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Global control for a class of chained nonholonomic systems with input saturations

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With widespread applications in areas such as two-wheeled robot systems, the control problems of nonholonomic systems have attracted significant attention. Among these, the chained nonholonomic system, a special class of nonholonomic systems, has been extensively studied since its introduction in [1]. Due to the inherent structural properties of nonholonomic systems, they do not satisfy Brockett's necessary condition [2]. As a result, stabilization can only be achieved through discontinuous time-invariant control, smooth time-varying control, or hybrid control strategies. For chained nonholonomic systems, various approaches have been proposed, including smooth time-varying feedback [1] and nonsmooth control laws [3].

Every actuator in a control system operates within inherent limitations [4]. For nonholonomic systems, input saturation is a critical issue, as it can significantly affect system stability and performance, underscoring the importance of addressing this challenge for practical implementation. In the context of nonholonomic mobile robots, the global tracking and stabilization control problem with unknown parameters was examined [5]. Additionally, switching control strategies to manage input saturation in chained nonholonomic systems were investigated in [3]. However, many of these saturation control approaches are either overly complex in design or heavily dependent on frequent switching between control laws, which pose significant challenges for practical engineering applications.

In this study, we investigate the global bounded time-varying control problem for a class of chained nonholonomic systems, which comprises an integer subsystem and a bilinear subsystem. For the scalar integer subsystem, a nonhomogeneous controller is proposed. In addressing the bilinear subsystem with a linear term, a cascade connection of saturation-function-based time-varying controllers is introduced, facilitated by a linear time-varying state transformation. The proposed methodology can be extended to second-order integer subsystems. The presented control strategies are rigorously shown to guarantee global attractivity of the origin point and local exponential convergence of the state to zero.

The technical contributions of this study are threefold. First, unlike the chained nonholonomic systems studied in [1, 3], i.e., $\dot{x}_0 = u_0$, $\dot{x}_i = u_0 x_{i+1}$, $i = 1, 2, \ldots, n-1$, $\dot{x}_n = u$, the nonholonomic system studied in this study includes a linear term, and the proposed method is also applicable to chained nonholonomic systems that incorporate a second-order integrator and a bilinear subsystem. Second, in contrast to traditional saturation control methods for nonholonomic systems, such as the switching control approaches in [3], the proposed controllers are locally smooth and can guarantee global attractivity of the origin point. Third, unlike traditional homogeneous controllers [3], the proposed controller employs a nonhomogeneous saturation control strategy, with a relatively simple design structure.

Problem introduction. Consider the following chained nonholonomic system with input saturations:

$$\begin{cases} \dot{x}_0(t) &= \mathrm{sat}_{u_{\max}}(u_0(t)), \\ \dot{x}_1(t) &= -\alpha x_1(t) + \mathrm{sat}_{u_{\max}}(u_0(t)) x_2(t), \\ \dot{x}_2(t) &= \mathrm{sat}_{u_{\max}}(u_1(t)), \end{cases} \tag{1}$$

where $\alpha > 0$, $\mathrm{sat}_{\varphi}(y) = \mathrm{sign}(y)\min\{|y|, \varphi\}$, $x_0(t)$ and $x(t) = [x_1(t), x_2(t)]^{\mathrm{T}}$ are system states, and $u_0(t) \in \mathbf{R}$ and $u_1(t) \in \mathbf{R}$ are control inputs, with $u_{\max}$ being a positive constant.

This study will design a continuous nonhomogeneous controller $u_0(t)$ and a continuous controller $u_1(t)$ for this system such that the states $x_0(t)$, $x_1(t)$, and $x_2(t)$ converge to zero globally.

Design of control law $u_0(t)$. A continuous nonhomogeneous controller $u_0(t)$ can be designed as follows.

Theorem 1. Let $\lambda > \alpha > 0$ and $\beta \neq 0$ be some constants. Consider the time-varying continuous nonhomogeneous controller

$$u_0(t) = -\lambda x_0(t) + \beta \mathrm{e}^{-\alpha t}. \tag{2}$$

Then, the origin point is globally attractive and locally converges to zero exponentially. In particular, there exists a positive constant T_0, such that, for $t \geqslant T_0$, $\mathrm{sat}_{u_{\max}}(u_0(t))$ can be expressed as

$$\mathrm{sat}_{u_{\max}}(u_0(t)) = \mathrm{e}^{-\alpha t}(\theta(t) + \delta), \tag{3}$$

where $\delta = -\alpha\beta/(\lambda - \alpha)$ and $\theta(t)$ is a time-varying function satisfying $\lim_{t\to\infty} \theta(t) = 0$.

The proof of Theorem 1 can be found in Appendix A.

Remark 1. This study introduces a nonhomogeneous term, $\beta \mathrm{e}^{-\alpha t}$, into the controllers (2). The purpose of this term is to ensure that $\mathrm{sat}_{u_{\max}}(u_0(t))$ maintains the form given in (3), even

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when $x_0(0) = 0$. Traditional control strategies (e.g., [3]) typically handle the scenario $x_0(0) = 0$ by employing switching control to drive $x_0(t)$ away from the origin. Once $x_0(t_1) \neq 0$ at some $t_1 > 0$, a predefined control strategy is applied. In contrast, the method proposed in this theorem is locally smooth and ensures that the origin point is globally attractive, with the state converging to zero exponentially in a local sense.

Design of control law $u_1(t)$. Take the linear time-varying state transformation

$$\begin{cases} z_1(t) &= \mathrm{e}^{\alpha t} x_1(t), \\ z_2(t) &= x_2(t). \end{cases} \tag{4}$$

By using (4), x-subsystem in (1) can be written as

$$\begin{cases} \dot{z}_1 &= \mathrm{e}^{\alpha t} \mathrm{sat}_{u_{\max}}(u_0) x_2, \\ \dot{z}_2 &= \mathrm{sat}_{u_{\max}}(u_1). \end{cases} \tag{5}$$

Considering $t \geqslant T_0$, then the controller $\mathrm{sat}_{u_{\max}}(u_0(t))$ can be expressed in the form shown in (3). Substituting $\mathrm{sat}_{u_{\max}}(u_0(t))$ shown in (3) into (5), we can get, for $t \geqslant T_0$,

$$\begin{cases} \dot{z}_1 &= \delta z_2 + \theta(t) z_2, \\ \dot{z}_2 &= \mathrm{sat}_{u_{\max}}(u_1). \end{cases} \tag{6}$$

Let

$$Y(t) = \begin{bmatrix} \frac{\lambda_2}{\delta} & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix}. \tag{7}$$

Then the time-derivative of $Y = Y(t) = [Y_1(t), Y_2(t)]^{\mathrm{T}}$ along (6) can be calculated as

$$\dot{Y} = A_{\mathrm{T},2} Y + \theta(t) \begin{bmatrix} 0 & \frac{\lambda_2}{\delta} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} + b_{\mathrm{T},2} \mathrm{sat}_{u_{\max}}(u_1), \tag{8}$$

where

$$A_{\mathrm{T},2} = \begin{bmatrix} 0 & \lambda_2 \\ 0 & 0 \end{bmatrix}, \quad b_{\mathrm{T},2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{9}$$

With the above preparations, we can get the following results.

Theorem 2. Let $\lambda_i > 0$, and $\varepsilon_i > 0$, $i = 1, 2$, be some constants satisfying

$$\varepsilon_1 + \varepsilon_2 \leqslant u_{\max}, \quad \varepsilon_2 > \varepsilon_1.$$

Consider the following time-varying feedback:

$$u_1(t) = -\varepsilon_2 \mathrm{sat}\left(\frac{\lambda_2 Y_2(t)}{\varepsilon_2}\right) - \varepsilon_1 \mathrm{sat}\left(\frac{\lambda_1 Y_1(t)}{\varepsilon_1}\right) \tag{10}$$

with $\mathrm{sat}(x) = \mathrm{sat}_1(x)$. Then, the origin point is globally attractive, and the state converges locally to zero exponentially.

The proof of Theorem 2 can be found in Appendix B.

As observed from (6), this study transforms the nonholonomic system into a saturation control problem for a linear integrator system with time-varying terms by employing a nonhomogeneous time-varying feedback (3). Subsequently, the method outlined in [4] is applied to design the saturation controller (10) step by step, following a bottom-up approach.

Unlike previous saturation control strategies for nonholonomic systems, such as those in [3], the control strategies (3) and (6) proposed in this study leverage the special structure of the nonholonomic system. They do not require switching and achieve locally smooth exponential convergence, making them simpler. Furthermore, an extension of these strategies to other nonholonomic systems is provided in Appendix C, while a numerical simulation demonstrating their effectiveness can be found in Appendix D.

Conclusion. In this work, we have studied the global time-varying control problem for a class of chained nonholonomic systems with input saturations. Nonhomogeneous controllers were constructed for both the scalar and second-order integer subsystems. For bilinear subsystems with a linear term, a cascade connection of saturation-function-based time-varying controllers was proposed by employing a linear time-varying state transformation. The proposed control strategies were rigorously proven to be globally attractive and locally exponentially convergent. Unlike existing saturation control strategies for nonholonomic systems, the proposed approach is locally smooth and linear. The effectiveness of the proposed approach has been demonstrated through a numerical example. In the future, we plan to extend the method studied in this work to nonholonomic systems with perturbation nonlinearities.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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