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Practical fixed-time intermittent control of coupled neutral complex networks

Fanchao KONG[1] & Jinde CAO[2*]

[1]*School of Mathematics and Statistics, Anhui Normal University, Wuhu 241000, China*
[2]*School of Mathematics, Southeast University, Nanjing 210096, China*

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Abstract This paper aims to investigate the practical fixed-time control of coupled neutral complex networks (CNCNs) with mismatched parameters. New intermittent fixed-time stability lemmas are established, formulated using indefinite functions and unified exponent conditions on the Lyapunov function. These lemmas incorporate and improve upon previous results. Taking into account the unavailability of global information in practical scenarios, intermittent-type practical fixed-time stability lemmas are derived, which extend and enhance the previously negative definite conditions. New estimations of the settling-time required to reach the residual set are provided. Based on the newly established stability lemmas, the fixed-time stability and practical fixed-time stability of the considered CNCNs are analyzed. The designed controllers incorporate the arctangent function, ensuring that the control values remain bounded. This approach addresses the issue of high control gains effectively. Finally, numerical simulations of a drilling system modeled by CNCNs with mismatched parameters are presented to validate the main results.

Keywords coupled neutral complex networks, mismatched parameters, intermittent control, bounded control, fixed-time stability, practical fixed-time stability

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1 Introduction

Intermittent control, as one of the discontinuous control methods, has gained popularity recently due to its cost-effectiveness and practicality in engineering applications. Many researchers have achieved significant results using this approach. Intermittent control can be categorized into periodically intermittent control and aperiodically intermittent control. For periodically intermittent control, both the work and rest time intervals are fixed. Several results on periodically intermittent control have been reported, for example, [1–6]. In contrast, aperiodically intermittent control allows for flexible adjustment of intervals, making it more effective and convenient for practical applications. In recent years, numerous outcomes related to aperiodically intermittent control have been achieved, including, but not limited to [7–13]. For instance, in [11–13], several intermittent fixed-time (FxT) stability lemmas have been established. These lemmas are significant because FxT stability ensures that the settling-time (ST) function is independent of initial conditions. To unify the exponents of the Lyapunov function and reduce additional parameter inputs, Li and Wang [11] proposed an intermittent FxT stability lemma, which was further improved by Qin et al. [12]. For a clear comparison, refer to Table 1.

Notably, the estimation of the ST in the aforementioned results relies on global information, which may limit their practical applicability. This is because the global information of many systems, such as large-scale swarms or power grids, is often unclear or unidentifiable. In recent years, practical finite-time control (see [14–16]) and practical FxT control (see [17–21]) have been extensively studied. For a detailed comparison, refer to Table 2.

Although there are fruitful results on studying the intermittent control issues, there are at least two pending problems.

The first one is the intermittent inequality conditions in stability lemmas. (i) From the intermittent inequality conditions summarized in Table 1, it is evident that Inequality No.1 in [11] and Inequality No.2 in [12] are both negative definite. However, these conditions have limitations in practical applications, as discussed in [22, 23]. Although Inequality No.3 in [13] is indefinite, it introduces two additional input parameters, p and q, and fails to

* Corresponding author (email: jdcao@seu.edu.cn)

Table 1 Previous intermittent FxT stability conditions.

No.	Intermittent-type FxT stability inequalities	Condition	Ref.
1	$\begin{cases}\dot{\Theta} \leqslant -\alpha(\Theta)^{\wp+\mathrm{sign}(\Theta-1)}, & \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta} \leqslant 0, & s_\ell \leqslant \varsigma < \varsigma_{\ell+1}\end{cases}$	$\alpha > 0,\ 1 \leqslant \wp < 2$	[11]
2	$\begin{cases}\dot{\Theta} \leqslant -\alpha_1\Theta - \alpha_2(\Theta)^{\wp+\mathrm{sign}(\Theta-1)}, & \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta} \leqslant \alpha_3\Theta, & s_\ell \leqslant \varsigma < \varsigma_{\ell+1}\end{cases}$	$\alpha_1, \alpha_2, \alpha_3 > 0,\ 1 \leqslant \wp < 2$	[12]
3	$\begin{cases}\dot{\Theta} \leqslant -I\Theta - \alpha\Theta^p - \beta\Theta^q, & \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta} \leqslant 0, & s_\ell \leqslant \varsigma < \varsigma_{\ell+1}\end{cases}$	I is indefinite, $\alpha, \beta > 0,\ 0 < p < 1 < q$	[13]

Table 2 Previous practical FxT stability conditions.

No.	Practical FxT stability inequalities	Condition	Ref.
1	$\dot{\Theta} \leqslant -\alpha_1\Theta^{2-\frac{p}{q}} - \alpha_2\Theta^{2-\frac{p}{q}} + \xi$	$\alpha_1, \alpha_2, \xi > 0,\ 0 < \frac{p}{q} < 1$	[17]
2	$\begin{cases}\dot{\Theta} \leqslant -\alpha_1\Theta^p - \alpha_2\Theta^q, & \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta} \leqslant 0, & s_\ell \leqslant \varsigma < \varsigma_{\ell+1}\end{cases}$	$\alpha_1, \alpha_2 > 0,\ 0 < p < 1 < q$	[20]
3	$\begin{cases}\dot{\Theta} \leqslant -\alpha_1\Theta^p - \alpha_2\Theta^q - \alpha_3 V, & \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta} \leqslant \alpha_4 V, & s_\ell \leqslant \varsigma < \varsigma_{\ell+1}\end{cases}$	$\alpha_1, \alpha_2, \alpha_3, \alpha_4 > 0,\ 0 < p < 1 < q$	[20]
4	$\begin{cases}\dot{\Theta} \leqslant -\alpha_1\Theta^p - \alpha_2\Theta^q - \alpha_3 V + \xi_1, & \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta} \leqslant \alpha_4 V + \xi_2, & s_\ell \leqslant \varsigma < \varsigma_{\ell+1}\end{cases}$	$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \xi_1, \xi_2 > 0,\ 0 < p < 1 < q$	[21]

unify the exponents as achieved in No.1 and No.2 using $\wp+\mathrm{sign}(\Theta-1)$. In practice, achieving FxT stability requires both internal mechanisms and external inputs. Naturally, minimizing external inputs is essential to reducing costs. (ii) The inequality condition $\dot{\Theta}(\varsigma) \leqslant 0$ ($s_\ell \leqslant \varsigma < \varsigma_{\ell+1}$) in [11, 13] (see Table 1), is proposed. Such a condition is too special to be general, which should be improved like that in No.2 (see Table 1). (iii) As for the intermittent practical inequality conditions in Table 2, one can see that all the inequalities are negative definite. And, all the exponents on the Θ are not unified. As pointed out in [24], too many parameter inputs will cause too much energy loss.

Consequently, the existing intermittent inequality conditions should be improved.

The second consideration is the control constraint. In real-life applications, control constraints must be taken into account because actuators have finite control capabilities. Therefore, incorporating input constraints into controller design is both vital and necessary. Numerous results have been reported on control design for systems with bounded inputs, including [25–27]. From the designed control laws for intermittent control in the literature, it is evident that several controllers include exponential terms of state variables, such as $|\cdot|$ and $|\cdot|^\theta$ ($\theta > 0$). However, the values of such controllers can exhibit exponential growth when $\theta \gg 1$. The control laws proposed in existing studies, such as FxT control in [11–13], as well as practical FxT control in [17–21], may result in large or even unbounded control inputs. Therefore, it is essential to design bounded control laws to avoid the aforementioned issues of unlimited control.

On the other hand, many physical processes can be modeled by function differential equations of neutral systems, such as partial element equivalent circuit, flexible systems, very large scale integration systems, and population ecology (see [28–30]). As for the unitary discontinuous system, the Filippov system, has been widely studied due to its vital applications in describing many biological, chemical, and physical models (see [31]). But for the continuous neutral systems, neutral systems with discontinuous perturbation functions, especially the coupled neutral Filippov systems, are rarely studied and there is no result on the FxT stability and practical FxT stability of coupled neutral complex networks (CNCNs) via intermittent control. Moreover, discontinuous networks with mismatched parameters described by Filippov systems have been studied, see [32,33]. But, there is also no result on the practical FxT stability of networks modeled by Filippov systems with mismatched parameters.

To sum up, inspired by the unsolved problems in the existing results analyzed above, in the paper, the FxT stability and practical FxT stability of CNCNs are considered via bounded intermittent control strategies and generalized FxT stability and practical FxT stability lemmas. The main contributions are stated as follows.

• New intermittent FxT stability lemmas containing indefinite function and unite exponent condition are given, which can improve and include the previous ones with negative definite conditions.

• Intermittent practical FxT stability lemmas with indefinite function and unite exponent condition are proposed for the first time. Detail analyses show that the residual set is closely related to the unite exponent. This also implies that the radius of the residual set is dependent on the states of the system. Some previous results are improved, such as [17, 20, 21].

• In the practical FxT stability, the dynamical behaviors of the states are also discussed after the state enters the residual set, which supplements results in [18, 20, 21].

• The bounded intermittent controllers are designed, which can solve the large values of the controllers containing the term $|\cdot|^{\theta}(\theta>0)$ in the related results, such as [7–10, 13, 20, 21].

2 Basic definitions and lemmas

Consider the following differential system:

$$\dot{\mathfrak{X}}(\varsigma)=f(\mathfrak{X}(\varsigma)),\ \text{a.e.}\ \varsigma>\varsigma_0>0, \tag{1}$$

where $\mathfrak{X}(\varsigma)=(\mathfrak{X}_1(\varsigma),\mathfrak{X}_2(\varsigma),\ldots,\mathfrak{X}_n(\varsigma))^{\top}$ denotes state vector, $\dot{\mathfrak{X}}(\varsigma)$ denotes the time derivative of $\mathfrak{X}$ and $f:\mathbb{R}^n\to\mathbb{R}^n$ is discontinuous and measurable and essentially locally bounded. System (1) is named as a Filippov system.

Definition 1 (See [34])**.** The origin of (1) is finite-time stable, if the following is satisfied.

(i) Lyapunov stability: For any $\varsigma>0$, there exists a $\delta=\delta(\varsigma_0,\varepsilon)>0$ such that for any $\mathfrak{X}_0\in\mathbb{B}(0,\delta)=\{\mathfrak{X}_0\in\mathbb{R}^n:\|\mathfrak{X}_0\|<\delta\}$, $\|\mathfrak{X}(\varsigma)\|<\varepsilon$ holds for $\varsigma\geqslant 0$.

(ii) Finite-time convergence: There exists a $0<T(\mathfrak{X}_0)<+\infty$ such that $\lim_{\varsigma\to T(\mathfrak{X}_0)}\mathfrak{X}(\varsigma)=0$ and $\mathfrak{X}(\varsigma)\equiv 0$ for all $\varsigma\geqslant T(\mathfrak{X}_0)$. Here, $T(\mathfrak{X}_0)$ is called the ST.

Definition 2 (See [34])**.** The origin of (1) is FxT stable, if it is finite-time stable and there is a constant $T_{\max}>0$ such that $T(\mathfrak{X}_0)\leqslant T_{\max}$, for any initial state-point $\mathfrak{X}_0\in\mathbb{R}^n$.

Definition 3. The state of (1) is said to be practical FxT stable if, for positive definite function $\Theta(\mathfrak{X})$, $\forall\mathfrak{X}_0$, there exists a estimation of the settling-time $T(\mathfrak{X}_0)$, i.e., $T_{\max}\in[0,+\infty)$, a constant $\gamma>0$ and a suitable control $u(\varsigma)$ such that

$$\begin{cases}\lim\limits_{\varsigma\to T_{\max}}\mathfrak{X}(\varsigma)\in\{\mathfrak{X}|\Theta(\mathfrak{X})\leqslant\gamma\},\\ \mathfrak{X}\in\{\mathfrak{X}(\varsigma)|\Theta(\mathfrak{X}(\varsigma))\leqslant\gamma\},\ \forall\varsigma>T_{\max},\\ \lim\limits_{\varsigma\to+\infty}|\mathfrak{X}|=0,\end{cases}$$

where $T_{\max}$ is irrelevant to $\mathfrak{X}_0$, and $\{\mathfrak{X}|\Theta(\mathfrak{X})\leqslant\gamma\}\triangleq\Omega$ is called a residual set.

Remark 1. The "practical" FxT stability defined in Definitions 2 means that the state disagreement will converge to a neighborhood of the origin (i.e., residual set) at T without dependence on initial states. Such a neighborhood can be adjusted to a desired level. The "practical" FxT stability is put forward to solve the problem that the estimate of the ST in FxT stability depends on the global information, which may usually not be available; see [17].

Lemma 1. There exist a number $o>0$ and parameters $0<\tau_1<\tau_2$ such that $\tau_1|\mathfrak{X}|\leqslant|\arctan(\tau_2 x)|$, $|\mathfrak{X}|\leqslant o$. For any number $r>0$, $\frac{\arctan(\tau_2 r)}{r}|\mathfrak{X}|\leqslant|\arctan(\tau_2 x)|$, $o<|\mathfrak{X}|\leqslant r$, and $\frac{\arctan(\tau_2 r)}{r}|\mathfrak{X}|<\tau_1|\mathfrak{X}|$, $0<o<r$.

Proof. Let $h(\mathfrak{X})=-|\arctan(\tau_2\mathfrak{X})|+\tau_1|\mathfrak{X}|$. Then

$$h(\mathfrak{X})=\begin{cases}-\arctan(\tau_2\mathfrak{X})+\tau_1\mathfrak{X}, & \mathfrak{X}>0,\\ \arctan(\tau_2\mathfrak{X})-\tau_1\mathfrak{X}, & \mathfrak{X}<0.\end{cases}$$

Since $h(0^+)<0$, $h(0^-)<0$, $h(+\infty)>0$, $h(-\infty)>0$, then there exists a point $o>0$ such that

$$\begin{aligned}&|\arctan(\tau_2\mathfrak{X})|\geqslant\tau_1|\mathfrak{X}|, \quad |\mathfrak{X}|\leqslant o,\\ &|\arctan(\tau_2\mathfrak{X})|<\tau_1|\mathfrak{X}|, \quad |\mathfrak{X}|>o.\end{aligned} \tag{2}$$

For any number $r>0$, when $o<|\mathfrak{X}|\leqslant r$, $\frac{|\mathfrak{X}|}{r}\leqslant 1$, we have $\frac{\arctan(\tau_2 r)}{r}|\mathfrak{X}|\leqslant|\arctan(\tau_2\mathfrak{X})|$, $o<|\mathfrak{X}|\leqslant r$, by (2), we can obtain $\frac{\arctan(\tau_2 r)}{r}|\mathfrak{X}|\leqslant|\arctan(\tau_2\mathfrak{X})|<\tau_1|\mathfrak{X}|$, $0<o<|\mathfrak{X}|<r$, namely, $\frac{\arctan(\tau_2 r)}{r}|\mathfrak{X}|<\tau_1|\mathfrak{X}|$, $0<o<r$. Therefore, the proof is complete.

Remark 2. Since the $\tau_1|\mathfrak{X}|$ and $|\mathfrak{X}|$ are even function, there exists a $-o<0$ such that $|\arctan(\tau_2\mathfrak{X})|<\tau_1|\mathfrak{X}|$, $\mathfrak{X}<-o$. For any point $-r<-o<0$, $\frac{|\arctan(\tau_2 r)|}{r}|\mathfrak{X}|<\tau_1|\mathfrak{X}|$, $-r<\mathfrak{X}<-o$.

Lemma 2 (See [8])**.** For aperiodically intermittent strategy, if $\Psi(\varsigma)=\frac{\varsigma-s_\ell}{\varsigma-\varsigma_\ell}$, $\varsigma\in(s_\ell,\varsigma_{\ell+1}]$, and $\ell\in\mathbb{D}$ is a strictly increasing function, one can get that $\Psi(\varsigma)\leqslant\frac{\varsigma_{\ell+1}-s_\ell}{\varsigma_{\ell+1}-\varsigma_\ell}\leqslant\lim\limits_{m\to\infty}\sup\frac{\varsigma_{\ell+1}-s_\ell}{\varsigma_{\ell+1}-\varsigma_\ell}=\Psi$. Then, one can derive $0\leqslant\Psi\leqslant 1$. Herein, $\mathbb{D}$ is a set of natural numbers.

Lemma 3 (See [35])**.** Let $z_1,z_2,\ldots,z_l\geqslant 0$, $0<q\leqslant 1$, $p>1$, the following two inequalities hold $\sum_{\iota=1}^{l}z_\iota^q\geqslant\big(\sum_{\iota=1}^{l}z_\iota\big)^q$, $\sum_{\iota=1}^{l}z_\iota^p\geqslant l^{1-p}\big(\sum_{\iota=1}^{l}z_\iota\big)^p$.

3 Intermittent stability lemma

In the following, two absolutely new intermittent stability lemmas with indefinite function and unified exponents shown by the sign function are given.

3.1 Intermittent FxT stability lemmas

Lemma 4. Assume that the function $\Theta(\varsigma)$ is non-negative for $\varsigma \in [\varsigma_0, +\infty)$. There are a function $\Gamma \in \mathcal{K}_\infty$, an indefinite function $\mathfrak{H}$ and a positive constant $\wp$, such that $\Gamma(\|\mathfrak{X}\|) \leqslant \Theta(\mathfrak{X}(\varsigma)) = \Theta(\varsigma)$, $\forall \mathfrak{X} \in \mathbb{R}^n \setminus \{0\}$, $\varsigma > \varsigma_0$, and

$$\begin{cases} \dot{\Theta}(\varsigma) \leqslant -\mathfrak{H}(\varsigma)\Theta(\varsigma) - \mathfrak{F}(\Theta(\varsigma))^{\wp+\mathrm{sign}(\Theta(\varsigma)-1)}, \varsigma_\ell \leqslant \varsigma < s_\ell, \\ \dot{\Theta}(\varsigma) \leqslant -\rho\Theta(\varsigma), s_\ell \leqslant \varsigma < \varsigma_{\ell+1}, \end{cases} \tag{3}$$

for a.e. $\varsigma > \varsigma_0$, where $\ell \in \mathbb{D}$, $1 \leqslant \wp < 2$, $\mathfrak{F} > 0$, $\rho \geqslant \sup\limits_{\varsigma > \varsigma_0}\{\mathfrak{H}(\varsigma)\}$ and function $\mathfrak{H}$ satisfies the following inequality:

$$N_1 \leqslant \int_{\varsigma_0}^{\varsigma} \mathfrak{H}(s)\mathrm{d}s \leqslant N_2, \tag{4}$$

N_1, N_2 are positive constants. Then $\lim\limits_{\varsigma \to T_{\max}} \Theta(\varsigma) = 0$ and $\Theta(\varsigma) \equiv 0$ if $\varsigma \geqslant T_{\max}$, where $T_{\max} = \frac{1}{\wp\mathfrak{F}\exp\{-\wp N_2\}(1-\Psi)} + \frac{1}{(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}(1-\Psi)}$; here Ψ is defined in Lemma 2.

Proof. The proof will be complete by the following three steps.

• Step 1. When $\varsigma_\ell \leqslant \varsigma < s_\ell$ from (3), it follows that $\dot{\Theta}(\varsigma) \leqslant -\mathfrak{H}(\varsigma)\Theta(\varsigma)$, multiplying the above inequality by $\mathrm{e}^{\int_{\varsigma_0}^{\varsigma}\mathfrak{H}(s)\mathrm{d}s}$, we obtain $\mathrm{e}^{\int_{\varsigma_0}^{\varsigma}\mathfrak{H}(s)\mathrm{d}s}\dot{\Theta}(\varsigma) \leqslant -\mathfrak{H}(\varsigma)\Theta(\varsigma)\mathrm{e}^{\int_{\varsigma_0}^{\varsigma}\mathfrak{H}(s)\mathrm{d}s}$, namely, $\mathrm{d}\big[\mathrm{e}^{\int_{\varsigma_0}^{\varsigma}\mathfrak{H}(s)\mathrm{d}s}\Theta(\varsigma)\big]/\mathrm{d}\varsigma \leqslant 0$, integrating it from ς_0 to ς gives $\mathrm{e}^{\int_{\varsigma_0}^{\varsigma}\mathfrak{H}(s)\mathrm{d}s}\Theta(\varsigma) - \Theta(\varsigma_0) \leqslant 0$ and by (4), we further have

$$\Theta(\varsigma) \leqslant \Theta(\varsigma_0)\mathrm{e}^{-\int_{\varsigma_0}^{\varsigma}\mathfrak{H}(s)\mathrm{d}s} \leqslant \Theta(\varsigma_0)\mathrm{e}^{-N_1}, \forall \varsigma \geqslant \varsigma_0.$$

Based on the fact that $\Theta(\varsigma)$ is continuous at $\mathfrak{X}_0$, $\Theta(0) = 0$, then $\forall \varepsilon > 0$ and $\forall \varsigma_0 \geqslant 0$, there exist a $\delta = \delta(\varepsilon) > 0$ and $\forall \mathfrak{X}_0 \in \{\mathfrak{X} \in \mathbb{R}^n : \|\mathfrak{X}\| < \delta\}$, such that $\Theta(\varsigma_0) \leqslant \frac{\Gamma(\varepsilon)}{\mathrm{e}^{-N_1}}$. Then, we can get

$$\|\mathfrak{X}\| \leqslant \Gamma^{-1}(\Theta(\varsigma)) \leqslant \Gamma^{-1}(\Theta(\varsigma_0)\mathrm{e}^{c_1\varsigma_0}) \leqslant \Gamma^{-1}\left(\frac{\Gamma(\varepsilon)}{\mathrm{e}^{-N_1}} \cdot \mathrm{e}^{-N_1}\right) = \varepsilon,$$

which means that the origin of (1) is Lyapunov stable according to the (i) in Definition 1. When $s_\ell \leqslant \varsigma < \varsigma_{\ell+1}$, the same consequence can be obtained since $\dot{\Theta}(\varsigma) \leqslant -\rho\Theta(\varsigma)$ is the special case of $\dot{\Theta}(\varsigma) \leqslant -\mathfrak{H}(\varsigma)\Theta(\varsigma)$. Thus, the origin of (1) is Lyapunov stable.

The following two steps will prove the finite-time convergence and estimate the settling-time.

• Step 2. When $\Theta(\varsigma) > 1$, $\varsigma_\ell \leqslant \varsigma < s_\ell$, $\ell \in \mathbb{D}$, Eq. (3) becomes

$$\begin{cases} \dot{\Theta}(\varsigma) \leqslant -\mathfrak{H}(\varsigma)\Theta(\varsigma) - \mathfrak{F}(\Theta(\varsigma))^{\wp+1}, \ \varsigma_\ell \leqslant \varsigma < s_\ell, \\ \dot{\Theta}(\varsigma) \leqslant -\rho\Theta(\varsigma), \ s_\ell \leqslant \varsigma < \varsigma_{\ell+1}. \end{cases}$$

Select $\varsigma_0 = 0$. For $\varsigma \in [0, s_0)$, define

$$\begin{aligned} &R_0(\varsigma) = S(\varsigma) - \tilde{M}_0, \quad \tilde{M}_0 = \Theta^{-\wp}(0), \\ &S(\varsigma) = \Theta^{-\wp}(\varsigma)\exp\left\{-\wp\int_0^{\varsigma}\mathfrak{H}(s)\mathrm{d}s\right\} - \wp\mathfrak{F}\exp\{-\wp N_2\}\varsigma. \end{aligned}$$

Clearly, $R_0(0) = 0$. And, we can obtain

$$\dot{R}_0(\varsigma) \geqslant \wp\mathfrak{F}\exp\left\{-\wp\int_0^{\varsigma}\mathfrak{H}(s)\mathrm{d}s\right\} - \wp\mathfrak{F}\exp\{-\wp N_2\} \geqslant 0.$$

Thus, $S(\varsigma) \geqslant \tilde{M}_0$, $\forall \varsigma \in [0, s_0)$.

For $\varsigma \in [s_0, \varsigma_1)$, let $\tilde{R}_0(\varsigma) = S(\varsigma) - \tilde{M}_0 + \wp\mathfrak{F}\exp\{\wp N_2\}(\varsigma - s_0)$. Obviously, we have $\tilde{R}_0(s_0) = S(s_0) - \tilde{M}_0 \geqslant 0$, and the derivative of $\tilde{R}_0(\varsigma)$ is

$$\begin{aligned}\dot{\tilde{R}}_0(\varsigma) =& (-\wp)\Theta^{-\wp-1}(\varsigma)\dot{\Theta}(\varsigma)\exp\left\{-\wp\int_0^\varsigma \mathfrak{H}(s)\mathrm{d}s\right\} - \Theta^{-\wp}(\varsigma)\wp\mathfrak{H}(\varsigma)\exp\left\{-\wp\int_0^\varsigma \mathfrak{H}(s)\mathrm{d}s\right\}\\ & - \wp\mathfrak{F}\exp\{-\wp N_2\} + \wp\mathfrak{F}\exp\{-\wp N_2\} \geqslant (\rho - \mathfrak{H}(\varsigma))\Theta^{-\wp}(\varsigma)\wp\exp\left\{-\wp\int_0^\varsigma \mathfrak{H}(s)\mathrm{d}s\right\} \geqslant 0.\end{aligned}$$

Therefore, $\tilde{R}_0(\varsigma) \geqslant 0$, that is $S(\varsigma) \geqslant \tilde{M}_0 - \wp\mathfrak{F}\exp\{-\wp N_2\}(\varsigma - s_0)$, $\varsigma \in [s_0, \varsigma_1)$. By mathematical induction, we assume that

$$\begin{aligned}S(\varsigma) \geqslant& \tilde{M}_0 - \wp\mathfrak{F}\exp\{-\wp N_2\}\sum_{k=1}^{\ell-1}(\varsigma_k - s_{k-1}), \varsigma_{\ell-1} \leqslant \varsigma < s_{\ell-1},\\ S(\varsigma) \geqslant& \tilde{M}_0 - \wp\mathfrak{F}\exp\{-\wp N_2\}\left[\sum_{k=1}^{\ell-1}(\varsigma_k - s_{k-1}) + \varsigma - s_{\ell-1}\right], s_{\ell-1} \leqslant \varsigma < \varsigma_\ell.\end{aligned}$$

In fact, when $\varsigma \in [\varsigma_\ell, s_\ell)$, define $R_\ell(\varsigma) = S(\varsigma) - \tilde{M}_0 + \wp\mathfrak{F}\exp\{-\wp N_2\}\sum_{k=1}^{\ell}(\varsigma_k - s_{k-1})$, then, we have that

$$\begin{aligned}R_\ell(\varsigma_\ell) =& S(\varsigma_\ell) - \tilde{M}_0 + \wp\mathfrak{F}\exp\{-\wp N_2\}\sum_{k=1}^{\ell}(\varsigma_k - s_{k-1}) \geqslant \tilde{M}_0 - \wp\mathfrak{F}\exp\{-\wp N_2\}\left[\sum_{k=1}^{\ell-1}(\varsigma_k - s_{k-1})\right.\\ & \left. + \varsigma_\ell - s_{\ell-1}\right] - \tilde{M}_0 + \wp\mathfrak{F}\exp\{-\wp N_2\}\sum_{k=1}^{\ell}(\varsigma_k - s_{k-1}) = 0.\end{aligned}$$

Moreover,

$$\begin{aligned}\dot{R}_\ell(\varsigma) =& (-\wp)\Theta^{-\wp-1}(\varsigma)\dot{\Theta}(\varsigma)\exp\left\{-\wp\int_0^\varsigma \mathfrak{H}(s)\mathrm{d}s\right\} - \Theta^{-\wp}(\varsigma)\wp\mathfrak{H}(\varsigma)\exp\left\{-\wp\int_0^\varsigma \mathfrak{H}(s)\mathrm{d}s\right\}\\ & - \wp\mathfrak{F}\exp\{-\wp N_2\} \geqslant \wp\mathfrak{F}\exp\left\{-\wp\int_0^\varsigma \mathfrak{H}(s)\mathrm{d}s\right\} - \wp\mathfrak{F}\exp\{-\wp N_2\} \geqslant 0.\end{aligned}$$

Thus, we obtain $S(\varsigma) \geqslant \tilde{M}_0 - \wp\mathfrak{F}\exp\{-\wp N_2\}\sum_{k=1}^{\ell}(\varsigma_k - s_{k-1})$, $\varsigma_\ell \leqslant \varsigma < s_\ell$.

Additionally, let

$$\tilde{R}_\ell(\varsigma) = S(\varsigma) - \tilde{M}_0 + \wp\mathfrak{F}\exp\{-\wp N_2\}\left[\sum_{k=1}^{\ell}(\varsigma_k - s_{k-1}) + \varsigma - s_\ell\right], s_\ell \leqslant \varsigma < \varsigma_{\ell+1}.$$

Using a similar proof as $\tilde{R}_\ell(s_\ell) \geqslant 0$, we get $\tilde{R}_\ell(\varsigma) \geqslant 0$, and

$$S(\varsigma) \geqslant M_0 - \wp\mathfrak{F}\exp\{-\wp N_2\}\left[\sum_{k=1}^{\ell}(\varsigma_k - s_{k-1}) + \varsigma - s_\ell\right], s_\ell \leqslant \varsigma < \varsigma_{\ell+1}.$$

Therefore, we can conclude that

$$\begin{cases}S(\varsigma) \geqslant \tilde{M}_0 - \wp\mathfrak{F}\exp\{-\wp N_2\}\sum_{k=1}^{\ell}(\varsigma_k - s_{k-1}), \varsigma_\ell \leqslant \varsigma < s_\ell,\\ S(\varsigma) \geqslant \tilde{M}_0 - \wp\mathfrak{F}\exp\{-\wp N_2\}[\sum_{k=1}^{\ell}(\varsigma_k - s_{k-1}) + \varsigma - s_\ell], s_\ell \leqslant \varsigma < \varsigma_{\ell+1}.\end{cases}$$

For $\varsigma \in [\varsigma_\ell, s_\ell)$, we have

$$\begin{aligned}S(\varsigma) \geqslant& \tilde{M}_0 - \wp\mathfrak{F}\exp\{-\wp N_2\}\sum_{k=1}^{\ell}\frac{\varsigma_k - s_{k-1}}{\varsigma_k - \varsigma_{k-1}}(\varsigma_k - \varsigma_{k-1}) \geqslant \tilde{M}_0 - \wp\mathfrak{F}\exp\{-\wp N_2\}\Psi\sum_{k=1}^{\ell}(\varsigma_k - \varsigma_{k-1})\\ =& \tilde{M}_0 - \wp\mathfrak{F}\exp\{-\wp N_2\}\Psi\varsigma_\ell \geqslant \tilde{M}_0 - \wp\mathfrak{F}\exp\{-\wp N_2\}\Psi\varsigma.\end{aligned}$$

For $\varsigma \in [s_\ell, \varsigma_{\ell+1})$, we get

$$\begin{aligned} S(\varsigma) \geqslant& \tilde{M}_0 - \wp\mathfrak{F}\exp\{-\wp N_2\}\left[\sum_{k=1}^{\ell}\frac{\varsigma_k - s_{k-1}}{\varsigma_k - \varsigma_{k-1}}(\varsigma_k - \varsigma_{k-1}) + \frac{\varsigma - s_\ell}{\varsigma - \varsigma_\ell}(\varsigma - \varsigma_\ell)\right] \\ \geqslant& \tilde{M}_0 - \wp\mathfrak{F}\exp\{-\wp N_2\}\Psi\varsigma. \end{aligned}$$

Hence, for all $\varsigma \in [\varsigma_0, +\infty)$, $S(\varsigma) \geqslant \tilde{M}_0 - \wp\mathfrak{F}\exp\{-\wp N_2\}\Psi\varsigma$ holds. Then,

$$\begin{aligned} \Theta^{-\wp}(\varsigma) \geqslant& \Theta^{-\wp}(\varsigma)\exp\left\{-\wp\int_0^\varsigma \mathfrak{H}(s)\mathrm{d}s\right\} \geqslant \Theta^{-\wp}(0) + \wp\mathfrak{F}\exp\{-\wp N_2\}(1-\Psi)\varsigma \\ \geqslant& \wp\mathfrak{F}\exp\{-\wp N_2\}(1-\Psi)\varsigma, \varsigma \in [0, +\infty), \end{aligned}$$

which implies that

$$\Theta^{\wp}(\varsigma) \leqslant \frac{1}{\wp\mathfrak{F}\exp\{-\wp N_2\}(1-\Psi)\varsigma}, \varsigma \in [0, +\infty). \tag{5}$$

Let $\tilde{\varphi}(\varsigma) = \frac{1}{\wp\mathfrak{F}\exp\{-\wp N_2\}(1-\Psi)\varsigma}$. It is easy to see that $\tilde{\varphi}(\varsigma)$ is a strictly decreasing continuous function of ς. Setting the right side of (5) to 1, we can get

$$T_1 = \frac{1}{\wp\mathfrak{F}\exp\{-\wp N_2\}(1-\Psi)}, \tag{6}$$

and $\lim\limits_{\varsigma\to T_1}\Theta^{\wp}(\varsigma) = 1$. It follows from (5), (6) and the monotonicity of $\tilde{\varphi}(\varsigma)$ that $\lim\limits_{t\to T_1}\Theta(\varsigma) = 1$ and $\Theta(\varsigma) \leqslant 1$ for all $\varsigma \geqslant T_1$.

• Step 3. When $\Theta(\varsigma) < 1$ for all $\varsigma > T_1$, by (3), we have

$$\begin{cases} \dot{\Theta} \leqslant -\mathfrak{H}(\varsigma)V - \mathfrak{F}\Theta^{\wp-1}, \ T_1 < \varsigma_\ell \leqslant \varsigma < s_\ell, \\ \dot{\Theta} \leqslant -\rho V, \ s_\ell \leqslant \varsigma < \varsigma_{\ell+1}. \end{cases} \tag{7}$$

When $\varsigma \in [T_1, s_0)$, let

$$\begin{aligned} &Q_0(\varsigma) = H(\varsigma) - M_0, \ M_0 = \Theta^{2-\wp}(T_1) \\ &H(\varsigma) = \Theta^{2-\wp}(\varsigma)\exp\left\{(2-\wp)\int_{T_1}^{\varsigma}\mathfrak{H}(s)\mathrm{d}s\right\} + (2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}\varsigma. \end{aligned}$$

Obviously, $Q_0(0) = 0$. Differentiate $Q_0(\varsigma)$ and based on (7), we can obtain

$$\begin{aligned} \dot{Q}_0(\varsigma) =& (2-\wp)\Theta^{1-\wp}(\varsigma)\dot{\Theta}(\varsigma)\exp\left\{(2-\wp)\int_{T_1}^{\varsigma}\mathfrak{H}(s)\mathrm{d}s\right\} \\ &+ \Theta^{2-\wp}(\varsigma)(2-\wp)\mathfrak{H}(\varsigma)\exp\left\{(2-\wp)\int_{T_1}^{\varsigma}\mathfrak{H}(s)\mathrm{d}s\right\} + (2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\} \\ \leqslant& -(2-\wp)\mathfrak{F}\exp\left\{(2-\wp)\int_{T_1}^{\varsigma}\mathfrak{H}(s)\mathrm{d}s\right\} + (2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\} \leqslant 0, \end{aligned}$$

based on which implies that $H(\varsigma) \leqslant M_0$, $\forall\varsigma \in [T_1, s_0)$.

When $\varsigma \in [s_0, \varsigma_1)$, let

$$\tilde{Q}_0(\varsigma) = H(\varsigma) - (2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}(\varsigma - s_0) - M_0.$$

When $\varsigma \in [s_0, \varsigma_1)$, it gives that $\tilde{Q}_0(s_0) = H(s_0) - M_0 \leqslant 0$.

Furthermore, since $\rho \geqslant \sup\limits_{\varsigma > \varsigma_0}\{\mathfrak{H}(\varsigma)\}$, it follows that

$$\dot{\tilde{Q}}_{T_1}(\varsigma) = (2-\wp)\Theta^{1-\wp}(\varsigma)\dot{\Theta}(\varsigma)\exp\left\{(2-\wp)\int_{T_1}^{\varsigma}\mathfrak{H}(s)\mathrm{d}s\right\}$$

$$
\begin{aligned}
&+\Theta^{2-\wp}(\varsigma)(2-\wp)\mathfrak{H}(\varsigma)\exp\left\{(2-\wp)\int_{T_1}^{\varsigma}\mathfrak{H}(s)\mathrm{d}s\right\}\\
&+(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}-(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}\\
=&(\mathfrak{H}(\varsigma)-\rho)\Theta^{2-\wp}(\varsigma)(2-\wp)\exp\left\{(2-\wp)\int_{T_1}^{\varsigma}\mathfrak{H}(s)\mathrm{d}s\right\}\leqslant 0,
\end{aligned}
$$

which implies that $H(\varsigma)\leqslant M_0+(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}(\varsigma-s_0), \forall\varsigma\in[s_0,\varsigma_1)$.

Similar to the proof process in Step 2, by mathematical induction, we can get

$$
\begin{cases}
H(\varsigma)\leqslant M_0+(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}\sum_{k=1}^{\ell}(\varsigma_k-s_{k-1}), \varsigma_\ell\leqslant\varsigma<s_\ell;\\
H(\varsigma)\leqslant M_0+(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}[\sum_{k=1}^{\ell}(\varsigma_k-s_{k-1})+\varsigma-s_{\ell-1}], s_\ell\leqslant\varsigma<\varsigma_{\ell+1}.
\end{cases}
\tag{8}
$$

For $\varsigma\in[\varsigma_\ell,s_\ell)$, from the first inequality of (8), we have

$$
\begin{aligned}
H(\varsigma)\leqslant& M_0+(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}\sum_{k=1}^{\ell}\frac{\varsigma_k-s_{k-1}}{\varsigma_k-\varsigma_{k-1}}(\varsigma_k-\varsigma_{k-1})\\
\leqslant& M_0+(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}\Psi\sum_{k=1}^{\ell}(\varsigma_k-\varsigma_{k-1})\\
=& M_0+(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}\Psi\varsigma_\ell\\
\leqslant& M_0+(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}\Psi\varsigma.
\end{aligned}
$$

For $\varsigma\in[s_\ell,\varsigma_{\ell+1})$, from the second inequality of (8), we have

$$
\begin{aligned}
H(\varsigma)\leqslant& M_0+(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}\left[\sum_{k=1}^{\ell}\frac{\varsigma_k-s_{k-1}}{\varsigma_k-\varsigma_{k-1}}\cdot(\varsigma_k-\varsigma_{k-1})+\frac{t-s_\ell}{\varsigma-\varsigma_\ell}(\varsigma-\varsigma_\ell)\right]\\
\leqslant& M_0+(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}\Psi\varsigma.
\end{aligned}
$$

Thus, for all $\varsigma\in[T_1,+\infty)$, $H(\varsigma)\leqslant M_0+(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}\Psi\varsigma$ holds. So,

$$
\begin{aligned}
&\Theta^{2-\wp}(\varsigma)\exp\left\{(2-\wp)\int_{T_1}^{\varsigma}\mathfrak{H}(s)\mathrm{d}s\right\}+(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}\varsigma\\
\leqslant&\ \Theta^{2-\wp}(T_1)+(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}\Psi\varsigma,\ \varsigma\in[0,+\infty).
\end{aligned}
$$

Then for all $\varsigma\in[T_1,+\infty)$, we have

$$
\begin{aligned}
\Theta^{2-\wp}(\varsigma)\leqslant&\ \Theta^{2-\wp}(\varsigma)\exp\left\{(2-\wp)\int_{0}^{\varsigma}\mathfrak{H}(s)\mathrm{d}s\right\}\\
\leqslant&\ \Theta^{2-\wp}(T_1)-(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}(1-\Psi)\\
=&\ 1-(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}(1-\Psi).
\end{aligned}
\tag{9}
$$

Let $\varphi(\varsigma)=1-(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}(1-\Psi)\varsigma$. It is easy to see that $\varphi(\varsigma)$ is a strictly decreasing and continuous function of ς. Letting the right side of $\varphi(\varsigma)$ to 0, we can obtain

$$
T_2=\frac{1}{(2-\wp)\mathfrak{F}\exp\{(\wp-2)N_1\}(1-\Psi)},
\tag{10}
$$

and $\lim\limits_{\varsigma\to T_{\max}}\Theta^{2-\wp}(\varsigma)=0$. It follows from (9) and the monotonicity of $\varphi(\varsigma)$ that $\lim\limits_{\varsigma\to T_{\max}}\Theta(\varsigma)=0$ and $\Theta(\varsigma)\equiv 0$ for all $\varsigma\geqslant T_{\max}$.

Consequently, from (6) and (10), we can see that $\lim\limits_{\varsigma\to T_{\max}}\Theta(\varsigma)=0$ and $\Theta(\varsigma)\equiv 0$ if

$$
\varsigma\geqslant T_{\max}=\frac{1}{\wp\mathfrak{F}\exp\{-\wp N_2\}(1-\Psi)}+\frac{1}{(2-\wp)\mathfrak{F}\exp\{(2-\wp)N_1\}(1-\Psi)},
$$

where Ψ is defined in Lemma 3. Therefore, the proof is complete.

Table 3 Previous indefinite function conditions.

No.	FxT stability inequalities	Condition	Ref.
1	$\dot{\Theta} \leqslant \alpha(\varsigma)\Theta - p_1\Theta^{\theta} - p_2\Theta^{\delta}$	$\int_{\varsigma_0}^{\varsigma} \mathfrak{H}\mathrm{d}s \leqslant -\lambda(\varsigma-\varsigma_0)+N$, λ, N are positive constants	[36]
2	$\dot{\Theta} \leqslant \alpha(\varsigma)\Theta - (p_1(\varsigma)\Theta^{\theta} + p_2(\varsigma)\Theta^{\delta})^k$	$\int_{\varsigma_0}^{\varsigma} \mathfrak{H}(s)\mathrm{d}s \leqslant -\lambda(\varsigma-\varsigma_0)+N,\lambda, N$ are positive constants	[37]

Table 4 Previous and this paper intermittent FxT stability conditions.

No.	Intermittent-type FxT stability inequalities	Condition	Ref.
1	$\begin{cases}\dot{\Theta} \leqslant -\alpha(\Theta)^{\wp+\mathrm{sign}(\Theta-1)}, & \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta} \leqslant 0, & s_\ell \leqslant \varsigma < \varsigma_{\ell+1}\end{cases}$	$\alpha > 0$, $1 \leqslant \wp < 2$	[11]
2	$\begin{cases}\dot{\Theta} \leqslant -\alpha_1\Theta - \alpha_2(\Theta)^{\wp+\mathrm{sign}(\Theta-1)}, & \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta} \leqslant \alpha_3\Theta, & s_\ell \leqslant \varsigma < \varsigma_{\ell+1}\end{cases}$	$\alpha_1, \alpha_2, \alpha_3 > 0$, $1 \leqslant \wp < 2$	[12]
3	$\begin{cases}\dot{\Theta} \leqslant -I\Theta - \alpha\Theta^p - \beta\Theta^q, & \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta} \leqslant 0, & s_\ell \leqslant \varsigma < \varsigma_{\ell+1}\end{cases}$	I is indefinite, $\alpha, \beta > 0$, $0 < p < 1 < q$	[13]
4	$\begin{cases}\dot{\Theta}(\varsigma) \leqslant -\mathfrak{H}(\varsigma)\Theta(\varsigma) - \mathfrak{F}(\Theta(\varsigma))^{\wp+\mathrm{sign}(\Theta(\varsigma)-1)}, \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta}(\varsigma) \leqslant -\rho\Theta(\varsigma), s_\ell \leqslant \varsigma < \varsigma_{\ell+1}\end{cases}$	$\mathfrak{H}(\varsigma)$ is indefinite, $\mathfrak{F}, \rho > 0$, $1 \leqslant \wp < 2$	This paper

Remark 3. Lemma 4 differs from those in [36, 37] which impose unbounded conditions on the indefinite function. Refer to Table 3 for particulars. Unbounded conditions may lead to unlimited growth of system states or control inputs, thereby compromising the stability of the system. Unbounded conditions can make the system more sensitive to external disturbances or parameter variations, reducing its robustness. Consequently, the FxT stability Lemma 4 offers more benefits. For more details, see Table 3.

Lemma 5. Assume that the function $\Theta(\varsigma)$ is non-negative for $\varsigma \in [\varsigma_0, +\infty)$. There are a function $\Gamma \in \mathcal{K}_\infty$, an indefinite function $\mathfrak{H}$ and a positive constant $\wp$, such that $\Gamma(\|\mathfrak{X}\|) \leqslant \Theta(\mathfrak{X}(\varsigma)) = \Theta(\varsigma)$, $\forall \mathfrak{X} \in \mathbb{R}^n \setminus \{0\}$, $\varsigma > \varsigma_0$, and

$$\begin{cases}\dot{\Theta}(\varsigma) \leqslant \begin{cases}-\mathfrak{H}(\varsigma)\Theta(\varsigma) - \mathfrak{F}_1(\Theta(\varsigma))^{\wp+1}, \Theta(\varsigma) > 1,\\ -\mathfrak{H}(\varsigma)\Theta(\varsigma) - \mathfrak{F}_2(\Theta(\varsigma))^{\wp-1}, \Theta(\varsigma) < 1,\end{cases} \quad \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta}(\varsigma) \leqslant -\rho\Theta(\varsigma), s_\ell \leqslant \varsigma < \varsigma_{\ell+1},\end{cases}$$

for a.e. $\varsigma > \varsigma_0$, where $\ell \in \mathbb{D}$, $\mathfrak{F}_1 > 0$, $\mathfrak{F}_2 > 0$, $1 \leqslant \wp < 2$, $\mathfrak{F} > 0$, $\rho \geqslant \sup_{\varsigma > \varsigma_0}\{\mathfrak{H}(\varsigma)\}$ and function $\mathfrak{H}$ satisfies the inequality (4). Then $\lim_{\varsigma \to T_{\max}} \Theta(\varsigma) = 0$ and $\Theta(\varsigma) \equiv 0$ if $\varsigma \geqslant T_{\max}$, where $T_{\max} = \frac{1}{\wp\mathfrak{F}_1 \exp\{-\wp N_2\}(1-\Psi)} + \frac{1}{(2-\wp)\mathfrak{F}_2 \exp\{(2-\wp)N_1\}(1-\Psi)}$ and Ψ is defined in Lemma 2.

Proof. The proof is similar to that in Lemma 4. We omit it.

Remark 4. The intermittent FxT stability Lemma 4 is formulated by an indefinite function and unite exponent condition $\wp + \mathrm{sign}(\Theta(\varsigma) - 1)$. Such inequality (see (3)) relaxes the negative definiteness of the derivative of Θ and reduces the system-independent parameter inputs. The problem that $\dot{\Theta}(\varsigma) \leqslant 0$ $(s_\ell \leqslant \varsigma < \varsigma_{\ell+1})$ is not general has been solved. It is easy to see that the intermittent inequality conditions in [11–13] are the special cases of (3), and so are the periodic ones in [1, 2, 4]. For more details, see Table 4.

3.2 Intermittent practical FxT stability lemmas

Lemma 6. Assume that the function $\Theta(\varsigma)$ is non-negative for $\varsigma \in [\varsigma_0, +\infty)$. There are a function $\Gamma \in \mathcal{K}_\infty$, an indefinite function $\mathfrak{H}$ and a positive constant $\wp$, such that $\Gamma(\|\mathfrak{X}\|) \leqslant \Theta(\mathfrak{X}(\varsigma)) = \Theta(\varsigma)$, $\forall \mathfrak{X} \in \mathbb{R}^n \setminus \{0\}$, $\varsigma > \varsigma_0$, and

$$\begin{cases}\dot{\Theta}(\varsigma) \leqslant -\mathfrak{H}(\varsigma)\Theta(\varsigma) - \mathfrak{F}(\Theta(\varsigma))^{\wp+\mathrm{sign}(\Theta(\varsigma)-1)} + \gamma, \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta}(\varsigma) \leqslant -\rho\Theta(\varsigma) + \kappa, s_\ell \leqslant \varsigma < \varsigma_{\ell+1},\end{cases} \tag{11}$$

for a.e. $\varsigma > \varsigma_0$, where $\ell \in \mathbb{D}$, $1 \leqslant \wp < 2$, $\mathfrak{F} > 0$, $\rho \geqslant \sup_{\varsigma > \varsigma_0}\{\mathfrak{H}(\varsigma)\}$ and function $\mathfrak{H}(\cdot)$ satisfies the inequality (4). Then, the state x of (1) achieves intermittent practical FxT stability with the residual set given as follows:

$$\Omega = \left\{\mathfrak{X} | \Theta(\mathfrak{X}) \leqslant \max\left\{\left(\frac{\gamma}{\mathfrak{F}(1-\phi)}\right)^{\frac{1}{\wp+1}}, \left(\frac{\gamma}{\mathfrak{F}(1-\phi)}\right)^{\frac{1}{\wp-1}}, \frac{\kappa}{\rho\phi}\right\}\right\}, \tag{12}$$

where $\phi \in (0, 1)$. And the estimation of ST to attain the residual set is

$$T_{\max} = \frac{1}{\wp\mathfrak{F}\phi\exp\{-\wp N_2\}(1-\Psi)} + \frac{1}{(2-\wp)\mathfrak{F}\phi\exp\{(2-\wp)N_1\}(1-\Psi)}, \tag{13}$$

where Ψ is defined in Lemma 2.

Proof. (1) For $t \in [t_r, s_r)$, from (11), it follows that

$$\begin{aligned}\dot{\Theta}(\varsigma) \leqslant & -\mathfrak{H}(\varsigma)\Theta(\varsigma) - \mathfrak{F}(\Theta(\varsigma))^{\wp+\mathrm{sign}(\Theta(\varsigma)-1)} + \gamma \\ \leqslant & -\mathfrak{H}(\varsigma)\Theta(\varsigma) - \mathfrak{F}\phi(\Theta(\varsigma))^{\wp+\mathrm{sign}(\Theta(\varsigma)-1)} - \mathfrak{F}(1-\phi)(\Theta(\varsigma))^{\wp+\mathrm{sign}(\Theta(\varsigma)-1)} + \gamma \\ \leqslant & -\mathfrak{H}(\varsigma)\Theta(\varsigma) - \mathfrak{F}\phi(\Theta(\varsigma))^{\wp+\mathrm{sign}(\Theta(\varsigma)-1)}, \text{ if } \Theta(\varsigma) \geqslant \left(\frac{\gamma}{\mathfrak{F}(1-\phi)}\right)^{\frac{1}{\wp+\mathrm{sign}(\Theta(\varsigma)-1)}}.\end{aligned} \tag{14}$$

(2) For $t \in [s_r, t_{r+1})$, we can obtain

$$\dot{\Theta}(\varsigma) \leqslant -\rho\Theta(\varsigma) + \kappa \leqslant -\rho(1-\phi)\Theta(\varsigma) - \phi\rho\Theta(\varsigma) + \kappa \leqslant -\rho(1-\phi)\Theta(\varsigma), \text{ if } \Theta(\varsigma) \geqslant \frac{\kappa}{\rho\phi}. \tag{15}$$

Based on Lemma 4, the state x of (1) achieves intermittent practical FxT stability with the residual set Ω, which is defined in (12). And, the estimation of ST to attain the residual set is given by (13).

The proof is completed.

Remark 5. The practical FxT stability Lemma 6 is more generalized than those in the previous studies, such as [20, 21] since the first inequality in (11) can include the previous ones. It should point out that, in the represent rest interval $s_\ell \leqslant \varsigma < \varsigma_{\ell+1}$, the $\dot{\Theta} \leqslant \alpha_4\Theta + \xi_2$ in [21] is not rigorous since function Θ should be decreasing. The use of an indefinite function can reduce the limitation, and most importantly, it can still ensure the decrease of the Θ. For details, see the proof in Lemma 6. The second inequality in (11) can ensure the decrease of the function Θ.

Lemma 7. Assume that the function $\Theta(\varsigma)$ is non-negative for $\varsigma \in [\varsigma_0, +\infty)$. There are a function $\Gamma \in \mathcal{K}_\infty$, an indefinite function $\mathfrak{H}$ and a positive constant $\wp$, such that $\Gamma(\|\mathfrak{X}\|) \leqslant \Theta(\mathfrak{X}(\varsigma)) = \Theta(\varsigma)$, $\forall \mathfrak{X} \in \mathbb{R}^n \setminus \{0\}$, $\varsigma > \varsigma_0$, and

$$\begin{cases}\dot{\Theta}(\varsigma) \leqslant \begin{cases}-\mathfrak{H}(\varsigma)\Theta(\varsigma) - \mathfrak{F}_1(\Theta(\varsigma))^{\wp+1} + \gamma, \\ -\mathfrak{H}(\varsigma)\Theta(\varsigma) - \mathfrak{F}_2(\Theta(\varsigma))^{\wp-1} + \gamma,\end{cases} & \varsigma_\ell \leqslant \varsigma < s_\ell, \\ \dot{\Theta}(\varsigma) \leqslant -\rho\Theta(\varsigma) + \kappa, s_\ell \leqslant \varsigma < \varsigma_{\ell+1}, & \end{cases}$$

for a.e. $\varsigma > \varsigma_0$, $\ell \in \mathbb{D}$, $1 \leqslant \wp < 2$, $\mathfrak{F}_1 > 0$, $\mathfrak{F}_2 > 0$, $\rho \geqslant \sup_{\varsigma > \varsigma_0}\{\mathfrak{H}(\varsigma)\}$ and function $\mathfrak{H}$ satisfies the inequality (4). Then, Eq. (1) achieves intermittent practical FxT stability with the residual set given as follows:

$$\Omega = \left\{\mathfrak{X} | \Theta(\mathfrak{X}) \leqslant \max\left\{\left(\frac{\gamma}{\mathfrak{F}_1(1-\phi)}\right)^{\frac{1}{\wp+1}}, \left(\frac{\gamma}{\mathfrak{F}_2(1-\phi)}\right)^{\frac{1}{\wp-1}}, \frac{\kappa}{\rho\phi}\right\}\right\},$$

where $\phi \in (0,1)$. And the estimation of the settling-time to attain the residual set is

$$T_{\max} = \frac{1}{\wp\mathfrak{F}_1\phi\exp\{-\wp N_2\}(1-\Psi)} + \frac{1}{(2-\wp)\mathfrak{F}_2\phi\exp\{(2-\wp)N_1\}(1-\Psi)},$$

where Ψ is defined in Lemma 2.

Remark 6. Practical FxT stability can overcome the limitations of needing the global information. In this respect, it has a wider application. Intermittent control can save more costs and is more practical in engineering. So the intermittent-type practical FxT stability takes more advantages. The results in [20, 21] have established the intermittent-type practical FxT stability lemmas, which can improve the continuous time practical FxT stability lemma in [17]. But the intermittent-type practical FxT stability lemmas in [20, 21] also have some limitations. Clearly, we can see that the proposed inequality (11) in Lemma 6 can include those in [17, 20, 21]. For more details, see Table 5.

3.3 Network description

Consider coupled neutral Filippov systems on networks with mismatched parameters as follows:

$$(\mathcal{D}\dot{\mathfrak{X}}_i)(\varsigma) = -C^i\mathfrak{X}_i(\varsigma) + A^i f(\mathfrak{X}_i(\varsigma)) + I^i + \sum_{j=1, j\neq i}^{N} b_{ij}(g(\mathfrak{X}_j(\varsigma)) - g(\mathfrak{X}_i(\varsigma))), \tag{16}$$

where $(\mathcal{D}\mathfrak{X}_i)(\varsigma) = \mathfrak{X}_i(\varsigma) - \Gamma^i\mathfrak{X}_i(\varsigma - \sigma(\varsigma))$, here, $i = 1, 2, \ldots, N$, $\mathfrak{X}_i(\varsigma) = (\mathfrak{X}_{i1}(\varsigma), \mathfrak{X}_{i2}(\varsigma), \ldots, \mathfrak{X}_{in}(\varsigma))^\top \in \mathbb{R}^n$ is the state vector of node i, the nonlinear term $f(\mathfrak{X}_i(\varsigma)) = (f_1(\mathfrak{X}_{i1}(\varsigma)), f_2(\mathfrak{X}_{i2}(\varsigma)), \ldots, f_n(\mathfrak{X}_{in}(\varsigma)))^\top \in \mathbb{R}^n$ represents

Table 5 Previous and this paper practical FxT stability conditions.

No.	Practical FxT stability inequalities	Condition	Ref.
1	$\dot{\Theta} \leqslant -\alpha_1\Theta^{2-\frac{p}{q}} - \alpha_2\Theta^{2-\frac{p}{q}} + \xi$	$\alpha_1, \alpha_2, \xi > 0,\ 0 < \frac{p}{q} < 1$	[17]
2	$\begin{cases}\dot{\Theta} \leqslant -\alpha_1\Theta^p - \alpha_2\Theta^q, & \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta} \leqslant 0, & s_\ell \leqslant \varsigma < \varsigma_{\ell+1}\end{cases}$	$\alpha_1, \alpha_2 > 0,\ 0 < p < 1 < q$	[20]
3	$\begin{cases}\dot{\Theta} \leqslant -\alpha_1\Theta^p - \alpha_2\Theta^q - \alpha_3 V, & \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta} \leqslant \alpha_4 V, & s_\ell \leqslant \varsigma < \varsigma_{\ell+1}\end{cases}$	$\alpha_1, \alpha_2, \alpha_3, \alpha_4 > 0,\ 0 < p < 1 < q$	[20]
4	$\begin{cases}\dot{\Theta} \leqslant -\alpha_1\Theta^p - \alpha_2\Theta^q - \alpha_3 V + \xi_1, & \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta} \leqslant \alpha_4 V + \xi_2, & s_\ell \leqslant \varsigma < \varsigma_{\ell+1}\end{cases}$	$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \xi_1, \xi_2 > 0,\ 0 < p < 1 < q$	[21]
5	$\begin{cases}\dot{\Theta}(\varsigma) \leqslant -\mathfrak{H}(\varsigma)\Theta(\varsigma) - \mathfrak{F}(\Theta(\varsigma))^{\wp+\mathrm{sign}(\Theta(\varsigma)-1)} + \gamma, \varsigma_\ell \leqslant \varsigma < s_\ell,\\ \dot{\Theta}(\varsigma) \leqslant -\rho\Theta(\varsigma) + \kappa, s_\ell \leqslant \varsigma < \varsigma_{\ell+1},\end{cases}$	$\mathfrak{H}(\varsigma)$ is indefinite, $\mathfrak{F}, \rho, \kappa, \gamma, > 0,\ 1 \leqslant \wp < 2$	This paper

discontinuous activation, and the nonlinear coupling function $g(\mathfrak{X}_j(\varsigma)) = (g_1(\mathfrak{X}_{j1}(\varsigma), g_2\mathfrak{X}_{j2}(\varsigma), \ldots, g_n\mathfrak{X}_{jn}(\varsigma)))^\top \in \mathbb{R}^n$ is continuous. The matrix $C^i = \mathrm{diag}\{c_1^i, c_2^i, \ldots, c_n^i\}$ is the neuron self-inhibition with $c_j^i > 0$ $(j = 1, 2, \ldots, n)$, $A^i = (a_{lk}^i) \in \mathbb{R}^{n\times n}$, $\Gamma^i = (\Gamma_{lk}^i) \in \mathbb{R}^{n\times n}$ $(l, k = 1, 2, \ldots, n)$ represents the connection weight matrix, vector $I^i = (I_1^i, I_2^i, \ldots, I_n^i)$ denotes the constant external input for different nodes $i = 1, 2, \ldots, N$, and σ denotes the delay.

In addition, outer coupling matrix $B = (b_{ij}) \in \mathbb{R}^{N\times N}$ denotes the topology structure satisfying: if node i receives information from node j, $b_{ij} > 0$; otherwise, $b_{ij} = 0$, and the diagonal elements are $b_{ii} = 0$. Therefore, the corresponding Laplacian matrix L can be defined as $l_{ij} = -b_{ij}$ for $j \neq i$ and diagonal elements $l_{ij} = \sum_{j=1, j\neq i} b_{ij}$ $(i = 1, 2, \ldots, N)$. Thus, the controlled network can be written as

$$(\mathcal{D}\dot{\mathfrak{X}}_i)(\varsigma) = -C^i\mathfrak{X}_i(\varsigma) + A^i f(\mathfrak{X}_i(\varsigma)) + I^i - \sum_{j=1}^N l_{ij} g(\mathfrak{X}_j(\varsigma)) + u_i(\varsigma),\ i = 1, 2, \ldots, N. \tag{17}$$

Remark 7. As was pointed out by Hale [38, pp. 24–26], the main reason for considering the neutral equation with the difference is that it will be included without imposing too many smoothness conditions on the initial data. So, the $(\mathcal{D}x)(\varsigma)$ is more generalized to show the neutral terms.

For convenience, the following assumptions are given.

Assumption 1. For each $i = 1, 2, \ldots, n$, $f_i : \mathbb{R}^n \to \mathbb{R}$ is piecewise continuous, $0 \in \overline{co}[f_i(0)]$, and there exist nonnegative constants l_i and m_i such that $\sup_{\xi\in\overline{co}[f_i(x)], \eta\in\overline{co}[f_i(y)]} |\xi - \eta| \leqslant l_i|x - y| + m_i$, where $\overline{co}[f_i(\theta)] = [\min\{f_i^+(\theta), f_i^-(\theta)\}, \max\{f_i^+(\theta), f_i^-(\theta)\}]$.

Assumption 2. For $k = 1, 2, \ldots, n$, there exists positive constant ρ_k such that for all $x \in \mathbb{R}$, we have $|g_k(x)| \leqslant \rho_k|x|$.

Lemma 8 (See [39])**.** If $\Gamma = \max_{i,l,k}\{\Gamma_{lk}^i\} < 1$, the inverse of $\mathcal{D}$, denoted by $\mathcal{D}^{-1}$ exists and satisfies $\sup_{\varsigma\in\mathbb{R}} |\mathcal{D}^{-1}(\varsigma)| \leqslant 1/(1-\Gamma)$.

From Lemma 8, we can see that $\mathcal{D}^{-1}$ exits. Let $(\mathcal{D}\mathfrak{X}_i)(\varsigma) = \eta_i(\varsigma)$, then $\mathfrak{X}_i(\varsigma) = (\mathcal{D}^{-1}\eta_i)(\varsigma)$. Thus, Eq. (17) could be translated into the following system. For convenience, let $\mathfrak{X}_i(\varsigma)$ denote state, that is,

$$\dot{\mathfrak{X}}_i(\varsigma) = -C^i\mathfrak{X}_i(\varsigma) - C^i\Gamma^i(\mathcal{D}^{-1}\mathfrak{X}_i)(\varsigma - \sigma(\varsigma)) + I^i + A^i f((\mathcal{D}^{-1}\mathfrak{X}_i)(\varsigma)) - \sum_{j=1}^N l_{ij} g((\mathcal{D}^{-1}\mathfrak{X}_j)(\varsigma)) + u_i(\varsigma).$$

Based on differential inclusion theory in [31] and the measurable selection theorem [40], there exists measurable function $\phi_i(\varsigma) \in \overline{co}[\mathbf{f}((\mathcal{D}^{-1}\mathfrak{X}_i)(\varsigma))]$. Then the controlled system can be described as

$$\dot{\mathfrak{X}}_i(\varsigma) = -C^i\mathfrak{X}_i(\varsigma) - C^i\Gamma^i(\mathcal{D}^{-1}\mathfrak{X}_i)(\varsigma - \sigma(\varsigma)) + A^i\phi_i(\varsigma) - \sum_{j=1}^N l_{ij} g((\mathcal{D}^{-1}\mathfrak{X}_j)(\varsigma)) + I^i + u_i(\varsigma), \tag{18}$$

where $u_i(\varsigma)$ is the controller to be designed.

Let $\hat{a} = \max_{i,k,l}\{|a_{lk}^i|\}$, $\overline{c} = \max_{ij}\{c_j^i\}$, $\underline{c} = \min_{ij}\{c_j^i\}$, $\hat{l} = \max_{ij}\{|l_{ij}|\}$, $\hat{I} = \max_{ij}\{|I_j^i|\}$, $\hat{L} = \max_i\{l_i\}$, $\hat{m} = \max_i\{m_i\}$, and $\hat{\rho} = \max_k\{\rho_k\}$.

4 FxT stability analysis of (18)

Based on the arctangent function, we design the following bounded controller:

$$\begin{cases} u_i(\varsigma) = -\mathrm{sign}(\mathfrak{X}_i(\varsigma))\Big[(n^2\hat{a}\hat{m}+n\hat{I}) + \big(-\underline{c}n+\frac{\hat{a}\hat{L}n}{1-\Gamma}-N\hat{l}\hat{\rho}\big)|\arctan(\tau_2\mathfrak{X}_i(\varsigma))| \\ \qquad +\mathfrak{H}(\varsigma)|\mathfrak{X}_i(\varsigma)| + \mathfrak{F}|\arctan(\tau_2\mathfrak{X}_i(\varsigma))|^{v} + \frac{\overline{c}\Gamma}{1-\Gamma}|\arctan(\tau_2\mathfrak{X}_i(\varsigma-\sigma(\varsigma)))|\Big], \varsigma_\ell \leqslant \varsigma < s_\ell, \\ u_i(\varsigma) = -\mathrm{sign}(\mathfrak{X}_i(\varsigma))\Big[(n^2\hat{a}\hat{m}+n\hat{I}) + \big(-\underline{c}n+\frac{\hat{a}\hat{L}n}{1-\Gamma}+N\hat{l}\hat{\rho}+\omega\big)|\arctan(\tau_2\mathfrak{X}_i(\varsigma))| \\ \qquad +\frac{\overline{c}\Gamma}{1-\Gamma}|\arctan(\tau_2\mathfrak{X}_i(\varsigma-\sigma(\varsigma)))|\Big], s_\ell \leqslant \varsigma < \varsigma_{\ell+1}, \end{cases} \tag{19}$$

where $\ell \in \mathbb{D}$, $\nu = \wp + \mathrm{sign}(V-1)$, $\mathfrak{H}(\varsigma)$ satisfies the inequality (4), and $\omega > 0$ is a constant.

4.1 FxT stability via Lemma 5

In this section, we will establish some criteria to guarantee FxT stability for (18) based on Lemma 5.

Theorem 1. If Assumptions 1 and 2 hold, the following inequality holds:

$$\rho \geqslant \max\left\{\sup_{\varsigma>\varsigma_0}\{\mathfrak{H}(\varsigma)\}\right\}. \tag{20}$$

Then the origin of (18) achieves FxT stability and the estimation of ST is

$$T_{\max} = \frac{1}{\wp\mathfrak{F}_1\exp\{-\wp\tau_1 N_2\}(1-\Psi)} + \frac{1}{(2-\wp)\mathfrak{F}_2\exp\{(2-\wp)\frac{\arctan(\tau_2 r)}{r}N_1\}(1-\Psi)}, \tag{21}$$

where

$$\begin{aligned} \mathfrak{F}_1 &= \min\left\{\mathfrak{F}\tau_1^{\wp+1}N^{-\wp}, \mathfrak{F}\Big(\frac{\arctan(\tau_2 r)}{r}\Big)^{\wp+1}N^{-\wp}\right\}, \\ \mathfrak{F}_2 &= \min\left\{\mathfrak{F}\tau_1^{\wp-1}, \mathfrak{F}\Big(\frac{\arctan(\tau_2 r)}{r}\Big)^{\wp-1}\right\},\ \rho = \min\left\{\omega\tau_1, \frac{\arctan(\tau_2 r)}{r}\omega\right\}, \end{aligned} \tag{22}$$

and Ψ is defined in Lemma 2.

Proof. Consider the following Lyapunov function.

$$\Theta(\varsigma) = \sum_{i=1}^{N}|\mathfrak{X}_i(\varsigma)| = \sum_{i=1}^{N}\sum_{j=1}^{n}|\mathfrak{X}_{ij}(\varsigma)|. \tag{23}$$

In view of the switching characteristics of intermittent controller, the proof will be divided into two parts, one is $\varsigma \in [\varsigma_\ell, s_\ell)$, the other is $\varsigma \in [s_\ell, \varsigma_{\ell+1})$, $\ell \in \mathbb{D}$.

First, for $\forall\varsigma \in [\varsigma_\ell, s_\ell)$, calculating the time derivative of $\Theta(\varsigma)$, by (18) and (19), it can deduce that

$$\begin{aligned} \dot{\Theta}(\varsigma) = &\sum_{i=1}^{N}\mathrm{sign}(\mathfrak{X}_i(\varsigma))\Bigg\{-C^i\mathfrak{X}_i(\varsigma) - C^i\Gamma^i(\mathcal{D}^{-1}\mathfrak{X}_i)(\varsigma-\sigma(\varsigma)) + A^i\phi_i(\varsigma) - \sum_{j=1}^{N}l_{ij}\mathbf{g}((\mathcal{D}^{-1}\mathfrak{X}_j)(\varsigma)) + I^i \\ &- \mathrm{sign}(\mathfrak{X}_i(\varsigma))\Bigg[\Bigg(-\underline{c}n + \frac{\hat{a}\hat{L}n}{1-\Gamma} + N\hat{l}\hat{\rho}\Bigg)\arctan(\tau_2\mathfrak{X}_i(\varsigma)) + \mathfrak{H}(\varsigma)|\mathfrak{X}_i(\varsigma)| \\ &+ \mathfrak{F}|\arctan(\tau_2\mathfrak{X}_i(\varsigma))|^{v-1} + \frac{\overline{c}\Gamma}{1-\Gamma}|\arctan(\tau_2\mathfrak{X}_i(\varsigma-\sigma(\varsigma)))| + (n^2\hat{a}\hat{m}+n\hat{I})\Bigg]\Bigg\}. \end{aligned} \tag{24}$$

Using Assumption 1 and Lemma 8, we have

$$\sum_{i=1}^{N}\mathrm{sign}(\mathfrak{X}_i(\varsigma))A^i\phi_i(\varsigma) \leqslant \sum_{i=1}^{N}\sum_{l=1}^{n}\sum_{k=1}^{n}|a_{lk}^i|(l_k|(\mathcal{D}^{-1}\mathfrak{X}_{ij})(\varsigma)| + m_k) \leqslant \frac{\hat{a}\hat{L}n}{1-\Gamma}\sum_{i=1}^{N}|\mathfrak{X}_i(\varsigma)| + \sum_{i=1}^{N}n^2\hat{a}\hat{m}. \tag{25}$$

Since $l_{ji} = l_{ij} < 0$, $j \neq i$, combining with Assumption 2, we can obtain

$$\begin{aligned} -\sum_{i=1}^{N}\sum_{j=1}^{N}\mathrm{sign}(\mathfrak{X}_i(\varsigma))l_{ij}g(\mathfrak{X}_j(\varsigma)) &\leqslant \sum_{i=1}^{N}\sum_{j=1}^{N}|l_{ij}|\sum_{k=1}^{n}|g_k(\mathfrak{X}_{jk}(\varsigma))| \\ &\leqslant \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{n}|l_{ij}\rho_k\mathfrak{X}_{jk}(\varsigma)| \leqslant N\hat{l}\hat{\rho}\sum_{i=1}^{N}|\mathfrak{X}_i(\varsigma)|. \end{aligned} \tag{26}$$

Meanwhile, note that

$$\begin{aligned} &-\sum_{i=1}^{N}\mathrm{sign}(\mathfrak{X}_i(\varsigma))C^i\mathfrak{X}_i(\varsigma) \leqslant -\underline{c}n\sum_{i=1}^{N}|\mathfrak{X}_i(\varsigma)|, \\ &\sum_{i=1}^{N}\mathrm{sign}(\mathfrak{X}_i(\varsigma))I^i \leqslant \sum_{i=1}^{N}\sum_{j=1}^{n}|I_j^i| \leqslant \sum_{i=1}^{N}n\hat{I}, \\ &-\sum_{i=1}^{N}\mathrm{sign}(\mathfrak{X}_i(\varsigma))C^i\Gamma^i(\mathcal{D}^{-1}\mathfrak{X}_i)(\varsigma-\sigma(\varsigma)) \leqslant \frac{\overline{c}\Gamma n}{1-\Gamma}\sum_{i=1}^{N}|\mathfrak{X}_i(\varsigma-\sigma(\varsigma))|. \end{aligned} \tag{27}$$

Thus, the following two cases should be discussed according to Lemma 1.

- **Case 1.** For $|\mathfrak{X}(\varsigma)| \leqslant o$, using (19), we get

$$\begin{aligned} &-\sum_{i=1}^{N}\mathrm{sign}(\mathfrak{X}_i(\varsigma))\mathrm{sign}(\mathfrak{X}_i(\varsigma))\bigg[(n^2\hat{a}\hat{m}+n\hat{I}) + \Big(-\underline{c}n+\frac{\hat{a}\hat{L}n}{1-\Gamma}+N\hat{l}\hat{\rho}\Big)\arctan(\tau_2\mathfrak{X}_i(\varsigma)) \\ &\qquad + \mathfrak{H}(\varsigma)|\mathfrak{X}_i(\varsigma)| + \mathfrak{F}|\arctan(\tau_2\mathfrak{X}_i(\varsigma))|^v + \frac{\overline{c}\Gamma}{1-\Gamma}|\arctan(\tau_2\mathfrak{X}_i(\varsigma-\sigma(\varsigma)))|\bigg] \\ &\leqslant -\sum_{i=1}^{N}\bigg(-\underline{c}n+\frac{\hat{a}\hat{L}n}{1-\Gamma}+N\hat{l}\hat{\rho}\bigg)\tau_1|\mathfrak{X}_i(\varsigma)| + \sum_{i=1}^{N}\mathfrak{H}(\varsigma)|\mathfrak{X}_i(\varsigma)| - \sum_{i=1}^{N}\mathfrak{F}\tau_1^v|\mathfrak{X}_i(\varsigma)|^v \\ &\qquad - \sum_{i=1}^{N}\tau_1\frac{\overline{c}\Gamma}{1-\Gamma}|\mathfrak{X}_i(\varsigma-\sigma(\varsigma))| - \sum_{i=1}^{N}(n^2\hat{a}\hat{m}+n\hat{I}). \end{aligned} \tag{28}$$

Substituting (25)–(27) and (28) into (24), it gives that $\dot{\Theta}(\varsigma) \leqslant \sum_{i=1}^{N}\mathfrak{H}(\varsigma)|\mathfrak{X}_i(\varsigma)| - \sum_{i=1}^{N}\mathfrak{F}\tau_1^v|\mathfrak{X}_i(\varsigma)|^v$. Consider the following cases.

(1) If $V > 1$, we can deduce from Lemma 3 that

$$\sum_{i=1}^{N}\mathfrak{H}(\varsigma)|\mathfrak{X}_i(\varsigma)| - \sum_{i=1}^{N}\mathfrak{F}\tau_1^v|\mathfrak{X}_i(\varsigma)|^v \leqslant \mathfrak{H}(\varsigma)V - \mathfrak{F}\tau_1^{\wp+1}N^{-\wp}\Theta^{\wp+1}. \tag{29}$$

(2) If $0 < V < 1$, we can deduce from Lemma 3 that

$$\sum_{i=1}^{N}\mathfrak{H}(\varsigma)|\mathfrak{X}_i(\varsigma)| - \sum_{i=1}^{N}\mathfrak{F}\tau_1^v|\mathfrak{X}_i(\varsigma)|^v \leqslant \mathfrak{H}(\varsigma)V - \mathfrak{F}\tau_1^{\wp-1}\Theta^{\wp-1}. \tag{30}$$

- **Case 2.** For $o < |\mathfrak{X}(\varsigma)| \leqslant r$, using (19), we get

$$\begin{aligned} &-\sum_{i=1}^{N}\mathrm{sign}(\mathfrak{X}_i(\varsigma))\mathrm{sign}(\mathfrak{X}_i(\varsigma))\bigg[(n^2\hat{a}\hat{m}+n\hat{I}) + \bigg(-\underline{c}n+\frac{\hat{a}\hat{L}n}{1-\Gamma}+N\hat{l}\hat{\rho}\bigg)\arctan(\tau_2\mathfrak{X}_i(\varsigma)) \\ &\qquad + \mathfrak{H}(\varsigma)|\mathfrak{X}_i(\varsigma)| + \mathfrak{F}|\arctan(\tau_2\mathfrak{X}_i(\varsigma))|^v + \frac{\overline{c}\Gamma}{1-\Gamma}|\arctan(\tau_2\mathfrak{X}_i(\varsigma-\sigma(\varsigma)))|\bigg] \\ &\leqslant -\sum_{i=1}^{N}\bigg(-\underline{c}n+\frac{\hat{a}\hat{L}n}{1-\Gamma}+N\hat{l}\hat{\rho}\bigg)\frac{\arctan(\tau_2 r)}{r}|\mathfrak{X}_i(\varsigma)| + \mathfrak{H}(\varsigma)|\mathfrak{X}_i(\varsigma)| - \sum_{i=1}^{N}(n^2\hat{a}\hat{m}+n\hat{I}) \\ &\qquad - \sum_{i=1}^{N}\frac{\overline{c}\Gamma}{1-\Gamma}\frac{\arctan(\tau_2 r)}{r}\sum_{i=1}^{N}|\mathfrak{X}_i(\varsigma-\sigma(\varsigma))| - \sum_{i=1}^{N}\mathfrak{F}\bigg(\frac{\arctan(\tau_2 r)}{r}\bigg)^v\sum_{i=1}^{N}|\mathfrak{X}_i(\varsigma)|^v. \end{aligned} \tag{31}$$

Substituting (25)–(27) and (31) into (24), it gives that $\dot{\Theta}(\varsigma) \leqslant \sum_{i=1}^{N} \mathfrak{H}(\varsigma)|\mathfrak{X}_i(\varsigma)| - \sum_{i=1}^{N} \mathfrak{F}\left(\frac{\arctan(\tau_2 r)}{r}\right)^{v} |\mathfrak{X}_i(\varsigma)|^{v}$. Then, it needs to make the following discussions.

(1) If $V > 1$, we can deduce from Lemma 3 that

$$\sum_{i=1}^{N} \mathfrak{H}(\varsigma)|\mathfrak{X}_i(\varsigma)| - \sum_{i=1}^{N} \mathfrak{F}\left(\frac{\arctan(\tau_2 r)}{r}\right)^{v} |\mathfrak{X}_i(\varsigma)|^{v} \leqslant \mathfrak{H}(\varsigma)V - \mathfrak{F}\left(\frac{\arctan(\tau_2 r)}{r}\right)^{\wp+1} N^{-\wp}\Theta^{\wp+1}. \tag{32}$$

(2) If $0 < V < 1$, we can deduce from Lemma 3 that

$$\sum_{i=1}^{N} \mathfrak{H}(\varsigma)|\mathfrak{X}_i(\varsigma)| - \sum_{i=1}^{N} \mathfrak{F}\left(\frac{\arctan(\tau_2 r)}{r}\right)^{v} |\mathfrak{X}_i(\varsigma)|^{v} \leqslant \mathfrak{H}(\varsigma)V - \mathfrak{F}\left(\frac{\arctan(\tau_2 r)}{r}\right)^{\wp-1} \Theta^{\wp-1}. \tag{33}$$

Thus, from (29), (30), (32) and (33), we can have

$$\dot{\Theta}(\varsigma) \leqslant \begin{cases} -\mathfrak{H}(\varsigma)\Theta(\varsigma) - \mathfrak{F}_1(\Theta(\varsigma))^{\wp+1}, \Theta(\varsigma) > 1, \\ -\mathfrak{H}(\varsigma)\Theta(\varsigma) - \mathfrak{F}_2(\Theta(\varsigma))^{\wp-1}, \Theta(\varsigma) < 1, \end{cases} \quad \varsigma_\ell \leqslant \varsigma < s_\ell. \tag{34}$$

On the other hand, for $\forall \varsigma \in [s_\ell, \varsigma_{\ell+1})$, it is easy to get that

$$\dot{\Theta}(\varsigma) \leqslant -\rho\Theta(\varsigma), \ \varsigma \in [s_\ell, \varsigma_{\ell+1}), \tag{35}$$

where ρ is well defined in (22).

Based on the Lemma 5, (34) and (35), we can conclude that Eq. (18) can achieve FxT stability under the bounded intermittent controller (19), and the estimation of ST is given in (21).

Up to now, the proof is complete.

Remark 8. From (19), it is clear to see that whether $\varsigma_\ell \leqslant \varsigma < s_\ell$ or $s_\ell \leqslant \varsigma < \varsigma_{\ell+1}$, $\mathbf{u}_i(\varsigma)$ is always bounded, which can guarantee the control constraint and solve the large value problems arising in the designed intermittent controllers, such as [7–10, 13, 20, 21].

Moreover, quantized intermittent control in [41] can ensure the control constraint, but it cannot capture the complete information of the system. The designed control strategy $\mathbf{u}_i(\varsigma)$ can ensure the global information of the system since there is no restriction on the state. Consequently, the designed control strategy (19) is effective and offers more advantages.

4.2 Practical FxT stability via Lemma 7

In this section, we will establish some criteria to guarantee practical FxT stability under intermittent control for (18) based on Lemma 7, which is an absolutely new result.

Design the following bounded intermittent controller:

$$\begin{cases} u_i(\varsigma) = -\mathrm{sign}(\mathfrak{X}_i(\varsigma))\Big[(n^2\hat{a}\hat{m} + n\hat{I} - \hat{\gamma}) + \big(-\underline{c}n + \frac{\hat{a}\hat{L}n}{1-\Gamma} + N\hat{l}\hat{\rho}\big)|\arctan(\tau_2\mathfrak{X}_i(\varsigma))| \\ \qquad + \frac{\overline{c}\Gamma}{1-\Gamma}|\arctan(\tau_2\mathfrak{X}_i(\varsigma-\sigma(\varsigma)))| + \mathfrak{H}(\varsigma)|\mathfrak{X}_i(\varsigma)| + \mathfrak{F}|\arctan(\tau_2\mathfrak{X}_i(\varsigma))|^{v}\Big], \varsigma_\ell \leqslant \varsigma < s_\ell, \\ u_i(\varsigma) = -\mathrm{sign}(\mathfrak{X}_i(\varsigma))\Big[(n^2\hat{a}\hat{m} + n\hat{I} - \hat{\kappa}) + \big(-\underline{c}n + \frac{\hat{a}\hat{L}n}{1-\Gamma} + N\hat{l}\hat{\rho} + \omega\big)|\arctan(\tau_2\mathfrak{X}_i(\varsigma))| \\ \qquad + \frac{\overline{c}\Gamma}{1-\Gamma}|\arctan(\tau_2\mathfrak{X}_i(\varsigma-\sigma(\varsigma)))|\Big], s_\ell \leqslant \varsigma < \varsigma_{\ell+1}, \end{cases} \tag{36}$$

where $\ell \in \mathbb{D}$, $\nu = \wp + \mathrm{sign}(\Theta(\varsigma) - 1)$, $\mathfrak{H}(\varsigma)$ satisfies the inequality (4), and $\omega, \kappa = N\hat{\kappa}, \gamma = N\hat{\gamma} > 0$ are constants.

Theorem 2. If Assumptions 1 and 2 hold, inequalities in (20) are satisfied. Then, we have the following results.

(i) CNCNs (18) achieves intermittent practical FxT stability with the residual set given as follows:

$$\Omega = \left\{\mathfrak{X}|\Theta(\mathfrak{X}) \leqslant \max\left\{\left(\frac{\gamma}{\mathfrak{F}_1(1-\phi)}\right)^{\frac{1}{\wp+1}}, \left(\frac{\gamma}{\mathfrak{F}_2(1-\phi)}\right)^{\frac{1}{\wp-1}}, \frac{\kappa}{\rho\phi}\right\}\right\},$$

where $\phi \in (0, 1)$. And the estimation of ST to attain the residual set is

$$T_{\max} = \frac{1}{\wp\mathfrak{F}_1\phi\exp\{-\wp\tau_1 N_2\}(1-\Psi)} + \frac{1}{(2-\wp)\mathfrak{F}_2\phi\exp\{(2-\wp)\frac{\arctan(\tau_2 r)}{r}N_1\}(1-\Psi)},$$

where Ψ is defined in Lemma 2, $\mathfrak{F}_1$, $\mathfrak{F}_2$ and ρ are defined in (22).

(ii) After entering the residual set Ω, Eq. (18) achieves asymptotic stability, i.e., $\lim_{\varsigma\to+\infty} |\mathfrak{X}(\varsigma)| = 0$.

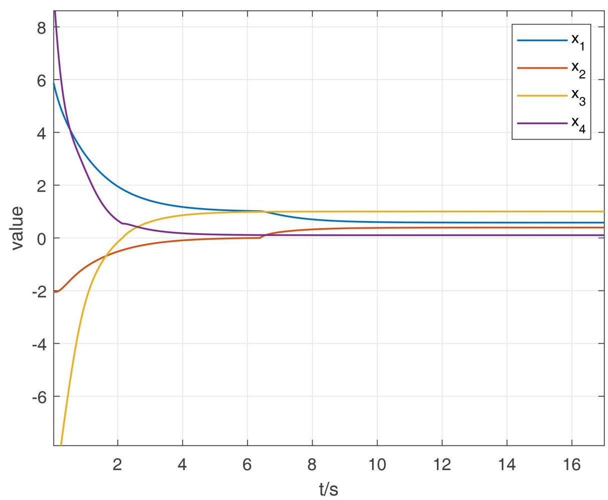

Figure 1 (Color online) Time evolution of states without controller.

Proof. First, construct the same Lyapunov function as that in (23). Based on the proof in Theorem 1, we can obtain that the result (i).

As for (ii), based on the proof in Theorem 1, we have $\dot{\Theta}(\varsigma) \leqslant \sum_{i=1}^{N} \mathfrak{H}(\varsigma)|\mathfrak{X}_i(\varsigma)| - \sum_{i=1}^{N} \mathfrak{F}\tau_1^{v}|\mathfrak{X}_i(\varsigma)|^{v}$. It is easy to see that $\dot{\Theta} \equiv 0$ requires $\mathfrak{X}_i = 0$. By the LaSalle's invariance principle, it follows that Eq. (18) with (36) achieves the asymptotic stability.

Up to now, the proof is complete.

Remark 9. From the previous practical FxT stability results in [20, 21], one can easily see that the stability lemmas cannot be applied to obtain the results in Theorems 1 and 2 since their stability lemmas are the special cases of Lemmas 6 and 7. Moreover, as pointed out in [17], after the state enters the residual set, the dynamical behaviors of the state should be discussed, but there is no related discussion in [20, 21]. From the proof in Theorem 2, one can see that, under the bounded intermittent control strategy (36), Eq. (18) achieves the asymptotic stability after the state enters the residual set, and $\varsigma \to +\infty$. The result is absolutely new.

5 Application and simulations

In [42], a drilling system described by a switched neutral type delay equation with nonlinear perturbations has been considered. But, the nonlinear perturbations were bounded and continuous. In the section, a drilling system described by the CNCNs (17) is considered, which is more generalized.

Example 1. Consider the drilling system described by CNCNs (17), let the coefficients be

$$C^1 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, C^2 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, C^3 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, A^1 = \begin{pmatrix} 2 & -0.1 \\ 4 & 4.2 \end{pmatrix}, A^2 = \begin{pmatrix} 2 & -0.14 \\ 4 & 4.3 \end{pmatrix},$$

$$A^3 = \begin{pmatrix} 2 & -0.12 \\ 4 & 4.1 \end{pmatrix}, \Gamma^1 = \begin{pmatrix} 0.2 & -0.1 \\ -0.1 & -0.2 \end{pmatrix}, \Gamma^2 = \begin{pmatrix} -0.3 & -0.1 \\ 0.3 & -0.2 \end{pmatrix}, \Gamma^3 = \begin{pmatrix} -0.1 & -0.2 \\ -0.1 & 0.4 \end{pmatrix},$$

$$I^1 = (0.3, 0.2)^{\top},\ I^2 = (0.1, 0.2)^{\top},\ I^3 = (0.2, 0.4)^{\top},$$

and the Laplacian matrix is $L = \begin{pmatrix} -1 & -1 \\ -1 & -2 \end{pmatrix}$. Moreover, let the discontinuous function be $f(\mathfrak{X}) = 0.1\tanh(\mathfrak{X}) + 0.1\mathrm{sign}(\mathfrak{X})$, and the nonlinear coupling function is defined as $g(v) = \begin{cases} 2v_1 + 0.3\sin v_1, \\ 1.8v_2 + 0.6\cos v_2. \end{cases}$ Select $\hat{L} = 0.2$, $\hat{m} = 0.2$ and $\hat{\rho} = 2.4$. Then, Assumptions 1 and 2 hold. Let $\varsigma_0 = 0$ and let the initial state be $\mathfrak{X}(0) = [6, -2, -10, 10]^{\top}$. Without control inputs, the states will not achieve stability at the origin within a fixed time, see Figure 1.

Let $\mathfrak{H}(\varsigma) = 0.5\cos(\varsigma) - \frac{1}{(1+\varsigma)^2}$, $\varsigma > 0$. Then we can choose $N_1 = 0.5$ and $N_2 = 1.5$. Select the control gains as $\hat{a}_1 = 4.2$, $\hat{a}_2 = 4.3$, $\hat{a}_3 = 4.1$, $\overline{c} = 0.1$, $\underline{c} = 0.1$, $\hat{l} = 2$, $\hat{I}_1 = 0.3$, $\hat{I}_2 = 0.2$, $\hat{I}_3 = 0.4$, $\Gamma = 0.5$, $\gamma = 1$, $\kappa = 0.6$, $\omega = 9.6$,

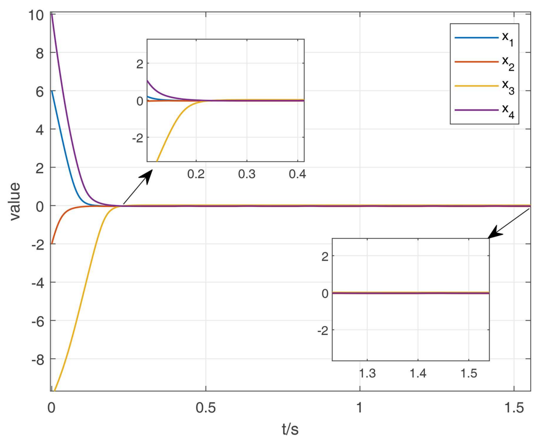

Figure 2 (Color online) Time evolution of states under bounded intermittent controller (37).

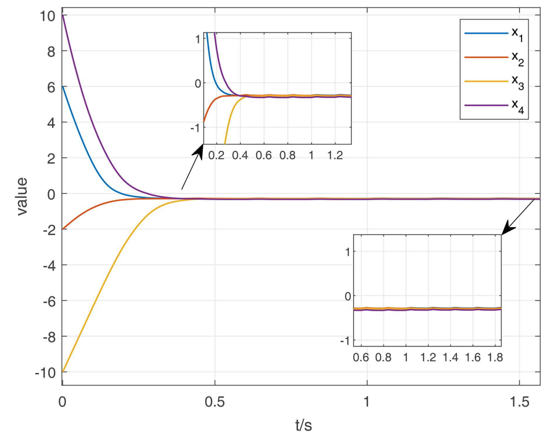

Figure 3 (Color online) Time evolution of states under bounded intermittent controller (38).

$\mathfrak{F} = 19.9$ and $\wp = 1.1$, $v = 1.1 + \mathrm{sign}(\sum_{i=1}^{3} |\mathfrak{X}_i(\varsigma)| - 1)$. Design the bounded control as follows:

$$\begin{cases} u_1(\varsigma) = -\mathrm{sign}(\mathfrak{X}_1(\varsigma))\Big[4.24 + 17.64|\arctan(\tau_2\mathfrak{X}_i(\varsigma))| + (0.5\cos(\varsigma) - \frac{1}{(1+\varsigma)^2})|\mathfrak{X}_1(\varsigma)| \\ \qquad +19.9|\arctan(\tau_2\mathfrak{X}_1(\varsigma))|^v + 0.1|\arctan(\tau_2\mathfrak{X}_1(\varsigma - \sigma(\varsigma)))|\Big], \varsigma_\ell \leqslant \varsigma < s_\ell, \\ u_1(\varsigma) = -\mathrm{sign}(\mathfrak{X}_1(\varsigma))\Big[4.24 + 27.24|\arctan(\tau_2\mathfrak{X}_1(\varsigma))| + 0.1|\arctan(\tau_2\mathfrak{X}_1(\varsigma - \sigma(\varsigma)))|\Big], s_\ell \leqslant \varsigma < \varsigma_{\ell+1}, \end{cases}$$
$$\begin{cases} u_2(\varsigma) = -\mathrm{sign}(\mathfrak{X}_2(\varsigma))\Big[4.24 + 17.64|\arctan(\tau_2\mathfrak{X}_2(\varsigma))| + (0.5\cos(\varsigma) - \frac{1}{(1+\varsigma)^2})|\mathfrak{X}_2(\varsigma)| \\ \qquad +19.9|\arctan(\tau_2\mathfrak{X}_2(\varsigma))|^v + 0.1|\arctan(\tau_2\mathfrak{X}_2(\varsigma - \sigma(\varsigma)))|\Big], \varsigma_\ell \leqslant \varsigma < s_\ell, \\ u_2(\varsigma) = -\mathrm{sign}(\mathfrak{X}_2(\varsigma))\Big[4.24 + 27.24|\arctan(\tau_2\mathfrak{X}_2(\varsigma))| + 0.1|\arctan(\tau_2\mathfrak{X}_2(\varsigma - \sigma(\varsigma)))|\Big], s_\ell \leqslant \varsigma < \varsigma_{\ell+1}, \end{cases} \tag{37}$$
$$\begin{cases} u_3(\varsigma) = -\mathrm{sign}(\mathfrak{X}_3(\varsigma))\Big[4.24 + 17.64|\arctan(\tau_2\mathfrak{X}_3(\varsigma))| + (0.5\cos(\varsigma) - \frac{1}{(1+\varsigma)^2})|\mathfrak{X}_3(\varsigma)| \\ \qquad +19.9|\arctan(\tau_2\mathfrak{X}_3(\varsigma))|^v + 0.1|\arctan(\tau_2\mathfrak{X}_3(\varsigma - \sigma(\varsigma)))|\Big], \varsigma_\ell \leqslant \varsigma < s_\ell, \\ u_3(\varsigma) = -\mathrm{sign}(\mathfrak{X}_3(\varsigma))\Big[4.24 + 27.24|\arctan(\tau_2\mathfrak{X}_3(\varsigma))| + 0.1|\arctan(\tau_2\mathfrak{X}_3(\varsigma - \sigma(\varsigma)))|\Big], s_\ell \leqslant \varsigma < \varsigma_{\ell+1}. \end{cases}$$

Lemma 1 holds with $\tau_1 = 1$, $\tau_2 = 5$, $o = 1.43$, $r = 1.5$. By direct calculation, we can get that the inequality in Theorem 1 holds and

$$T_{\max} = \frac{1}{\wp\mathfrak{F}_1 \exp\{-\wp\tau_1 N_2\}(1 - \Psi)} + \frac{1}{(2-\wp)\mathfrak{F}_2 \exp\{(2-\wp)\frac{\arctan(\tau_2 r)}{r} N_1\}(1 - \Psi)} \approx 1.137 \text{ (s)},$$

where

$$\mathfrak{F}_1 = \min\left\{\mathfrak{F}\tau_1^{\wp+1} N^{-\wp}, \mathfrak{F}\left(\frac{\arctan(\tau_2 r)}{r}\right)^{\wp+1} N^{-\wp}\right\} \approx 5.441,$$
$$\mathfrak{F}_2 = \min\left\{\mathfrak{F}\tau_1^{\wp-1}, \mathfrak{F}\left(\frac{\arctan(\tau_2 r)}{r}\right)^{\wp-1}\right\} \approx 18.218,\ \rho = \min\left\{\omega\tau_1, \frac{\arctan(\tau_2 r)}{r}\omega\right\} \approx 9.205.$$

Moreover, by making the use of Theorem 1, it can yield that Eq. (16) achieves FxT stability via Lemma 5 with the intermittent controller (19). See Figure 2 for details.

Furthermore, select $\gamma = 1$, $\kappa = 0.6$, and design the following bounded control:

$$\begin{cases} u_1(\varsigma) = -\mathrm{sign}(\mathfrak{X}_1(\varsigma))\Big[3.91 + 17.64|\arctan(\tau_2\mathfrak{X}_1(\varsigma))| + 0.1|\arctan(\tau_2\mathfrak{X}_1(\varsigma - \sigma(\varsigma)))| \\ \qquad +(0.5\cos(\varsigma) - \frac{1}{(1+\varsigma)^2})|\mathfrak{X}_1(\varsigma)| + 19.9|\arctan(\tau_2\mathfrak{X}_1(\varsigma))|^v\Big], \varsigma_\ell \leqslant \varsigma < s_\ell, \\ u_1(\varsigma) = -\mathrm{sign}(\mathfrak{X}_1(\varsigma))\Big[4.04 + 27.24|\arctan(\tau_2\mathfrak{X}_1(\varsigma))| + 0.1|\arctan(\tau_2\mathfrak{X}_1(\varsigma - \sigma(\varsigma)))|\Big], s_\ell \leqslant \varsigma < \varsigma_{\ell+1}, \end{cases}$$

$$\begin{cases} u_2(\varsigma) = -\mathrm{sign}(\mathfrak{X}_2(\varsigma))\Big[3.91 + 17.64|\arctan(\tau_2\mathfrak{X}_2(\varsigma))| + 0.1|\arctan(\tau_2\mathfrak{X}_2(\varsigma - \sigma(\varsigma)))| \\ \qquad +(0.5\cos(\varsigma) - \frac{1}{(1+\varsigma)^2})|\mathfrak{X}_2(\varsigma)| + 19.9|\arctan(\tau_2\mathfrak{X}_2(\varsigma))|^v\Big], \varsigma_\ell \leqslant \varsigma < s_\ell, \\ u_2(\varsigma) = -\mathrm{sign}(\mathfrak{X}_2(\varsigma))\Big[4.04 + 27.24|\arctan(\tau_2\mathfrak{X}_2(\varsigma))| + 0.1|\arctan(\tau_2\mathfrak{X}_2(\varsigma - \sigma(\varsigma)))|\Big], s_\ell \leqslant \varsigma < \varsigma_{\ell+1}, \end{cases}$$

$$\begin{cases} u_3(\varsigma) = -\mathrm{sign}(\mathfrak{X}_3(\varsigma))\Big[3.91 + 17.64|\arctan(\tau_2\mathfrak{X}_3(\varsigma))| + 0.1|\arctan(\tau_2\mathfrak{X}_3(\varsigma - \sigma(\varsigma)))| \\ \qquad +(0.5\cos(\varsigma) - \frac{1}{(1+\varsigma)^2})|\mathfrak{X}_3(\varsigma)| + 19.9|\arctan(\tau_2\mathfrak{X}_3(\varsigma))|^v\Big], \varsigma_\ell \leqslant \varsigma < s_\ell, \\ u_3(\varsigma) = -\mathrm{sign}(\mathfrak{X}_3(\varsigma))\Big[4.04 + 27.24|\arctan(\tau_2\mathfrak{X}_3(\varsigma))| + 0.1|\arctan(\tau_2\mathfrak{X}_3(\varsigma - \sigma(\varsigma)))|\Big], s_\ell \leqslant \varsigma < \varsigma_{\ell+1}. \end{cases} \tag{38}$$

Then, by direct calculation, we can get that the conditions in Theorem 2 hold. Let $\phi = 0.95$ in Theorem 2. Then Eq. (16) achieves practical FxT stability via Lemma 7 and intermittent controller (38) with the residual set given as follows: $\Omega = \{\mathfrak{X}|\Theta(\mathfrak{X}) \leqslant 0.4964\}$, where $\max\left\{\left(\frac{\gamma}{\mathfrak{F}_1(1-\phi)}\right)^{\frac{1}{\wp+1}}, \left(\frac{\gamma}{\mathfrak{F}_2(1-\phi)}\right)^{\frac{1}{\wp-1}}, \frac{\kappa}{\rho\phi}\right\} \approx 0.4964$. And the estimation of ST to attain the residual set is $T_{\max} \approx 1.197(s)$, see Figure 3 for details. As pointed out in Remark 1, the residual set can be adjusted to the desired level by adjusting the parameter ϕ.

Comparisons and discussions.

• The construction of the intermittent FxT stability lemma is of paramount importance. Since the intermittent intervals are discontinuous, this renders the intermittent FxT stability lemma considerably more complex than its continuous-time counterpart. Nonetheless, the negative definiteness of the derivative of the V-function is an immutable attribute. Existing literature has sought to refine the inequality of the V-function's derivative; yet it has overlooked the necessity to preserve the negative definiteness of the V-function's derivative. In comparison to the lemmas in [12, 20, 21], the negative derivative of the Lyapunov function is guaranteed in the newly established intermittent FxT stability lemmas in the paper, which provides the theoretical underpinning for the closed-loop system to ultimately achieve FxT stability and practical FxT stability, see Figure 2.

• In the study of FxT stability, the use of bounded feedback is very rare. Bounded control is an indispensable aspect of control system design, as it ensures the system operates stably, safely, and efficiently under various working conditions by restricting the range of control inputs. Different from the FxT control in [11–13], and practical FxT control in [17–21], in the paper, the bounded control has been designed. When the FxT stability is achieved, from (37) and (38), the controllers converge to zero, which signifies the accomplishment of the control objective and the cessation of the control operation.

6 Conclusion

This paper investigates the FxT stability and practical FxT stability of CNCNs with mismatched parameters. New intermittent stability lemmas are proposed. The indefinite function and unified exponent condition on the Lyapunov function are utilized, which can encompass and improve upon previous results. As highlighted in many studies, the unavailability of global information is inevitable. Therefore, practical FxT stability is more realistic. Intermittent practical FxT stability lemmas are also derived. More generalized inequality conditions are provided. New estimations of the ST to reach the residual set are presented, which are determined by the constant parameters of the system. Based on the newly established stability lemmas, the FxT stability and practical FxT stability of the considered CNCNs are studied using bounded controllers.

Time delay is an unavoidable factor in networked control systems, making the investigation of practical FxT stability for mismatched CNCNs with time delays both a challenging and fascinating problem.

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