BFD problem, then reviews BFD strategies in the literature.

4.1 Mathematical formulation

At the vehicle level, a vehicle is modeled as a single-point control system in (1), whereas a multi-point model is required at the carriage level, with each point representing a carriage [97, 98]. Define the set of carriages as $\mathbb{I} = \{1, \dots, N\}$, where $\mathbb{I} = \mathbb{I}^m \cup \mathbb{I}^t$, with \mathbb{I}^m and \mathbb{I}^t denoting the sets of MCs and TCs, respectively. A railway vehicle with $N \geq 2$ carriages is modeled as

$$\begin{aligned} x_i[k+1] &= x_i[k] + t_s v_i[k], \\ v_i[k+1] &= v_i[k] + m_i^{-1} t_s (u_i[k] - F_i[k]), \quad i \in \mathbb{I}. \end{aligned} \quad (9)$$

Similar to (1), m_i is the mass of carriage i , while $x_i \in \mathbb{R}$, $v_i \in \mathbb{R}$, and $u_i \in \mathbb{R}$ represent its position, velocity, and braking force, respectively. Unlike (1), a discrete-time model with sampling period $t_s > 0$ is used to facilitate BFD algorithms. All trajectory parameters, $x_i[k]$, $v_i[k]$, $u_i[k]$, and $F_i[k]$, are expressed as functions of time instants $k = 0, 1, 2, \dots$. Additionally, let $a_i[k]$ denote the acceleration of carriage i .

This modeling approach effectively captures key longitudinal interactions within multi-carriage systems, including individual carriage masses and inter-carriage force transmission mechanisms, which are fundamental to the BFD problem formulation. Moreover, the discrete-time system representation aligns with practical considerations for real-time onboard controllers, which inherently operate on sampled-data principles with fixed processing intervals for measurement acquisition and control command generation.

The external force F_i is given by

$$F_i[k] = F_i^r[k] + F_i^g[k] + F_i^c[k], \quad i \in \mathbb{I},$$

where $F_i^r[k]$ and $F_i^g[k]$ are similar to those in (2) and (3), respectively. At the carriage level, the in-train coupler forces F_i^c are crucial for preventing coupler failures and enhancing passenger comfort. Let $F_{i,i+1}^c[k]$ represent the

coupling force between adjacent carriages i and $i + 1$. Then, F_i^c is the sum of $-F_{i-1,i}^c$ and/or $F_{i,i+1}^c$. The coupling force follows a spring-damping model [7]:

$$F_{i,i+1}^c[k] = k_i(x_i[k] - x_{i+1}[k] - l) + b_i(v_i[k] - v_{i+1}[k]), \quad i = 1, \dots, N - 1,$$

where k_i and b_i are the stiffness and damping coefficients, respectively, and l is the nominal distance between adjacent carriages.

The braking forces applied to all carriages ensure the vehicle follows the optimal braking trajectories $t(x)$, $v(x)$, $u^{\text{elec}}(x)$, and $u^{\text{air}}(x)$, obtained from the solution to Problem 1 in BTP. Additionally, the optimal acceleration can be denoted as $a(x)$. These reference trajectories are expressed as

$$x^*[k], v(x^*[k]), u^{\text{elec}}(x^*[k]), u^{\text{air}}(x^*[k]), a(x^*[k]), \quad (10)$$

where the time index k is used as the independent variable. The trajectories in (10) provide the reference for BFD design at the carriage level.

Define the velocity tracking error for each carriage as

$$e_i[k] = v_i[k] - v(x^*[k]), \quad i \in \mathbb{I}.$$

The acceleration tracking error and jerk are given by

$$\begin{aligned} \zeta_i[k] &= a_i[k] - a(x^*[k]), \\ \eta_i[k] &= t_s^{-1}(a_i[k] - a_i[k - 1]), \quad i \in \mathbb{I}. \end{aligned}$$

The objective of BFD is to distribute braking forces $u_i[k]$ to minimize the state-output vector $z_i[k] = [e_i[k + 1], \zeta_i[k], \eta_i[k]]^T$, $i \in \mathbb{I}$, ensuring precise trajectory tracking and passenger comfort. The first two terms govern braking curve tracking, while the third enhances passenger comfort [99]. Accordingly, the optimal control problem is formulated with the penalty function

$$J(z[k]) = z^T[k]Qz[k],$$

where z is the concatenated vector of z_i , $i \in \mathbb{I}$, and Q is a positive definite weight matrix.

The challenge in this optimization problem lies in the complex constraints. First, the braking forces applied to all carriages must sum to the total braking force for the vehicle. The braking force u_i for each MC and TC can be expressed in terms of the EB u_i^{elec} and AB u_i^{air} as follows:

$$\begin{aligned} u_i[k] &= u_i^{\text{elec}}[k] + u_i^{\text{air}}[k], \quad i \in \mathbb{I}^m, \\ u_i[k] &= u_i^{\text{air}}[k], \quad i \in \mathbb{I}^t. \end{aligned} \quad (11)$$

The constraints are then given by

$$\begin{aligned} \sum_{i \in \mathbb{I}^m} u_i^{\text{elec}}[k] &= u^{\text{elec}}(x^*[k]), \\ \sum_{i \in \mathbb{I}} u_i^{\text{air}}[k] &= u^{\text{air}}(x^*[k]). \end{aligned} \quad (12)$$

Second, in practical railway systems, braking forces and vehicle states must satisfy specific constraints:

$$\begin{aligned} 0 &\leq u_i^{\text{elec}}[k] \leq \sigma_i^{\text{elec}}[k], \quad i \in \mathbb{I}^m, \\ 0 &\leq u_i^{\text{air}}[k] \leq \sigma_i^{\text{ad}}[k] - \sigma_i^{\text{elec}}[k], \quad i \in \mathbb{I}^m, \\ 0 &\leq u_i[k] \leq \sigma_i^{\text{ad}}[k], \quad i \in \mathbb{I}^t, \\ -v(x^*[k]) &\leq e_i[k] \leq \sigma_v(x^*[k]) - v(x^*[k]), \quad i \in \mathbb{I}, \\ m_i \mathbf{a} + F_i[k] &\leq u_i \leq F_i[k], \quad i \in \mathbb{I}. \end{aligned} \quad (13)$$

Here, σ_i^{ad} , σ_i^{elec} , and σ_i^{air} correspond to those in (6) and (7), but for individual carriages. The last two constraints in (13) enforce velocity and acceleration limits:

$$0 \leq v_i[k] \leq \sigma_v(x^*[k]), \quad \mathbf{a} \leq a_i[k] \leq 0, \quad i \in \mathbb{I}.$$

Here, σ_v is the velocity protection profile defined in (5), and \mathbf{a} is the maximum braking acceleration.

The BFD problem is then formulated as follows, with μ representing all control inputs u_i^{elec} and u_i^{air} for $i \in \mathbb{I}$.

Problem 2 (Braking force distribution). Given the carriage model (9) and the reference trajectories (10), solve

$$\min_{\mu} \sum_k J(z[k]) \quad \text{s.t. (12) and (13)}. \quad (14)$$

This mathematical formulation does not explicitly incorporate the force prioritization principle as a constraint. Instead, it is enforced in the optimization problem with a necessary trade-off. A typical BFD first distributes u_i^{elec} for $i \in \mathbb{I}^m$, followed by u_i^{air} for $i \in \mathbb{I}^t$, and then u_i^{air} for $i \in \mathbb{I}^m$. The advantage of EB u_i^{elec} is its ability to convert kinetic energy into electrical energy, which can be used to power onboard equipment. Notably, this prioritization is also considered in BTP at the vehicle level. When the total EB force is insufficient, AB forces from TCs are applied first, followed by those from MCs if necessary. This priority-based strategy is widely adopted in railway vehicles [100].

4.2 Methods

Solving the optimization problem while fully implementing the prioritization principle is challenging. Various BFD strategies have been developed to balance this trade-off. We first review methods that focus on force distribution under specific prioritization rules. Then, we examine approaches that emphasize optimal performance.

(1) **BFD with prioritization.** The primary objective of this method is to design a feasible BFD that regulates speed while maximizing the use of EB mode. Although all these methods employ centralized braking control with an EB-first strategy, they differ in how they allocate AB forces. For example, Ref. [101] introduced two AB force distribution methods, equal wear and equal adhesion, demonstrating that both achieved similar performance. To refine the average and creep difference distribution methods, Ref. [102] proposed a BFD approach based on the proportional slope of the tangent force coefficient. Additionally, Ref. [103] developed a BFD method for fault conditions, accounting for adhesion limitations in MCs and TCs.

The BFD strategies must also adhere to the braking force constraints of each carriage to keep forces within adhesion limits and prevent wheelset sliding. To achieve this at the control layer of the carriage level, wheel-slip prevention (WSP) devices are applied to adjust braking forces in real time, preventing wheelset sliding or locking [104]. From a control perspective, WSP devices typically use curve parameterization to identify the optimal slip value, ensuring effective braking while maintaining vehicle stability.

Several model-based control schemes have been proposed to optimize WSP device performance, as shown in [105, 106]. These methods enable real-time calculation of optimal operating points (or maximum adhesion force), allowing BFD strategies to dynamically incorporate anti-slip constraints. In the BFD problem at the carriage level, the highly nonlinear interaction between the track and wheels is simplified into a linear constraint to facilitate the anti-slip function. A more detailed analysis of this nonlinear relationship at the wheel level will be presented in Section 5.

This simplification enables braking force adjustments based on current conditions to prevent excessive wheelset slip. For example, a BFD strategy based on the positive proportion of adhesion force was developed in [107] to optimize braking by leveraging available wheel-rail adhesion. Similarly, the load-inverse proportional BFD strategy in [40] aimed to mitigate sliding faults by distributing braking forces inversely to carriage load. More recently, distributed BFD algorithms have been introduced in [108] to enhance train operation robustness and reduce passenger discomfort from sudden bumps.

While these strategies focus on sliding prevention and adherence to adhesion limits, they rely solely on priority rules and overlook critical factors such as tracking accuracy, passenger comfort, and physical constraints. These factors are essential for achieving optimal braking performance, especially under abnormal operating conditions.

(2) **Optimal BFD algorithms.** In contrast, optimal BFD algorithms focus on maximizing system performance by minimizing energy consumption, reducing tracking error, and ensuring smooth deceleration. While these aspects have been studied, the integration of BFD strategies remains largely overlooked in both research and engineering applications.

For example, Ref. [109] proposed a computationally efficient design to enhance speed tracking. Similarly, Ref. [110] introduced a method for tracking the recommended speed profile using a MAX-MIN ant system. Additionally, adaptive control designs have been developed to improve vehicle tracking performance, as shown in [111–113]. An optimal BFD function that maximizes total adhesion utilization was established in [114], accounting for axle load transfer. This approach enables coordinated BFD optimization based on a vehicle's total adhesion utilization ratio.

Many studies have aimed to enhance braking performance and passenger comfort using methods like fuzzy control and expert systems. These intelligent control approaches integrate driving knowledge into domain-specific rules. For example, Ref. [115] applied predictive fuzzy control to railway vehicles, defining fuzzy sets for performance

Table 2 Comparison of different braking force distribution techniques.

Refs.	Objectives	Pros	Cons
(1) BFD with prioritization			
[101]	<ul style="list-style-type: none"> ○ Implement EB-prioritized braking ○ Evaluate equal adhesion strategies 	<ul style="list-style-type: none"> ○ Improved braking force uniformity ○ Multiple AB distribution 	<ul style="list-style-type: none"> ○ Comm-dependent centralized scheme ○ Comfort and tracking not considered
[102]	<ul style="list-style-type: none"> ○ Implement EB-prioritized braking ○ Optimize AB force distribution 	<ul style="list-style-type: none"> ○ Sensitive to adhesion curve features ○ Improved anti-slip under low adhesion 	<ul style="list-style-type: none"> ○ Comm-dependent centralized scheme ○ Lack of comfort and energy trade-offs
[103]	<ul style="list-style-type: none"> ○ Ensure stability under failure ○ Maximize adhesion utilization 	<ul style="list-style-type: none"> ○ Enhanced braking robustness ○ Suppression of fault propagation/slip 	<ul style="list-style-type: none"> ○ Comm-dependent centralized scheme ○ Sensitive to fault detection accuracy
[104]	<ul style="list-style-type: none"> ○ Maximize adhesion utilization ○ Prevent wheel slip and lock-up 	<ul style="list-style-type: none"> ○ Real-time slip correction ○ Robust to track condition variations 	<ul style="list-style-type: none"> ○ Comm-dependent centralized scheme ○ Sensitive to identification accuracy
[105, 106]	<ul style="list-style-type: none"> ○ Maximize adhesion utilization 	<ul style="list-style-type: none"> ○ Predictive anti-slip capability 	<ul style="list-style-type: none"> ○ Comm-dependent centralized scheme
[107]	<ul style="list-style-type: none"> ○ Maximize adhesion utilization 	<ul style="list-style-type: none"> ○ Robust to track condition variations 	<ul style="list-style-type: none"> ○ Comm-dependent centralized scheme
[108]	<ul style="list-style-type: none"> ○ Implement EB-prioritized braking ○ Eliminate communication reliance 	<ul style="list-style-type: none"> ○ Fully decentralized operation ○ Robust to abnormal scenarios 	<ul style="list-style-type: none"> ○ May sacrifice global optimality ○ Lack of comfort and energy trade-offs
(2) Optimal BFD algorithms			
[109]	<ul style="list-style-type: none"> ○ Enhanced tracking performance 	<ul style="list-style-type: none"> ○ Reduced computational load 	<ul style="list-style-type: none"> ○ Comfort and robustness ignored
[110]	<ul style="list-style-type: none"> ○ Enhanced tracking performance ○ Rule-based BFD optimization 	<ul style="list-style-type: none"> ○ Robust global search capabilities ○ Model-free optimization 	<ul style="list-style-type: none"> ○ Lack of interpretability ○ Convergence sensitive to setup
[111–113]	<ul style="list-style-type: none"> ○ Robust tracking performance 	<ul style="list-style-type: none"> ○ Robust to uncertainties 	<ul style="list-style-type: none"> ○ Lack of comfort and energy trade-offs
[114]	<ul style="list-style-type: none"> ○ Maximize total adhesion usage 	<ul style="list-style-type: none"> ○ Adaptive to axle load transfer 	<ul style="list-style-type: none"> ○ Dependence on load model accuracy
[115]	<ul style="list-style-type: none"> ○ Balance comfort, energy, tracking ○ Predictive braking rule selection 	<ul style="list-style-type: none"> ○ Embeds domain knowledge ○ Robustness to uncertainties 	<ul style="list-style-type: none"> ○ Rule design complexity ○ Lack of interpretability
[121]	<ul style="list-style-type: none"> ○ Balance tracking and comfort 	<ul style="list-style-type: none"> ○ UIC/GB standards compliance 	<ul style="list-style-type: none"> ○ Not integrated with full BFD logic

indices such as passenger comfort, tracking accuracy, and energy consumption. The method predicts the outcome of each control command and selects the optimal braking rule based on these fuzzy sets. More studies can be found in [116–118].

Considerations for jerk, as outlined in the European Union’s International Union of Railways (UIC) Leaflet 513-1994 [119] and implemented in China’s railway systems [120], have received increasing attention. In [121], a cascaded controller was introduced to track reference trajectories while accounting for jerk limitations. Despite these advancements, the integration of BFD strategies with jerk and other factors remains largely unexplored in current research.

The aforementioned methods are also summarized in Table 2. Based on optimization performance, implementation complexity, real-time feasibility, and robustness, appropriate methods can be selected for specific scenarios. Systems with adequate computational resources benefit from optimal BFD algorithms, particularly model predictive control (MPC)-based and adaptive control architectures, which excel in prioritizing force distribution, trajectory tracking, adhesion utilization, and passenger comfort, though they require high-fidelity models, accurate state estimation, and significant processing power. In contrast, for dynamic environments or real-time constraints, simpler methods such as force-proportional, adhesion-based, or fuzzy-rule-based controllers offer more practical performance due to their rapid response, robustness to uncertainty, and ease of integration, despite lower optimization precision.

5 Braking torque distribution

The braking forces for each carriage are calculated at the carriage level: $u_i[k] = u_i^{\text{elec}}[k] + u_i^{\text{air}}[k]$ for $i \in \mathbb{I}^m$, and $u_i[k] = u_i^{\text{air}}[k]$ for $i \in \mathbb{I}^t$. These forces are achieved through braking torques actuated on the wheels. The objective at the wheel level is to control these torques, estimated by the following equations:

$$\begin{aligned} T_i^{\text{elec}}[k] &= T(u_i^{\text{elec}}[k]), \quad i \in \mathbb{I}^m, \\ T_i^{\text{air}}[k] &= T(u_i^{\text{air}}[k]), \quad i \in \mathbb{I}, \end{aligned} \quad (15)$$

where T is an empirical function [122]. Here, $T_i^{\text{elec}}[k]$ represents the total EB torque exerted by all axles within a single MC, while $T_i^{\text{air}}[k]$ represents the total AB torque in either an MC or a TC. Although the required torques are determined through the empirical relationships in (15), they represent the total torque generated by the wheelset of each carriage. The primary task at the wheel level is to distribute the torques, $T_i^{\text{elec}}[k]$ and $T_i^{\text{air}}[k]$, among the various wheel actuators. This section introduces the formulation of the torque distribution problem and the methods relevant to its solution.

5.1 Mathematical formulation

Suppose there are $N_w \geq 2$ wheelsets on a carriage, either an MC or a TC, and define $\mathbb{I}_w = \{1, \dots, N_w\}$. Let $T_{ij} \in \mathbb{R}$ represent the braking torque for the j -th wheelset of the i -th carriage. Specifically, for an MC, the braking torque $T_{ij}[k]$ consists of both the EB component $T_{ij}^{\text{elec}}[k]$ and the AB component $T_{ij}^{\text{air}}[k]$. On the other hand, for a TC, only the AB component $T_{ij}^{\text{air}}[k]$ is used, as expressed by

$$\begin{aligned} T_{ij}[k] &= T_{ij}^{\text{elec}}[k] + T_{ij}^{\text{air}}[k], \quad i \in \mathbb{I}^m, \\ T_{ij}[k] &= T_{ij}^{\text{air}}[k], \quad i \in \mathbb{I}^t. \end{aligned} \quad (16)$$

Given $T_i^{\text{elec}}[k]$ and $T_i^{\text{air}}[k]$ from the empirical equations in (15), we aim to calculate $T_{ij}^{\text{elec}}[k]$ and $T_{ij}^{\text{air}}[k]$ such that

$$\begin{aligned} \sum_{j=1}^{N_w} T_{ij}^{\text{elec}}[k] &= T_i^{\text{elec}}[k], \quad i \in \mathbb{I}^m, \\ \sum_{j=1}^{N_w} T_{ij}^{\text{air}}[k] &= T_i^{\text{air}}[k], \quad i \in \mathbb{I}. \end{aligned} \quad (17)$$

A simple calculation follows the principle of equal torque distribution. However, this approach lacks robustness to environmental changes and may lead to slip faults in individual wheelsets. To address this issue, another key principle is to mitigate slip by dynamically redistributing braking torques. In other words, this layer works in conjunction with WSP devices to prevent wheelset locking, thereby enhancing overall braking control performance [123]. Based on this principle, a specific optimization problem is formulated below.

The braking torque generated by the actuators regulates the wheelset speed and produces the necessary adhesive force through the contact between the wheelsets and the rail. Let $F_{ij}^a[k]$ denote the adhesive force generated by the j -th wheelset, and let the total adhesive force of a carriage be given by

$$F_i^a[k] = \sum_{j \in \mathbb{I}_w} F_{ij}^a[k], \quad i \in \mathbb{I}. \quad (18)$$

This adhesive force acts as the braking force $u_i[k]$ in (9), i.e., $u_i[k] = F_i^a[k]$. As a result, the model (9), together with the wheelset dynamics, can be expressed as the following discrete-time system:

$$\begin{aligned} x_i[k+1] &= x_i[k] + t_s v_i[k], \\ v_i[k+1] &= v_i[k] + m_i^{-1} t_s (F_i^a[k] - F_i[k]), \\ \omega_{ij}[k+1] &= \omega_{ij}[k] + J_{ij}^{-1} t_s (T_{ij}[k] - b \omega_{ij}[k] - r F_{ij}^a[k]), \quad i \in \mathbb{I}, \quad j \in \mathbb{I}_w, \end{aligned} \quad (19)$$

where $\omega_{ij} \in \mathbb{R}$ represents the wheelset's angular speed, b is the viscous friction coefficient, J_{ij} is the wheelset inertia, and r is the wheelset radius.

Moreover, the adhesive force $F_{ij}^a[k]$ is characterized by

$$F_{ij}^a[k] = \mu(\lambda_{ij}[k]) m_{ij} g, \quad (20)$$

where m_{ij} is the axle mass, and $\mu(\lambda_{ij}[k])$ is the adhesion coefficient. The adhesion coefficient is assumed to be computable through an empirical model [124]:

$$\mu(\lambda_{ij}[k]) = - \left(\psi_1 \left(1 - e^{\psi_2 \lambda_{ij}[k]} \right) + \frac{\lambda_{ij}[k]}{\psi_3} \right), \quad (21)$$

for some nonlinear parameterized coefficients ψ_1, ψ_2, ψ_3 related to the rail-surface environment. The slip ratio $\lambda_{ij}[k]$ is defined as the relative difference between the carriage speed $v_i[k]$ and the wheelset angular speed $\omega_{ij}[k]$:

$$\lambda_{ij}[k] = \frac{r \omega_{ij}[k] - v_i[k]}{v_i[k]}. \quad (22)$$

Additional models for adhesive force can be found in [125, 126].

The modeling approach adopted here captures the fundamental coupling between wheelset angular dynamics, applied braking torques, and adhesive force generation at the wheel-rail interface, enabling explicit analysis of how actuator commands affect longitudinal deceleration through wheel-rail interactions. To maintain computational tractability and ensure compatibility with practical controller synthesis, the model deliberately excludes higher-order physical phenomena, such as detailed contact patch deformation, thermal effects on adhesion, micro-slip region dynamics, and nonlinearities from rail surface irregularities or contamination. These secondary effects have a negligible impact on overall braking performance under nominal operating conditions.

The prolonged operation of a vehicle on complex railway lines, such as flat and ramped sections, can lead to wheelset wear, with severe wear potentially causing slipping faults [127]. Furthermore, the vehicle's operating environment is complex and variable, resulting in differing levels of severity for each occurrence of slipping faults [128]. Thus, adhesion control at the wheel level aims to distribute the appropriate torque in response to slipping faults and other uncertainties. For instance, the slip ratio $\lambda_{ij}[k]$ during vehicle operation must be maintained within a creep region Γ_λ^c to prevent wheelset skidding. Conversely, the slip region Γ_λ^s indicates evident wheel skid when the slip ratio falls within this region.

The sets Γ_λ^c and Γ_λ^s are defined as [129]

$$\begin{aligned}\Gamma_\lambda^c &= \{\lambda : |\lambda_{ij}| < \lambda_{ij}^{\text{opt}}, i \in \mathbb{I}, j \in \mathbb{I}_w\}, \\ \Gamma_\lambda^s &= \{\lambda : \lambda_{ij}^{\text{opt}} \leq |\lambda_{ij}| \leq 1, i \in \mathbb{I}, j \in \mathbb{I}_w\}.\end{aligned}$$

To ensure the anti-skid function of the wheels, the following condition must be satisfied:

$$\lambda_{ij} \in \Gamma_\lambda^c, i \in \mathbb{I}, j \in \mathbb{I}_w. \quad (23)$$

Let the optimal slip ratio $\lambda_{ij}^{\text{opt}}$ correspond to the maximum adhesive coefficient, defined as $\lambda_{ij}^{\text{opt}} = \arg \max\{\mu(\lambda_{ij})\}$. Then, a cost function I is defined as

$$I = \sum_k \sum_{i \in \mathbb{I}} \sum_{j \in \mathbb{I}_w} \frac{\mu(\lambda_{ij}[k+1])}{\mu(\lambda_{ij}^{\text{opt}}[k+1])}, \quad (24)$$

which represents the maximization of the adhesion utilization rate. Based on this, the braking torque distribution problem can be formulated as follows.

Problem 3 (Braking torque distribution). Given the wheel model (19) and the total torques $T_i^{\text{elec}}[k]$ and $T_i^{\text{air}}[k]$, solve

$$\max_{T_{ij}^{\text{elec}}[k], T_{ij}^{\text{air}}[k], i \in \mathbb{I}, j \in \mathbb{I}_w} I \quad \text{s.t. (17) and (23)}. \quad (25)$$

5.2 Methods

Maximum vehicle deceleration occurs when the slip ratio reaches its optimal value, ensuring maximum adhesion between the wheelset and rail. Exceeding this value risks wheelset locking, reducing braking force and causing potential damage. Anti-slip control relies on the online identification of the optimal working point $\lambda_{ij}^{\text{opt}}$. However, this is a challenging task since the adhesion curve $\mu(\lambda_{ij})$ depends on wheelset and rail properties, which are influenced by contaminants such as water, oil, and sand [130]. Environmental factors, including weather, surface temperature, and friction modifiers, further impact online identification [131].

Given the difficulty of direct identification, various studies propose observers and alternative estimation methods to approximate adhesion force. However, these methods must consider the complex wheel-rail geometry, torsional oscillations [132], and uneven load distribution over small contact patches [133]. Additional factors such as vehicle speed, rail self-cleaning mechanisms [134], track irregularities, and axle load distribution further complicate adhesion estimation. Consequently, adhesion depends on multiple parameters, many of which pose analytical challenges.

Although accurately identifying the optimal slip ratio or estimating adhesion force remains challenging with current technology, BTD strategies based on these procedures continue to evolve. This survey specifically examines adhesion control in the context of BTD. In [135–138], a BTD strategy in the adhesion control layer is typically divided into two tasks: re-adhesion control and adhesion optimization control. The former, a form of corrective maintenance, is activated when the wheelset begins to slip, while the latter aims to maintain the wheelset state near the optimal working point. The techniques addressing these BTD tasks can be broadly classified into three categories: signal-based methods, model-based methods, and intelligent control methods.

(1) **Signal-based methods.** These methods adjust braking torque based directly on sensor signals (e.g., speed or acceleration) without requiring identification or estimation techniques. Typically, the highest state value among

a carriage's axles serves as the reference, and deviations of the state values of other axles from this reference are compared to a preset threshold to detect slipping wheelsets [139]. Once identified, anti-skid valves incrementally reduce braking torque on the slipping axle until the slip condition is resolved, following proportional control principles [140].

Building on this approach, several studies have improved anti-slip performance by extending proportional control to proportional-integral-derivative control [141–143]. However, early-stage slip faults may produce signals too weak to trigger detection. Additionally, on inclined tracks, wheelset acceleration signals may resemble those of early slipping faults, increasing the risk of false detections and incorrect torque distribution.

While signal-based methods do not rely on dynamic wheel models, their preset thresholds are typically derived from field tests and driver experience [144], making the identification of slipping brake axles somewhat subjective. Moreover, a key limitation of these methods is that the re-adhesion controller may not optimally utilize adhesion under varying rail surface conditions.

(2) **Model-based methods.** To improve vehicle adaptability across varying track surface conditions, model-based methods have become a widely adopted approach, typically implemented in a two-step strategy.

The first step involves parameterizing the adhesion curve as defined in (21). Once established, model-based schemes enable online estimation of the optimal working point. For instance, Ref. [145] identified a single varying parameter. However, a single parameter may not fully capture the complexity of the orbital environment when estimating multiple parameters. To address this, Ref. [146] employed recursive least squares for adhesion coefficient estimation.

Additionally, Ref. [147] proposed a differentiator filter to address the estimation challenge. Similarly, Ref. [148] introduced an extended Kalman filter for identifying the optimal working point, while later studies [149, 150] proposed an unscented Kalman filter. Although both Kalman filters enhance estimation accuracy, they are prone to divergence if not properly tuned [151], highlighting the importance of precise parameter selection. An alternative, parameter-free approach was presented in [104, 152] using extremum seeking techniques, eliminating the need for system parameterization.

The second step involves determining the appropriate torque distribution based on braking requirements. To ensure adequate torque during a slip fault, it is essential to coordinate and optimize the power output of the remaining wheelsets [153]. A fast model predictive control method for energy-saving under slipping conditions was proposed in [154], which fully considers the energy-saving potential of slip faults and employs the generalized minimal residual algorithm for rapid optimization. Another linear time-varying model predictive controller, introduced in [155], addresses vehicle safety and dynamic power distribution for torque distribution. However, these methods are typically tailored to single operating conditions. When transitioning to inclined tracks, axle load distribution can shift [156]. Without incorporating axle load transfer mechanisms in multi-wheelset friction optimization, inconsistent wheelset speeds may occur, leading to increased wear and energy losses due to slipping.

Adaptive adhesion control for BTD addresses scenarios with unknown parameters and external disturbances, reflecting real-world conditions. For instance, an adaptive friction sliding mode optimal controller was designed in [157] using Lyapunov stability theory, enhancing robustness and ensuring stability despite uncertainties. In [158], a robust prediction-based controller was introduced, using a nonlinear predictive approach to design an optimal control law and adaptively approximate unknown uncertainties with a radial basis function neural network.

Since accurately estimating the optimal working point remains challenging, various methods have been developed for adhesion force estimation, including disturbance observers [159] and maximum likelihood identification [160], to address the BTD problem. However, most existing observers operate in an open-loop structure, lacking feedback compensation to correct errors, which compromises precision and robustness, leading to steady-state fluctuations and significant initial errors that limit practical applicability.

Although model-based methods provide a more precise approach to torque distribution by directly utilizing estimated adhesion parameters, they require accurate parameterization and considerable computational resources, especially under complex rail conditions. These requirements limit their practical application in railway vehicles.

(3) **Intelligent control methods.** Intelligent control methods, such as fuzzy logic, neural networks, expert systems, and data mining algorithms, apply heuristic rules to dynamically adjust torque distribution based on real-time observations. By leveraging observational data rather than relying on a detailed braking dynamics model, these methods provide a flexible alternative to traditional approaches.

Early studies applied fuzzy controllers to first-order linear systems [161] and real-time slip-ratio- and pressure-based braking torque adaptation [162]. To enhance braking system stability, a self-learning fuzzy controller combined with a sliding mode controller was proposed in [163]. In rule-based BTD strategies, key design aspects include selecting control criteria, such as speed difference, deceleration, and slip ratio, and determining thresholds. A slip-ratio-based criterion and a time-to-wheel-lock threshold were introduced to optimize re-adhesion control [164],

while finite state machine-based control logic was used to simplify programming complexity [165]. Fuzzy logic has proven effective in threshold optimization, with hybrid fuzzy-rule architectures improving adaptability to varying rail conditions [166,167]. Although thresholds can be derived empirically through field tests [168], this approach demands significant resources.

Some examples of other intelligent control methods are provided below. In [169], BTD is managed using a genetic neural fuzzy control approach, where a neural optimizer identifies the optimal wheelset slip, and the fuzzy logic component calculates the required braking torque to maintain this slip. In [170], a hybrid control system incorporating a recurrent neural network observer was developed for adhesion control. Similarly, Ref. [171] employed two parallel neural networks to implement neuro-sliding mode control. Additionally, Ref. [172] proposed a particle swarm optimization algorithm with an efficient energy recovery control strategy to optimize torque distribution between the EB and AB modes. It is important to note that the parameters of data-driven intelligent control schemes are often sensitive to noise, and identification accuracy may be insufficient unless model parameters are updated to account for changing environmental conditions.

The BTD strategies discussed in the three methods above focus on distributing braking torque calculated from BFD at the carriage level. An alternative approach, known as adhesive tracking control [124,129,173,174], bypasses BFD and directly links BTP to BTD. This method employs an active adhesion controller to maintain the braking system on a predefined trajectory while keeping the slip rate within a controllable range, effectively providing a preventive anti-skid function. Unlike re-adhesion control and adhesion optimization control in BTD, adhesive tracking control does not rely on corrective maintenance or require the wheelset to remain at an optimal operating point. While an optimal operating point is critical in the emergency braking mode, where minimizing braking distance and time by maximizing adhesion force is the priority [126], it is not always necessary in the common braking mode.

Among the BTD strategies discussed in this section and summarized in Table 3, the selection of an appropriate approach is primarily guided by operational requirements such as adhesion variability, environmental disturbances, model availability, and computational resources. For high-performance applications like HSTs and heavy-haul trains, model-based BTD methods such as MPC and adaptive estimation offer precise torque control and efficient energy recovery via online adhesion estimation, though they require significant computational power and high-fidelity sensing. In contrast, urban transit systems benefit more from signal-based or fuzzy control approaches, which, despite lower adhesion utilization, provide fast response, robust disturbance rejection, and easy integration with minimal computational demand. Intelligent methods offer adaptable, model-free solutions suitable for uncertain rail conditions, though they demand careful tuning, making them ideal where robustness and flexibility are prioritized over strict optimality.

6 Braking system validation platforms

A validation platform is essential for testing theoretical approaches before deploying them in practical railway systems. A key requirement is the ability to evaluate braking systems under harsh conditions, where components are prone to failure.

The use of physical validation platforms offers the most direct means of replicating operational scenarios and is primarily utilized for functionality testing of equipment. Recent collaborations between Chinese high-speed railway manufacturers and academic institutions have significantly advanced the development of such platforms. Notably, a full-scale platform (constructed at a 1:1 scale with respect to production vehicles) has been implemented at the China Railway Rolling Stock Corporation for evaluating fault diagnosis algorithms [175]. This platform allows the simulation of various fault scenarios, such as brake cylinder pressure loss, component degradation, and sensor malfunction, through auxiliary component installations. However, its reliance on manual fault injection introduces limitations in fault severity control precision and raises potential operational safety concerns.

To address these limitations and enable more flexible, cost-effective testing, virtual simulation and hardware-in-the-loop simulation platforms have been developed as complementary validation tools. These environments support system-level design verification, analysis of operational behavior, and fault-oriented safety assessment under a range of simulated conditions. For instance, a China Railway High-speed 2 (CRH2) HST simulation platform was developed to optimize travel time, maximum speed, and energy consumption [176]. Further applications include harmonic suppression analysis using a CRH6 platform [177], accident scenario reconstruction through cellular automata modeling [178], and evaluation of fault-tolerant control systems via a virtual train operation platform spanning over 70 km of high-speed railway infrastructure [179].

Among various non-physical simulation platforms, the fault injection technique has emerged as a practical and

Table 3 Comparison of different braking torque distribution techniques.

Refs.	Objectives	Pros	Cons
(1) Signal-based methods			
[139, 140]	<ul style="list-style-type: none"> ○ Wheel slip detection ○ Rapid slip suppression 	<ul style="list-style-type: none"> ○ Simple proportional control ○ Model-free optimization 	<ul style="list-style-type: none"> ○ Experience-dependent threshold ○ No adhesion optimization
[141–143]	<ul style="list-style-type: none"> ○ Precise slip deviation tracking ○ Anti-slip correction ○ Control stability assurance 	<ul style="list-style-type: none"> ○ Faster correction, error elimination ○ Strong robustness to disturbances ○ Precise torque modulation accuracy 	<ul style="list-style-type: none"> ○ Increased tuning complexity ○ Delay/overreaction due to weak signals
[144]	<ul style="list-style-type: none"> ○ Multi-threshold and rule-based slip detection 	<ul style="list-style-type: none"> ○ Rule-based implementation ○ Logical structure simplicity 	<ul style="list-style-type: none"> ○ Manual rule design dependency ○ Limited solution interpretability
(2) Model-based methods			
[153–155]	<ul style="list-style-type: none"> ○ Energy consumption minimization ○ Coordinated BTP design 	<ul style="list-style-type: none"> ○ Robust to slipping, energy-saving ○ Energy-efficient optimization 	<ul style="list-style-type: none"> ○ Real-time computation requirement ○ Model uncertainty sensitivity
[157, 158]	<ul style="list-style-type: none"> ○ Adhesion online estimation ○ Stability under uncertainties 	<ul style="list-style-type: none"> ○ Guaranteed convergence and tracking ○ Learning of unknown dynamics 	<ul style="list-style-type: none"> ○ High computational complexity ○ High implementation complexity
[159, 160]	<ul style="list-style-type: none"> ○ Online adhesion estimation ○ Stability under uncertainties 	<ul style="list-style-type: none"> ○ Compensation for disturbances ○ Rigorous theoretical foundation 	<ul style="list-style-type: none"> ○ Model accuracy sensitivity ○ High implementation complexity
(3) Intelligent control methods			
[161–165]	<ul style="list-style-type: none"> ○ Slip-based torque modulation ○ Stability and re-adhesion control 	<ul style="list-style-type: none"> ○ Simple implementation ○ Slip variation adaptability 	<ul style="list-style-type: none"> ○ Expertise required for rule tuning ○ Threshold selection sensitivity
[166–168]	<ul style="list-style-type: none"> ○ Dynamic fuzzy rule adaptation ○ Handle uncertain rail conditions 	<ul style="list-style-type: none"> ○ Operational condition adaptability ○ Simple fuzzy logic structure 	<ul style="list-style-type: none"> ○ Heavy tuning effort ○ Potential computational delay
[170, 171]	<ul style="list-style-type: none"> ○ Robust adhesion control ○ Approximate nonlinear adhesion 	<ul style="list-style-type: none"> ○ Superior environmental adaptability ○ Robustness under adverse conditions 	<ul style="list-style-type: none"> ○ High computational cost ○ Noise-sensitive training procedures
[172]	<ul style="list-style-type: none"> ○ RBE maximization ○ Optimize AB force distribution 	<ul style="list-style-type: none"> ○ Global search capability ○ Gradient-free optimization suitability 	<ul style="list-style-type: none"> ○ Slow convergence ○ Parameter configuration sensitivity

influential solution [180]. It enables the deliberate introduction of faults into a system to analyze its behavior under failure conditions, offering a cost-effective alternative to physical fault testing. Notable developments include a parametric library for simulating pneumatic braking systems [181], detailed pneumatic plant models [182], and simulation frameworks for electric braking faults [183].

Recently, Central South University developed a validation platform for extensive braking system testing, available at the website¹⁾. The platform includes a fault library with physical and signal models [184]. Physical models replicate electric and pneumatic braking actuators, while signal models simulate sensor signals and enable fault injection. This allows for testing two main fault types: electrical actuator faults (e.g., issues with the traction control unit, motor, and inverter) and air actuator faults (e.g., valve and cylinder malfunctions). The platform particularly emphasizes air actuator faults, including EP valve failures, train pipe leaks, brake cylinder faults, and sensor failures affecting air braking system performance.

In this platform, electric actuators are simulated in Simulink, whereas pneumatic actuators (e.g., valves and pipes) are modeled using advanced modeling environment for simulation (AMESim). The co-simulation between Simulink and AMESim increases model complexity and introduces data conversion and communication delays. These challenges were mitigated in [185], which proposed a rapid simulation method using equivalent models.

The developed validation platform implements two primary functionalities: fault injection and algorithm evaluation. The fault injection module enables precise specification of fault characteristics, including type, subsystem location, temporal activation parameters, and severity, via a structured user interface. Once configured, the simulation environment autonomously executes the specified fault scenarios, facilitating comprehensive analysis of system dynamic responses and performance degradation metrics. The algorithm evaluation module supports integration of user-designed diagnostic algorithms, which operate concurrently with reference algorithms during simulation. Real-time visualization interfaces display critical performance metrics such as detection rate, false alarm rate, and missed detection probability, enabling quantitative comparative assessments.

Despite the platform's extensive fault injection and algorithm evaluation capabilities, its development encountered several significant technical challenges to ensure simulation fidelity, scalability, and real-time performance. (i) High-resolution physical component modeling improves simulation accuracy but significantly increases computational complexity and co-simulation latency. Addressing this requires the development of reduced-order equivalent models that preserve essential fault-related characteristics while enhancing computational efficiency. (ii) Complex fault dynamics, such as valve obstructions and pressure leakage, exhibit nonlinear and time-varying behavior that

1) <https://github.com/CSU-IILab/BS-FIB.git>.

cannot be adequately captured using linear or static models. Hybrid modeling approaches are needed to integrate fundamental physical principles with empirical adjustments. (iii) Co-simulation integration across multiple simulation tools presents challenges in maintaining temporal synchronization and data consistency in heterogeneous environments.

7 Opportunities and challenges

As the demand for railway transport and vehicle speeds continues to increase, ongoing research into braking systems and control techniques is anticipated. This section explores new opportunities and challenges in developing braking control methods.

7.1 Decentralized architectures

Communication security is crucial for ensuring reliable braking performance, as centralized railway vehicle operations depend heavily on real-time communication networks [186]. Maintaining data integrity and robust cybersecurity is vital for system reliability. Key measures include advanced encryption, intrusion detection systems, and fail-safe protocols. Emerging distributed architectures offer significant promise in addressing this challenge. Decentralized edge nodes allow for localized processing of safety-critical braking data, reducing vulnerabilities in central servers while improving resilience and lowering operational latency [187]. Modern Internet-of-Things architectures further support these advantages through validated real-time communication-computation co-design frameworks [188].

Despite their proven potential in railway applications, decentralized architectures still face fundamental challenges. In control-theoretic algorithm design, achieving system stability and optimal performance under specific priority settings remains challenging, especially when multiple controllers and actuators interact locally. The difficulties lie in formulating control objectives with specific priorities, performance metrics, and constraints, as well as solving the complex optimal control problem. While distributed control strategies localize data processing, inter-controller communication delays can impact real-time controller design, adding further complexity to an already intricate process.

Decentralized architectures also face significant challenges in edge computing implementation. One such challenge is limited computational resources. Edge devices in railway systems, such as trackside sensors and onboard controllers, often have constrained processing power, memory, and energy. This limitation restricts their ability to handle complex tasks like real-time data analysis or running advanced algorithms. However, progress has been made with the deployment of domain-specific hardware accelerators [189,190]. A promising approach to address this is the use of model compression techniques, such as pruning, quantization, and knowledge distillation, which reduce the computational load and enable the efficient execution of lightweight AI models on resource-constrained devices.

Another challenge lies in data integration and real-time processing within railway edge architectures, particularly in harmonizing heterogeneous data streams and ensuring ultra-low-latency processing for safety-critical operations [191,192]. First, the seamless integration of multi-source data (e.g., onboard sensors, maintenance logs) requires standardized architectures and protocols to address inconsistencies in formats and communication, which can undermine the reliability of artificial intelligence (AI)-driven predictive maintenance models. Second, resource-constrained edge devices struggle to meet the real-time demands of applications like automated braking or anomaly detection. Developing new techniques to address these issues presents a promising avenue for future research.

Advanced intelligence-driven control strategies demonstrate significant potential for enhancing decentralized braking system efficiency. Distributed computing paradigms including federated learning and edge intelligence enable local data processing on embedded controllers while maintaining global coordination integrity. Large-scale distributed learning frameworks trained across multiple computational nodes facilitate real-time adaptation capabilities without centralizing sensitive operational data, thereby improving system autonomy and operational resilience.

7.2 Energy efficiency advancements

Carbon performance, defined by greenhouse gas emissions, has become an important metric in railway systems across Europe and Asia [193]. It is closely linked to energy consumption, with RBE serving as a key recyclable resource [194]. Traditional braking control methods focus on maximizing regenerative energy in the common braking mode. However, real-world implementation at the trajectory planning layer faces significant challenges due to idealized assumptions, such as 100% efficiency of electric motors and generators, as well as neglecting the practical dynamics of both regenerative and mechanical braking systems. For example, air braking systems may

experience delays due to pressurization time, and electric braking performance may differ from theoretical models. Additionally, the effectiveness of these traditional methods can be compromised in extreme gradient scenarios, leading to non-monotonic candidate speeds.

The work in [195] developed a network model for trains equipped with onboard energy storage devices to calculate the total energy recovered and proposed a practical method for generating optimal braking trajectories that minimize energy consumption. Researchers have also explored maximizing the use of RBE by incorporating additional equipment, such as onboard or wayside energy storage devices or reversible substations [196].

However, due to the high cost and limited capacity of energy storage devices, most trains in real-world applications are not equipped with these systems. A preferred approach is to utilize the RBE generated by a braking train to accelerate another train within the same substation, thus eliminating the need for energy storage devices [197]. If this regenerative energy is not used immediately, it is dissipated as heat through resistors. Therefore, collaboration among multiple vehicles to maximize RBE utilization is an intriguing research topic.

Advanced data analytics methodologies, integrated with physics-informed machine learning frameworks, enable optimized utilization of RBE. Reinforcement learning-based adaptive control strategies dynamically modulate BFD to simultaneously maximize energy recovery efficiency and minimize mechanical component degradation. Physics-informed neural network architectures further enhance predictive modeling precision by incorporating fundamental physical principles into optimization frameworks.

7.3 Robustness in emergency scenarios

While this paper primarily reviews control technologies in the common braking mode, emergency events still present significant challenges that can severely impact train performance and passenger safety [198]. Emergency events in the context of vehicle braking can include abnormal conditions (e.g., mechanical faults [199] or loss of braking forces [200] within the braking system), sudden environmental changes, or even malicious network attacks that compromise data integrity. These situations differ from operational scheduling emergencies, which typically involve network-level adjustments and logistical coordination, such as sudden changes in train schedules or network congestion [201].

Although advanced control methods for railway vehicles have been explored in the literature, several challenges and future considerations remain. A primary challenge in managing emergency events is the rapid and accurate detection of faults. Mechanical failures and sudden loss of braking forces require real-time monitoring systems capable of quickly identifying abnormalities and implementing corrective actions. Developing robust algorithms that can adaptively respond to unforeseen faults remains a critical research area.

In addition, braking systems must incorporate adaptive control mechanisms that can swiftly adjust braking force (or torque) and distribution to maintain safety during sudden environmental changes, such as adverse weather conditions. This necessitates the use of advanced sensor networks and predictive models to forecast environmental shifts and optimize braking performance.

As the railway industry evolves, a proactive approach to addressing these challenges in emergency scenarios will be essential to ensuring the safety and efficiency of modern railway transportation. Intelligence-enhanced predictive models demonstrate capability for early fault detection and optimal emergency response determination in real-time operational environments. Deep learning architectures trained on comprehensive historical datasets enable dynamic braking force redistribution to maintain operational stability under degraded adhesion conditions or component failure scenarios.

7.4 Next-generation verification platforms

The current braking system platform primarily focuses on developing a fault benchmark and a braking dynamics model to validate fault diagnosis and control algorithms. However, it lacks the capability for performance testing or comparative analysis of different methods. Additionally, considering the harsh and dynamic operating environment, along with the complexity of braking conditions, there is considerable potential to enhance the braking dynamics model and perform a more comprehensive analysis of braking scenarios using braking characteristics data.

The development of new verification platforms with the following features presents promising research directions. (i) Expanding the fault model library to include additional fault types and higher-precision fault levels across braking systems to improve coverage. (ii) Implementing a dynamic parameter adjustment mechanism based on feedback to better align fault injection methods with testing requirements and optimizing strategies through comprehensive result analysis. (iii) Integrating an evaluation function for fault diagnosis and control algorithms, with performance indicators tailored to braking systems, and assessing algorithm adaptability and real-time processing capabilities in dynamic environments. (iv) Incorporating environmental interference factors, such as tunnels, curves, and extreme

weather, to enhance model realism. (v) Adding trajectory planning algorithms, wheel dynamics models, and anti-skid control strategies to the platform to improve its functionality.

The development of intelligence-powered digital twin technologies offers advanced capabilities for braking system simulation and validation. Foundation models trained on multi-modal railway operational datasets facilitate comprehensive simulation across diverse operational and fault scenarios, enabling thorough system evaluation without dependency on resource-intensive physical testing methodologies. With the growing deployment of railway vehicles in urban transit systems worldwide, the next-generation verification platform is expected to bridge theoretical research on fault diagnosis and control algorithms with real-world railway applications. It will provide guidance on equipment maintenance, troubleshooting, repair, and system optimization, fostering collaboration between academia and industry in the transportation sector.

8 Conclusion

As railway transportation evolves to meet increasing demand and higher operational speeds, braking systems and control techniques must advance to ensure safety, efficiency, and reliability. This review has explored the key techniques developed in the field over the past decades, focusing on their theoretical architectures, and outlined potential directions for future research. More broadly, the rapid growth of AI technology presents new opportunities to reshape the future of braking control. By integrating cutting-edge AI models, big data analytics, large-scale learning frameworks, and large language models, braking systems can achieve enhanced intelligence, efficiency, and safety. However, challenges such as AI decision explainability, ethical deployment, and the standardization of hybrid architectures require ongoing long-term research.

Acknowledgements This work was supported in part by National Natural Science Foundation of China (Grant No. U20A20186), Postgraduate Scientific Research Innovation Project of Central South University (Grant No. 2023ZZTS0620), and the Science and Technology Innovation Program of Hunan Province (Grant No. 2024RC9015).

Open Access funding enabled and organized by CAUL and its Member Institutions.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

- Kim D S, Yoon W C. An accident causation model for the railway industry: application of the model to 80 rail accident investigation reports from the UK. *Saf Sci*, 2013, 60: 57–68
- Hood C. *Shinkansen: from bullet train to symbol of modern Japan*. London: Routledge, 2006
- Yin J, Tang T, Yang L, et al. Research and development of automatic train operation for railway transportation systems: a survey. *Transp Res Part C-Emerging Technol*, 2017, 85: 548–572
- Whyman F. Railway electric braking systems. *Proc IEE Part IA-Electric Railway Traction*, 1950, 97: 276–283
- Xiao Y, Luo X, Liu J, et al. Dynamic response of railway bridges under heavy-haul freight trains. *Adv Civ Eng*, 2020, 2020: 7486904
- Zhuan X, Xia X. Cruise control scheduling of heavy haul trains. *IEEE Trans Contr Syst Technol*, 2006, 14: 757–766
- Chou M, Xia X, Kayser C. Modelling and model validation of heavy-haul trains equipped with electronically controlled pneumatic brake systems. *Control Eng Pract*, 2007, 15: 501–509
- Hamada A T, Orhan M F. An overview of regenerative braking systems. *J Energy Storage*, 2022, 52: 105033
- Lv Z, Shang W. Impacts of intelligent transportation systems on energy conservation and emission reduction of transport systems: a comprehensive review. *Green Technol Sustain*, 2023, 1: 100002
- Chen J, Hu H, Ge Y, et al. An energy storage system for recycling regenerative braking energy in high-speed railway. *IEEE Trans Power Deliver*, 2020, 36: 320–330
- Sharma R C, Dhingra M, Pathak R K. Braking systems in railway vehicles. *Int J Eng Res Technol*, 2015, 4: 206–211
- Wu Q, Spiryagin M, Cole C. Longitudinal train dynamics: an overview. *Vehicle Syst Dyn*, 2016, 54: 1688–1714
- Günay M, Korkmaz M E, Özmen R. An investigation on braking systems used in railway vehicles. *Eng Sci Tech an Int J*, 2020, 23: 421–431
- Pretagostini F, Ferranti L, Berardo G, et al. Survey on wheel slip control design strategies, evaluation and application to antilock braking systems. *IEEE Access*, 2020, 8: 10951–10970
- Aulia Putra M R, Pratama P S, Prabowo A R. Failure of friction brake components against rapid braking process: a review on potential challenges and developments. *Transpation Res Procedia*, 2021, 55: 653–660
- Visakan M M, Kumar U M, Sivadasan V. Review on braking system in railways. *Int Res J Eng Technol*, 2021, 8: 2333–2339
- Cunillera A, Bevsinovic N, Lentink R M, et al. A literature review on train motion model calibration. *IEEE Trans Intell Transp Syst*, 2023, 24: 3660–3677
- Wu Q, Cole C, Spiryagin M, et al. Freight train air brake models. *Int J Rail Transp*, 2023, 11: 1–49
- Khodaparastan M, Mohamed A A, Brandauer W. Recuperation of regenerative braking energy in electric rail transit systems. *IEEE Trans Intell Transp Syst*, 2019, 20: 2831–2847

- 20 Tang R, de Donato L, Besinović N, et al. A literature review of artificial intelligence applications in railway systems. *Transp Res Part C-Emerg Technol*, 2022, 140: 103679
- 21 Liu Y, Zhou Y, Su S, et al. An analytical optimal control approach for virtually coupled high-speed trains with local and string stability. *Transp Res Part C-Emerg Technol*, 2021, 125: 102886
- 22 Mo P, Yang L, D Ariano A, et al. Energy-efficient train scheduling and rolling stock circulation planning in a metro line: a linear programming approach. *IEEE Trans Intell Transp Syst*, 2019, 21: 3621–3633
- 23 Liu H, Yang L, Yang H. Cooperative optimal control of the following operation of high-speed trains. *IEEE Trans Intell Transp Syst*, 2022, 23: 17744–17755
- 24 Zhang K, Gao J, Xu Z, et al. Headway compression oriented trajectory optimization for virtual coupling of heavy-haul trains. *Control Eng Pract*, 2024, 143: 105784
- 25 Song H, Shangguan W, Qiu W, et al. Two-stage optimal trajectory planning based on resilience adjustment model for virtually coupled trains. *IEEE Trans Intell Transp Syst*, 2023, 24: 15219–15235
- 26 Ying P, Zeng X, Shen T, et al. Partial train speed trajectory optimization. In: *Proceedings of the 25th International Conference on Intelligent Transportation Systems (ITSC)*, 2022. 548–553
- 27 Lu S, Weston P, Hillmansen S, et al. Increasing the regenerative braking energy for railway vehicles. *IEEE Trans Intell Transp Syst*, 2014, 15: 2506–2515
- 28 Lu S, Xue F, Ting T O, et al. Speed trajectory optimisation for electric vehicles in eco-approach and departure using linear programming. In: *Proceedings of IET International Conference on Intelligent and Connected Vehicles*, 2016
- 29 Zhou M, Liu Y, Wang H, et al. Trajectory optimization for heavy-haul trains considering cyclic braking under complex operating conditions. *J Rail Transp Planning Manage*, 2024, 31: 100462
- 30 Cai L, Huang D. Trajectory tracking control of heavy haul train in whole operation procedure. *IEEE Trans Veh Technol*, 2024, 73: 16225–16237
- 31 Oldknow K, Mulligan K, McTaggart-Cowan G. The trajectory of hybrid and hydrogen technologies in North American heavy haul operations. *Rail Eng Sci*, 2021, 29: 233–247
- 32 Wang X, Tang T, He H. Optimal control of heavy haul train based on approximate dynamic programming. *Adv Mech Eng*, 2017, 9: 1687814017698110
- 33 Xu Z. Simulation and study on air brake system of EMU based on AMESim. Dissertation for Ph.D. Degree. Chengdu: Southwest Jiaotong University, 2013
- 34 Spiryagin M, Lee K S, Yoo H H. Control system for maximum use of adhesive forces of a railway vehicle in a tractive mode. *Mech Syst Signal Process*, 2008, 22: 709–720
- 35 Yi J, Alvarez L, Horowitz R. Adaptive emergency braking control with underestimation of friction coefficient. *IEEE Trans Contr Syst Technol*, 2002, 10: 381–392
- 36 Su S, Wang X, Tang T, et al. Energy-efficient operation by cooperative control among trains: a multi-agent reinforcement learning approach. *Control Eng Pract*, 2021, 116: 104901
- 37 Su S, Li X, Tang T, et al. A subway train timetable optimization approach based on energy-efficient operation strategy. *IEEE Trans Intell Transp Syst*, 2013, 14: 883–893
- 38 Wang Y, Ning B, van den Boom T, et al. Optimal trajectory planning for a single train. In: *Optimal Trajectory Planning and Train Scheduling for Urban Rail Transit Systems*. Berlin: Springer, 2016. 23–51
- 39 Hoang H, Polis M, Haurie A. Reducing energy consumption through trajectory optimization for a metro network. *IEEE Trans Automat Contr*, 1975, 20: 590–595
- 40 Zhu Q, Bao S, Tan X, et al. Research on optimization of electro-pneumatic braking cooperative control for E-MU. *Comput Eng*, 2013, 39: 17–21
- 41 Robinson M, Schut D. Rail as the sustainable backbone of the energy efficient transport chain—a world view. *OIDA Int J Sustain Dev*, 2014, 7: 19–30
- 42 Adinolfi A, Lamedica R, Modesto C, et al. Experimental assessment of energy saving due to trains regenerative braking in an electrified subway line. *IEEE Trans Power Deliver*, 1998, 13: 1536–1542
- 43 Zhang Z, Cheng X, Xing Z, et al. Pareto multi-objective optimization of metro train energy-saving operation using improved NSGA-II algorithms. *Chaos Solitons Fractals*, 2023, 176: 114183
- 44 Corlu C G, de la Torre R, Serrano-Hernandez A, et al. Optimizing energy consumption in transportation: literature review, insights, and research opportunities. *Energies*, 2020, 13: 1115
- 45 Dai X, Zhao H, Yu S, et al. Dynamic scheduling, operation control and their integration in high-speed railways: a review of recent research. *IEEE Trans Intell Transp Syst*, 2021, 23: 13994–14010
- 46 Ding Y, Liu H, Bai Y, et al. A two-level optimization model and algorithm for energy-efficient urban train operation. *J Transp Syst Eng Inf Tech*, 2011, 11: 96–101
- 47 Kondo K. Recent energy saving technologies on railway traction systems. *IEEJ Trans Elec Engng*, 2010, 5: 298–303
- 48 Ghaviha N, Campillo J, Bohlin M, et al. Review of application of energy storage devices in railway transportation. *Energy Procedia*, 2017, 105: 4561–4568
- 49 Zhou L, Tong L C, Chen J, et al. Joint optimization of high-speed train timetables and speed profiles: a unified modeling approach using space-time-speed grid networks. *Transp Res Part B-Methodol*, 2017, 97: 157–181
- 50 Shafiq M, Khoo S, Kouzani A Z. Modelling and simulation of regeneration in AC traction propulsion system of electrified railway. *IET Electrical Syst Trans*, 2015, 5: 145–155
- 51 Yang X, Chen A, Li X, et al. An energy-efficient scheduling approach to improve the utilization of regenerative energy for metro systems. *Transp Res Part C-Emerg Technol*, 2015, 57: 13–29
- 52 Gupta S D, Tobin J K, Pavel L. A robust mixed integer optimization model to utilize regenerative energy of trains in a railway network. [ArXiv:1507.01646](https://arxiv.org/abs/1507.01646)
- 53 Gupta S D, Pavel L, Tobin J K. An optimization model to utilize regenerative braking energy in a railway network. In: *Proceedings of American Control Conference (ACC)*, 2015. 5919–5924
- 54 Tian Z, Hillmansen S, Roberts C, et al. Energy evaluation of the power network of a DC railway system with regenerating trains. *IET Electrical Syst Trans*, 2016, 6: 41–49
- 55 Naseri F, Farjah E, Ghanbari T. An efficient regenerative braking system based on battery/supercapacitor for electric, hybrid and plug-in

- hybrid electric vehicles with BLDC motor. *IEEE Trans Veh Technol*, 2017, 66: 3724–3738
- 56 Wang Y, Zhu S, D'Ariano A, et al. Energy-efficient timetabling and rolling stock circulation planning based on automatic train operation levels for metro lines. *Transp Res Part C-Emerg Technol*, 2021, 129: 103209
- 57 Gill P E, Murray W, Saunders M A. SNOPT: an SQP algorithm for large-scale constrained optimization. *SIAM Rev*, 2005, 47: 99–131
- 58 Gallo M, Marinelli M. Sustainable mobility: a review of possible actions and policies. *Sustainability*, 2020, 12: 7499
- 59 Wächter A, Biegler L T. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Math Program*, 2006, 106: 25–57
- 60 Xiao Z, Wang Q, Sun P, et al. Modeling and energy-optimal control for high-speed trains. *IEEE Trans Transp Electrific*, 2020, 6: 797–807
- 61 Lai Q, Liu J, Haghani A, et al. Energy-efficient speed profile optimization for medium-speed maglev trains. *Transp Res Part E-Logist Transp Rev*, 2020, 141: 102007
- 62 Wang Y, de Schutter B, van den Boom T J J, et al. Optimal trajectory planning for trains—a pseudospectral method and a mixed integer linear programming approach. *Transp Res Part C-Emerg Technol*, 2013, 29: 97–114
- 63 Wang Y, de Schutter B, van den Boom T J J, et al. Optimal trajectory planning for trains under fixed and moving signaling systems using mixed integer linear programming. *Control Eng Pract*, 2014, 22: 44–56
- 64 Wang J, Han D, Yan Y, et al. Energy-saving and punctuality combined velocity planning for the autonomous-rail rapid tram with enhanced pseudospectral method. *Chin J Mech Eng*, 2023, 36: 82
- 65 Xu S, Deng K, Li S E, et al. Legendre pseudospectral computation of optimal speed profiles for vehicle eco-driving system. In: *Proceedings of IEEE Intelligent Vehicles Symposium Proceedings*, 2014. 1103–1108
- 66 Lin Q, Li S E, Du X, et al. Minimize the fuel consumption of connected vehicles between two red-signalized intersections in urban traffic. *IEEE Trans Veh Technol*, 2018, 67: 9060–9072
- 67 Lu S, Hillmansen S, Ho T K, et al. Single-train trajectory optimization. *IEEE Trans Intell Transp Syst*, 2013, 14: 743–750
- 68 Haahr J T, Pisinger D, Sabbaghian M. A dynamic programming approach for optimizing train speed profiles with speed restrictions and passage points. *Transp Res Part B-Methodol*, 2017, 99: 167–182
- 69 Calderaro V, Galdi V, Graber G, et al. An algorithm to optimize speed profiles of the metro vehicles for minimizing energy consumption. In: *Proceedings of International Symposium on Power Electronics, Electrical Drives, Automation and Motion*, 2014. 813–819
- 70 Cao Y, Zhang Z, Cheng F, et al. Trajectory optimization for high-speed trains via a mixed integer linear programming approach. *IEEE Trans Intell Transp Syst*, 2022, 23: 17666–17676
- 71 Wang Y, Ning B, Tang T, et al. Efficient real-time train scheduling for urban rail transit systems using iterative convex programming. *IEEE Trans Intell Transp Syst*, 2015, 16: 3337–3352
- 72 Luan X, Wang Y, de Schutter B, et al. Integration of real-time traffic management and train control for rail networks—part 1: optimization problems and solution approaches. *Transp Res Part B-Methodol*, 2018, 115: 41–71
- 73 Wang X, Xiao Z, Chen M, et al. Energy-efficient speed profile optimization and sliding mode speed tracking for metros. *Energies*, 2020, 13: 6093
- 74 Zhang H, Jia L, Wang L, et al. Energy consumption optimization of train operation for railway systems: algorithm development and real-world case study. *J Cleaner Production*, 2019, 214: 1024–1037
- 75 Guo X, Sun H, Wu J, et al. Multiperiod-based timetable optimization for metro transit networks. *Transp Res Part B-Methodol*, 2017, 96: 46–67
- 76 Bocharnikov Y V, Tobias A M, Roberts C, et al. Optimal driving strategy for traction energy saving on DC suburban railways. *IET Electr Power Appl*, 2007, 1: 675–682
- 77 Chang C S, Sim S S. Optimising train movements through coast control using genetic algorithms. *IEE Proc Electr Power Appl*, 1997, 144: 65–73
- 78 Yang X, Chen A, Ning B, et al. A stochastic model for the integrated optimization on metro timetable and speed profile with uncertain train mass. *Transp Res Part B-Methodol*, 2016, 91: 424–445
- 79 Su S, Wang X, Cao Y, et al. An energy-efficient train operation approach by integrating the metro timetabling and eco-driving. *IEEE Trans Intell Transp Syst*, 2019, 21: 4252–4268
- 80 Wang P, Goverde R M P. Two-train trajectory optimization with a green-wave policy. *Transp Res Record-J Transp Res Board*, 2016, 2546: 112–120
- 81 Yin J, Chen D, Li L. Intelligent train operation algorithms for subway by expert system and reinforcement learning. *IEEE Trans Intell Transp Syst*, 2014, 15: 2561–2571
- 82 Bryson A E. *Applied Optimal Control: Optimization, Estimation and Control*. London: Routledge, 2018
- 83 Pontryagin L S. *Mathematical Theory of Optimal Processes*. London: Routledge, 2018
- 84 Howlett P G, Milroy I P, Pudney P J. Energy-efficient train control. *Control Eng Pract*, 1994, 2: 193–200
- 85 Khmelnitsky E. On an optimal control problem of train operation. *IEEE Trans Automat Contr*, 2000, 45: 1257–1266
- 86 Liu R R, Golovitcher I M. Energy-efficient operation of rail vehicles. *Transp Res Part A-Policy Pract*, 2003, 37: 917–932
- 87 Albrecht A, Howlett P, Pudney P, et al. The key principles of optimal train control-part 2: existence of an optimal strategy, the local energy minimization principle, uniqueness, computational techniques. *Transp Res Part B-Methodol*, 2016, 94: 509–538
- 88 Albrecht A, Howlett P, Pudney P, et al. The key principles of optimal train control—part 1: formulation of the model, strategies of optimal type, evolutionary lines, location of optimal switching points. *Transp Res Part B-Methodol*, 2016, 94: 482–508
- 89 Betts J T. Survey of numerical methods for trajectory optimization. *J Guidance Control Dyn*, 1998, 21: 193–207
- 90 Malyuta D, Yu Y, Elango P, et al. Advances in trajectory optimization for space vehicle control. *Annu Rev Control*, 2021, 52: 282–315
- 91 Kelly M. An introduction to trajectory optimization: how to do your own direct collocation. *SIAM Rev*, 2017, 59: 849–904
- 92 Ko H, Koseki T, Miyatake M. Application of dynamic programming to the optimization of the running profile of a train. 2004. <https://www.witpress.com/Secure/elibrary/papers/CR04/CR04011FU.pdf>
- 93 Vasak M, Baotić M, Perić N, et al. Optimal rail route energy management under constraints and fixed arrival time. In: *Proceedings of European Control Conference (ECC)*, 2009. 2972–2977
- 94 Ye H, Liu R. A multiphase optimal control method for multi-train control and scheduling on railway lines. *Transp Res Part B-Methodol*, 2016, 93: 377–393
- 95 Cormen T H, Leiserson C E, Rivest R L, et al. *Introduction to Algorithms*. Cambridge: MIT Press, 2022
- 96 Zhao N, Roberts C, Hillmansen S, et al. A multiple train trajectory optimization to minimize energy consumption and delay. *IEEE Trans Intell Transp Syst*, 2015, 16: 2363–2372

- 97 Astolfi A, Menini L. Input/output decoupling problems for high-speed trains. In: Proceedings of American Control Conference, 2002. 549–554
- 98 Zhuang X, Xia X. Speed regulation with measured output feedback in the control of heavy haul trains. *Automatica*, 2008, 44: 242–247
- 99 Liu H E, Yang H, Cai B G. Optimization for the following operation of a high-speed train under the moving block system. *IEEE Trans Intell Transp Syst*, 2017, 19: 3406–3413
- 100 Ma T, Wu M, Tian C. A general method of braking process simulation for flexible marshalling EMUs. *Int J TDI*, 2018, 2: 353–361
- 101 Wen M, Shun D. Research on air braking force distribution mode of urban rail train. *Rolling Stock*, 2015, 53: 9–12
- 102 Qin J. Study on the braking force redistribution of high-speed train in the slip. Dissertation for Ph.D. Degree. Beijing: Beijing Jiaotong University, 2013
- 103 Chen P, Gao S, Meng Q. Braking force distribution strategy of EMU. *Railway Locomotor Car*, 2018, 38: 24–29
- 104 Ariyur K B, Krstic M. Real-time Optimization by Extremum-seeking Control. Hoboken: John Wiley & Sons, 2003
- 105 Chen Y, Dong H, Lu J, et al. A super-twisting-like algorithm and its application to train operation control with optimal utilization of adhesion force. *IEEE Trans Intell Transp Syst*, 2016, 17: 3035–3044
- 106 Watanabe T. Anti-slip readhesion control with presumed adhesion force—method of presuming adhesion force and running test results of high-speed shinkansen train. *Q Rep RTRI*, 2000, 41: 32–36
- 107 Zhang C, Ying X, Liu J, et al. Research on optimal control algorithm for braking force distribution of high-speed train. *J Electron Meas Instrum*, 2018, 32: 80–87
- 108 Wang J, Chen Z, Chen Z, et al. A decentralised braking force distribution strategy for high-speed trains. *IET Intell Trans Sys*, 2025, 19: e70100
- 109 Song Q, Song Y, Tang T, et al. Computationally inexpensive tracking control of high-speed trains with traction/braking saturation. *IEEE Trans Intell Transp Syst*, 2011, 12: 1116–1125
- 110 Ke B R, Lin C L, Lai C W. Optimization of train-speed trajectory and control for mass rapid transit systems. *Control Eng Pract*, 2011, 19: 675–687
- 111 Song Y D, Song Q, Cai W C. Fault-tolerant adaptive control of high-speed trains under traction/braking failures: a virtual parameter-based approach. *IEEE Trans Intell Transp Syst*, 2013, 15: 737–748
- 112 Gao S, Dong H, Chen Y, et al. Approximation-based robust adaptive automatic train control: an approach for actuator saturation. *IEEE Trans Intell Transp Syst*, 2013, 14: 1733–1742
- 113 Hou Z, Wang Y, Yin C, et al. Terminal iterative learning control based station stop control of a train. *Int J Control*, 2011, 84: 1263–1274
- 114 Guo F, He J. Optimal allocation method of electric/air braking force of high-speed train considering axle load transfer. *High-speed Railway*, 2024, 2: 77–84
- 115 Yasunobu S, Miyamoto S, Ihara H. Fuzzy control for automatic train operation system. *IFAC Proc Volumes*, 1983, 16: 33–39
- 116 Yasunobu S, Miyamoto S. Automatic train operation system by predictive control. In: *Industrial Applications of Fuzzy Control*. Amsterdam: Elsevier, 1985
- 117 van der Meulen R. Selected heavy-haul insights: some south african perspectives. In: *Proceedings of the Conference on Rolling Stock of the 21st Century*, 2008. 117–122
- 118 Dong H, Gao S, Ning B, et al. Extended fuzzy logic controller for high speed train. *Neural Comput Applic*, 2013, 22: 321–328
- 119 UIC. Leaflet 513-1994: guidelines for evaluating passenger comfort in relation to vibration in railway vehicles. <https://shop.uic.org/en/51-running-and-suspension-gear/951-guidelines-for-evaluating-passenger-comfort-in-relation-to-vibration-in-railway-vehicles.html>
- 120 National Railway Administration of People's Republic of China. Specification for dynamic performance assessment and testing verification of rolling stock. GB/T 5599-2019. <http://c.gb688.cn/bzgk/gb/showGb?type=online&hcno=FDCDB3ECBAFBC8B39CD35A739AB67F2>
- 121 Brochard A, Pasillas-Lépine W, Demaya B. Cascaded train speed regulation: robustness to feedback delay and measurement filtering. *IFAC-PapersOnLine*, 2022, 55: 126–131
- 122 Yin S, Peng T, Yang C, et al. Dynamic hybrid observer-based early slipping fault detection for high-speed train wheelsets. *Control Eng Pract*, 2024, 142: 105736
- 123 UIC. Appendix F—brakes specifications for the construction of various brake parts—wheel slide protection device (WSP). UIC541-05. <https://standards.globalspec.com/std/10161886/uic-541-05-e>
- 124 Huang D, Yang W, Huang T, et al. Iterative learning operation control of high-speed trains with adhesion dynamics. *IEEE Trans Contr Syst Technol*, 2021, 29: 2598–2608
- 125 Polach O. Creep forces in simulations of traction vehicles running on adhesion limit. *Wear*, 2005, 258: 992–1000
- 126 Picasso B, Caporale D, Colaneri P. Braking control in railway vehicles: a distributed preview approach. *IEEE Trans Automat Contr*, 2017, 63: 189–195
- 127 Shi L B, Ma L, Guo J, et al. Influence of low temperature environment on the adhesion characteristics of wheel-rail contact. *Tribol Int*, 2018, 127: 59–68
- 128 Trummer G, Buckley-Johnstone L E, Voltr P, et al. Wheel-rail creep force model for predicting water induced low adhesion phenomena. *Tribol Int*, 2017, 109: 409–415
- 129 Cai W C, Li D Y, Song Y D. A novel approach for active adhesion control of high-speed trains under antiskid constraints. *IEEE Trans Intell Transp Syst*, 2015, 16: 3213–3222
- 130 Iwnicki S. *Handbook of Railway Vehicle Dynamics*. Boca Raton: CRC Press, 2006
- 131 Charles G, Goodall R, Dixon R. Model-based condition monitoring at the wheel-rail interface. *Vehicle Syst Dyn*, 2008, 46: 415–430
- 132 Zhang T, Chen Z, Zhai W, et al. Establishment and validation of a locomotive-track coupled spatial dynamics model considering dynamic effect of gear transmissions. *Mech Syst Signal Process*, 2019, 119: 328–345
- 133 Guo F, Qin J, Wen C. Research on braking force distribution in axle controlled braking control system based on axle load transfer. *Locomot Rolling Stock Technol*, 2023, 1–4
- 134 Spiriyagin M, Wolfs P, Wu Q, et al. Rail cleaning process and its influence on locomotive performance. In: *Proceedings of the ASME/IEEE Joint Rail Conference*, 2017. 1–1
- 135 Uyulan C, Gokasan M, Bogosyan S. Comparison of the re-adhesion control strategies in high-speed train. *Proc IMechE Part I: J Syst Control Eng*, 2018, 232: 92–105
- 136 Shrestha S, Wu Q, Spiriyagin M. Review of adhesion estimation approaches for rail vehicles. *Int J Rail Transp*, 2019, 7: 79–102
- 137 Abouzeid A F, Guerrero J M, Lejarza-Lasuen L, et al. Advanced maximum adhesion tracking strategies in railway traction drives. *IEEE*

Trans Transp Electrific, 2023, 10: 3645–3660

- 138 Zhangpeng N, Bing W, Guangwen X, et al. Wheel-rail adhesion control model by integrating neural network and direct torque control during traction under low adhesion. *J Vib Control*, 2025, 31: 2328–2339
- 139 Ivanov V, Savitski D, Shyrokau B. A survey of traction control and antilock braking systems of full electric vehicles with individually controlled electric motors. *IEEE Trans Veh Technol*, 2014, 64: 3878–3896
- 140 Zou S, He X, Hu L, et al. Characteristics of the velocity field in slipstream induced by a CR 400 high-speed train lead-carriage. *Measurement*, 2022, 196: 111205
- 141 Kawabe T. Slip suppression of electric vehicles using model predictive PID controller. *Int J World Acad Sci Eng Technol*, 2012, 67: 524–529
- 142 Chen H, Lian X, Lyu C. Piecewise integral-proportional wheel slip control for an in-wheel motor driven vehicle. *Vehicle Syst Dyn*, 2023, 61: 3181–3201
- 143 Vo-Duy T, Nguyen B H, Trovão J P F, et al. A unified anti-slip cruise control strategy for electric vehicles. *Nonlinear Dyn*, 2024, 112: 18357–18375
- 144 Cheok A D, Shiomi S. A fuzzy logic based anti-skid control system for railway applications. In: *Proceedings of the 2nd International Conference Knowledge-Based Intelligent Electronic Systems (KES)*, 1998. 1–1
- 145 Tyukin I. *Adaptation in Dynamical Systems*. Cambridge: Cambridge University Press, 2011
- 146 Rajamani R, Phanomchoeng G, Piyabongkarn D, et al. Algorithms for real-time estimation of individual wheel tire-road friction coefficients. *IEEE ASME Trans Mechatron*, 2011, 17: 1183–1195
- 147 Villagra J, d'Andréa-Novel B, Fliess M, et al. A diagnosis-based approach for tire-road forces and maximum friction estimation. *Control Eng Pract*, 2011, 19: 174–184
- 148 Baffet G, Charara A, Lechner D. Estimation of vehicle sideslip, tire force and wheel cornering stiffness. *Control Eng Pract*, 2009, 17: 1255–1264
- 149 Antonov S, Fehn A, Kugi A. Unscented Kalman filter for vehicle state estimation. *Vehicle Syst Dyn*, 2011, 49: 1497–1520
- 150 Hamann H, Hedrick J K, Rhode S, et al. Tire force estimation for a passenger vehicle with the unscented Kalman filter. In: *Proceedings of IEEE Intelligent Vehicles Symposium Proceedings*, 2014. 814–819
- 151 Fitzgerald R. Divergence of the Kalman filter. *IEEE Trans Automat Contr*, 1971, 16: 736–747
- 152 Oliveira T R, Hsu L, Peixoto A J. Output-feedback global tracking for unknown control direction plants with application to extremum-seeking control. *Automatica*, 2011, 47: 2029–2038
- 153 Wei H, Zhang N, Liang J, et al. Deep reinforcement learning based direct torque control strategy for distributed drive electric vehicles considering active safety and energy saving performance. *Energy*, 2022, 238: 121725
- 154 Zhang C, Lin Z, Yang S X, et al. Total-amount synchronous control based on terminal sliding-mode control. *IEEE Access*, 2017, 5: 5436–5444
- 155 Wei H, Ai Q, Zhao W, et al. Modelling and experimental validation of an EV torque distribution strategy towards active safety and energy efficiency. *Energy*, 2022, 239: 121953
- 156 Liu Y, Jiang T, Zhao X, et al. Effects of axle load transfer on wheel rolling contact fatigue of high-power AC locomotives with oblique traction rods. *Int J Fatigue*, 2020, 139: 105748
- 157 Park S H, Kim J S, Choi J J, et al. Modeling and control of adhesion force in railway rolling stocks. *IEEE Control Syst Mag*, 2008, 28: 44–58.
- 158 Mirzaeinejad H. Robust predictive control of wheel slip in antilock braking systems based on radial basis function neural network. *Appl Soft Comput*, 2018, 70: 318–329
- 159 Schwarz C, Brembeck J, Heckmann B. Dynamics observer for the longitudinal behavior of a wheelset on a roller rig. *Proc Inst Mech Eng Part F-J Rail Rapid Transit*, 2019, 233: 1112–1119
- 160 He J, Liu G, Zhang C, et al. Maximum likelihood identification method for adhesion performance parameters of heavy duty locomotive. *J Electron Meas Instrum*, 2017, 31: 170–177.
- 161 Layne J R, Passino K M, Yurkovich S. Fuzzy learning control for antiskid braking systems. *IEEE Trans Contr Syst Technol*, 1993, 1: 122–129
- 162 Mauer G F. A fuzzy logic controller for an ABS braking system. *IEEE Trans Fuzzy Syst*, 1995, 3: 381–388
- 163 Lin C M, Hsu C F. Self-learning fuzzy sliding-mode control for antilock braking systems. *IEEE Trans Contr Syst Technol*, 2003, 11: 273–278
- 164 Nakazawa S. Development of a new wheel slide protection system using a new detection algorithm. *QR RTRI*, 2011, 52: 136–140
- 165 Chen P, Zhu W, Yu C, et al. Research on train braking model by improved Polach model considering wheel-rail adhesion characteristics. *IET Intell Trans Sys*, 2023, 17: 2432–2443
- 166 Mousavi A, Markazi A H D, Masoudi S. Adaptive fuzzy sliding-mode control of wheel slide protection device for ER24PC locomotive. *Lat Am J Solids Struct*, 2017, 14: 2019–2045
- 167 Yi L Z, Jiang W B, Yi Y, et al. Freight train speed active disturbance rejection tracking and wheel anti-slip based on fuzzy neural network with DBO optimization. *Electr Eng*, 2024, 106: 631–644
- 168 Challa A, Ramakrushnan K, Gaurkar P V, et al. A 3-phase combined wheel slip and acceleration threshold algorithm for anti-lock braking in heavy commercial road vehicles. *Vehicle Syst Dyn*, 2022, 60: 2312–2333
- 169 Lee Y, Zak S H. Genetic neural fuzzy control of anti-lock brake systems. In: *Proceedings of American Control Conference*, 2001. 671–676
- 170 Lin C M, Hsu C F. Neural-network hybrid control for antilock braking systems. *IEEE Trans Neural Netw*, 2003, 14: 351–359
- 171 Tsai C H, Chung H Y, Yu F M. Neuro-sliding mode control with its applications to seesaw systems. *IEEE Trans Neural Netw*, 2004, 15: 124–134
- 172 Li L, Zhang Y, Yang C, et al. Model predictive control-based efficient energy recovery control strategy for regenerative braking system of hybrid electric bus. *Energy Convers Manage*, 2016, 111: 299–314
- 173 Chen B, Huang Z, Zhang R, et al. Adaptive slip ratio estimation for active braking control of high-speed trains. *ISA Trans*, 2021, 112: 302–314
- 174 Moaveni B, Rashidi Fathabadi F, Molavi A. Fuzzy control system design for wheel slip prevention and tracking of desired speed profile in electric trains. *Asian J Control*, 2022, 24: 388–400
- 175 Ji H, He X, Sai H, et al. Fault detection of EMU brake cylinder. In: *Proceedings of the 35th Chinese Control Conference (CCC)*, 2016. 6668–6672

- 176 Fang X, Yang Z, Lin F. Virtual development platform of high-speed train traction drive system in view of top-level goals. In: Proceedings of IEEE Vehicle Power and Propulsion Conference (VPPC), 2013. 1–5
- 177 Feng J, Cao J G, Wu Z H. Analysis and study of the influence on the train side harmonic as a SMES fabricated into the EMU converter system. *IEEE Trans Appl Supercond*, 2019, 29: 1–5
- 178 Zhou Y, Tao X, Luan L, et al. Revisiting the 7/23 train accident using computer reconstruction simulation for causation and prevention analysis. *Math Comput Simul*, 2018, 148: 1–15
- 179 Gao S, Dong H, Ning B, et al. Adaptive fault-tolerant automatic train operation using RBF neural networks. *Neural Comput Applic*, 2015, 26: 141–149
- 180 Johnson B W, Smith D T, DeLong T A. A Survey of Fault Simulation, Fault Grading and Test Pattern Generation Techniques with Emphasis on the Feasibility of VHDL based Fault Simulation. Technical Report 19980040087. 1997
- 181 Pugi L, Malvezzi M, Allotta B, et al. A parametric library for the simulation of a Union Internationale des Chemins de Fer (UIC) pneumatic braking system. *Proc Inst Mech Eng Part F-J Rail Rapid Transit*, 2004, 218: 117–132
- 182 Pugi L, Palazzolo A, Fioravanti D. Simulation of railway brake plants: an application to SAADKMS freight wagons. *Proc Inst Mech Eng Part F-J Rail Rapid Transit*, 2008, 222: 321–329
- 183 Yang C, Peng T, Tao H W, et al. Review of recent research on fault injection for high-speed train information control systems (in Chinese). *Sci Sin Inform*, 2020, 50: 465–482
- 184 Chen Z, Peng L, Fan J, et al. EPBS.FIDMV: a fault injection and diagnosis methods validation benchmark for EPBS of EMU. *Control Eng Pract*, 2024, 145: 105873
- 185 Yang C, Chen Z, Wang J, et al. Rapid simulation method for electro-pneumatic composite braking system based on equivalent modeling. In: Proceedings of the 6th International Conference on Industrial Artificial Intelligence (IAI), 2024. 1–6
- 186 Hu J, Liu G, Li Y, et al. Off-network communications for future railway mobile communication systems: challenges and opportunities. *IEEE Commun Mag*, 2022, 60: 64–70
- 187 Zhu L, Liang Y, Li Y. Toward optimal train control: an edge computing approach with adaptive computation offloading. *IEEE Int Things J*, 2025, 12: 10601–10612
- 188 Singh P, Elmi Z, Krishna Meriga V, et al. Internet of things for sustainable railway transportation: past, present, and future. *Cleaner Logist Supply Chain*, 2022, 4: 100065
- 189 Jeong S, Kim H, Kim L W. Optimizing edge AI solutions through hardware and software co-design. In: Proceedings of the IEEE International Conference on Consumer Electronics-Asia (ICCE-Asia), 2023. 1–3
- 190 Rachmanto R D, Sukma Z, Nabhaan A N, et al. Characterizing deep learning model compression with post-training quantization on accelerated edge devices. In: Proceedings of the IEEE International Conference on Edge Computing and Communications (EDGE), 2024. 110–120
- 191 Zhu L, Gong T, Liang H, et al. Application of edge intelligence in rail transit: prospects and future outlook. *J Electron Inf Technol*, 2023, 45: 1514–1528
- 192 Wang S. Edge computing: applications, state-of-the-art and challenges. *Adv Netw*, 2019, 7: 8–15
- 193 Nadine Levick M. White Paper for: “Toward Zero Deaths: A National Strategy on Highway Safety”. Technical Report, National Strategy on Highway Safety Initiative, 2010
- 194 Yang X, Li X, Ning B, et al. A survey on energy-efficient train operation for urban rail transit. *IEEE Trans Intell Transp Syst*, 2015, 17: 2–13
- 195 Dominguez M, Fernandez-Cardador A, Cucala A P, et al. Energy savings in metropolitan railway substations through regenerative energy recovery and optimal design of ATO speed profiles. *IEEE Trans Automat Sci Eng*, 2012, 9: 496–504
- 196 Scheepmaker G M, Goverde R M P, Kroon L G. Review of energy-efficient train control and timetabling. *Eur J Oper Res*, 2017, 257: 355–376
- 197 González-Gil A, Palacin R, Batty P, et al. A systems approach to reduce urban rail energy consumption. *Energy Convers Manage*, 2014, 80: 509–524
- 198 Li Y, Yang X, Wu J, et al. Discrete-event simulations for metro train operation under emergencies: a multi-agent based model with parallel computing. *Phys A-Stat Mech its Appl*, 2021, 573: 125964
- 199 Cheng C, Wang J, Chen H, et al. A review of intelligent fault diagnosis for high-speed trains: qualitative approaches. *Entropy*, 2020, 23: 1
- 200 Wang J, Chen Z, Chen Z, et al. An optimization control strategy for braking system of high-speed trains under partial loss of braking force. In: Proceedings of the 17th International Conference on Control & Automation (ICCA), 2022. 109–114
- 201 Ma J, Ma S, Hu W, et al. Scheduling-measure dependent modelling of delay propagation on a single railway line: a matrix transformation approach. *IET Intell Trans Sys*, 2021, 15: 1429–1439