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Resource allocation for high-order multiagent systems with uncertainties from non-neighboring nodes

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Abstract This paper studies the distributed resource allocation problems of high-order multiagent systems with nonlinear uncertainties. The nonlinear uncertainties consist of the external time-varying disturbances, the uncertain dynamics of agents, and the interferences from the neighboring and non-neighboring nodes of agents. To accomplish the distributed resource allocation under various uncertainties, this paper proposes a new algorithm based on actively estimating and compensating for the lumped uncertainty. By considering the output-feedback situation, the algorithm is constructed based on a full-order extended state observer that offers the estimates of lumped uncertainty and unmeasured states. For the uncertainties with nonlinear growth rates, the convergence analysis of the proposed algorithm is given. The proposed theoretical results illustrate that the resource allocation task can be practically achieved with a tunable optimization error. Finally, numerical simulations show the effectiveness of the proposed algorithms.

Keywords distributed resource allocation, nonlinear uncertainty, multiagent system, extended state observer, active disturbance rejection control

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1 Introduction

Distributed optimization has attracted great interest from both the research community and industrial sectors in recent years. There are various types of distributed optimization problems, including resource allocation [1, 2], source localization [3], and economic dispatch problems [4].

Among these, the resource allocation problem is an important issue in massive engineering applications, such as smart grids [5], communication networks [6], and wireless networks [7]. For the distributed resource allocation problem, each agent has a local cost function and a local resource demand. The objective of multiagent systems is to minimize the total cost function consisting of all local cost functions subject to the total resource constraint. Through a communication network, the agents exchange information with neighbors only. Over the last few years, a number of resource allocation algorithms have been proposed for various dynamics of agents [8, 9], different optimal objectives [10, 11], and diverse constraints [12–14]. For instance, to solve the resource allocation problem in power systems, Wang et al. [8] proposed a second-order continuous-time algorithm based on the saddle point dynamics. Furthermore, Deng [9] developed a resource allocation algorithm for agents with high-order dynamics under an undirected communication topology. Moreover, by considering the differential resource allocation problem under connected digraphs, Chen and Li [10] proposed a projected gradient-based distributed algorithm. Based on the subgradient method and local Lipschitz continuous functions, a distributed algorithm was proposed by Wei et al. [11] to solve nonsmooth convex consensus optimization problems. In addition, by considering different optimization constraints, a distributed algorithm based on the Lagrangian method was developed by Doan and Beck [12] under equality network resource constraints. Moreover, Deng and Chen [13] studied a distributed resource allocation problem under inequality network resource constraints and local inequality constraints. To solve the resource allocation problem with local feasibility constraints for high-order multiagent systems, Zhang et al. [14] proposed an innovative integrated event-triggered strategy. To deal with the optimization problem under constraints coupled with

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equalities and inequalities, Falsone and Prandini [15] proposed a novel augmented Lagrangian algorithm. Li et al. [16] investigated an algorithm based on the Newton-Raphson method, which can successfully reduce communication costs. In addition, a quantized event-triggered communication mechanism was used to reduce the communication expenditure in a study by Li et al. [17], which considers the limited communication capacity. Although the research on resource allocation problems has made great progress on different types of optimal objectives and constraints, the study on agents with uncertain dynamics is still in its infancy, and further investigation is needed.

In engineering applications, disturbances and uncertainties always interfere with the actual effect of the designed algorithm. Hence, it is a significant issue to tackle the distributed resource allocation problem with disturbances and uncertainties. To accomplish the distributed optimization tasks with disturbances, some modified optimization algorithms have been proposed. For instance, an optimal steady-state regulation problem with internal disturbances was solved by an embedded design based on the internal model principle in the work by Tang [18]. To deal with the internal continuous disturbances in distributed high-order multiagent systems, an algorithm based on an adaptive optimal protocol was proposed by Yu et al. [19]. Based on finite-time disturbance observer techniques, a control framework was proposed by Wang et al. [20] to solve finite-time optimization problems with unknown external disturbances. Furthermore, by using the internal model principle and adaptive technique, a consensus optimization problem for a class of nonlinear dynamical systems with the disturbances generated by external systems was solved [21]. To tackle the mismatched disturbances, Wang et al. [22] proposed a nonsmooth control framework based on finite-time design. Moreover, Zhang et al. [23] investigated the distributed optimization problem for multiagent systems disturbed by general noise under a weight-balanced and strongly connected directed graph. The time-varying external disturbances and internal uncertainties generated by known external systems have been fully studied in existing research [18–23]. However, nonlinear uncertainties widely exist in multiagent systems, including nonlinear interferences from neighboring agents and non-neighboring agents. To the best of the authors knowledge, the problem of resource allocation with nonlinear uncertainties from non-neighboring nodes has not been solved yet.

In this paper, the resource allocation for high-order multiagent systems with nonlinear uncertainties from non-neighboring nodes is investigated. The output-feedback case is considered in this paper, and the proposed algorithms are based on an extended state observer [24–26], which is a powerful online estimator for lumped uncertainty, as shown in marine systems [27], power systems [28], and unmanned driving systems [29]. Furthermore, based on the gradient descent method, the algorithm takes into account the guarantee of the equality constraints as well as the privacy preservation of the agents. The main contributions of this paper are presented as follows.

(1) Compared with previous research [18–23], this paper considers high-order multiagent systems, wherein the closed-loop stability is challenging to analyze due to the complexity of high-order dynamics. Furthermore, it is difficult to perform the convergence analysis when only the outputs of the agents are acquired.

(2) In this paper, the considered nonlinear uncertainties include the unmodeled dynamics of agents and the interference from other agents. More specifically, the nonlinear interference from non-neighboring agents is considered. For the nonlinear coupling uncertainties dependent on the states of all agents, it is a formidable task to design the distributed algorithm to achieve resource allocation. To handle nonlinear coupling uncertainties, this paper proposes a new algorithm based on actively estimating and compensating for the lumped uncertainty.

(3) In contrast to the assumption of bounded disturbances [19, 20], the preassumed bounded condition is not rational for the nonlinear coupling uncertainties. Moreover, the uncertainties in this paper exhibit a nonlinear growth rate related to the states of all agents rather than Lipschitz continuity. Based on a weakened assumption for nonlinear uncertainties, this paper rigorously analyzes the closed-loop stability. The rest of the paper is presented as follows. In Section 2, the preliminaries and problem formulation are presented. In Section 3, the algorithms for the state-feedback case and output-feedback case are proposed. The convergence of the proposed algorithms is studied in Section 4, and the numerical results and conclusion are shown in Sections 5 and 6.

2 Preliminaries and problem formulation

2.1 Preliminaries

Some basic notations are introduced in this section. Here, $\mathbb{R}^n$ and $\mathbb{R}$ denote the n-dimensional Euclidean space and the set of real numbers, respectively. The symbol $\otimes$ represents the Kronecker product, whereas $\|\cdot\|$ denotes the spectral norm of a vector or a matrix. In addition, $\mathbf{I}_n$ denotes the $n \times n$ dimensional identity matrix. $\mathrm{diag}\{a_1, a_2, \ldots, a_n\}$ represents a diagonal matrix, where $a_i \in \mathbb{R}$ for $i = 1, 2, \ldots, n$. Let $\mathbf{1}_N = [1, 1, \ldots, 1]^{\mathrm{T}}$ and $\mathbf{0}_N = [0, 0, \ldots, 0]^{\mathrm{T}}$. For vectors $x_1, \ldots, x_n$, define $\mathrm{col}(x_1, \ldots, x_n) = [x_1^{\mathrm{T}}, \ldots, x_n^{\mathrm{T}}]^{\mathrm{T}}$.

Some concepts of graph theory are explained in the following. An undirected graph is defined as $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ and $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ are the vertex set and edge set, respectively. If node j could receive the information from node i, then node j is one neighbor of node i. The graph $\mathcal{G}$ is connected if there exists a directed path between any two nodes i and j. Moreover, graph $\mathcal{G}$ is an undirected connected graph if and only if there are $(i, j) \in \mathcal{V}$ and $(j, i) \in \mathcal{V}$. Let $\mathbf{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ denote the weighted adjacency matrix, where $a_{ii} = 0$, $a_{ij} \geqslant 0$ for $(i, j) \in \mathcal{E}$; otherwise, $a_{ij} = 0$. The Laplacian matrix of $\mathcal{G}$ is defined as $\mathbf{L} = \mathbf{D} - \mathbf{A}$, where $\mathbf{D} = \operatorname{diag}\left\{\sum_{j=1}^{N} a_{1j}, \ldots, \sum_{j=1}^{N} a_{Nj}\right\}$. Let $\lambda_1, \ldots, \lambda_N$ be the eigenvalues of $\mathbf{L}$, where $\lambda_i \leqslant \lambda_j$ if $i \leqslant j$. According to Deng [9], communication topology $\mathcal{G}$ is connected if and only if $\lambda_2 > 0$.

Let $\aleph_i = \{k | (k, i) \in \mathcal{E}\}$ denote the neighboring set of agent i, and let $\bar{\aleph}_i = \{k | (k, i) \notin \mathcal{E}\}$ denote the non-neighboring set of agent i. Next, the following concepts of convex and Lipschitz functions are introduced [30].

Definition 1. A function f: $\mathbb{R}^n \to \mathbb{R}$ is convex, if

$$f(\varsigma x + (1 - \varsigma) y) \leqslant \varsigma f(x) + (1 - \varsigma) f(y), \ \forall x, y \in \mathbb{R}^n, \ \forall \varsigma \in [0, 1]. \tag{1}$$

Definition 2. A differentiable function $f: \mathbb{R}^n \to \mathbb{R}$ is β-strongly convex with $\beta > 0$, if

$$(x - y)^{\mathrm{T}} (\nabla f(x) - \nabla f(y)) \geqslant \beta \|x - y\|^2, \ \forall x, y \in \mathbb{R}^n. \tag{2}$$

Definition 3. A function f: $\mathbb{R}^n \to \mathbb{R}$ is θ-Lipschitz with $\theta > 0$, if

$$\|f(x) - f(y)\| \leqslant \theta \|x - y\|, \ \forall x, y \in \mathbb{R}^n. \tag{3}$$

2.2 Problem formulation

Considering multiagent systems composed of N agents on the communication graph $\mathcal{G}$, the agent $i \in \mathcal{V}$ satisfies the following high-order nonlinear dynamics:

$$\begin{cases} \dot{x}_{i1}(t) = x_{i2}(t), \\ \quad \vdots \\ \dot{x}_{in-1}(t) = x_{in}(t), \\ \dot{x}_{in}(t) = u_i(t) + v_i(x_i(t)) + \displaystyle\sum_{j \in \aleph_i} g_{ij}(x_j(t)) + \sum_{k \in \bar{\aleph}_i} g_{ik}(x_k(t)) + q_i(t), \end{cases} \tag{4}$$

where $x_{il}(t) \in \mathbb{R}$ is the decision from agent i for $l \in \{1, 2, \ldots, n\}$; $u_i(t) \in \mathbb{R}$ is the control input of agent i; $v_i(x_i(t)) \in \mathbb{R}$ represents the internal unmodeled dynamics of agent i; $g_{ij}(x_j(t)) \in \mathbb{R}$ with $j \in \aleph_i$ represents the disturbances from the neighboring nodes of agent i; $g_{ik}(x_k(t)) \in \mathbb{R}$ with $k \in \bar{\aleph}_i$ denotes the disturbances from the non-neighboring node of agent i; and $q_i(t) \in \mathbb{R}$ represents the external time-varying disturbance affecting agent i.

Remark 1. Let $\sum_{j \in \aleph_i} g_{ij}(x_j(t))$ denote the lumped uncertainties from all neighboring nodes of agent i. Let $\sum_{k \in \bar{\aleph}_i} g_{ik}(x_k(t))$ denote the lumped uncertainties from all non-neighboring nodes of agent i. Compared with existing studies [18, 21, 31], this paper not only considers the external disturbances but also addresses the internal nonlinear uncertainties originating from the non-neighboring nodes. The main challenge lies in effectively handling the internal uncertainties originating from non-neighboring nodes.

The distributed resource allocation of multiagent systems can be summarized by the following problem:

$$\min_{\tilde{x} \in R^N} f(\tilde{x}(t)), \ f(\tilde{x}(t)) = \sum_{i=1}^{N} f_i(x_{i1}(t)), \quad \text{subject to } \sum_{i=1}^{N} x_{i1}(t) = \sum_{i=1}^{N} d_i, \tag{5}$$

where $d_i \in \mathbb{R}$ is the local resource demand of agent i, $f_i(x_{i1}(t))$ is the local cost function for agent i, $f(\tilde{x}(t))$ is the total cost function, and $\tilde{x}(t) = \operatorname{col}(x_{11}(t), \ldots, x_{N1}(t))$. The objective of resource allocation is to design an algorithm based on local information to ensure that each agent can converge to the optimal solution of the problem (5) with the high-order dynamics (4).

The following basic assumptions for graph $\mathcal{G}$ and the local cost function f_i are introduced. From Assumption 2, there exists a unique optimal solution of the optimization problem (5), and the optimal solution is defined as $\tilde{x}^* = \operatorname{col}(\eta_{11}^*, \ldots, \eta_{N1}^*)$ for $\eta_{i1}^* \in R$ and $i = 1, 2, \ldots, N$.

Assumption 1. The undirected graph $\mathcal{G}$ is connected.

Assumption 2 ([30]). The local cost function f_i is β_i-strongly convex for some $\beta_i > 0$, and $f_i'(x_{i1})$ is θ_i-Lipschitz for some $\theta_i > 0$.

Let $x(t) = \mathrm{col}(x_1(t), \ldots, x_n(t))$ and $x_i(t) = \mathrm{col}(x_{i1}(t), \ldots, x_{in}(t))$. The notation $g_i(x,t) = v_i(x_i(t)) + \sum_{j\in\aleph_i} g_{ij}(x_j(t)) + \sum_{k\in\bar{\aleph}_i} g_{ik}(x_k(t)) + q_i(t)$ represents the lumped uncertainty of agent i. The following assumption for the lumped uncertainty g_i is presented, which allows the nonlinear growth rate of uncertainties.

Assumption 3. The unknown function $g_i(x,t)$ is differentiable. There exists a known function $\varphi(x)$, such that

$$\sup_{t\geqslant t_0}\left\{|g_i(x,t)|, \left|\frac{\partial g_i}{\partial x_j}(x,t)\right|, \left|\frac{\partial g_i}{\partial t}(x,t)\right|\right\} \leqslant \varphi(x). \tag{6}$$

Remark 2. By defining $\phi(\alpha) = \sup_{\|x\|\leqslant\alpha} \varphi(x)$, the following inequality is satisfied for $\|x\| \leqslant \alpha$:

$$\sup_{t\geqslant t_0}\left\{|g_i(x,t)|, \left|\frac{\partial g_i}{\partial x_j}(x,t)\right|, \left|\frac{\partial g_i}{\partial t}(x,t)\right|\right\} \leqslant \phi(\alpha). \tag{7}$$

Since the lumped nonlinear uncertainties depend on the states of all agents, it cannot be directly assumed that $g_i(x,t)$ is bounded for $t \geqslant t_0$. It is necessary to prove that all states are bounded and then infer that the lumped uncertainty $g_i(x,t)$ and the derivative of $g_i(x,t)$ are bounded. Compared with the bounded assumption [18–23], Assumption 3 generalizes the assumption for uncertainties.

Remark 3. Since the design and analysis progress for addressing the nonlinear coupling uncertainties $g_i(x,t)$ are highly complex, this paper simplifies this analysis progress by considering the optimization problem (5) over an undirected graph. The research results in this paper can be naturally generalized to a weight-balanced, directed, and strongly connected graph based on the properties of directed graphs given in the work by Deng et al. [32].

3 Resource allocation

3.1 Algorithm design

The algorithms are shown in this section. The decision $x_{i1}(t)$ of the agent i is measurable, whereas the decision x_{il} of the agent i is unmeasurable for $l = 2, 3, \ldots, n$, and a full-order extended state observer of agent i is established to actively estimate the unmeasurable decision $x_{il}(t)$ and lumped uncertainty $g_i(t)$, as follows:

$$\begin{cases} \dot{\hat{x}}_{i1}(t) = \hat{x}_{i2}(t) - \beta_1(\hat{x}_{i1}(t) - x_{i1}(t)), \\ \vdots \\ \dot{\hat{x}}_{i(n-1)}(t) = \hat{x}_{in}(t) - \beta_{n-1}(\hat{x}_{i1}(t) - x_{i1}(t)), \\ \dot{\hat{x}}_{in}(t) = u_i(t) + \hat{g}_i(t) - \beta_n(\hat{x}_{i1}(t) - x_{i1}(t)), \\ \dot{\hat{g}}_i(t) = -\beta_{n+1}(\hat{x}_{i1}(t) - x_{i1}(t)), \end{cases} \tag{8}$$

where $\hat{x}_{il}(t)$ is the estimated value of $x_{il}(t)$, $\hat{g}_i(t)$ is the estimated value of $g_i(t)$, $\beta_i = \phi_i\omega_o^i$ for $i = 1, 2, \ldots, n+1$, and $\omega_o > 0$. The constants ϕ_i are selected, such that the following matrix $\bar{\mathbf{A}}_1$ is Hurwitz, and a feasible choice is $\phi_i = \frac{(n+1)!}{i!(n+1-i)!}$ [25]:

$$\bar{\mathbf{A}}_1 = \begin{bmatrix} -\phi_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\phi_n & 0 & \cdots & 1 \\ -\phi_{n+1} & 0 & \cdots & 0 \end{bmatrix}.$$

Based on the estimation from the observer (8), the following resource allocation algorithm composed of active disturbance compensation is designed for the system (4):

$$u_i(t) = -\,y_i(t) - f_i'(x_{i1}(t)) - \sum_{l=2}^{n} \varepsilon^{n-l+1} k_{l-1}\hat{x}_{il}(t) - \hat{g}_i(t), \tag{9a}$$

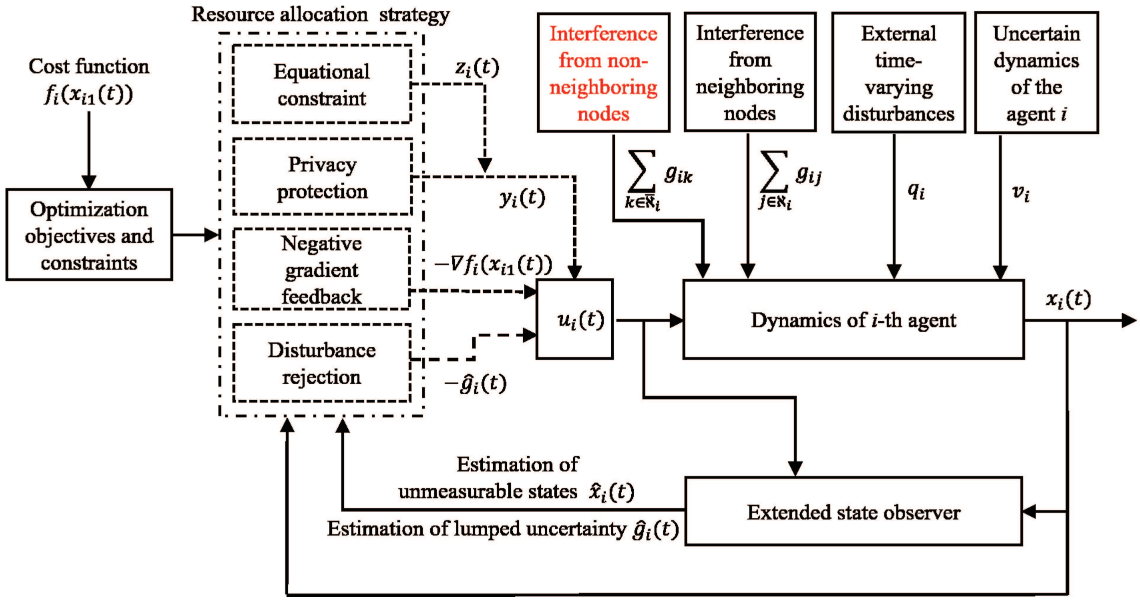

Figure 1 (Color online) Proposed design based on disturbance compensation.

$$\dot{y}_i(t) = \frac{k_1}{\varepsilon^{n-1}}\left(\sum_{j=1}^{N} a_{ij}(z_i(t)-z_j(t)) - d_i + x_{i1}(t)\right) - \frac{2\varepsilon}{\lambda_N}\sum_{j=1}^{N} a_{ij}(y_i(t)-y_j(t)) + \sum_{l=2}^{n}\varepsilon^{2-l}k_{l-1}\hat{x}_{il}(t), \tag{9b}$$

$$\dot{z}_i(t) = -\frac{k_1}{\varepsilon^{n-1}}\sum_{j=1}^{N} a_{ij}(y_i(t)-y_j(t)) - \frac{k_1}{\varepsilon^{2n-2}}\sum_{j=1}^{N} a_{ij}(\hat{x}_{in}(t)-\hat{x}_{jn}(t)), \tag{9c}$$

where $y_i(t) \in \mathbb{R}$ and $z_i(t) \in \mathbb{R}$ are used to satisfy the privacy protection and constraint conditions, respectively; $f'(x_{i1}(t)) \in \mathbb{R}$ is utilized for seeking the optimal solution; the constant λ_N represents the maximum eigenvalue of the Laplace matrix L; and the parameters ε and k_i are the variables to be tuned for $i = 1, 2, \ldots, N$. The detailed discussion on the selection of parameters ε and k_i is given in Subsection 3.2. The design framework for the distributed optimization anti-disturbance algorithm (9) is shown in Figure 1.

Remark 4. For the output-feedback situation, only the information x_{i1} can be obtained to design the distributed resource allocation algorithm, which adds new challenges to the algorithm design and theoretical analysis. Then, a full-order extended state observer is employed to estimate the unmeasurable state variables and the lumped uncertainty, including the disturbances from the non-neighboring nodes.

3.2 Parameter selection

A detailed introduction of parameters is given in this subsection. The constant k_1 is chosen, such that $k_1 > \frac{2(\theta^2+1)}{\beta}$, where $\theta = \max\{\theta_1, \ldots, \theta_N\}$ and $\beta = \min\{\beta_1, \ldots, \beta_N\}$. Then, the constants $k_2, \ldots, k_{n-1}$ are selected, such that the following matrix is Hurwitz:

$$\mathbf{A}_k = \begin{bmatrix} 0_{n-2} & I_{n-2} \\ -k_1 & [-k_2, \ldots, -k_{n-1}] \end{bmatrix}.$$

Since A_k is Hurwitz, there exists a positive define matrix P_k satisfying the following equation:

$$\mathbf{A}_k^{\mathrm{T}}\mathbf{P}_k + \mathbf{P}_k\mathbf{A}_k = -\mathbf{I}_{n-1}.$$

Finally, the constant ε is designed as follows:

$$\varepsilon > \sqrt[n]{\frac{(n-1)\bar{p}(\theta^2+1)}{\beta k_1} + (2n-1)\bar{p} + \frac{2k_1}{\beta}}, \tag{10}$$

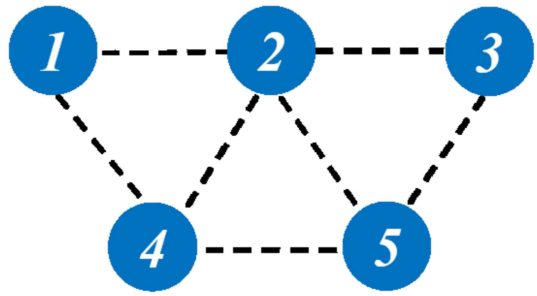

Figure 2 (Color online) Communication topology.

where

$$\bar{p} = \max\left\{(2p_{1(n-1)} + k_2)^2, \ldots, (2p_{(n-2)(n-1)} + k_{n-1})^2, (2p_{(n-1)(n-1)} + 1)^2, (2p_{(n-1)(n-1)} + 2)^2\right\}, \tag{11}$$

and $p_{i(n-1)}$ is the last element of the i-th row vector of the positive define matrix $\mathbf{P}_k$.

4 Convergence analysis

In this section, the convergence results of the algorithms for the output-feedback case are presented. The following theorem shows the convergence result for the output-feedback case.

Theorem 1. Consider the system (4) with the resource allocation algorithm (9) under Assumptions 1–3. If $u_i(t) = 0$ for $t \in [t_0, t_u)$, and $u_i(t)$ satisfies (9) for $t \in [t_u, +\infty)$, then there exist positive constants $\omega_o^* > 1$ and t_T, such that, for any $\omega_o \geqslant \omega_o^*$ and $t \geqslant t_T$, the following inequalities hold for the i-th agent:

$$\|x_{i1}(t) - \eta_{i1}^*\| \leqslant \sqrt{\mathrm{e}^{\frac{-\sigma}{\lambda_{\max}}(t-t_u)}\frac{\lambda_{\max}}{\lambda_{\min}}\rho_{1,v} + \frac{\tilde{\sigma}}{\lambda_{\min}}\max\left\{\frac{1}{\omega_o^2}, \frac{ln\omega_o}{\omega_o}\right\}}, \tag{12}$$

$$\|x_{il}(t) - \hat{x}_{il}(t)\| \leqslant \gamma_1\left(\mathrm{e}^{-\gamma_2\omega_o(t-t_u)} + \frac{1}{\omega_o}\right), \quad l = 1, 2, \ldots, n, \tag{13}$$

$$\|g_i(x,t) - \hat{g}_i(t)\| \leqslant \gamma_1\left(\mathrm{e}^{-\gamma_2\omega_o(t-t_u)} + \frac{1}{\omega_o}\right), \tag{14}$$

where $t_u = t_0 + 2n\bar{c}_{12}\frac{\ln(\tilde{\rho}_0^{\frac{1}{n}}\omega_o)}{\omega_o}$; the positive constant $\tilde{\rho}_0$ satisfies $\sqrt{\sum_{i=1}^{N}\sum_{l=1}^{n}|x_{il}(t_0) - \hat{x}_{il}(t_0)|^2} \leqslant \tilde{\rho}_0$; $\bar{c}_{12}$ is the maximum eigenvalue of $\bar{\mathbf{P}}_1$; the parameters $\lambda_{\max}$ and $\lambda_{\min}$ represent the maximum and minimum eigenvalues of the positive definite matrix $\mathbf{\Phi} = \mathrm{diag}\{\mathbf{\Phi}_1, \mathbf{\Phi}_2\}$, respectively; the definitions of $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$ are given in Appendix C; and the definitions of the positive constants $\rho_{1,v}$, σ, $\tilde{\sigma}$, γ_1, γ_2 are shown in Appendix B.

Remark 5. Compared with previous research [18–23], this paper considers the lumped uncertainties with a nonlinear growth rate. To address the nonlinear lumped uncertainties, a new distributed optimization algorithm is designed based on an extended state observer. Moreover, it is complicated to analyze the closed-loop stability in detail for the lumped uncertainties dependent on the states of all agents.

The proofs of Theorem 1 are given in Appendix C.

5 Simulation

In this section, the simulation for an economic dispatch problem of electric power grids is presented. Consider a smart grid consisting of five generation systems. The communication topology of generators is given in Figure 2.

The economic dispatch problem is described as follows:

$$\min_{W(t)\in R^5} f(W(t)) = \sum_{i=1}^{5} f_i(W_i(t)), \quad \text{subject to} \sum_{i=1}^{5} W_i(t) = \sum_{i=1}^{5} d_i, \tag{15}$$

Table 1 Performance indicators for the proposed algorithm and the algorithm in [9].

Agent	Integral of squared tracking error		Max steady-state error ($t > 10$ s)	
	Algorithms (8) and (9)	Algorithm in [9]	Algorithms (8) and (9)	Algorithm in [9]
1	10.2786	5.5572×10^3	0.0574	80.5737
2	38.7611	1.8162×10^3	0.0292	44.2881
3	18.5238	28.8739	0.1278	4.1642
4	4.7146	541.3599	0.0467	24.4902
5	12.5985	21.3743	0.1583	3.8134

where $W_i(t) \in \mathbb{R}$ is the output power of the i-th generator; $f_i(W_i(t))$ is the cost function of the i-th generator; d_i is the local constraint of the i-th generator; and $W(t) = \mathrm{col}(W_1(t), W_2(t), \ldots, W_5(t))$. According to Sharifian and Abdi [33], the cost function of the i-th generator is given by the following quadratic function:

$$f_i(W_i) = O_i W_i^2 + S_i W_i + Q_i, \tag{16}$$

where O_i, S_i, and Q_i are constant parameters. Based on the work by Guo et al. [34], the i-th generation system can be written as follows:

$$\begin{cases} \dot{W}_i(t) = -\frac{1}{U_i} W_i(t) + \frac{C_i}{U_i} Y_i(t), \\ \dot{Y}_i(t) = -\frac{1}{F_i} Y_i + \frac{1}{F_i} u_i(t) + g_i(W(t), t), \end{cases} \tag{17}$$

where $u_i(t)$ is the control input of the i-th generator, and $g_i(W(t), t)$ is the nonlinear uncertainties of the i-th generator. The uncertainty $g_i(W(t), t)$ includes the interference from non-neighboring agents.

Let $x_{i1}(t) = W_i(t)$ and $x_{i2}(t) = -\frac{1}{U_i} W_i(t) + \frac{C_i}{U_i} Y_i(t)$; then, Eq. (17) is rewritten as follows:

$$\begin{cases} \dot{x}_{i1}(t) = x_{i2}(t), \\ \dot{x}_{i2}(t) = N_i(x, t) + \frac{C_i}{U_i F_i} u_i(t), \end{cases} \tag{18}$$

where $x_i(t) = \mathrm{col}(x_{i1}(t), x_{i2}(t))$ is the state vector, and $N_i(x, t) = -\frac{1}{F_i U_i} x_{i1}(t) - (\frac{1}{U_i} + \frac{1}{F_i}) x_{i2}(t) + \frac{C_i}{U_i} g_i(x, t)$ is the total disturbance of the i-th generator. Let $\mathbf{Q} = \mathrm{col}(Q_1, \ldots, Q_5)$, $\mathbf{S} = \mathrm{col}(S_1, \ldots, S_5)$, $\mathbf{O} = \mathrm{col}(O_1, \ldots, O_5)$, $\mathbf{U} = \mathrm{col}(U_1, \ldots, U_5)$, $\mathbf{F} = \mathrm{col}(F_1, \ldots, F_5)$, and $\mathbf{C} = \mathrm{col}(C_1, \ldots, C_5)$. The system parameters in the simulation are shown as

$$\begin{cases} \mathbf{Q} = \mathrm{col}(15.00, 25.00, 35.00, 45.00, 55.00), \\ \mathbf{S} = \mathrm{col}(24.30, 12.10, 20.60, 28.40, 30.60), \\ \mathbf{O} = \mathrm{col}(0.30, 0.28, 0.26, 0.25, 0.10), \\ \mathbf{U} = \mathrm{col}(0.34, 0.37, 0.50, 0.43, 0.40), \\ \mathbf{F} = \mathrm{col}(0.12, 0.27, 0.23, 0.17, 0.30), \\ \mathbf{C} = \mathrm{col}(1.13, 1.42, 1.26, 1.33, 2.00). \end{cases} \tag{19}$$

The parameters in the proposed algorithm are selected as $k_1 = 3$, $a_{ij} = 1$, $\varepsilon = 0.7$, $\lambda_N = 5$, $\omega_0 = 200$. The local demand is chosen as $\mathbf{d} = \mathrm{col}(25, 17, 12, 30, 20)$, and the initial value of $\mathbf{W(0)}$ is selected as $\mathbf{W(0)} = \mathrm{col}(1, 1, 1, 1, 1)$. Furthermore, the nonlinear uncertainties are chosen as follows:

$$\begin{cases} g_1(x, t) = \sin(x_{11}(t)) + \sin(t) + 2x_{21}(t) + 3x_{31}^2(t), \\ g_2(x, t) = \cos(x_{21}(t)) + \cos(t) + 3x_{11}(t) + 2\log_{10}(1 + |x_{41}(t)|), \\ g_3(x, t) = \cos(x_{31}(t)) + 4x_{41}(t) + 3x_{21}(t) + 2x_{51}^2(t) + 66, \\ g_4(x, t) = \cos(x_{41}(t)) + 5x_{51}(t) + 3x_{31}(t) + 4, \\ g_5(x, t) = \cos(x_{51}(t)) + \sin(t) + 2x_{11}(t) + 4x_{21}(t). \end{cases} \tag{20}$$

To quantitatively describe the property of the proposed algorithm, some indices are given, as shown in Table 1. Compared with the method by Deng [9], the proposed algorithm can address the distributed resource allocation problem despite nonlinear uncertainties.

Comparing the algorithm in [9], the simulation results are shown in Figures 3 and 4. From Figure 3, the output power of each generator can converge to the optimal output based on the proposed algorithm, whereas the output

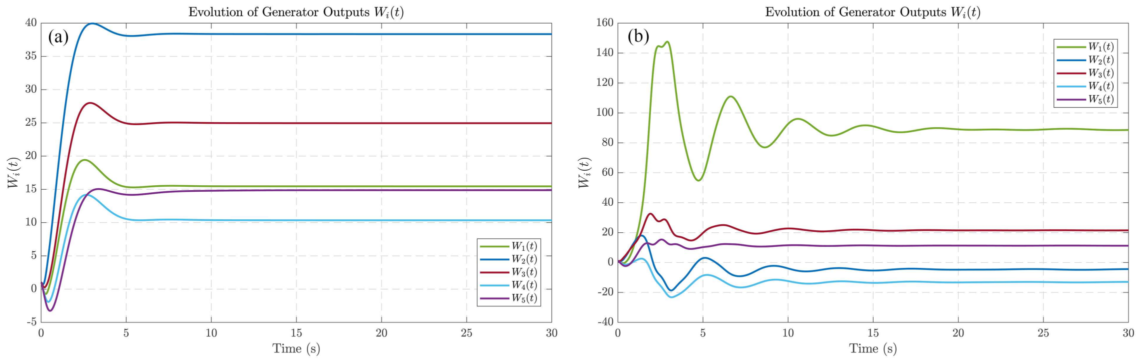

Figure 3 (Color online) (a) The power outputs $W_i(t)$ based on algorithms (8) and (9); (b) the power outputs $W_i(t)$ according to algorithm in [9].

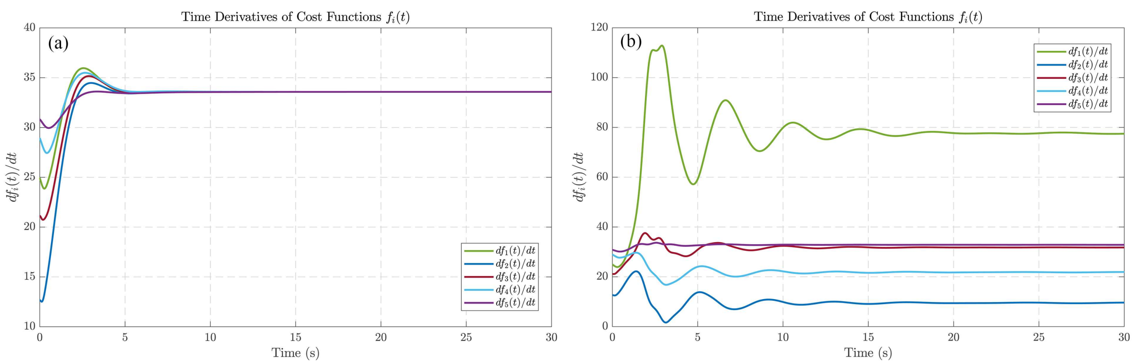

Figure 4 (Color online) (a) The gradient term of $f_i(t)$ based on algorithms (8) and (9); (b) the gradient term of $f_i(t)$ according to algorithm in [9].

power based on the algorithm in [9] is oscillating. As shown in Figure 4, the gradient terms $\dot{f}_i(t), i = 1, 2, \ldots, 5$ of the proposed algorithm can converge to the same value. From the work by Deng [9], the optimal problem (5) is solved if and only if the gradient terms $\dot{f}_i(t), i = 1, 2, \ldots, 5$ are equal to each other. Combining with Figure 4(a), the optimal problem (5) under nonlinear uncertainties is solved by the proposed algorithm. As shown in Figure 4(b), the algorithm in [9] cannot deal with the optimal problem (5) under nonlinear uncertainties.

6 Conclusion

This paper has investigated the resource allocation problem of high-order multiagent systems with nonlinear uncertainties. To accomplish this resource allocation problem under lumped uncertainty, the algorithms have been proposed on the basis of a privacy protection protocol, gradient descent method, and disturbance rejection design. More specifically, a disturbance rejection strategy based on an extended state observer has been proposed. By effectively estimating and compensating for the lumped uncertainties, the proposed method ensures privacy protection and the constraints of resource demand. Based on the Lyapunov stability theory, it is rigorously proven that the outputs of agents can exponentially converge to a narrow region around the optimal solution. Finally, a simulation of smart grids illustrates the effectiveness of the proposed algorithm in solving the resource allocation problem with uncertainties of non-neighboring agents.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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