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Asynchronous and aperiodic sampled-data control for general linear multiagent systems

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Abstract In this paper, an asynchronous and aperiodic sampled-data control scheme for continuous-time multiagent systems (MASs) with intermittent communication is investigated. We propose a novel time-based and aperiodic sampling scheme for data transmissions among agents. In distributed MASs, the accurate timing alignment of different agents is difficult to guarantee; thus, data-sampling time sequences are naturally asynchronous for different agents. Initially, a minimum allowable sampling period is enforced after each data sampling to eliminate the Zeno behavior. The maximum allowable sampling periods play a crucial role in ensuring the consensus of MASs. We focus on prolonging the intersampling periods while ensuring the consensus of MASs. A reverse average dwell time condition is introduced, which can significantly improve the maximum allowable sampling periods. Moreover, additional dynamic clock variables are introduced to characterize the sampling intervals. Based on hybrid system theories, some sufficient conditions that guarantee the consensus of MASs are given. The results indicate that there exist certain trade-offs between the maximum allowable sampling period and reverse average dwell time. Finally, some numerical simulations are provided to demonstrate the effectiveness of the obtained theoretical results.

Keywords asynchronous sampling, hybrid systems, multiagent systems, reverse average dwell time

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1 Introduction

In recent years, the cooperative control of multiagent systems (MASs) has received considerable attention in various fields, such as vehicle platooning [1–3], formation of mobile robots [4–6], reconnaissance of unmanned aerial vehicles [7, 8], and games [9]. An important task in the cooperative control of MASs is to design distributed control laws to achieve collaborative control goals [10]. In traditional consensus control, information interaction among agents is generally assumed to be continuous in time. However, the computation and communication resources are limited in practical systems; thus, continuous-time communication is unrealistic. To cope with the limitations of communication resources in practical systems, time-based [11–15] and event-based [16–18] communication strategies have been widely adopted in MASs and networked control systems. The main idea of event-based communications is that communications are triggered whenever a predefined state-dependent criterion is satisfied. In event-triggered communication schemes, the sampling instants can be determined based on the real-time fluctuation of agent states [19], which usually requires the real-time monitoring of agent states. In contrast to event-based communication schemes, time-based communication schemes can be state-independent.

The time-based data-sampling schemes mainly include periodic and aperiodic sampling strategies. In periodic sampling strategies, data sampling is activated after a fixed elapsed time [20]. In [21], a time-based sampling scheme with variable sampling periods was designed to solve leader-following consensus problems. In [22, 23], the consensus problems of linear MASs with a probabilistic periodic sampling strategy were investigated. The sampling periods were assumed to switch between two different values. However, the explicit upper bounds of sampling periods were not provided in the existing sampling strategies with variable periods [21–24].

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In the aforementioned literature, some sufficient conditions in terms of maximum allowable sampling periods were given to ensure the achievement of the control objectives. Technically, as long as the upper bounds of the sampling periods satisfy some specified conditions, the control objectives can be achieved. Therefore, we focus on how to improve the estimates of the maximum allowable sampling periods while ensuring the consensus of MASs. In [25–27], the use of the reverse average dwell time condition improves the estimates of the maximum allowable sampling periods or impulsive intervals. However, the entire system is still required to respond synchronously at sampling time instants. Different from traditional networked control systems, multiple agents cannot ensure clock synchronization and accurate timing alignment in distributed MASs [28,29]. Notably, asynchronous behavior among agents is ubiquitous in many large-scale networks of MASs [30–35]. Therefore, asynchronous sampling schemes for MASs are more relevant in real scenarios. However, because of the complexity of coupled MASs, the theoretical analysis of asynchronous MASs is more challenging than that of synchronous MASs.

In this study, we focus on the design of asynchronous and aperiodic sampled-data schemes for continuous-time MASs with general linear dynamics. Different from traditional periodic sampling strategies [20], we propose a novel time-based and aperiodic sampling strategy for MASs. However, one technical challenge in the event-triggered sampling scheme is the exclusion of potential Zeno behavior. In our framework, a minimum allowable intersampling period is enforced for each agent after each sampling instant, which rules out the Zeno behavior. Notably, the enforcement of a minimum allowable intersampling period can negatively affect the convergence of systems [12], whereas asymptotic convergence is guaranteed in this study. Moreover, the maximum allowable sampling periods play a crucial role in ensuring the consensus of MASs. Naturally, the estimates of the maximum allowable sampling periods could be as large as possible to relax the system consensus conditions. A reverse average dwell time condition is obtained for each agent, and it can significantly improve the estimates of the maximum allowable sampling periods. Therefore, the asynchronous and aperiodic sampling time sequence of each agent is subject to the conditions of the minimum, maximum, and average allowable sampling periods. Compared with event-based communication schemes [36–39], the proposed time-based sampling scheme is state-independent and the communication time instants are precomputed offline, which means that the real-time monitoring of system states is not required. In an asynchronous MAS, each agent has its sampling time sequence and each agent sends the sampled data to its neighbors only at sampling instants. However, the analysis of the restriction on the "average" in asynchronous and coupled MASs is technically challenging. We use the tools from hybrid systems to analyze and solve this problem. The conditions for generating the sampling time sequences are state-independent, which leads to difficulties in consensus analysis. To record the elapsed time after each sampling action, extra clock variables are introduced. Then, a hybrid system model, which integrates system states, observer states, and two clock variables, is established. The conditions that guarantee the consensus of MASs are derived from the designed Lyapunov function.

This study analyzes the design and implementation of both aperiodic sampling and distributed control schemes in asynchronous continuous-time MASs with intermittent communication. The main contributions of this study are outlined as follows.

(1) For the investigated asynchronous sampled-data MASs with general linear dynamics, a novel asynchronous and aperiodic sampling scheme is proposed to reduce communication costs. To improve the estimates of the maximum allowable sampling periods on the premise of state consensus, the concept of the average allowable sampling period under reverse average dwell time conditions, which requires that, on average, there is at least one sampling action in the time interval of a length equal to the average allowable sampling period, is introduced. In addition, the minimum allowable sampling period is enforced after each sampling action to naturally eliminate the Zeno behavior. The proposed sampling scheme not only prevents the conservatism of traditional fixed-period sampling schemes but also improves the estimates of the maximum allowable sampling periods.

(2) To solve the consensus problem of asynchronous sampled-data MASs, a distributed observer-based control protocol is designed for each agent. In the case of unmeasurable system states, a state observer is designed to observe the system states of each agent. Agents only transmit the state of their observer to neighbor agents at sampling instants. In contrast to the sample-and-hold strategies [23], the sampled data of observers are indirectly used in the consensus protocol, and the appropriate state estimation in terms of the sampled data is employed to estimate the real-time states of observers.

(3) Some sufficient conditions in terms of the minimum and maximum allowable sampling periods and the reverse average dwell time are derived to ensure consensus. To handle the analytical difficulties of the restriction on the "average" in asynchronous and coupled MASs, technically, two additional dynamic clock variables are introduced in the hybrid system model to assist in measuring the intersampling times. Then, a novel Lyapunov function, which includes system states, observer states, and two prior unknown functions for clock variables, is designed to analyze the consensus conditions of MASs. The results of this study show the existence of a trade-off between the maximum

allowable sampling period and the reverse average dwell time.

The remainder of this paper is organized as follows. Some basic concepts are given in Section 2. The considered problem is formulated and the hybrid system model is established in Section 3. The main results are discussed in Section 4. A simulation example is given in Section 5. Finally, the conclusion and future work are presented in Section 6.

2 Preliminaries

2.1 Notations

$\mathbb{N}$ and $\mathbb{R}$ denote the sets of natural and real numbers, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. $\mathbf{0}$ ($\mathbf{1}$) denotes the appropriate dimensional vector with all elements being 0 (1). I_m denotes the $m\times m$ identity matrix. $\|\cdot\|$ denotes the 2-norm. $\langle\cdot,\cdot\rangle$ and $\otimes$ denote the inner and Kronecker products, respectively. $\lambda_{\max}(\Gamma)$ and $\lambda_{\min}(\Gamma)$ denote the maximum and minimum eigenvalues of the symmetric matrix Γ, respectively. For the set $\mathcal{S}$, $\overline{\mathcal{S}}$ denotes the closure of the set $\mathcal{S}$.

2.2 Graph theory

Consider an MAS containing m nodes and M edges. The communication topology among agents is described as an undirected graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$ with a set of nodes $\mathcal{V}=\{1,2,\dots,m\}$ and a set of edges $\mathcal{E}=\{1,2,\dots,M\}$. $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{m\times m}$ denotes the adjacency matrix of graph $\mathcal{G}$, where $a_{ij}>0$ if $(j,i)\in\mathcal{E}$, and $a_{ij}=0$ otherwise. Node j is called the neighbor of node i if $a_{ij}\neq 0$. The set of neighbors of node i is represented by $\mathcal{N}_i=\{j\in\mathcal{V}:(j,i)\in\mathcal{E}\}$. $\mathcal{L}=[l_{ij}]_{m\times m}$ denotes the Laplacian matrix of graph $\mathcal{G}$, where $l_{ij}=\sum_{j\in\mathcal{N}_i}a_{ij}$ if $j=i$ and $l_{ij}=-a_{ij}$ if $j\neq i$. $D=[d_{i\ell}]\in\mathbb{R}^{m\times M}$ denotes the incidence matrix of graph $\mathcal{G}$, where $d_{i\ell}=1$ if node i is the head of the ℓ-th edge, $d_{i\ell}=-1$ if node i is the tail of the ℓ-th edge, and $d_{i\ell}=0$ otherwise. In an undirected graph, $\mathcal{L}=DD^{\mathrm{T}}$. For each graph $\mathcal{G}$, the eigenvalues of $\mathcal{L}$ can be listed in increasing order, as follows: $0\leqslant\lambda_1(\mathcal{L})\leqslant\lambda_2(\mathcal{L})\leqslant\cdots\leqslant\lambda_m(\mathcal{L})$.

Assumption 1. The communication graph $\mathcal{G}$ is undirected and connected.

2.3 Hybrid systems

A hybrid system $\mathcal{H}=(\mathcal{C},F,\mathcal{D},G)$ is a tuple composed of a flow set $C\in\mathbb{R}^n$, a jump set $\mathcal{D}\in\mathbb{R}^n$, a flow map $F:\mathbb{R}^n\to\mathbb{R}^n$, and a jump map $G:\mathbb{R}^n\rightrightarrows\mathbb{R}^n$. When $x\in\mathcal{C}$, the system can flow continuously. When $x\in\mathcal{D}$, the system can jump discontinuously. We recall some definitions related to hybrid systems [40].

Definition 1 (Hybrid time domains [40])**.** A set $E\subset\mathbb{R}\times\mathbb{N}$ is called a compact hybrid time domain if $E=\cup_{k=0}^{S-1}([t_k,t_{k+1}],k)$ for a finite sequence of times $0\leqslant t_0\leqslant t_1\leqslant\cdots\leqslant t_S$. A set E is called a hybrid time domain if $E\cap([0,T]\times\{0,1,\dots S\})$, $\forall(T,S)\in E$ is a compact hybrid time domain.

Definition 2 (Solutions to hybrid systems [40])**.** A hybrid arc $\varphi:\mathrm{dom}(\varphi)\to\mathbb{R}^n$ is a solution to $\mathcal{H}$ if the following conditions are satisfied: (i) $\varphi(0,0)\in\mathcal{C}\cup\mathcal{D}$; (ii) $\varphi(t,k)\in\mathcal{C}$ for any $k\in\mathbb{N}$ and $\dot\varphi(t,k)\in F(\varphi(t,k))$ for almost all $t\in\{t:(t,k)\in\mathrm{dom}(\varphi)\}$; (iii) $\varphi(t,k)\in\mathcal{D}$ and $\varphi(t,k+1)\in G(\varphi(t,k))$ for all $(t,k)\in\mathrm{dom}(\varphi)$ and $(t,k+1)\in\mathrm{dom}(\varphi)$.

3 Problem formulation

3.1 MAS model

Consider a continuous-time MAS consisting of m agents. The dynamics of agent i is described as follows:

$$\begin{cases}\dot x_i(t)=Ax_i(t)+Bu_i(t),\\ y_i(t)=Cx_i(t),\ i=1,2,\dots,m,\end{cases}\tag{1}$$

where $x_i(t)\in\mathbb{R}^{n_x}$ and $y_i(t)\in\mathbb{R}^{n_y}$ are the state vector and the measured output of agent i, respectively; $u_i(t)\in\mathbb{R}^{n_u}$ is the control input of agent i; and A, B, and C are constant matrices. We assume that (A,B) is stabilizable and (C,A) is observable. Then, the following Riccati equations hold for the positive definite matrices P and Q:

$$A^{\mathrm{T}}P+PA-PBB^{\mathrm{T}}P+I=0,\tag{2}$$

and

$$QA^{\mathrm{T}}+AQ-QC^{\mathrm{T}}CQ+I=0.\tag{3}$$

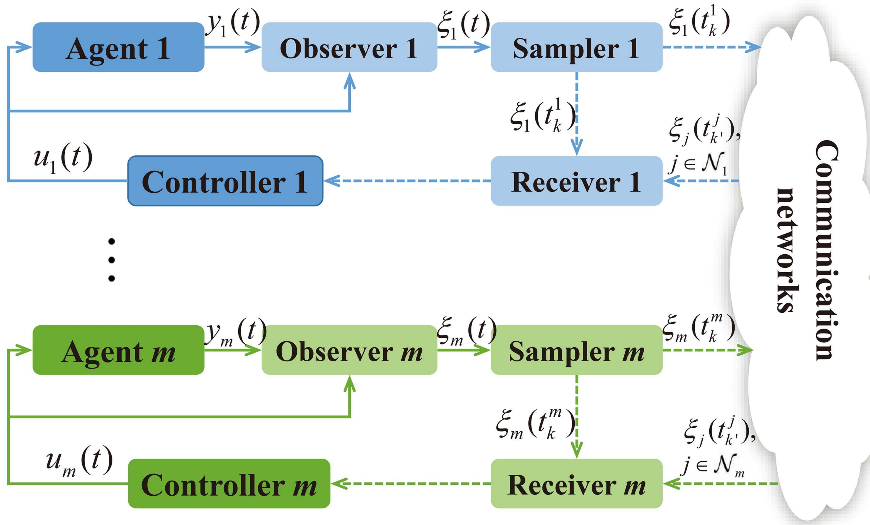

Figure 1 (Color online) Illustration of the information interaction (the solid line denotes the transmission of continuous-time signals, and the dashed line denotes the transmission of discrete-time signals).

Because the state $x_i(t)$ is indirectly measurable and only the output $y_i(t)$ are available, a state observer for agent i is designed as follows:

$$\dot{\xi}_i(t) = A\xi_i(t) + Bu_i(t) + F(C\xi_i(t) - y_i(t)), \tag{4}$$

where $\xi_i(t)$ is the observer state associated with agent i and $F = -\frac{1}{2}QC^{\mathrm{T}}$ is the observer gain matrix.

In this study, we consider asynchronous time-based sampling schemes. We denote the sampling time sequence as $\{t_k^i\}_{k\in\mathbb{N}} \triangleq \{t_0^i, t_1^i, \ldots\}$, where t_k^i is the k-th sampling instant of agent i. The basic illustration of the information interaction of MASs considered in this study is shown in Figure 1. For agent i, each observer observes the corresponding agent state, and the sampled state $\xi_i(t_k^i)$, $k = 0, 1, 2, \ldots$, is transmitted to its neighbors at sampling instants. The distributed consensus protocol $u_i(t)$ is expressed as follows:

$$u_i(t) = -K \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{\xi}_i(t) - \hat{\xi}_j(t)), \tag{5}$$

where $K = B^{\mathrm{T}}P$ is the feedback gain matrix and $\hat{\xi}_i(t)$ is the state estimation of $\xi_i(t)$ with the following dynamics:

$$\begin{cases} \dot{\hat{\xi}}_i(t) = A\hat{\xi}_i(t), \ t \neq t_k^i, \\ \hat{\xi}_i(t) = \xi_i(t), \quad t = t_k^i. \end{cases} \tag{6}$$

The objective of this study is to design a distributed observer-based control protocol under asynchronous sampling strategies for MASs (Eq. (1)) to achieve a consensus, i.e., $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \forall i, j \in \mathcal{V}$. $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \forall i, j \in \mathcal{V}$.

Remark 1. In Figure 1, each agent broadcasts its observer states to its neighbors at its sampling instants and receives the observer states of the neighbors at the sampling instants of the neighbors via networks. In addition, the received discrete-time observer states are indirectly used in the consensus protocol. By contrast, the state estimation $\hat{\xi}_i(t)$ is used. It follows from (6) that $\hat{\xi}_i(t)$ is implemented as an open-loop estimate during $t \in [t_k^i, t_{k+1}^i)$ and the estimation values are reset to the sampling state at discrete time t_k^i. Therefore, $u_i(t)$ is piecewise continuous.

3.2 Asynchronous sampling strategy

As shown in Figure 1, the data are sampled and broadcasted via networks at sampling instants. For different agents, the data may be transmitted via different networks (e.g., wired/wireless networks). In our framework, the data sampling of different agents is independent and asynchronous. For agent i, let $h_k^i = t_{k+1}^i - t_k^i$, where $i \in \mathcal{V}$ and $k \in \mathbb{N}$. For all agents, Assumption 2 restricts the sampling intervals.

Assumption 2. For agent i with $i \in \mathcal{V}$, there exist the constants $\tau_{\max}^i > 0$ and $\tau_{\min}^i \in (0, \tau_{\max}^i)$ such that

$$\tau_{\min}^i \leqslant h_k^i \leqslant \tau_{\max}^i, \forall k \in \mathbb{N}. \tag{7}$$

In Assumption 2, $\tau^i_{\max} > 0$ is called the maximum allowable sampling period and $\tau^i_{\min} > 0$ is called the minimum allowable sampling period. Notably, Assumption 2 not only restricts the maximum sampling period $\tau^i_{\max}$ to ensure system stability but also restricts the minimum sampling period $\tau^i_{\min}$. Thus, the Zeno behavior is naturally excluded. In the periodic [20] and aperiodic [21, 41] sampling, the key issue is how to determine the maximum sampling interval so that the stability of a closed-loop sampled-data system is guaranteed. However, the estimated value of the maximum allowable sampling period $\tau^i_{\max}$ in (7) is preferred to be as large as possible to relax the system consensus conditions. The utilization of the reverse average dwell time in [25, 26] provides the possibility to prolong the maximum allowable sampling period $\tau^i_{\max}$.

Remark 2. In Assumption 2, there exists a minimum allowable sampling period $\tau^i_{\min}$ between any two sampling instants. The minimum allowable sampling period $\tau^i_{\min}$ can be determined by physical platforms if the finite computing capacity of the hardware is considered in practical systems [28, 38]. The minimum allowable sampling period $\tau^i_{\min}$ can theoretically have an arbitrarily small value if it is only to prevent the potential Zeno behavior.

In [25,26], the reverse average dwell time can characterize the impulsive interval in impulsive systems. However, in most existing studies (see [25–27]), there exists a requirement that the response of the system should be synchronous, which is unsuitable for the MASs with asynchronous sampled-data control investigated in this study. Motivated by the reverse average dwell time concept in [25–27], the following assumption is imposed upon the average allowable sampling period.

Assumption 3. Given any $t,\, T \in \mathbb{R}_{\geqslant 0}$ with $T \geqslant t$, let $N_i(t, T)$, $i \in \mathcal{V}$ denote the number of samples in the time interval $[t, T)$. For agent i with $i \in \mathcal{V}$, there exist the constants τ^i_R and $\bar{N}_i \geqslant 1$ such that

$$N_i(t,T) \geqslant \frac{T-t}{\tau^i_R} - \bar{N}_i \tag{8}$$

for all $t,\, T \in \mathbb{R}_{\geqslant 0}$ with $T \geqslant t$.

An inequality constraint condition of asynchronous sampling for individual agents is given in Assumption 3. Intuitively, to ensure the desired control performance, the number $N_i(t, T)$ of samples cannot be too few in a certain time interval $[t, T)$. In Assumption 3, τ^i_R is called the reverse average dwell time of agent i, which provides the maximum allowable sampling period in the condition expressed in (8). If sampling does not occur, then we derive the expression $0 \geqslant (T-t)/\tau^i_R - \bar{N}_i$; thus, the maximum allowable sampling period is in the form of $\tau^i_R \bar{N}_i$. We can choose $\bar{N}_i \geqslant \tau^i_{\max}/\tau^i_R$ with $\tau^i_R \leqslant \tau^i_{\max}$ such that the condition expressed in (8) ensures that there exists an upper bound smaller than or equal to the maximum allowable sampling period $\tau^i_{\max}$ on the sampling intervals $[t^i_k, t^i_{k+1})$.

Remark 3. Compared with that reported in [25, 26], there are two main differences. First, the reverse average dwell time is used to characterize the impulsive interval in impulsive systems in [25, 26], which requires that the response of the entire impulsive system is synchronous. However, the reverse average dwell time is introduced for each agent, and asynchronous sampling schemes for MASs are designed in this study because it is difficult for multiple agents to ensure clock synchronization in distributed MASs. Second, the width of the impulsive interval is determined only under the reverse average dwell time condition in [25, 26], whereas the length of the sampling interval is not only subject to the reverse average dwell time condition but also subject to the minimum and maximum allowable sampling period conditions.

In this study, we consider the aperiodic sampled-data scheme satisfying Assumptions 2 and 3, i.e., the sampling time instants $t^i_0, t^i_1, \ldots$ of agent i are subject to both (7) and (8). However, Assumptions 2 and 3 are given for individual agents. The analysis of the overall MAS in an asynchronous setting is quite challenging. The theory of hybrid systems provides a powerful modeling and analytical tool. In our framework, the consensus protocol $u_i(t)$ is piecewise continuous and could produce jumps only at sampling instants. Therefore, each sampling action of agents can be regarded as a jump action in the modeled hybrid system. To characterize the sampling intervals of agent i described in Assumptions 2 and 3, we introduce two timer variables, i.e., $\tau_i(t, k) \in \mathbb{R}_{\geqslant 0}$ and $s_i(t, k) \in \mathbb{R}_{\geqslant 0}$ with $\tau(0, 0) \in [0, \tau^i_{\max}]$ and $s_i(0, 0) \in [0, \tau^i_{\max} - \tau^i_{\min}]$. The hybrid system model can be established as follows:

$$\begin{cases} \left.\begin{aligned} &\dot{\tau}_i(t,k) = 1, \\ &\dot{s}_i(t,k) = 0, \end{aligned}\right. \quad \text{for } \tau_i(t,k) \in [0, \tau^i_{\max}], \\ \left.\begin{aligned} &\tau_i^+(t,k+1) = \max\{0, \tau_i(t,k) - \tau^i_R\}, \\ &s_i^+(t,k+1) = \max\{0, \tau_i(t,k) - \tau^i_R\}, \end{aligned}\right. \quad \text{for } \tau_i(t,k) \in [s_i(t,k) + \tau^i_{\min}, \tau^i_{\max}]. \end{cases} \tag{9}$$

It follows from (9) that $\tau_i(t, k)$ and $s_i(t, k)$ are not always reset to zero when a jump occurs, which is determined based on the sampling conditions expressed in (7) and (8). The concept of the average allowable sampling period

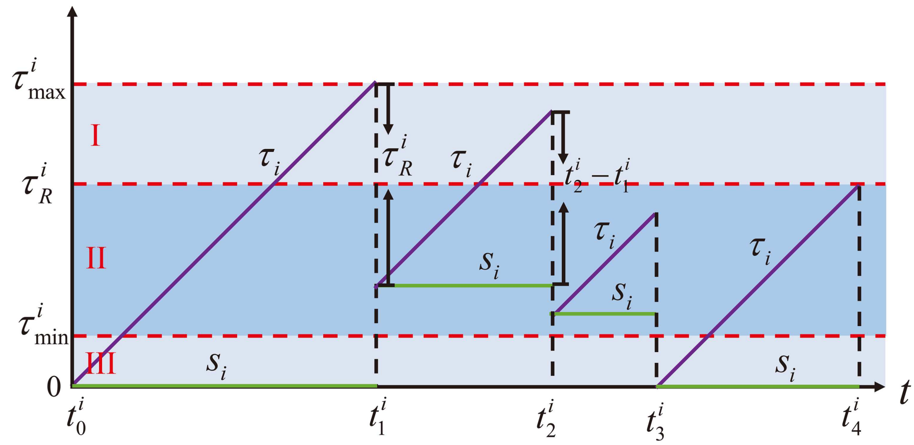

Figure 2 (Color online) Trajectories of the timer variables.

under the reverse average dwell time condition expressed in (8), which requires that, on average, there is at least one sampling action in the time interval of a length equal to the average allowable sampling period τ^i_R, is introduced. Eq. (9) shows that $\tau_i(t,k)$ is at most decreased by τ^i_R time units and $s_i(t,k)$ is reset to the same value as $\tau_i(t,k)$ when a jump occurs. If the timer variables are reset to zero when a jump occurs [42], then the elapsed time for the subsequent jump could always be $\tau^i_{\max}$, which directly results in the jump of the timer variables in a fixed period. Therefore, $\tau_i(t,k)$ and $s_i(t,k)$ are not always reset to zero after jumps. Notably, $\dot{s}_i(t,k)=0$ and $\dot{\tau}_i(t,k)=1$ between two consecutive jumps; thus, $s_i(t,k)$ records the last jump value of $\tau_i(t,k)$, which is significant for calculating the elapsed time from the k-th sampling instant to the current instant. Notably, $\tau_i(t,k)-s_i(t,k)$, $t\in[t^i_k,t^i_{k+1})$ is the elapsed time from the k-th sampling instant to the current instant. Assumption 2 specifies the minimum sampling period $\tau^i_{\min}$, which indicates that the subsequent jump is allowed at least after $\tau^i_{\min}$ time units when a jump occurs. $s_i(t,k)$ is introduced to precisely characterize the limitation of the minimum sampling period $\tau^i_{\min}$.

Figure 2 shows an example of possible jump scenarios of the timer variables. Figure 2 also shows that $\tau_i(t,k+1)$ and $s_i(t,k+1)$ are reset to $\tau_i(t,k)-\tau^i_R$ when the jump occurs in Area I and $\tau_i(t,k+1)$ and $s_i(t,k+1)$ are reset to zero when the jump occurs in Area II. In other words, $\tau^+_i(t,k+1)=s^+_i(k+1)=\tau_i(t,k)-\tau^i_R$. In addition, the jump of the timer variables cannot occur in Area III, which is attributed to the enforcement of a minimum allowable intersampling period $\tau^i_{\min}$ after each sampling instant. Intuitively, two consecutive jumps of the timer variables are separated by at least $\tau^i_{\min}$ time units, and the time between jumps should not be greater than τ^i_R time units. For example, a jump occurs at $\tau_i(t,k)=\tau^i_{\max}$ and $\tau_i(t,k+1)$ is reset to zero after the jump expressed in (9), which directly leads to the fact that the elapsed time for the subsequent jump could be $\tau^i_{\max}$. The reverse average dwell time τ^i_R in Assumption 3 is ineffective if the aforementioned process is always repeatedly performed. Therefore, $\tau_i(t,k)$ is at most decreased by τ^i_R time units rather than always reset to zero after jumps.

Lemma 1 describes the relationship between the hybrid time domain $E_i=\mathrm{dom}(\tau_i(t,k),s_i(t,k))$ for the solution $(\tau_i(t,k),s_i(t,k))$ to (9) and its jump instant t^i_k satisfies (7) and (8).

Lemma 1 (Proposition 1 in [27])**.** For the solution $(\tau_i(t,k),s_i(t,k))$ with initial state set $\mathcal{I}_0=[0,\tau^i_{\max}]\times[0,\tau^i_{\max}-\tau^i_{\min}]$ in (9), E_i is a hybrid time domain of $(\tau_i(t,k),s_i(t,k))$ if and only if the jump time sequence $\{t^i_k\}$ satisfies (7) and

$$N_i(t,T)\geqslant\frac{(T-t)-\tau^i_{\max}}{\tau^i_R},\tag{10}$$

where $i\in\mathcal{V}$, $\tau^i_{\min}\leqslant\tau^i_R\leqslant\tau^i_{\max}$, and $N_i(t,T)$ is the jump number of the hybrid system expressed in (9) in the time interval $[t,T)$.

Remark 4. In Lemma 1, the condition expressed in (10) is equivalent to (8) with $\bar{N}_i=\tau^i_{\max}/\tau^i_R$. The sampling/jump time sequence of different agents is independent; thus, Proposition 1 in [27] can be used here for agent i. However, the MAS is not a decoupled system, which will be shown subsequently in the hybrid formulation of MASs.

3.3 Hybrid systems formulation of MASs

We construct the continuous-time MAS model with discrete-time communication in networks based on the hybrid system framework. The agent and observer states of agent i remain unchanged at sampling instant t^i_k, i.e., $x^+_i(t,k+$

$1) = x_i(t,k)$ and $\xi_i^+(t,k+1) = \xi(t,k)$, but the state estimation is reset at sampling instant t_k^i, i.e., $\hat{\xi}_i^+(t,k+1) = \xi(t,k)$. Combined with the clock variables defined in (9), a hybrid system for agent i can be formally defined as follows:

$$\begin{cases} \dot{x}_i(t,k) = Ax_i(t,k) + Bu_i(t,k), \\ \dot{\xi}_i(t,k) = A\xi_i(t,k) + Bu_i(t,k) + F(C\xi_i(t,k) - y_i(t,k)), \\ \dot{\hat{\xi}}_i(t,k) = A\hat{\xi}_i(t,k), & \text{for } \tau_i(t,k) \in [0, \tau_{\max}^i], \\ \dot{\tau}_i(t,k) = 1, \\ \dot{s}_i(t,k) = 0, \\ x_i^+(t,k+1) = x_i(t,k), \\ \xi_i^+(t,k+1) = \xi_i(t,k), \\ \hat{\xi}_i^+(t,k+1) = \xi_i(t,k), & \text{for } \tau_i(t,k) \in [s_i(t,k) + \tau_{\min}^i, \tau_{\max}^i]. \\ \tau_i^+(t,k+1) = \max\{0, \tau_i(t,k) - \tau_R^i\}, \\ s_i^+(t,k+1) = \max\{0, \tau_i(t,k) - \tau_R^i\}, \end{cases}$$

Let $x = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \ldots, x_m^{\mathrm{T}}]^{\mathrm{T}}$, $\xi = [\xi_1^{\mathrm{T}}, \xi_2^{\mathrm{T}}, \ldots, \xi_m^{\mathrm{T}}]^{\mathrm{T}}$, $\hat{\xi} = [\hat{\xi}_1^{\mathrm{T}}, \hat{\xi}_2^{\mathrm{T}}, \ldots, \hat{\xi}_m^{\mathrm{T}}]^{\mathrm{T}}$, $\tau = [\tau_1, \tau_2, \ldots, \tau_m]^{\mathrm{T}}$, $s = [s_1, s_2, \ldots, s_m]^{\mathrm{T}}$, and $\eta = [x^{\mathrm{T}}, \xi^{\mathrm{T}}, \hat{\xi}^{\mathrm{T}}, \tau^{\mathrm{T}}, s^{\mathrm{T}}]^{\mathrm{T}}$. Then, the overall hybrid system is modeled as follows:

$$\begin{cases} \dot{\eta} = F(\eta), & \text{for } \eta \in \mathcal{C}, \\ \eta^+ \in G(\eta), & \text{for } \eta \in \mathcal{D}. \end{cases} \tag{11}$$

For the hybrid system expressed in (11), the flow set $\mathcal{C}$ is derived as follows:

$$\mathcal{C} = \mathbb{R}^{mn_x} \times \mathbb{R}^{mn_x} \times \mathbb{R}^{mn_x} \times [0, \tau_{\max}^1] \times \cdots \times [0, \tau_{\max}^m] \times [0, \tau_{\max}^1] \times \cdots \times [0, \tau_{\max}^m],$$

and the flow map $F(\eta)$ is obtained as follows:

$$\begin{aligned} F(\eta) = &[(I_m \otimes A)x - (\mathcal{L} \otimes BK)\xi - (\mathcal{L} \otimes BK)e, (I_m \otimes A)\xi - (\mathcal{L} \otimes BK)\xi - (\mathcal{L} \otimes BK)e \\ &+ FC(\xi - x), (I_m \otimes A)\hat{\xi}, \mathbf{1}, \mathbf{0}]^{\mathrm{T}}, \end{aligned}$$

where $e = [e_1^{\mathrm{T}}, e_2^{\mathrm{T}}, \ldots, e_m^{\mathrm{T}}]^{\mathrm{T}}$ and $e_i = \hat{\xi}_i - \xi_i$, $i \in \mathcal{V}$. The jump set $\mathcal{D}$ is derived as follows:

$$\mathcal{D} = \mathbb{R}^{mn_x} \times \mathbb{R}^{mn_x} \times \mathbb{R}^{mn_x} \times [s_1 + \tau_{\min}^1, \tau_{\max}^1] \times \cdots \times [s_i + \tau_{\min}^m, \tau_{\max}^m] \times \mathbb{R}^m,$$

and the jump map $G(\eta)$ is obtained as follows:

$$G(\eta) = \{\cup G_i(\eta) : i \in \mathcal{V} \text{ and } \tau_i \in [s_i + \tau_{\min}^i, \tau_{\max}^i]\},$$

where $G_i(\eta) = [x, \xi, \hat{\xi}_1^{\mathrm{T}}, \ldots, \hat{\xi}_{i-1}^{\mathrm{T}}, \xi_i^{\mathrm{T}}, \hat{\xi}_{i+1}^{\mathrm{T}}, \hat{\xi}_m^{\mathrm{T}}, \tau_1 \ldots, \tau_{i-1}, \max\{0, \tau_i - \tau_R^i\}, \tau_{i+1}, \ldots, \tau_m, s_1 \ldots, s_{i-1}, \max\{0, \tau_i - \tau_R^i\}, s_{i+1}, \ldots, s_m]^{\mathrm{T}}$.

The states of the hybrid system expressed in (11) include the agent, observer, and estimator states and two timer variables. Based on the definitions of flow set $\mathcal{C}$ and jump set $\mathcal{D}$, the timer variable τ determines whether the system jumps or not. Moreover, the hybrid system expressed in (11) generates a jump if agent i exists such that τ_i jumps. Specifically, if t_k^i is the k-th sampling instant of agent i, then the jump map $G(\eta)$ only updates the estimator state of agent i and resets the timer variables τ_i and s_i. In other words, the hybrid system expressed in (11) undergoes a jump whenever agent i exists such that τ_i jumps. Therefore, Eq. (11) can be rewritten as follows:

$$\begin{cases} \dot{\eta} = F(\eta), & \forall i \in \mathcal{V}, \tau_i \in [0, \tau_{\max}^i], \\ \eta^+ \in G(\eta), & \exists i \in \mathcal{V}, \tau_i \in [s_i + \tau_{\min}^i, \tau_{\max}^i]. \end{cases}$$

Remark 5. In the hybrid system expressed in (11), as long as one clock variable jumps, the system expressed in (11) generates a jump. In this setting, the time sequence of jumps for the system expressed in (11) is an integration of the sampling time sequence of all agents. Notably, we can redefine the jump time sequence of the system expressed in (11). Let $\{\tilde{t}_s\}_{s \in \mathbb{Z}_{\geqslant 0}} \triangleq \{t_k^i, i \in \mathcal{V}, k \in \mathbb{N}\}$ with $\tilde{t}_{s+1} > \tilde{t}_s$ denote the set of total sampling instants for MASs (Eq. (1)). Then, $\{\tilde{t}_s\}$ is the set of jump instants of the hybrid system expressed in (11). Similar definitions can be found in [29]. An example is shown in Figure 3.

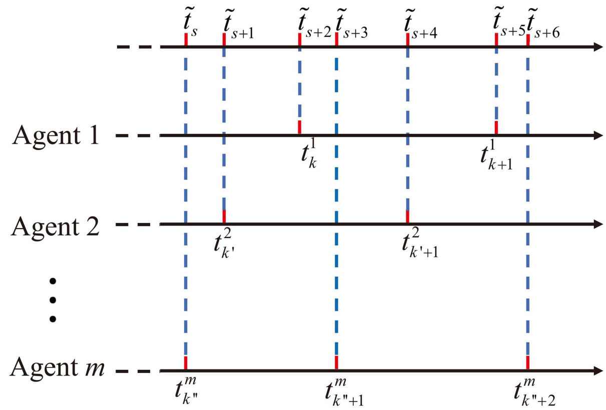

Figure 3 (Color online) Sampling instants of the hybrid system expressed in (11).

4 Main results

In this section, the theory of hybrid systems [40] is used to analyze MASs (Eq. (1)). We state that the function $V(\eta)$ is a Lyapunov function for the hybrid system expressed in (11) if $V(\eta)$ is positive definite, locally Lipschitz, and radially unbounded to η and satisfies the following relations:

$$\langle \nabla V(\eta), F(\eta)\rangle \leqslant -\delta V(\eta), \forall i \in \mathcal{V}, \tau_i \in [0, \tau^i_{\max}], \tag{12a}$$

$$V(G(\eta)) - V(\eta) \leqslant 0, \exists i \in \mathcal{V}, \tau_i \in [s_i + \tau^i_{\min}, \tau^i_{\max}], \tag{12b}$$

where $\delta > 0$ is a constant.

Remark 6. The Lyapunov function V of the hybrid system expressed in (11) is not required to be continuous for $t \in \mathbb{R}_{\geqslant 0}$. However, two conditions need to be satisfied [40], i.e., (i) $\overline{\mathcal{C}} \cup \mathcal{D} \cup G(\mathcal{D}) \subset \mathrm{dom}(V)$ and (ii) V is continuously differentiable on an open set containing $\overline{\mathcal{C}}$. In event-triggered control, the appropriate and predetermined Lyapunov function for MASs was designed to guarantee that the relations expressed in (12a) and (12b) are satisfied (see [38,43]). However, two prior unknown and state-independent functions are contained in the Lyapunov function $V(\eta)$ in this study, which will be reflected in the proof of Theorem 1.

We now establish the main result.

Theorem 1. Consider MASs with the dynamics expressed in (1) and the observer-based control protocol expressed in (5). Suppose that Assumptions 1–3 hold. The MAS (Eq. (1)) achieves a consensus if the reverse average dwell time τ^i_R and the maximum allowable sampling period $\tau^i_{\max}$ with $\tau^i_{\min} \leqslant \tau^i_R \leqslant \tau^i_{\max}$ satisfy the following relations:

$$\tau^i_R < \left| \frac{-(\tilde{\delta} - \delta_i)\tau^i_{\min} + \ln(\varepsilon^2)}{2\sqrt{p_i q_i} + b_i} \right|, \tag{13}$$

$$\tau^i_{\max} = \begin{cases} \frac{4\sqrt{\sigma_i}}{\mu_i^2 - \sigma_i}, & \text{if } b_i^2 = 4p_i q_i, \\ \frac{2}{\sqrt{-\tilde{\sigma}_i}} \arctan \frac{2\sqrt{\sigma_i}\sqrt{-\tilde{\sigma}_i}}{\mu_i^2 - \tilde{\sigma}_i - \sigma_i}, & \text{if } b_i^2 < 4p_i q_i, \\ \frac{2}{\sqrt{\tilde{\sigma}_i}} \operatorname{arctanh} \frac{2\sqrt{\sigma_i \sqrt{\tilde{\sigma}_i}}}{\mu_i^2 - \tilde{\sigma}_i - \sigma_i}, & \text{if } b_i^2 > 4p_i q_i, \end{cases} \tag{14}$$

where $\tilde{\delta} = \min\{\bar{\alpha}\lambda^{-1}_{\max}(P), \tilde{\alpha}\lambda^{-1}_{\max}(Q^{-1})\}$, $0 < \delta_i < \tilde{\delta}$, $p_i = \gamma_i \lambda_{\max}(P)(\beta_2^{-1}\lambda_{\max}(\Gamma) + \beta_4)$, $b_i = \tilde{\delta} + \lambda^{-1}_{\max}(P)(\lambda_{\max}(\Gamma) + 2\lambda^2_m(\mathcal{L})\lambda_{\max}(\Gamma) - 1) > 0$, $q_i = (\beta_1\gamma_i)^{-1}\lambda^{-1}_{\max}(P)\lambda^2_m(\mathcal{L})\lambda_{\max}(\Gamma)$, $\sigma_i = (b_i + \mu_i)^2 - 4p_i q_i$, $\tilde{\sigma}_i = b_i^2 - 4p_i q_i$, $\mu_i = \frac{-(\tilde{\delta} - \delta_i)\tau^i_{\min} + \ln(\varepsilon^2)}{\tau^i_R}$, $\bar{\alpha} = 1 - (1 - (2 - \beta_1 - \beta_2)\lambda_2(\mathcal{L}))\lambda_{\max}(\Gamma) > 0$, $\tilde{\alpha} = \alpha\lambda_{\min}(Q^{-1}Q^{-1}) - (\beta_3^{-1} + \beta_4^{-1})\|PFC\|^2$, $\alpha > 0$, and $\beta_1, \beta_2, \beta_3, \beta_4 \in (0, 1)$ are constants.

Proof. See Appendix A.

The aperiodic sampling scheme proposed in this study only requires that the sampling time sequence of each agent satisfy (7) and (8). In Theorem 1, the reverse average dwell time τ^i_R and the maximum allowable sampling period $\tau^i_{\max}$ rely on the communication graph, system matrices, and several designed parameters. Moreover, on the premise of excluding the Zeno behavior and disregarding the hardware constraints, the minimum allowable sampling period $\tau^i_{\min}$ can be set to be arbitrarily small, which facilitates the improvement in the reverse average dwell time τ^i_R and the maximum allowable sampling period $\tau^i_{\max}$.

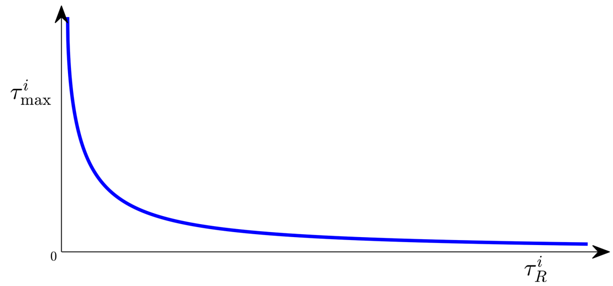

Figure 4 (Color online) Relationship between the values of τ^i_R and $\tau^i_{\max}$ in Theorem 1 with $b_i^2 = 4p_iq_i$ (the scale ratio of the horizontal and vertical coordinates is 1 to 10). The relationship between the values of τ^i_R and $\tau^i_{\max}$ also follows a similar change tendency for the scenarios of $b_i^2 > 4p_iq_i$ and $b_i^2 < 4p_iq_i$.

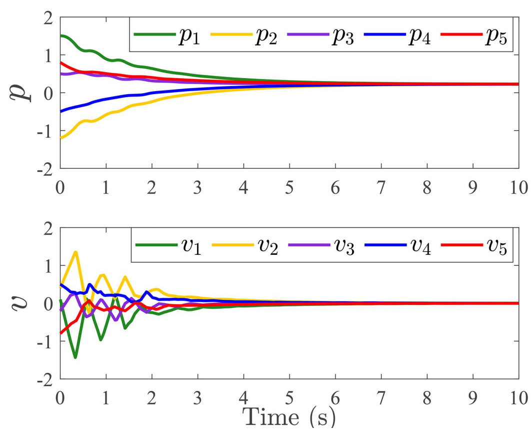

Figure 5 (Color online) Trajectories of the system states of the five agents.

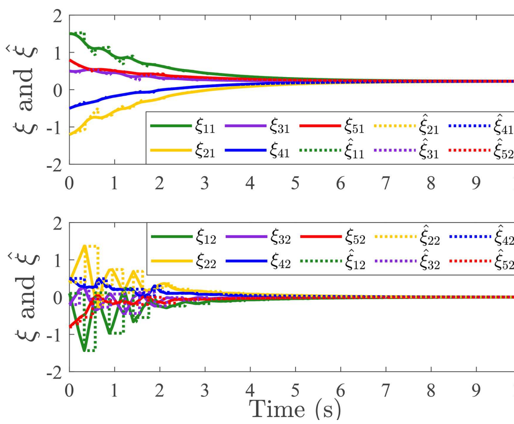

Figure 6 (Color online) Trajectories of the observer and estimator states of the five agents.

Notably, some coupled conditions in terms of the minimum allowable sampling period $\tau^i_{\min}$, reverse average dwell time τ^i_R, and maximum allowable sampling period $\tau^i_{\max}$ are presented in Theorem 1. Intuitively, Theorem 1 shows that the reverse average dwell time τ^i_R is influenced by the minimum allowable sampling period $\tau^i_{\min}$, and the maximum allowable sampling period $\tau^i_{\max}$ is influenced by both the minimum allowable sampling period $\tau^i_{\min}$ and the reverse average dwell time τ^i_R. It follows from (13) in Theorem 1 that, for a given minimum allowable sampling period $\tau^i_{\min}$, the estimated value of the maximum allowable sampling period $\tau^i_{\max}$ increases as the reverse average dwell time τ^i_R decreases. Figure 4 exemplifies this characterization. Therefore, there exists a trade-off between maximum allowable sampling period $\tau^i_{\max}$ and reverse average dwell time τ^i_R. In general, if the minimum allowable sampling period $\tau^i_{\min}$ is preselected, then choosing a smaller τ^i_R value can improve the estimate of the maximum sampling period $\tau^i_{\max}$.

Remark 7. In the event-triggered communication strategies [34, 37], the online monitoring of the real-time states of agents is usually required. However, the online monitoring of the real-time states of agents requires sensors, controllers, and/or actuators to collect data on agents. In practice, imperfections in network communications, such as packet losses [44], are inevitable such that the real-time states of the system are not continuously monitored. Notably, the communication time instants are generated offline in the time-based communication strategy proposed in this study, which avoids the online monitoring of the real-time states of agents.

Remark 8. In event-triggered sampling schemes [37, 38], events could occur intensively in some time intervals. However, the sampling instants can be uniformly located over the entire time domain for each agent in our designed time-based sampling schemes. Notably, the sampling time sequence of agents needs to satisfy (7) and (8) in the proposed sampling scheme. Our designed aperiodic sampling scheme can be considered an event-triggering scheme. The sampling instants are determined whenever Eqs. (7) and (8) are violated, which indicates that Eqs. (7) and (8) can be considered the event-triggered conditions. Different from the state-dependent event-triggered conditions in event-triggered schemes [45, 46], the time-based sampling conditions expressed in (7) and (8) are state-dependent, and only the intersampling times and the number of samples are restricted. Therefore, our proposed aperiodic sampling scheme not only avoids continuous monitoring of system states but also can be implemented in an event-

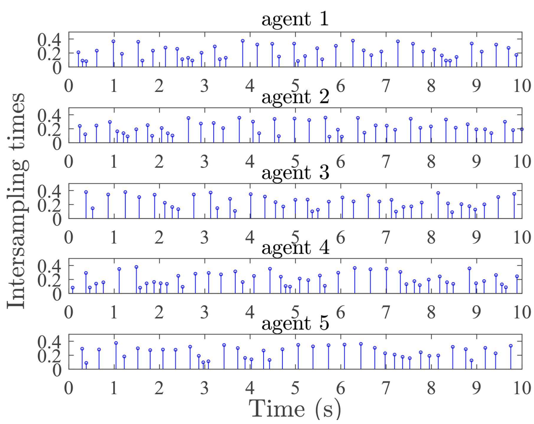

Figure 7 (Color online) Communication instants and intersampling times of the five agents.

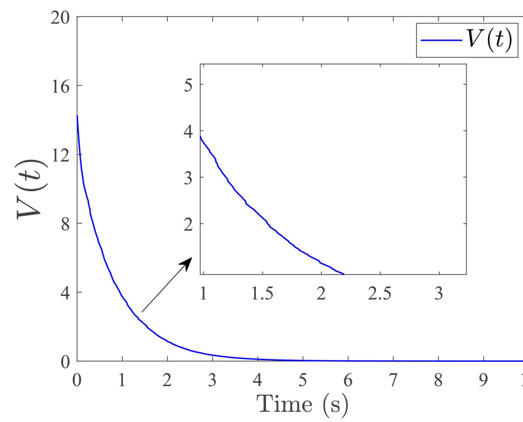

Figure 8 (Color online) Trajectories of the Lyapunov function $V(\eta)$ in (A1).

Table 1 Comparison of different sampling schemes.

		Agent 1	Agent 2	Agent 3	Agent 4	Agent 5
Aperiodic sampling in this study	Number of samples	45	45	41	47	39
	Maximum intersampling times (s)	0.37	0.36	0.37	0.38	0.37
	Minimum intersampling times (s)	0.08	0.09	0.09	0.08	0.09
	Average intersampling times (s)	0.22	0.22	0.24	0.21	0.26
Periodic sampling	Number of samples	45	45	47	47	47
	Fixed intersampling times (s)	0.22	0.22	0.21	0.21	0.21

triggered manner.

5 Numerical example

Consider a second-order MAS, which has been widely used in the formation of mobile robots [4] and vehicle platooning [3], with the following dynamics:

$$\begin{bmatrix} \dot{p}_i(t) \\ \dot{v}_i(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_i(t) \\ v_i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i(t),$$
$$y_i(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} p_i(t) \\ v_i(t) \end{bmatrix},$$

where $p_i(t) \in \mathbb{R}$ and $v_i(t) \in \mathbb{R}$ are the position and velocity, respectively. The Laplacian matrix is $\mathcal{L} = [3, -1, -1, -1, 0; -1, 3, 0, -1, -1; -1, 0, 2, -1, 0; -1, -1, -1, 4, -1; 0, -1, 0, -1, 2]$.

Based on Theorem 1, the controller and observer gains are $K = [1.0000,\ 1.7321]$ and $F = [-1.7321,\ -0.5000]^{\mathrm{T}}$, respectively. The initial states of five vehicles are $[p_1(0),\ v_1(0)]^{\mathrm{T}} = [1.5,\ 0.1]^{\mathrm{T}}$; $[p_2(0),\ v_2(0)]^{\mathrm{T}} = [-1.2,\ 0.4]^{\mathrm{T}}$; $[p_3(0),\ v_3(0)]^{\mathrm{T}} = [0.5,\ -0.2]^{\mathrm{T}}$; $[p_4(0),\ v_4(0)]^{\mathrm{T}} = [-0.5,\ 0.5]^{\mathrm{T}}$; and $[p_5(0),\ v_5(0)]^{\mathrm{T}} = [0.8,\ -0.8]^{\mathrm{T}}$. Figure 5 shows the trajectories of the system states of the five agents. Notably, the states of all agents achieve a consensus, which overcomes the bounded consensus in [12]. Figure 6 shows the trajectories of the observer and estimator states of the five agents. Figure 6 indicates that the estimator states can track the observer states. The communication instants and intersampling times of each agent are presented in Figure 7. Figure 8 shows the trajectories of the Lyapunov function $V(\eta)$ in (A1), where $\gamma_i = 0.5$, $i \in \{1, 2, 3, 4, 5\}$, $\chi_i(\tau_i)$ is governed by (A8) and $\psi_i(\tau_i)$ is governed by (A10). Notably, the Lyapunov function $V(\eta)$ satisfies (12a) and (12b) and converges to zero. Therefore, the communication and control strategies designed in this study are feasible.

Figure 9 shows the trajectories of the system states of the five agents in the fixed periodic sampling with $h_1 = h_2 = 0.22$ s and $h_3 = h_4 = h_5 = 0.21$ s, and Figure 10 shows the trajectories of the system states of the five agents in the fixed periodic sampling with $h_1 = h_2 = 0.22$ s and $h_3 = h_4 = h_5 = 0.23$ s. Notably, all agents cannot achieve a consensus when $h_1 = h_2 = 0.22$ s and $h_3 = h_4 = h_5 = 0.23$ s. However, the maximum intersampling times are

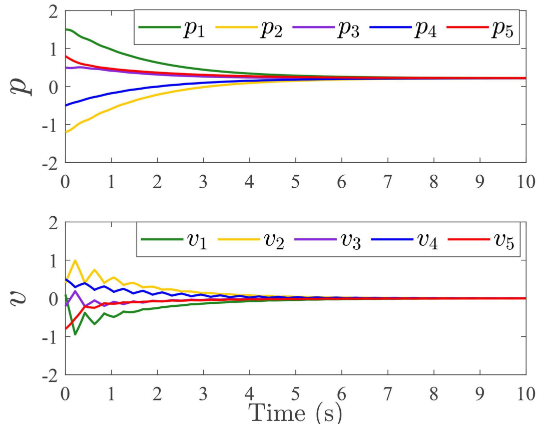

Figure 9 (Color online) Trajectories of the system states of the five agents with the sampling periods $h_1 = h_2 = 0.22$ s and $h_3 = h_4 = h_5 = 0.21$ s.

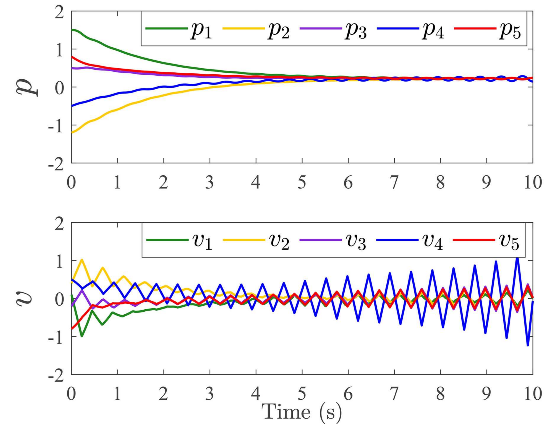

Figure 10 (Color online) Trajectories of the system states of the five agents with the sampling periods $h_1 = h_2 = 0.22$ s and $h_3 = h_4 = h_5 = 0.23$ s.

greater than 0.23 s in the aperiodic sampling scheme designed in this study; see Table 1 for the details. The results of the comparison between the aperiodic sampling scheme designed in this study and the fixed periodic sampling scheme are given in Table 1, in which the maximum allowable sampling period in this study is greater than the fixed sampling period. Furthermore, Figure 10 shows that agents cannot achieve a consensus even if the fixed sampling period is less than the maximum allowable sampling periods. Therefore, the aperiodic sampling scheme proposed in this study provides a significant improvement in the maximum allowable sampling periods.

6 Conclusion

In this study, the time-based asynchronous sampled-data control for general linear MASs was investigated. In the case of time-based communication, the concept of the average allowable sampling period under the reverse average dwell time condition was introduced. The proposed sampling criterion was state-independent, which could improve the maximum allowable sampling period of agents while ensuring a consensus. Based on the hybrid system theory, some sufficient conditions were derived to guarantee the consensus of MASs. In future work, we will consider a combination of time-based and event-based sampling strategies for MSAs.

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Appendix A

Proof of Theorem 1. Based on Assumption 2, for agent i with $i \in \mathcal{V}$, a minimum allowable sampling period $\tau^i_{\min} > 0$ is enforced after each sampling; thus, the Zeno behavior is naturally excluded for all agents. We consider the following candidate Lyapunov function for the hybrid system expressed in (11):

$$V(\eta) = \sum_{i=1}^{m} \chi_i(\tau_i) \left(V_P + \gamma_i \psi_i(\tau_i) V_C\right), \tag{A1}$$

where γ_i is a positive constant and $\chi_i(\tau_i), \psi_i(\tau_i) : [0, \tau^i_{\max}] \to \mathbb{R}_{\geqslant 0}$ are functions with strictly positive lower and upper bounds, i.e., there exist the constants $\underline{\chi}_i, \overline{\chi}_i \in \mathbb{R}_{>0}, \underline{\psi}_i, \overline{\psi}_i \in \mathbb{R}_{>0}$ such that

$$\underline{\chi}_i \leqslant \chi_i(\tau_i) \leqslant \overline{\chi}_i, \tag{A2a}$$

$$\underline{\psi}_i \leqslant \psi_i(\tau_i) \leqslant \overline{\psi}_i, \tag{A2b}$$

for all $\tau_i \in [0, \tau^i_{\max}]$ and $i \in \mathcal{V}$. Moreover, let $V_P = \xi^{\mathrm{T}}(t)(\mathcal{L} \otimes P)\xi(t) + \alpha\zeta^{\mathrm{T}}(t)(I_m \otimes Q^{-1})\zeta(t)$ and $V_C = e^{\mathrm{T}}(t)(I_m \otimes P)e(t)$, where α is a positive constant and $\zeta(t) = \xi(t) - x(t)$.

When $\eta \in \mathcal{C}$, the derivative of $V(\eta)$ is obtained as follows:

$$\begin{aligned}\langle V(\eta), F(\eta)\rangle = &\sum_{i=1}^{m}\Big(\dot\chi_i(\tau_i)\big(\xi^{\mathrm{T}}(\mathcal{L}\otimes P)\xi + \alpha\zeta^{\mathrm{T}}(I_m\otimes Q^{-1})\zeta + \gamma_i\psi_i(\tau_i)(\tau_i)e^{\mathrm{T}}(I_m\otimes P)e\big) + \chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\otimes A^{\mathrm{T}}P)\xi\\ &+ \chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\otimes PA)\xi - 2\chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\mathcal{L}\otimes\Gamma)\xi - 2\chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\mathcal{L}\otimes PBK)e + 2\chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\otimes PFC)\zeta\\ &+ \alpha\chi_i(\tau_i)\zeta^{\mathrm{T}}(I_m\otimes(A^{\mathrm{T}}Q^{-1}+Q^{-1}A))\zeta + 2\alpha\chi_i(\tau_i)\zeta^{\mathrm{T}}(I_m\otimes Q^{-1}FC)\zeta + \gamma_i\chi_i(\tau_i)\dot\psi_i(\tau_i)e^{\mathrm{T}}(I_m\otimes P)e\\ &+ \gamma_i\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(I_m\otimes(A^{\mathrm{T}}P+PA))e + 2\gamma_i\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(\mathcal{L}\otimes\Gamma)\xi + 2\gamma_i\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(\mathcal{L}\otimes\Gamma)e\\ &- 2\gamma_i\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(I_m\otimes PFC)\zeta\Big),\end{aligned}\tag{A3}$$

where $\Gamma = PBB^{\mathrm{T}}P$. Notably,

$$-2\chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\mathcal{L}\otimes\Gamma)e \leqslant \beta_1\chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\mathcal{L}\otimes\Gamma)\xi + \beta_1^{-1}\chi_i(\tau_i)e^{\mathrm{T}}(\mathcal{L}\mathcal{L}\otimes\Gamma)e,\tag{A4a}$$

$$2\gamma_i\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(\mathcal{L}\otimes\Gamma)\xi \leqslant \beta_2\chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\mathcal{L}\otimes\Gamma)\xi + \beta_2^{-1}\gamma_i^2\chi_i(\tau_i)\psi_i^2(\tau_i)e^{\mathrm{T}}(I_m\otimes\Gamma)e,\tag{A4b}$$

$$2\chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\otimes PFC)\zeta \leqslant \beta_3\chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\mathcal{L}\otimes I_{n_x})\xi + \beta_3^{-1}\chi_i(\tau_i)\zeta^{\mathrm{T}}(I_m\otimes PFC)^2\zeta,\tag{A4c}$$

$$-2\gamma_i\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(I_m\otimes PFC)\zeta \leqslant \beta_4\gamma_i^2\chi_i(\tau_i)\psi_i^2(\tau_i)e^{\mathrm{T}}e + \beta_4^{-1}\chi_i(\tau_i)\zeta^{\mathrm{T}}(I_m\otimes PFC)^2\zeta,\tag{A4d}$$

where $\beta_1,\beta_2,\beta_3,\beta_4 \in (0,1)$ are constants. Then, the substitution of (A4a) to (A4d) into (A3) yields the following expression:

$$\begin{aligned}\langle\nabla V(\eta), F(\eta)\rangle \leqslant &\sum_{i=1}^{m}\Big(\dot\chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\otimes P)\xi + \chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\otimes A^{\mathrm{T}}P)\xi + \chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\otimes PA)\xi - (2-\beta_1-\beta_2)\chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\mathcal{L}\otimes\Gamma)\xi\\ &+ \beta_3\chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\mathcal{L}\otimes I_{n_x})\xi + \alpha\dot\chi_i(\tau_i)\zeta^{\mathrm{T}}(I_m\otimes Q^{-1})\zeta + \alpha\chi_i(\tau_i)\zeta^{\mathrm{T}}(I_m\otimes(A^{\mathrm{T}}Q^{-1}+Q^{-1}A))\zeta + 2\alpha\chi_i(\tau_i)\\ &\otimes\zeta^{\mathrm{T}}(I_mQ^{-1}FC)\zeta + (\beta_3^{-1}+\beta_4^{-1})\chi_i(\tau_i)\zeta^{\mathrm{T}}\|PFC\|^2\zeta + \gamma_i\chi_i(\tau_i)\dot\psi_i(\tau_i)e^{\mathrm{T}}(I_m\otimes P)e + \gamma_i\dot\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(I_m\\ &\otimes P)e + \gamma_i\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(I_m\otimes(A^{\mathrm{T}}P+PA))e + 2\gamma_i\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(\mathcal{L}\otimes\Gamma)e + \beta_1^{-1}\chi_i(\tau_i)e^{\mathrm{T}}(\mathcal{L}\mathcal{L}\otimes\Gamma)e\\ &+ \beta_2^{-1}\gamma_i^2\chi_i(\tau_i)\psi_i^2(\tau_i)e^{\mathrm{T}}(I_m\otimes\Gamma)e + \beta_4\gamma_i^2\chi_i(\tau_i)\psi_i^2(\tau_i)e^{\mathrm{T}}e\Big).\end{aligned}\tag{A5}$$

Based on (2) and (3) and $F = -\frac{1}{2}QC^{\mathrm{T}}$, we derive the following expression:

$$\begin{aligned}\langle\nabla V(\eta), F(\eta)\rangle \leqslant &\sum_{i=1}^{m}\Big(\dot\chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\otimes P)\xi - \chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\otimes I_{n_x})\xi + \chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\otimes\Gamma)\xi - (2-\beta_1-\beta_2)\chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\mathcal{L}\otimes\Gamma)\xi\\ &+ \alpha\dot\chi_i(\tau_i)\zeta^{\mathrm{T}}(I_m\otimes Q^{-1})\zeta - \alpha\chi_i(\tau_i)\zeta^{\mathrm{T}}(I_m\otimes Q^{-1}Q^{-1})\zeta + \left(\frac{1}{\beta_3}+\frac{1}{\beta_4}\right)\chi_i(\tau_i)\zeta^{\mathrm{T}}\|PFC\|^2\zeta\\ &+ \gamma_i\chi_i(\tau_i)\dot\psi_i(\tau_i)e^{\mathrm{T}}(I_m\otimes P)e + \gamma_i\dot\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(I_m\otimes P)e + \gamma_i\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(I_m\otimes\Gamma)e\\ &- \gamma_i\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}e + 2\gamma_i\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(\mathcal{L}\otimes\Gamma)e + \frac{1}{\beta_1}\chi_i(\tau_i)e^{\mathrm{T}}(\mathcal{L}\mathcal{L}\otimes\Gamma)e\\ &+ \frac{1}{\beta_2}\gamma_i^2\chi_i(\tau_i)\psi_i^2(\tau_i)e^{\mathrm{T}}(I_m\otimes\Gamma)e + \beta_4\gamma_i^2\chi_i(\tau_i)\psi_i^2(\tau_i)e^{\mathrm{T}}e\Big).\end{aligned}\tag{A6}$$

To satisfy (12a), let

$$\begin{aligned}&\dot\chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\otimes P)\xi - \chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\otimes I_{n_x})\xi + \chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\otimes\Gamma)\xi - (2-\beta_1-\beta_2)\chi_i(\tau_i)\xi^{\mathrm{T}}(\mathcal{L}\mathcal{L}\otimes\Gamma)\xi\\ &+ \alpha\dot\chi_i(\tau_i)\zeta^{\mathrm{T}}(I_m\otimes Q^{-1})\zeta - \alpha\chi_i(\tau_i)\zeta^{\mathrm{T}}(I_m\otimes Q^{-1}Q^{-1})\zeta + \left(\frac{1}{\beta_3}+\frac{1}{\beta_4}\right)\chi_i(\tau_i)\zeta^{\mathrm{T}}\|PFC\|^2\zeta\\ &\leqslant -\delta_i\chi_i(\tau_i)V_P.\end{aligned}\tag{A7}$$

Thus, it follows that $\chi_i(\tau_i) : [0, \tau^i_{\max}] \to \mathbb{R}_{\geqslant 0}$ is governed by the following relation:

$$\dot\chi_i(\tau_i) = (\tilde\delta - \delta_i)\chi_i(\tau_i),\tag{A8}$$

where $0 < \delta_i < \tilde\delta$, $\tilde\delta = \min\{\bar\alpha\lambda^{-1}_{\max}(P), \tilde\alpha\lambda^{-1}_{\max}(Q^{-1})\}$, $\bar\alpha = 1 - (1 - (2-\beta_1-\beta_2)\lambda_2(\mathcal{L}))\lambda_{\max}(\Gamma) > 0$, $\tilde\alpha = \alpha\lambda_{\min}(Q^{-1}Q^{-1}) - (\beta_3^{-1} + \beta_4^{-1})\|PFC\|^2 > 0$. Similarly, let

$$\begin{aligned}&\gamma_i\chi_i(\tau_i)\dot\psi_i(\tau_i)e^{\mathrm{T}}(I_m\otimes P)e + \gamma_i\dot\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(I_m\otimes P)e + \gamma_i\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(I_m\otimes\Gamma)e - \gamma_i\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}e\\ &+ 2\gamma_i\times\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(\mathcal{L}\otimes\Gamma)e + \frac{1}{\beta_1}\chi_i(\tau_i)e^{\mathrm{T}}(\mathcal{L}\mathcal{L}\otimes\Gamma)e + \frac{1}{\beta_2}\gamma_i^2\chi_i(\tau_i)\psi_i^2(\tau_i)e^{\mathrm{T}}(I_m\otimes\Gamma)e + \beta_4\gamma_i^2\chi_i(\tau_i)\psi_i^2(\tau_i)e^{\mathrm{T}}e\\ &\leqslant -\delta_i\gamma_i\chi_i(\tau_i)\psi_i(\tau_i)V_C.\end{aligned}\tag{A9}$$

Thus, it also follows that $\psi_i(\tau_i) : [0, \tau^i_{\max}] \to \mathbb{R}_{\geqslant 0}$ is governed by the following nonlinear differential equation:

$$\dot{\psi}_i(\tau_i) = -p_i\psi_i^2(\tau_i) - b_i\psi_i(\tau_i) - q_i, \tag{A10}$$

where $p_i = \gamma_i\lambda_{\max}(P)(\beta_2^{-1}\lambda_{\max}(\Gamma)+\beta_4) > 0$, $b_i = \tilde{\delta}+\lambda^{-1}_{\max}(P)(\lambda_{\max}(\Gamma)+2\lambda^2_m(\mathcal{L})\lambda_{\max}(\Gamma)-1) > 0$, and $q_i = (\beta_1\gamma_i)^{-1}\lambda^{-1}_{\max}(P)\lambda^2_m(\mathcal{L})\times \lambda_{\max}(\Gamma) > 0$. Based on (A6) to (A10), we obtain the following expression:

$$\langle \nabla V(\eta), F(\eta)\rangle \leqslant -\sum_{i=1}^{m}\delta_i\left(\chi_i(\tau_i)\left(V_P + \gamma_i\psi_i(\tau_i)V_C\right)\right) \leqslant -\delta V(\xi), \tag{A11}$$

where $\delta = \min\{\delta_1, \dots, \delta_m\}$. It follows from (A11) that Eq. (12a) is satisfied.

When $\eta \in \mathcal{D}$, $V_P(G(\xi)) = V_P(\xi)$ because ξ and ζ in V_P do not undergo jumps. Moreover, $e_i(t) = 0$ when a jump occurs. In the asynchronous communication strategy, not all $e_i(t)$, $i \in \mathcal{V}$ are reset to 0 simultaneously. Generally, there exists the constant $\varepsilon \in [0, 1]$ such that $e^{+\mathrm{T}}e^{+} \leqslant \varepsilon^2 e^{\mathrm{T}}e$. If a jump occurs, then there exists $i \in \mathcal{V}$ such that $\tau_i \in [s_i + \tau^i_{\min}, \tau^i_{\max}]$. To formally simplify the analysis, we suppose that a unique agent i performs the sampling action when a jump occurs. A similar analysis can be performed when multiple agents perform the sampling action at the same time. Hence, we obtain the following expression:

$$\begin{aligned} V(G(\eta)) = &\sum_{j\in\mathcal{V}\setminus\{i\}}\Big(\chi_j(\tau_j)\big(\xi^{\mathrm{T}}(\mathcal{L}\otimes P)\xi + \alpha\zeta^{\mathrm{T}}(I_m\otimes Q^{-1})\zeta + \gamma_j\psi_j(\tau_j)\varepsilon^2 e^{\mathrm{T}}(I_m\otimes P)e\big)\Big) + \chi_i(\max\{0,\tau^i_R\})\Big(\xi^{\mathrm{T}}(\mathcal{L}\otimes P)\xi \\ &+ \alpha\zeta^{\mathrm{T}}(I_m\otimes Q^{-1})\zeta + \gamma_i\psi_i(\max\{0,\tau^i_R\})\varepsilon^2 e^{\mathrm{T}}(I_m\otimes P)e\Big), \end{aligned} \tag{A12}$$

where $i \in \mathcal{V}$. Based on (A8), we obtain $\chi_i(\max\{0, \tau^i_R\}) \leqslant \mathrm{e}^{-(\tilde{\delta}-\delta_i)\tau^i_{\min}}\chi_i(\tau_i)$. Because $\tau_i \geqslant \tau^i_{\min}$ and $\tau^i_{\min} \leqslant \tau^i_R$ in Assumptions 2 and 3, we obtain the following expression:

$$\mathrm{e}^{-(\tilde{\delta}-\delta_i)\tau^i_{\min}}\chi_i(\tau_i) \leqslant \chi_i(\tau_i). \tag{A13}$$

Based on (A12) and (A13), we derive the following expression:

$$\begin{aligned} V(G(\eta)) - V(\eta) = &\sum_{j\in\mathcal{V}\setminus\{i\}}\Big(\gamma_j(\varepsilon^2-1)\chi_j(\tau_j)\psi_j(\tau_j)e^{\mathrm{T}}(I_m\otimes P)e\Big) + \chi_i(\max\{0,\tau^i_R\})\Big(\xi^{\mathrm{T}}(\mathcal{L}\otimes P)\xi + \alpha\zeta^{\mathrm{T}}(I_m\otimes Q^{-1})\zeta \\ &+ \gamma_i\varepsilon^2\psi_i(\max\{0,\tau^i_R\})e^{\mathrm{T}}(I_m\otimes P)e\Big) - \chi_i(\tau_i)\Big(\xi^{\mathrm{T}}(\mathcal{L}\otimes P)\xi + \alpha\zeta^{\mathrm{T}}(I_m\otimes Q^{-1})\zeta + \gamma_i\psi_i(\tau_i)e^{\mathrm{T}}(I_m\otimes P)e\Big) \\ \leqslant &\mathrm{e}^{-(\tilde{\delta}-\delta_i)\tau^i_{\min}}\chi_i(\tau_i)\Big(\xi^{\mathrm{T}}(\mathcal{L}\otimes P)\xi + \alpha\zeta^{\mathrm{T}}(I_m\otimes Q^{-1})\zeta + \gamma_i\varepsilon^2\psi_i(\max\{0,\tau^i_R\})e^{\mathrm{T}}(I_m\otimes P)e\Big) \\ &- \chi_i(\tau_i)\Big(\xi^{\mathrm{T}}(\mathcal{L}\otimes P)\xi + \alpha\zeta^{\mathrm{T}}(I_m\otimes Q^{-1})\zeta + \gamma_i\varepsilon^2\psi_i(\tau_i)e^{\mathrm{T}}(I_m\otimes P)e\Big) \\ \leqslant &\mathrm{e}^{-(\tilde{\delta}-\delta_i)\tau^i_{\min}}\gamma_i\varepsilon^2\chi_i(\tau_i)\psi_i(\max\{0,\tau^i_R\})e^{\mathrm{T}}(I_m\otimes P)e - \gamma_i\chi_i(\tau_i)\psi_i(\tau_i)e^{\mathrm{T}}(I_m\otimes P)e. \end{aligned} \tag{A14}$$

Based on (A14), to satisfy (12b), we should have

$$\mathrm{e}^{-(\tilde{\delta}-\delta_i)\tau^i_{\min}}\varepsilon^2\psi_i(\max\{0,\tau^i_R\}) \leqslant \psi_i(\tau_i). \tag{A15}$$

To satisfy (A15), the decreasing value of $\psi_i(\tau_i)$ in any time interval of length less than or equal to τ^i_R should not be greater than $e^{-(\tilde{\delta}-\delta_i)\tau^i_{\min}}\varepsilon^2$. We ensure that Eq. (A15) holds by limiting the rate of change of $\psi_i(\tau_i)$. Let

$$\dot{\psi}_i(\tau_i) \geqslant \mu_i\psi_i(\tau_i), \tag{A16}$$

where μ_i is the bound of the derivative of ψ_i. It holds for any $\tau^1_i, \tau^2_i \in [0, \tau^i_{\max}]$ with $0 < \tau^2_i - \tau^1_i \leqslant \tau^i_R$ that

$$\psi_i(\tau^2_i) \geqslant \mathrm{e}^{\mu_i(\tau^2_i-\tau^1_i)}\psi_i(\tau^1_i) \geqslant \mathrm{e}^{\mu_i\tau^i_R}\psi_i(\tau^1_i). \tag{A17}$$

Let $\tau^1_i = \tau_i$ and $\tau^2_i = \max\{0, \tau^i_R\}$. Based on (A15) and (A17), we derive $\mathrm{e}^{\mu_i\tau^i_R} = \mathrm{e}^{-(\tilde{\delta}-\delta_i)\tau^i_{\min}}\varepsilon^2$. Thus, we obtain the following expression:

$$\mu_i = \frac{-(\tilde{\delta}-\delta_i)\tau^i_{\min} + \ln(\varepsilon^2)}{\tau^i_R}. \tag{A18}$$

Hence, if Eq. (A16) holds, then we can derive $V(G(\eta)) - V(\eta) \leqslant 0$ for $\eta \in \mathcal{D}$. Based on (A10) and (A16), we obtain the following expression:

$$p_i\psi_i^2(\tau_i) + (b_i + \mu_i)\psi_i(\tau_i) + q_i \leqslant 0. \tag{A19}$$

Notably, the left side of (A19) is a quadratic trinomial of $\psi_i(\tau_i)$. We determine that the inequality expressed in (A19) has a nonempty solution set when the inequality $|b_i + \mu_i| > 2\sqrt{p_iq_i}$ holds. Furthermore, the lower bound $\underline{\psi}_i$ and upper bound $\overline{\psi}_i$ of the function $\psi_i(\tau_i)$

are calculated as $\underline{\psi}_i = -\frac{1}{2p_i}(\sqrt{(b_i+\mu_i)^2-4p_iq_i}+(b_i+\mu_i))$ and $\overline{\psi}_i = \frac{1}{2p_i}(\sqrt{(b_i+\mu_i)^2-4p_iq_i}-(b_i+\mu_i))$. To ensure a strictly positive upper bound $\overline{\psi}_i$, $b_i+\mu_i<0$ must hold. Hence, we obtain the following expression:

$$\tau_R^i < \left|\frac{-(\tilde{\delta}-\delta_i)\tau_{\min}^i+\ln(\varepsilon^2)}{2\sqrt{p_iq_i}+b_i}\right|. \tag{A20}$$

Because $\underline{\psi}_i = \frac{q_i}{p_i}(\overline{\psi}_i)^{-1}$, the lower bound of ψ_i is strictly positive, indicating that if τ_R^i satisfies (A20), then the condition expressed in (A2b) holds. Therefore, the bound of $\tau_{\max}^i$ can be determined based on the time it takes for $\psi_i(\tau_i)$ to evolve from $\psi_i(0)=\overline{\psi}_i$ to $\psi_i(\tau_{\max}^i)=\underline{\psi}_i$. Based on (A10), we derive $\tau_{\max}^i = \int_{\overline{\psi}_i}^{\underline{\psi}_i} -\frac{\mathrm{d}\psi_i}{p_i\psi_i^2+b_i\psi_i+q_i}$, where $p_i=\gamma_i\lambda_{\max}(P)(2\lambda_{\max}(\Gamma)+\frac{1}{2})$, $b_i=\tilde{\delta}+4\gamma_i\lambda_m\lambda_{\max}(\Gamma)$, and $q_i=\frac{2}{\gamma_i}\lambda_{\max}(P)\lambda_m^2\lambda_{\max}(\Gamma)$. Furthermore, we derive the following expression:

$$\tau_{\max}^i = \begin{cases} \frac{4\sqrt{\sigma_i}}{\mu_i^2-\sigma_i}, & \text{if } b_i^2=4p_iq_i,\\ \frac{2}{\sqrt{-\tilde{\sigma}_i}}\arctan\frac{2\sqrt{\sigma_i}\sqrt{-\tilde{\sigma}_i}}{\mu_i^2-\tilde{\sigma}_i-\sigma_i}, & \text{if } b_i^2<4p_iq_i,\\ \frac{2}{\sqrt{\tilde{\sigma}_i}}\operatorname{arctanh}\frac{2\sqrt{\sigma_i}\sqrt{\tilde{\sigma}_i}}{\mu_2-\sigma_i-\tilde{\sigma}_i}, & \text{if } b_i^2>4p_iq_i, \end{cases}$$

where $\sigma_i=(b_i+\mu_i)^2-4p_iq_i$ and $\tilde{\sigma}_i=b_i^2-4p_iq_i$.

Finally, we prove that the MAS (Eq. (1)) achieves a consensus based on the invariance principle of hybrid systems [40]. We first prove that any maximal solution to (11) is nontrivial. Assume that $\eta\in\mathcal{C}\setminus\mathcal{D}$. If η is the interior of $\mathcal{C}$, then we obtain $T_{\mathcal{C}}(\eta)=\mathbb{R}^{4m+1}$, where $T_{\mathcal{C}}(\eta)$ is the tangent cone in $\mathcal{C}$. If $\eta\in\mathcal{C}\setminus\mathcal{D}$ and η is not in the interior of $\mathcal{C}$, then there exists $i\in\mathcal{V}$ such that a jump occurs. In this case, $T_{\mathcal{C}}(\xi)=\mathbb{R}^{3mn_x}\times\mathbb{R}^{i-1}\times[0,+\infty)\times\mathbb{R}^{m-i}\times\mathbb{R}^{i-1}\times[0,+\infty)\times\mathbb{R}^{m-i}$ and $F(\xi)\in T_{\mathcal{C}}(\xi)$. Therefore, any maximal solution to (11) is nontrivial (Proposition 6.10 in [40]). Because the solution of the system expressed in (11) cannot be guaranteed to be bounded, we introduce an auxiliary system to cope with this issue. $z=[z_1,z_2,\ldots,z_M]$ with $z_p=\xi_i-\xi_j$, $p\in\{1,2,\ldots,M\}$, and M is the number of the edges of graph $\mathcal{G}$. Let $z=(D^{\mathrm{T}}\otimes I_{n_x})\xi$, where D is the incidence matrix of graph $\mathcal{G}$. Consider the following hybrid systems:

$$\begin{cases} \dot{z}=(D^{\mathrm{T}}\otimes I_{n_x})((I_m\otimes A)\xi-(\mathcal{L}\otimes BK)e+(I_m\otimes FC)(\xi-x)),\\ \dot{\zeta}=(I_m\otimes(A+FC))\zeta,\\ \dot{e}=(I_m\otimes A+\mathcal{L}\otimes BK)e+(\mathcal{L}\otimes BK)\xi-(I_m\otimes FC)\zeta, & \forall\tau_i\in[0,\tau_{\max}^i],\\ \dot{\tau}=\mathbf{1},\\ \dot{s}=\mathbf{0},\\ \begin{bmatrix} z^+\\ \zeta^+\\ e^+\\ \tau^+\\ s^+ \end{bmatrix}\in\tilde{G},\ \exists\tau_i\in[s_i+\tau_{\min}^i,\tau_{\max}^i], \end{cases} \tag{A21}$$

where $\tilde{G}=\{\cup\tilde{G}_i: i\in\mathcal{V}$ and $\tau_i\in[s_i+\tau_{\min}^i,\tau_{\max}^i]\}$, and $\tilde{G}_i=[z,\ \zeta,\ e_1^{\mathrm{T}},\ \ldots,\ e_{i-1}^{\mathrm{T}},\ 0,\ e_{i+1}^{\mathrm{T}},\ e_m^{\mathrm{T}},\ \tau_1\ldots,\ \tau_{i-1},\max\{0,\tau_i-\tau_R^i\},\ \tau_{i+1},\ \ldots,\ \tau_m,\ s_1\ldots,s_{i-1},\max\{0,\tau_i-\tau_R^i\},\ s_{i+1},\ \ldots,\ s_m]^{\mathrm{T}}$. We denote

$$\begin{cases} \dot{\tilde{\eta}}=\tilde{F}(\tilde{\eta}), & \text{for } \tilde{\eta}\in\tilde{\mathcal{C}},\\ \tilde{\eta}^+\in\tilde{G}(\tilde{\eta}), & \text{for } \tilde{\eta}\in\tilde{\mathcal{D}}, \end{cases} \tag{A22}$$

where $\tilde{\eta}=[z,\zeta^{\mathrm{T}},e^{\mathrm{T}},\tau^{\mathrm{T}},s^{\mathrm{T}}]^{\mathrm{T}}$, $\tilde{\mathcal{C}}=\{\tilde{\eta}:\forall i\in\mathcal{V},\tau_i\in[0,\tau_{\max}^i]\}$, and $\tilde{\mathcal{D}}=\{\tilde{\eta}:\exists i\in\mathcal{V},\tau_i\in[s_i+\tau_{\min}^i,\tau_{\max}^i]\}$.

Let $\tilde{V}=V$ be the Lyapunov function of the system expressed in (A22). Based on the previously presented analysis, we derive the following expression:

$$\begin{aligned} &\langle\nabla\tilde{V}(\tilde{\eta}),\tilde{F}(\tilde{\eta})\rangle\leqslant u_c,\\ &\tilde{V}(\tilde{G}(\tilde{\eta}))-\tilde{V}(\tilde{\eta})\leqslant u_d, \end{aligned}$$

where

$$u_c=\begin{cases} -\delta\sum\limits_{i=1}^m(\chi_i(\tau_i)(V_P+\gamma_i\psi_i(\tau_i)V_C)), & \text{if } \tilde{\eta}\in\tilde{\mathcal{C}},\\ -\infty, & \text{otherwise}, \end{cases}$$

$$u_d=\begin{cases} 0, & \text{if } \tilde{\eta}\in\tilde{\mathcal{D}},\\ -\infty, & \text{otherwise}. \end{cases}$$

Based on (A11), the function u_c can be considered the upper bound of the rate of change of $\tilde{V}$ if $\eta\in\mathcal{C}$. Similarly, the function u_d can be considered the upper bound of the rate of change of $\tilde{V}$ if $\eta\in\mathcal{D}$. Therefore, the rate of change $\tilde{V}$ along any solution is bounded by u_c and u_d.

Let $\tilde{\eta}$ be a maximal solution to (A22). Because $\tilde{G}(\tilde{\mathcal{D}}) \in \tilde{\mathcal{C}}$ and $\tilde{\eta}$ is bounded, $\tilde{\eta}$ does not grow infinitely in a finite time interval. Thus, $\tilde{\tilde{\eta}}$ is complete, i.e., $\mathrm{dom}(\eta)$ is unbounded (Proposition 6.10 in [40]). Notably, u_c and u_d are nonpositive, and we have proven that any maximal solution to (A22) is precompact. Therefore, any maximal solution to (A22) approaches the largest weakly invariant subset $\mathcal{S}'$ of

$$\tilde{V}^{-1}(r) \cap U \cap \left[\overline{u_c^{-1}(0)} \cup \left(u_d^{-1}(0) \cap \tilde{G}\left(u_d^{-1}(0)\right)\right)\right],$$

where $U = \mathbb{R}^{(2n_x+2)m}$ and $r \in \tilde{V}(U)$. Because $\overline{u_c^{-1}(0)} = \{\tilde{\eta} : \tilde{\eta} \in \tilde{\mathcal{C}}, (\mathcal{L} \otimes I_{n_x})\xi = \mathbf{0}\ , \zeta = \mathbf{0},$ and $e = \mathbf{0}\}$ and $u_d^{-1}(0) = \tilde{\mathcal{D}}$, the aforementioned set can be rewritten as follows:

$$\tilde{V}^{-1}(r) \cap \left[\{\tilde{\eta} : \tilde{\eta} \in \tilde{\mathcal{C}}, \xi_i - \xi_j = \mathbf{0}, \zeta_i = \mathbf{0}, e_i = \mathbf{0}, \forall i, j \in \mathcal{V}\} \cup (\tilde{\mathcal{D}} \cap \tilde{G}(\tilde{\mathcal{D}}))\right].$$

Because $\mathcal{S}'$ is weakly forward invariant, $q \in \mathcal{S}'$ such that $\tilde{\eta}(0,0) = q$ and $\tilde{\eta}(t,k) \in \mathcal{S}'$ for any $(t,k) \in \mathrm{dom}(\tilde{\eta})$. Suppose that $q \notin \overline{u_c^{-1}(0)}$. Then, we have $q \in \tilde{\mathcal{D}} \cap G(\tilde{\mathcal{D}})$. The solution $\tilde{\eta}$ undergoes a finite number of jumps ℓ' until all of the clocks are reset. After jumps, $\tilde{\eta}(0,\ell') \in \tilde{\mathcal{C}} \setminus \tilde{\mathcal{D}}$, which indicates that $\tilde{\eta}(0,\ell') \in \overline{u_c^{-1}(0)}$. Therefore, $q \in \overline{u_c^{-1}(0)}$. Because ζ and ξ are unaffected by system jumps and must remain in $\overline{u_c^{-1}(0)}$ almost all of the time on flows, we have

$$\zeta(t,k) = \zeta(0,0), \xi(t,k) = \xi(0,0)$$

for any $(t,k) \in \mathrm{dom}(\tilde{\eta})$. This result has two consequences. The first consequence is that the domain of $\tilde{\eta}$ is unbounded. Hence, there exists a finite number of jumps $k' \in \mathbb{N}$, after which all of the clock variables and the sampled state variables have undergone a jump, i.e., $(t',k') \in \mathrm{dom}(\tilde{\eta})$ such that $\hat{\xi}(t,k) = \xi(0,0)$ for all $(t,k) \in \mathrm{dom}(\xi)$ with $t' + k' \geqslant t + k$. Hence, we can obtain $V_C = 0$ for any $(t',k') \in \mathrm{dom}(\xi)$ with $t' + k' \geqslant t + k$. The second consequence is that, for almost all $t \geqslant t'$ such that $(t,k) \in \mathrm{dom}(\xi)$ and $t' + k' \geqslant t + k$, $u(t,k) = 0$, V_P always equals to 0 along $\tilde{\eta}$. Then, $r = 0$ if $\tilde{V}(\tilde{\eta}(t,k)) = r$ for any $(t,k) \in \mathrm{dom}(\xi)$. Therefore, the set $\tilde{V}^{-1}(0)$ is globally attractive. Notably, $\tilde{V}^{-1}(0) = \{\eta \in \mathbb{R}^{m(3n_x+2)} | \xi_i - \xi_j = \mathbf{0}, \zeta_i = \mathbf{0}, e_i = \mathbf{0}, \forall i, j \in \mathcal{V}\}$. Consequently, the MAS (Eq. (1)) with observer-based control protocol (Eq. (5)) achieves a consensus.

Remark 9. In the proof of Theorem 1, prior unknown and state-independent functions $\chi_i(\tau_i)$ and $\psi_i(\tau_i)$ are contained in the Lyapunov function $V(\eta)$, which plays a significant role in deriving (13) and (14). These functions have strictly positive lower and upper bounds. A challenging issue in the proof is to construct the functions $\chi_i(\tau_i)$ and $\psi_i(\tau_i)$ to ensure that Eqs. (12a) and (12b) and Eqs. (A2a) and (A2b) are satisfied. In Theorem 1, the maximum allowable sampling period is determined based on the time it takes for the function $\psi_i(\tau_i)$ to evolve from the upper bound to the lower bound. In literature (see [36, 43]), a similar idea was applied in event-triggered control schemes. In the existing event-triggered schemes [36, 43], an event occurs if the auxiliary clock variable evolves to the set value. However, the auxiliary clock variable is state dependent in event-triggered control. In this paper, the sampling time sequence is state independent. On the premise of consensus, we derive the relationship among the minimum allowable sampling period $\tau_{\min}^i$, reverse average dwell time τ_R^i, and maximum allowable sampling period $\tau_{\max}^i$ by developing the evolution rules of the auxiliary functions $\chi_i(\tau_i)$ and $\psi_i(\tau_i)$.