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Delay-dependent stability analysis of load frequency control of microgrid based on the matrix injection method

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With the widespread adoption of open communication networks in microgrid (MG), load frequency control (LFC) in MG is inevitably affected by time-varying delays. These delays can degrade system performance and even lead to instability, posing a serious threat to the safe and stable operation of MG [1].

To assess the impact of time-varying delays on the stability of MG's LFC, numerous researchers have conducted extensive studies on stability criteria for delay-dependent MG systems. Ramakrishnan et al. [2] constructed a Lyapunov-Krasovskii functional (LKF) with double integral terms, using auxiliary function-based integral inequalities to estimate the derivatives of the functional to analyze the stability of MG's LFC under time-varying delays. He et al. [3] applied a mixed convex combination technique, and a less conservative delay stability criterion was introduced. Subsequently, Wei et al. [4] employed inverse convex inequalities and the cubic negative definiteness lemma to handle the delay inverse convex terms in the LKF derivative, thereby achieving a further reduction in the conservatism of delay estimation.

Nevertheless, these methods treat the reciprocal and higher-order terms of delays in the LKF derivatives separately, which introduces a degree of conservatism. Less conservative criteria provide a larger delay margin, guiding controller design and thereby enhancing the system tolerance to communication failures. Therefore, to further reduce the conservatism of the stability criteria, it is crucial to develop a unified approach capable of handling both the reciprocal and higher-order terms of time delays in stability analysis.

Driven by this motivation, this study addresses the stability of the MG system with time-varying delays. To analyze the stability of the MG's LFC system, a delay-dependent MG model is developed. Furthermore, by employing a matrix injection method, a unified framework is proposed to deal with the reciprocal terms and higher-order terms of time delay in the LKF derivative. The effectiveness and advantages of the proposed criterion are demonstrated through numerical examples.

Dynamic model. The frequency deviation caused by the power balance relationship in the MG can be expressed as

$$\begin{aligned}\Delta \dot{f}(t) =& \frac{1}{M}\left(\Delta P_{\mathrm{mt}}(t) + \Delta P_{\mathrm{fc}}(t)\right.\\ &\left.-\Delta P_{\mathrm{es}}(t) + \Delta P_{\mathrm{fess}}(t) - D\Delta f(t) + \Delta P_{\mathrm{dis}}(t)\right), \qquad (1)\end{aligned}$$

where Δf is the frequency deviation, ΔP_{mt} is the power deviation of micro-turbine, ΔP_{fc} is the power deviation of fuel cell system, ΔP_{es} is the power deviation of electrolyzer system, and ΔP_{fess} is the power deviation of flywheel energy storage system. The above variables satisfy the following conditions:

$$\begin{aligned}\Delta \dot{P}_{\mathrm{mt}} =& \alpha_{22}\Delta P_{\mathrm{mt}}(t) + \alpha_{23}\Delta P_{\mathrm{fc}}(t) + \alpha_{24}\Delta P_{\mathrm{es}}(t) \qquad (2)\\ &+ \alpha_{25}\Delta P_{\mathrm{fess}}(t) + \alpha_{26}\Delta f(t)\\ &+ \alpha_{d,21}K_I\int \Delta f(t-d(t))dt + \alpha_{d,22}\Delta P_{\mathrm{mt}}(t-d(t))\\ &+ \alpha_{d,23}\Delta P_{\mathrm{fc}}(t-d(t)) + \alpha_{d,24}\Delta P_{\mathrm{es}}(t-d(t))\\ &+ \alpha_{d,25}\Delta P_{\mathrm{fess}}(t-d(t)) + \alpha_{d,26}\Delta f(t-d(t))\\ &- \alpha K_{pl}\Delta \dot{P}_{\mathrm{dis}}(t) - \alpha K_{il}\Delta P_{\mathrm{dis}}(t), \qquad (3)\end{aligned}$$

$$\Delta \dot{P}_{\mathrm{fc}}(t) = -\frac{1}{T_{\mathrm{fc}}}\Delta P_{\mathrm{fc}}(t) + \frac{R_{\mathrm{fc}}}{T_{\mathrm{fc}}}\Delta f(t), \qquad (4)$$

$$\Delta \dot{P}_{\mathrm{es}}(t) = -\frac{1}{T_{\mathrm{es}}}\Delta P_{\mathrm{es}}(t) + \frac{R_{\mathrm{es}}}{T_{\mathrm{es}}}\Delta f(t), \qquad (5)$$

$$\Delta \dot{P}_{\mathrm{fess}}(t) = -\frac{1}{T_{\mathrm{fess}}}\Delta P_{\mathrm{fess}}(t) + \frac{R_{\mathrm{fess}}}{T_{\mathrm{fess}}}\Delta f(t). \qquad (6)$$

Defining $x(t) = [K_I\int \Delta f dt, \Delta P_{\mathrm{mt}}(t), \Delta P_{\mathrm{fc}}(t), \Delta P_{\mathrm{es}}(t), \Delta P_{\mathrm{fess}}(t), \Delta f(t)]^{\mathrm{T}}$. This study considers the internal stability and does not consider the external disturbance, and the state space model of MG can be obtained

$$\dot{x}(t) = Ax(t) + A_d x(t-d(t)), \qquad (7)$$

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where

$$A=\begin{bmatrix}0&0&0&0&0&K_I\\0&\alpha_{22}&\alpha_{23}&\alpha_{24}&\alpha_{25}&\alpha_{26}\\0&0&-\frac{1}{T_{\rm fc}}&0&0&\frac{R_{\rm fc}}{T_{\rm fc}}\\0&0&0&-\frac{1}{T_{\rm es}}&0&\frac{R_{\rm es}}{T_{\rm es}}\\0&0&0&0&-\frac{1}{T_{\rm fess}}&\frac{R_{\rm fess}}{T_{\rm fess}}\\0&\frac{1}{M}&\frac{1}{M}&-\frac{1}{M}&-\frac{1}{M}&-\frac{D}{M}\end{bmatrix},$$

$$A_d=\begin{bmatrix}0&0&0&0&0&0\\\alpha_{d,21}&\alpha_{d,22}&\alpha_{d,23}&\alpha_{d,24}&\alpha_{d,25}&\alpha_{d,26}\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix},$$

and $d(t)$ is the delay generated by the transmission of the signal in an open communication network, satisfying $0\leqslant d_1\leqslant d(t)\leqslant d_2$. d_1 is the lower bound of delay and d_2 is the upper bound of delay. To simplify the representation, let $d=d(t), d_{1d}=d-d_1, d_{2d}=d_2-d$, and $d_{12}=d_2-d_1$.

The MG LFC model and the symbols not explicitly defined in the main text are listed in Appendix A.

A stability analysis criterion. This study proposes a framework using the matrix injection method to uniformly manage delay derivative terms and higher-order terms, and derives the corresponding delay stability criterion, as shown in Theorem 1.

Theorem 1. For given scalars d_i, $i=1,2$, system (7) is asymptotically stable if there exist $P=\{P_{ij}\}_{5\times5}\in\mathbb{S}_+^{5n}, \{Q,R_i\}\in\mathbb{S}_+^n$, $i=1,2$, $Z=\{Z_{ij}\}_{4\times4}\in\mathbb{S}_+^{4n}$, any matrices $\{L_1,N_1\}\in\mathbb{R}^{3n\times10n}$, $\{L_2,N_2\}\in\mathbb{R}^{3n\times2n}, S_1\in\mathbb{R}^{2n\times2n}$, and $S_2\in\mathbb{R}^{2n\times10n}$ such that the following is valid for $d\in\{d_1,d_2\}$.

$$\begin{bmatrix}\Psi_1 & S_2^{\rm T}-d(E_A^{\rm T}S_1-E_B^{\rm T}\Phi_2^{\rm T}) & d_{1d}N_1^{\rm T}+d_{2d}L_1^{\rm T}\\ * & d\Phi_1+{\rm He}\{S_1\} & d_{1d}N_1^{\rm T}+d_{2d}L_2^{\rm T}\\ * & * & -d_{12}^2\hat{R}_2\end{bmatrix}<0. \quad (8)$$

Note that the detailed proof of Theorem 1 is discussed in Appendix B.

According to Theorem 1, the stability margin of the time-delay MG under different controller gains can be obtained. This study adopts a matrix injection method to create a unified framework for handling the reciprocal and higher-order terms of the time delay, addressing the high conservatism resulting from separate treatment in previous methods. By using the matrix injection method, the need to introduce additional negative definiteness conditions is avoided, thereby reducing the conservatism of the criterion, providing a more accurate delay-dependent assessment.

A case study. The parameters of system (1) are $M=10, D=1, R_{\rm mt}=0.04, R_{\rm es}=R_{\rm fess}=R_{\rm fc}=1, T_{\rm es}=T_{\rm fess}=1, T_{\rm fc}=4, K_{\rm pl}=K_{\rm il}=1$ [4].

In the MG system, different controller parameters yield different delay margins. Table 1 compares the results of this study with those from [4]. Based on the results of Table 1, the time delay margin calculated using the stability criterion proposed in this study is greater than the margins obtained using the method in references [4]. This demonstrates that the criterion introduced in this study is less conservative and provides a more accurate estimation of the system's stability.

To verify the validity of the above theoretical results, simulation validation is conducted in this study. However, due to space limitations, the details are presented in Appendix C.

Table 1 Upper bound of the time-varying delay under different controller gains and comparison with other methods.

K_I	Method	K_P			
		3	5	7	9
0.1	[4]	15.77	10.01	7.18	5.57
	Theorem 1	15.88	10.13	7.31	5.68
0.2	[4]	12.61	9.50	7.04	5.50
	Theorem 1	12.62	9.57	7.14	5.62
0.4	[4]	8.20	8.16	6.67	5.36
	Theorem 1	8.20	8.17	6.72	5.45
0.6	[4]	6.01	6.81	6.20	5.20
	Theorem 1	6.02	6.81	6.22	5.26

Conclusion. This study investigates the stability of a delay-dependent MG's LFC system. To address the reciprocal and higher-order terms of the time-varying delay in the LKF derivative, a matrix-injection method is proposed. This method obtains a framework for the unified treatment of two types of delay terms by injecting a few matrices and obtains the stability criterion of the delay MG's LFC with less conservatism. This approach can offer more accurate delay estimations and larger delay upper bounds, thereby guiding controller design and enhancing the system's tolerance to communication failures.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Yuan Z L, Zhang C K, Shangguan X C, et al. Stability analysis of load frequency control for shipboard microgrids with occasional large delays. IEEE Trans Circ Syst II, 2022, 69: 2161–2165
2 Ramakrishnan K, Vijeswaran D, Manikandan V. Stability analysis of networked micro-grid load frequency control system. J Anal, 2019, 27: 567–581
3 He J, Liang Y, Hao X H, et al. A quadratic convex framework with bigger freedom for the stability analysis of a cyber-physical microgrid system. Sci China Inf Sci, 2023, 66: 122202
4 Wei C G, Shangguan X C, He Y, et al. Delay-dependent stability evaluation for temperature control load participating in load frequency control of microgrid. IEEE Trans Ind Electron, 2025, 72: 449–459