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• **Supplementary File** •

Resource allocation for high-order multi-agent systems with uncertainties from non-neighboring nodes

Junlong He[1], Sen Chen[1*] & Wenchao Xue[2,3*]

[1]*School of Mathematics and Statistics, Shaanxi Normal University, Xi'an 710119, China*
[2]*State Key Laboratory of Mathematical Sciences, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China*
[3] *School of Mathematical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China*

Appendix A Introduction and proof of Lemma

To simplify the proof of Theorem 1, the following two lemmas are introduced.

Lemma A1. [9] Consider the following system.

$$\begin{cases} \dot{\eta}_1(t) = \eta_2(t), \\ \quad \vdots \\ \dot{\eta}_{n-1}(t) = \eta_n(t), \\ \dot{\eta}_n(t) = -\sum\limits_{l=2}^{n} \varepsilon^{n-l+1} k_{l-1}\eta_l(t) - y(t) - \tilde{f}(\eta_1(t)), \\ \dot{y}(t) = \frac{k_1}{\varepsilon^{n-1}}(Lz(t) - d + \eta_1(t)) - \frac{2\varepsilon}{\lambda_N} Ly + \sum\limits_{l=2}^{n} \varepsilon^{2-l} k_{l-1}\eta_l(t), \\ \dot{z}(t) = -\frac{k_1}{\varepsilon^{n-1}} Ly(t) - \frac{k_1}{\varepsilon^{2n-2}} L\eta_n(t), \end{cases} \tag{A1}$$

where $\eta_l(t) = \mathrm{col}(\eta_{1l}(t), \cdots, \eta_{Nl}(t))$, $y(t) = \mathrm{col}(y_1(t), \cdots, y_N(t))$, $\eta_{il}(t)$, $y_i(t)$, $z_i(t) \in R$, $\mathbf{d} = \mathrm{col}(d_1, \cdots, d_N)$, and $\tilde{f}(\eta_1(t)) = \mathrm{col}(f_1'(\eta_{11}(t)), \cdots, f_N'(\eta_{N1}(t)))$.

Denoting $\eta_1^* = \mathrm{col}(\eta_{11}^*, \cdots, \eta_{N1}^*)$, $y^* = \mathrm{col}(y_1^*, \cdots, y_N^*)$, $z^* = \mathrm{col}(z_1^*, \cdots, z_N^*)$ and supposing that Assumptions 1-2 are satisfied, then the following statements hold.

If (η^*, y^*, z^*) represents the equilibrium point of the system (A1), and $\eta^* = \mathrm{col}(\eta_1^*, \cdots, \eta_n^*)$, $\eta_l^* = 0 \in R^N$ for $l = 2, \cdots, n$, then η_1^* is the optimal solution for the resource allocation problem (5).

Let (η^*, y^*, z^*) be the equilibrium point of the system (A1) and define the following error variables.

$$\begin{cases} \bar{x}_{il} = x_{il} - \eta_{il}^*, \ \bar{y}_i = y_i - y_i^*, \\ \bar{z}_i = z_i - z_i^*, \ m_{il} = x_{il} - \hat{x}_{il}, \ l \in (1, \cdots, n), \\ \varphi_i(t) = g_i(x, t) - \hat{g}_i(t), \\ h_i(t) = f_i'(x_{i1}) - f_i'(\eta_{i1}^*), \ i \in (1, \cdots, N). \end{cases} \tag{A2}$$

Let $\xi(t) = (\mathbf{T} \otimes \mathbf{I}_N)\begin{pmatrix} m(t) \\ \varphi(t) \end{pmatrix}$, $\mathbf{T} = \mathrm{diag}\{\omega_o^n, \cdots, \omega_o, 1\}$, $m(t) = \mathrm{col}(m_1(t), \cdots, m_n(t))$, and $m_l(t) = \mathrm{col}(m_{1l}(t), \cdots, m_{Nl}(t))$ for $l \in (1, \cdots, n)$.

Lemma A2. Considering the system (4) with the algorithm (9), if $\|x(t)\| \leqslant \rho_{1,x}$, $\|y(t)\| \leqslant \rho_{1,y}$, $\|\xi(t)\| \leqslant \rho_{1,\xi}$, there exists a function $\tilde{\Phi}(x)$ such that $\|\frac{\mathrm{d}g}{\mathrm{d}t}\| \leqslant \tilde{\Phi}(\rho_{1,x})$.

Lemma A2 illustrates that the derivative of lumped uncertainty is bounded if the states are bounded. The proof of Lemma A2 is given as follows.

Proof. Combining the system (4) with the algorithm (9) and the Assumption 3, one can conclude that $\frac{d}{dt}g_i = \sum_{j=1}^{N} \frac{\partial g_i}{\partial x_j}\dot{x}_j + \frac{\partial g_i}{\partial t}$, $\dot{x}_j = \mathbf{A}x_j + \mathbf{B}_1(g_j + u_j)$, and $u_j(t) = -y_j(t) - f_j'(x_{j1}(t)) - \sum_{l=2}^{n} \varepsilon^{n-l+1} k_{l-1}\hat{x}_{jl}(t) - \hat{g}_j(t)$ for $j = 1, 2, \cdots, N$.

* Corresponding author (email: chensen14@mails.ucas.ac.cn, wenchaoxue@amss.ac.cn)

Since $\|y_j(t)\| \leqslant \rho_{1,y}$, and $\|g_j(t)-\hat{g}_j(t)\| \leqslant \|\xi(t)\|$, then $\|g_j(t)-\hat{g}_j(t)\| \leqslant \rho_{1,\xi}$. Based on $f_j'(\cdot)$ is θ_j-Lipschitz continuous, it can be concluded that $\|f_j'(x_{j1}(t)) - f_j'(0)\| \leqslant \theta\|x_{j1}(t)\|$, then $\|f_j'(x_{j1}(t))\| \leqslant \|f_j'(0)\| + \theta\rho_{1,x}$, where parameter $\theta = \max(\theta_1, \theta_2, \cdots, \theta_N)$. Furthermore, the following inequality is established.

$$\|\dot{x}_j\| \leqslant \|\mathbf{A}\|\rho_{1,x} + \rho_{1,y} + |f'(0)| + \theta\rho_{1,x} + \tilde{\varepsilon}\bar{k}(n-1)(\rho_{1,x}+\rho_{1,\xi}) + \rho_{1,\xi}. \tag{A3}$$

Finally, combining the inequality (A3) with the Assumption 3, the following result is yielded.

$$\|\frac{d}{dt}g(x,t)\| \leqslant \phi(\rho_{1,x})(N(\|\mathbf{A}\|\rho_{1,x} + \rho_{1,y} + |f'(0)| + \theta\rho_{1,x} + \tilde{\varepsilon}\bar{k}(n-1)(\rho_{1,x}+\rho_{1,\xi}) + \rho_{1,\xi}) + 1). \tag{A4}$$

$$\tilde{\Phi}(\rho_{1,x}) \triangleq N\|\bar{\mathbf{P}}_1\|\phi(\rho_{1,x})(N(\|\mathbf{A}\|\rho_{1,x} + \rho_{1,y} + |f'(0)| + \theta\rho_{1,x} + \tilde{\varepsilon}\bar{k}(n-1)(\rho_{1,x}+\rho_{1,\xi}) + \rho_{1,\xi}) + 1). \tag{A5}$$

This completes the proof of Lemma A2.

Appendix B Parameters defined in the proof of Theorem 1

$$\bar{k} = \max\{k_1, k_2, \cdots, k_{n-1}\}. \tag{B1}$$

$$\rho_0 = \max_{l\in\{1,2,\cdots,n-1\}} \left\{ \frac{(n+3)k_l^2\varepsilon^{n+1}}{4\lambda_2 k_1} + \frac{\varepsilon}{2}k_l \right\}. \tag{B2}$$

$$\tilde{\rho}_1 \triangleq \rho_0 + \frac{k_1\lambda_N^2(n+3)}{4\lambda_2\varepsilon^{n-1}} + \frac{\varepsilon}{2}k_{n-1} + \frac{\lambda_N k_1}{2\varepsilon^{n-1}}. \tag{B3}$$

$$\bar{\varepsilon} = \min\{\varepsilon, \varepsilon^2, \cdots, \varepsilon^{n-1}, 1\}. \tag{B4}$$

$$\tilde{\varepsilon} = \max\{\varepsilon, \varepsilon^2, \cdots, \varepsilon^{n-1}, 1\}. \tag{B5}$$

$$\begin{aligned}\sigma = \min\Big\{ & \frac{1}{2}(\varepsilon - (\frac{(n-1)\bar{p}(\theta^2+1)}{\beta k_1} + (2n-1)\bar{p} + \frac{2k_1}{\beta})\frac{1}{\varepsilon^{n-1}} - 2\mu\tilde{\rho}_1), \\ & \frac{\frac{1}{2}k_1\beta - \theta^2}{2\varepsilon^{n-1}} - \frac{1}{2\varepsilon^{n-1}} - \frac{\mu k_1(n+3+2\lambda_2)}{4\lambda_2\varepsilon^{n-1}}, \quad \frac{-M_0k_1\lambda_2}{2(n+3)\varepsilon^{n-1}} + \frac{\mu k_1\lambda_2}{2\varepsilon^{n-1}(n+3)}, \\ & \frac{1}{2n\varepsilon^{n-1}} - \mu(\frac{k_1(3\lambda_N+2)}{2\varepsilon^{n-1}} + \frac{(n+3)\varepsilon^{n+1}}{\lambda_2 k_1} + \varepsilon(\sum_{l=1}^{n-1}k_l - \frac{2\lambda_2}{\lambda_N}))\Big\}.\end{aligned} \tag{B6}$$

$$\begin{aligned}\pi_0 = & \mu\left(\frac{(n+3)^2\bar{k}^2\varepsilon^{n+1}}{k_1\lambda_2} + \frac{\bar{k}\varepsilon}{2} + \frac{(n+3)\lambda_N^2k_1}{\lambda_2\varepsilon^{n-1}} + \frac{k_1\lambda_N^2}{2\varepsilon^{n-1}}\right)\frac{1}{\bar{\varepsilon}} \\ & + \frac{k_1(n+3)\lambda_N^2}{2M_0\lambda_2\varepsilon^{n-1}}\frac{1}{\bar{\varepsilon}} + \frac{\beta k_1}{2\varepsilon^{n-1}} + \frac{k_1^2+n}{\varepsilon^{n-1}} + \left(\frac{(n-1)\varepsilon^{n+1}\bar{k}^2}{2} + (n-1)\varepsilon^{n+1}k_1^2\bar{k}^2)\right)\frac{1}{\bar{\varepsilon}}.\end{aligned} \tag{B7}$$

$$\rho_{1,v} \triangleq \frac{1}{\bar{\varepsilon}}(\rho_{1,x} + \|x^*\|^2) + \rho_{\bar{y}} + \rho_{\bar{z}}. \tag{B8}$$

$$\rho_{v_k} = \max\left\{\lambda_{\max}\rho_{1,v}, \frac{2\lambda_{\max}\pi_0\bar{c}_{12}\rho_{2,\xi}^2}{\sigma\bar{c}_{11}}\right\}. \tag{B9}$$

$$\begin{cases}\gamma_1 = \max\left\{\frac{\sqrt{\bar{c}_{12}}}{\sqrt{\bar{c}_{11}}}\rho_{3,\xi}; \frac{2\bar{c}_{12}}{\bar{c}_{11}}\tilde{\Phi}(\sqrt{\frac{\tilde{\varepsilon}\rho_{v_k}}{\lambda_{\min}} + \|x^*\|^2})\right\}. \\ \gamma_2 = \frac{1}{2\bar{c}_{12}}.\end{cases} \tag{B10}$$

$$\tilde{\sigma} = \frac{\lambda_{\max}\cdot\pi_0\cdot\bar{c}_{12}\cdot\rho_{2,\xi}^2}{\gamma_2\sigma\bar{c}_{11}} + \frac{4\pi_0\lambda_{\max}(\gamma_1)^2}{\sigma}. \tag{B11}$$

Appendix C Proof of Theorem 1

Before the detailed proof of Theorem 1, the error system of the optimization problem (5) is analyzed. With the combination of (A1) and (A2), the following dynamics of error variables is obtained by substituting the control algorithm (9) into the high-order multi-agent system (4).

$$\begin{cases} \dot{\bar{x}}_1(t) = \bar{x}_2(t), \\ \quad \vdots \\ \dot{\bar{x}}_{n-1}(t) = \bar{x}_n(t), \\ \dot{\bar{x}}_n(t) = -\bar{y}(t) - h(t) + \varphi(t) - \sum_{l=2}^{n} \varepsilon^{n-l+1} k_{l-1} \bar{x}_l(t) + \sum_{l=2}^{n} \varepsilon^{n-l+1} k_{l-1} m_l(t), \\ \dot{\bar{y}}(t) = \frac{k_1}{\varepsilon^{n-1}} (\mathbf{L}\bar{z}(t) + \bar{x}_1) - \frac{2\varepsilon}{\lambda_N} \mathbf{L}\bar{y}(t) + \sum_{l=2}^{n} \varepsilon^{2-l} k_{l-1} \bar{x}_l(t) - \sum_{l=2}^{n} \varepsilon^{2-l} k_{l-1} m_l(t), \\ \dot{\bar{z}}(t) = -\frac{k_1}{\varepsilon^{n-1}} \mathbf{L}\bar{y}(t) - \frac{k_1}{\varepsilon^{2n-2}} \mathbf{L}\bar{x}_n(t) + \frac{k_1}{\varepsilon^{2n-2}} \mathbf{L} m_n(t), \end{cases} \tag{C1}$$

where $\bar{x}_l(t) = \mathrm{col}(\bar{x}_{1l}(t), \cdots, \bar{x}_{Nl})$, $\bar{y}(t) = \mathrm{col}(\bar{y}_1(t), \cdots, \bar{y}_N(t))$, $\bar{z}(t) = \mathrm{col}(\bar{z}_1(t), \cdots, \bar{z}_N(t))$, and $h(t) = \mathrm{col}(h_1(t), \cdots, h_N(t))$ for $l = 1, 2, \cdots, n$.

Based on the error system (C1), the proof of Theorem 1 is divided into three sections. Firstly, the states of the system (4) are proved to be bounded for $t \in [t_0, t_u)$. Secondly, the boundedness analysis of tracking and estimate error is given for $t \in [t_u, +\infty)$. Thirdly, the convergence of the error system (C1) is analyzed for $t \in [t_u, +\infty)$.

Step 1: Analyzing the boundedness of variables for $t \in [t_0, t_u)$.

In the first step, the input of the agent i is designed as $u_i(t) = 0$. Substituting $u_i(t) = 0$ into the considered high-order multi-agent system, the dynamics of the agent i is shown as follows.

$$\begin{cases} \dot{x}_i(t) = \mathbf{A}x_i(t) + \mathbf{B}_1 g_i(x, t), \\ \dot{\bar{y}}(t) = \frac{k_1}{\varepsilon^{n-1}} (\mathbf{L}\bar{z}(t) + \bar{x}_1(t)) - \frac{2\varepsilon}{\lambda_N} \mathbf{L}\bar{y}(t) + \sum_{l=2}^{n} \varepsilon^{2-l} k_{l-1} \bar{x}_l(t) - \sum_{l=2}^{n} \varepsilon^{2-l} k_{l-1} m_l(t), \\ \dot{\bar{z}}(t) = -\frac{k_1}{\varepsilon^{n-1}} \mathbf{L}\bar{y}(t) - \frac{k_1}{\varepsilon^{2n-2}} \mathbf{L}\bar{x}_n(t) + \frac{k_1}{\varepsilon^{2n-2}} \mathbf{L} m_n(t), \end{cases} \tag{C2}$$

where $\mathbf{B}_1 = [0_{n-1}^T, 1]^T$, $\mathbf{A} = \begin{pmatrix} 0_{n-1} & I_{n-1} \\ 0 & 0_{n-1}^T \end{pmatrix}$.

If $\tilde{\rho}_0^{\frac{1}{n}} \omega_o > 1$, then $\lim_{\omega_o \to +\infty} \frac{\ln(\tilde{\rho}_0^{\frac{1}{n}} \omega_o)}{\omega_o} = 0$. Since $x_i(t)$, $\bar{y}(t)$ and $\bar{z}(t)$ have a bounded value at the initial time t_0, then there exist positive constants η_d, ρ_{1,x_i}, $\rho_{\bar{y}}$ and $\rho_{\bar{z}}$ for $\omega_1^* > 1$, such that

$$\begin{cases} \sup_{t_0 \leqslant t < t_u} \|x_i(t)\| \leqslant \rho_{1,x_i}, \\ \sup_{t_0 \leqslant t < t_u} \|\bar{y}(t)\| \leqslant \rho_{\bar{y}}, \\ \sup_{t_0 \leqslant t < t_u} \|\bar{z}(t)\| \leqslant \rho_{\bar{z}}, \\ t_u \leqslant t_0 + \frac{\eta_d}{2}. \end{cases} \tag{C3}$$

By defining $\rho_{1,x} = \sqrt{\sum_{i=1}^{N} \rho_{1,x_i}^2}$, then $\|x(t)\| \leqslant \rho_{1,x}$. Since $x(t)$, $\bar{y}(t)$ and $\bar{z}(t)$ are continuous at t_u, we have $\|x(t_u)\| \leqslant \rho_{1,x}$, $\|\bar{y}(t_u)\| \leqslant \rho_{\bar{y}}$, and $\|\bar{z}(t_u)\| \leqslant \rho_{\bar{z}}$ for $\omega_o \geqslant \omega_1^*$.

Based on the system (4) with the extended state observer (8), the estimation error equation is written as follows.

$$\begin{pmatrix} \dot{m}(t) \\ \dot{\varphi}(t) \end{pmatrix} = (\mathbf{B}_\beta \otimes \mathbf{I}_N) \begin{pmatrix} m(t) \\ \varphi(t) \end{pmatrix} + (\mathbf{B}_0 \otimes \mathbf{I}_N) \dot{g}(t), \tag{C4}$$

where $g(t) = \mathrm{col}(g_1(x, t), \cdots, g_N(x, t))$, $\mathbf{B}_0 = \mathrm{col}(0_n^T, 1)$,

$$\mathbf{B}_\beta = \begin{bmatrix} -\beta_1 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ -\beta_n & 0 & 0 & \cdots & 0 & 1 \\ -\beta_{n+1} & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}. \tag{C5}$$

To analyze the stability of the estimation error equation (C4), the following Lyapunov function is constructed.

$$V_1(t) = \xi^T (\bar{\mathbf{P}}_1 \otimes \mathbf{I}_N) \xi, \tag{C6}$$

where the positive definite matrix $\bar{\mathbf{P}}_1$ satisfying $\bar{\mathbf{A}}_1^T\bar{\mathbf{P}}_1 + \bar{\mathbf{P}}_1\bar{\mathbf{A}}_1 = -\mathbf{I}_{n+1}$.

Since $\|x(t)\| \leqslant \rho_{1,x}$ for $t \in [t_0, t_u]$, the equation (C2) and Assumption 3 indicate that $\dot{x}_i(t)$ and $\dot{g}_i(t)$ are bounded and the bounds of $\dot{x}_i(t)$ and $\dot{g}_i(t)$ are obtained easily for $t \in [t_0, t_u]$. Based on the Lyapunov function (C6), we have $\bar{c}_{11}\|\xi(t)\|^2 \leqslant V_1(t) \leqslant \bar{c}_{12}\|\xi(t)\|^2$, where $\bar{c}_{11}$ and $\bar{c}_{12}$ are the minimum and maximum eigenvalues of $\bar{P}_1$, respectively. Then the derivative of (C6) is obtained.

$$\frac{d}{dt}\sqrt{V_1(t)} \leqslant \frac{-\omega_o}{2\bar{c}_{12}}\sqrt{V_1(t)} + \frac{\tilde{\phi}}{\sqrt{\bar{c}_{11}}}, \tag{C7}$$

where $\tilde{\phi} = \|\bar{\mathbf{P}}_1\| N\phi(\rho_{(1,x)})(\|\mathbf{A}\|\rho_{(1,x)} + \phi(\rho_{(1,x)}) + 1)$.

Combining Bellman-Grownwall inequality with the bounds of $\xi(t_0)$ and $\|m(t_0)\| \leqslant \tilde{\rho}_0$, the boundedness of $\sqrt{V_1(t_u)}$ for $\omega_o \geqslant \omega_1^*$ is shown as $\sqrt{V_1(t_u)} \leqslant \frac{\sqrt{\bar{c}_{12}}}{\tilde{\rho}_0}(\tilde{\rho}_0 + \phi(\rho_{1,x}) + \|\hat{g}(t_0)\|) + \frac{2\bar{c}_{12}}{\omega_1^*}\frac{\tilde{\phi}}{\sqrt{\bar{c}_{11}}}$. Let $\tilde{\eta}_{1,\xi}^* \triangleq \frac{\sqrt{\bar{c}_{12}}}{\tilde{\rho}_0}(\tilde{\rho}_0 + N\phi(\rho_{1,x}) + \|\hat{g}(t_0)\|) + \frac{2\bar{c}_{12}}{\omega_1^*}\frac{\tilde{\phi}}{\sqrt{\bar{c}_{11}}}$, then $\|\xi(t_u)\| \leqslant \frac{\tilde{\eta}_{1,\xi}^*}{\sqrt{\bar{c}_{11}}}$.

If $\tilde{\rho}_0^{\frac{1}{n}}\omega_o \leqslant 1$, the initial value $x(t_0)$ exists and is bounded, then $\xi(t_0) \leqslant 1 + N\phi(\rho_{1,x}) + \|\hat{g}(t_0)\|$. Denote $\tilde{\eta}_{2,\xi}^* \triangleq \sqrt{\bar{c}_{12}}(1 + N\phi(\rho_{1,x}) + \|\hat{g}(t_0)\|)$, then $\|\xi(t_0)\| \leqslant \frac{\tilde{\eta}_{2,\xi}^*}{\sqrt{\bar{c}_{11}}}$.

Remark 1. Based on the definition of t_u, it can be concluded that $\lim_{\omega_o \to +\infty} t_u = t_0$, and t_u can approach t_0 by regulating the parameter ω_o. Thus, the controller has the similar performance with the commonly designed controller whose initial value is zero. Moreover, the selection of t_u is to avoid the effect of the peaking phenomenon.

Step 2: Analyzing the boundedness of variables for $t \in [t_u, +\infty)$.

In this step, by utilizing coordinate transformation and Lyapunov function, the tracking error and estimation error are proved to be bounded. Define $\tilde{m}_{il}(t) = \frac{1}{\varepsilon^{l-1}}m_{il}(t)$ and $\tilde{x}_{il}(t) = \frac{1}{\varepsilon^{l-1}}\bar{x}_{il}(t)$ for $l = 1, 2, \cdots, n$, and denote $\tilde{m}_l(t) = \mathrm{col}(\tilde{m}_{1l}(t), \tilde{m}_{2l}(t)\cdots, \tilde{m}_{Nl}(t)) \in \mathbb{R}^N$, $\tilde{x}_l(t) = \mathrm{col}(\tilde{x}_{1l}(t), \tilde{x}_{2l}(t)\cdots, \tilde{x}_{Nl}(t)) \in \mathbb{R}^N$, and $\tilde{m}(t) = \mathrm{col}(\tilde{m}_2(t), \tilde{m}_3(t)\cdots, \tilde{m}_n(t)) \in \mathbb{R}^{(n-1)N}$.

Define the following coordinate transformations.

$$H_1(t) = \mathrm{col}(H_{11}(t), H_{21}(t)) = [\mathbf{r}\ \mathbf{R}]^T m_1(t), \tag{C8}$$
$$H_l(t) = \mathrm{col}(H_{1l}(t), H_{2l}(t)) = [\mathbf{r}\ \mathbf{R}]^T \tilde{m}_l(t), \tag{C9}$$
$$\tau = \mathrm{col}(\tau_1, \tau_2) = [\mathbf{r}\ \mathbf{R}]^T \bar{x}_1, \tag{C10}$$
$$\tau_{l-1} = \mathrm{col}(\tau_{1(l-1)}, \tau_{2(l-1)}) = [\mathbf{r}\ \mathbf{R}]^T \tilde{x}_l, \tag{C11}$$
$$\zeta = \mathrm{col}(\zeta_1, \zeta_2) = [\mathbf{r}\ \mathbf{R}]^T \bar{y}, \tag{C12}$$
$$\varpi = \mathrm{col}(\varpi_1, \varpi_2) = [\mathbf{r}\ \mathbf{R}]^T \bar{z}, \tag{C13}$$
$$\psi = \mathrm{col}(\psi_1, \psi_2) = [\mathbf{r}\ \mathbf{R}]^T \varphi(t), \tag{C14}$$

where $H_{11}(t) \in \mathbb{R}$, $H_{1l}(t) \in \mathbb{R}$, $\tau_1 \in \mathbb{R}$, $\tau_{1(l-1)} \in \mathbb{R}$, $\zeta_1 \in \mathbb{R}$, $\varpi_1 \in \mathbb{R}$, $\psi_1 \in \mathbb{R}$, $H_{21}(t) \in \mathbb{R}^{N-1}$, $H_{2l}(t) \in \mathbb{R}^{N-1}$, $\tau_2 \in \mathbb{R}^{N-1}$, $\tau_{2(l-1)} \in \mathbb{R}^{N-1}$, $\zeta_2 \in \mathbb{R}^{N-1}$, $\varpi_2 \in \mathbb{R}^{N-1}$, $\psi_2 \in \mathbb{R}^{N-1}$ for $l \in \{2, 3, \cdots, n\}$.

The vector $\mathbf{r} = \frac{1}{\sqrt{N}}\mathbf{1}_N$ satisfies that $\mathbf{r}^T\mathbf{R} = \mathbf{0}_{N-1}^T$, $\mathbf{R}^T\mathbf{R} = \mathbf{I}_{N-1}$, and $\mathbf{R}\mathbf{R}^T = \mathbf{I}_N - \frac{1}{N}\mathbf{1}_N\mathbf{1}_N^T$. Hence, $\mathbf{L}\mathbf{r} = \mathbf{0}_N$ and $\mathbf{r}^T\mathbf{L} = \mathbf{0}_N^T$.

Let

$$\tilde{H}_1 = \mathrm{col}(H_{12}, W_{13}, \cdots, H_{1n}), \quad \tilde{H}_2 = \mathrm{col}(H_{22}, H_{23}, \cdots, H_{2n}), \tag{C15}$$

$$\tilde{\tau}_1 = \mathrm{col}(\tau_{11}, \tau_{12}, \cdots, \tau_{1(n-1)}), \quad \tilde{\tau}_2 = \mathrm{col}(\tau_{21}, \tau_{22}, \cdots, \tau_{2(n-1)}). \tag{C16}$$

Substituting (C1) and (C4) into (C8)–(C16), the following equations hold.

$$\dot{\tau}_1 = \varepsilon\tau_{11}, \tag{C17a}$$
$$\dot{\tau}_2 = \varepsilon\tau_{21}, \tag{C17b}$$
$$\dot{\zeta}_1 = \frac{k_1}{\varepsilon^{n-1}}\tau_1 + \sum_{l=2}^{n}\varepsilon k_{l-1}\tau_{1(l-1)} - \sum_{l=2}^{n}\varepsilon k_{l-1}H_{1l}, \tag{C17c}$$
$$\dot{\zeta}_2 = \frac{k_1}{\varepsilon^{n-1}}\mathbf{R}^T\mathbf{L}\mathbf{R}\varpi_2 - \frac{2\varepsilon}{\lambda_N}\mathbf{R}^T\mathbf{L}\mathbf{R}\zeta_2 + \frac{k_1}{\varepsilon^{n-1}}\tau_2 + \sum_{l=2}^{n}\varepsilon k_{l-1}\tau_{2(l-1)} - \sum_{l=2}^{n}\varepsilon k_{l-1}H_{2l}, \tag{C17d}$$
$$\dot{\varpi}_1 = 0, \tag{C17e}$$
$$\dot{\varpi}_2 = -\frac{k_1}{\varepsilon^{n-1}}\mathbf{R}^T\mathbf{L}\mathbf{R}\zeta_2 - \frac{k_1}{\varepsilon^{n-1}}\mathbf{R}^T\mathbf{L}\mathbf{R}\tau_{2(n-1)} + \frac{k_1}{\varepsilon^{n-1}}\mathbf{R}^T\mathbf{L}\mathbf{R}H_{2n}, \tag{C17f}$$
$$\dot{\tau}_{1(n-1)} = -\sum_{l=2}^{n}\varepsilon k_{l-1}\tau_{1(l-1)} - \frac{1}{\varepsilon^{n-1}}(\zeta_1 + \mathbf{r}^T h) + \frac{1}{\varepsilon^{n-1}}\psi_1(t) + \sum_{l=2}^{n}\varepsilon k_{l-1}H_{1l}, \tag{C17g}$$

$$\dot{\tau}_{2(n-1)} = -\sum_{l=2}^{n} \varepsilon k_{l-1}\tau_{2(l-1)} - \frac{1}{\varepsilon^{n-1}}(\zeta_2 + \mathbf{R}^T h) + \frac{1}{\varepsilon^{n-1}}\psi_2(t) + \sum_{l=2}^{n} \varepsilon k_{l-1} H_{2l}, \tag{C17h}$$

$$\dot{\tilde{\tau}}_1 = \varepsilon \mathbf{A}_k \tilde{\tau}_1 - \frac{1}{\varepsilon^{n-1}}\mathbf{B}_2(\zeta_1 + \mathbf{r}^T h) + \frac{1}{\varepsilon^{n-1}}\mathbf{B}_2\psi_1(t) + \mathbf{B}_2(\sum_{l=2}^{n} \varepsilon k_{l-1} H_{1l}), \tag{C17i}$$

$$\begin{aligned}\dot{\tilde{\tau}}_2 =& \varepsilon(\mathbf{A}_k \otimes \mathbf{I}_{N-1})\tilde{\tau}_2 - \frac{1}{\varepsilon^{n-1}}(\mathbf{B}_2 \otimes \mathbf{I}_{N-1})(\zeta_2 + \mathbf{R}^T h) \\ &+ \frac{1}{\varepsilon^{n-1}}(\mathbf{B}_2 \otimes \mathbf{I}_{N-1})\psi_2(t) + (\mathbf{B}_2 \otimes \mathbf{I}_{N-1})(\sum_{l=2}^{n} \varepsilon k_{l-1} H_{2l}),\end{aligned} \tag{C17j}$$

where $\mathbf{B}_2 = [0_{n-2}^T, 1]^T$.

Based on the above coordinate transformation (C8)-(C16), to prove the convergence of the error system (C1), the stability of the dynamics (C17) will be analyzed next.

According to the equations in (C17), the Lyapunov function $V(t)$ is similar to one given in [9]. The parameter μ in the reference [9] satisfies the following condition.

$$\begin{aligned}0 < M_0 < \mu < \min\Bigg\{ & \frac{1}{2\tilde{\rho}_1}(\varepsilon - (\frac{(n-1)\bar{p}(\theta^2+1)}{\beta k_1} + (2n-1)\bar{p} + \frac{2k_1}{\beta})\frac{1}{\varepsilon^{n-1}}), \\ & \frac{1}{n((3\lambda_N+2)k_1 + \frac{(2n+6)\varepsilon^{2n}}{\lambda_2 k_1} + 2\varepsilon^n|\sum_{l=1}^{n-1} k_l - \frac{2\lambda_2}{\lambda_N}|)}, \frac{\lambda_2(k_1\beta - 2\theta^2 - 2)}{k_1(n+3+2\lambda_2)}\Bigg\},\end{aligned} \tag{C18}$$

The Lyapunov function $V(t)$ can be rewritten as $V(t) = \vartheta^T \mathbf{\Phi} \vartheta$, where $\vartheta = \mathrm{col}(\tau_1, \tilde{\tau}_1, \zeta_1, \tau_2, \tilde{\tau}_2, \zeta_2, \varpi_2)$, and $\mathbf{\Phi} = \mathrm{diag}\{\mathbf{\Phi}_1, \mathbf{\Phi}_2\}$. Parameters $\lambda_{\min}, \lambda_{\max}$ are the minimum and maximum characteristic root of $\mathbf{\Phi}$, respectively. The definitions of $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$ can be found in [9].

By combining the orthogonal transformation specified in (C8) with the consideration of the convexity of cost functions and the Lipschitz property of the gradients of cost functions, along with the Schur Complement Lemma and the connectedness of undirected topology, the derivative of $V(t)$ can be written as follows.

$$\begin{aligned}\frac{d}{dt}V(t) \leqslant & -\frac{1}{2}(\varepsilon - (\frac{(n-1)\bar{p}(\theta^2+1)}{\beta k_1} + (2n-1)\bar{p} + \frac{2k_1}{\beta})\frac{1}{\varepsilon^{n-1}} - 2\mu\tilde{\rho}_1)\|\tilde{\tau}\|^2 \\ & - \left(\frac{\frac{1}{2}k_1\beta - \theta^2}{2\varepsilon^{n-1}} - \frac{1}{2\varepsilon^{n-1}} - \frac{\mu k_1(n+3+2\lambda_2)}{4\lambda_2\varepsilon^{n-1}}\right)\|\tau\|^2 \\ & - \left(\frac{-M_0 k_1 \lambda_2}{2(n+3)\varepsilon^{n-1}} + \frac{\mu k_1 \lambda_2}{2\varepsilon^{n-1}(n+3)}\right)\|\varpi_2\|^2 \\ & - \left(\frac{1}{2n\varepsilon^{n-1}} - \mu(\frac{k_1(3\lambda_N+2)}{2\varepsilon^{n-1}} + \frac{(n+3)\varepsilon^{n+1}}{\lambda_2 k_1} + \varepsilon(\sum_{l=1}^{n-1} k_l - \frac{2\lambda_2}{\lambda_N}))\right)\|\zeta\|^2 \\ & + \Bigg(\mu(\frac{(n+3)^2\bar{k}^2\varepsilon^{n+1}}{k_1\lambda_2} + \frac{\bar{k}\varepsilon}{2} + \frac{(n+3)\lambda_N^2 k_1}{\lambda_2\varepsilon^{n-1}} + \frac{k_1\lambda_N^2}{2\varepsilon^{n-1}})\frac{1}{\bar{\varepsilon}} \\ & + \frac{k_1(n+3)\lambda_N^2}{2M_0\lambda_2\varepsilon^{n-1}}\frac{1}{\bar{\varepsilon}} + \frac{\beta k_1}{2\varepsilon^{n-1}} + \frac{k_1^2+n}{\varepsilon^{n-1}} + (\frac{(n-1)\varepsilon^{n+1}\bar{k}^2}{2} + (n-1)\varepsilon^{n+1}k_1^2\bar{k}^2)\frac{1}{\bar{\varepsilon}}\Bigg)\|\xi(t)\|^2.\end{aligned} \tag{C19}$$

where $\tilde{\tau} = \mathrm{col}(\tilde{\tau}_1, \tilde{\tau}_2)$, $\tilde{H} = \mathrm{col}(\tilde{H}_1, \tilde{H}_2)$. Further, the inequality (C19) can be rewritten as

$$\frac{d}{dt}V(t) \leqslant -\sigma\|\vartheta(t)\|^2 + \pi_0\|\xi(t)\|^2, \tag{C20}$$

where σ and π_0 are positive constants presented in Appendix B.

Combining the definition of $V(t) = \vartheta^T \mathbf{\Phi} \vartheta$ with the continuity of $\bar{y}$, $\bar{z}$ and $\bar{x}$ for $t \in [t_0, t_u)$, there has $\|\vartheta(t_u)\|^2 \leqslant \frac{1}{\bar{\varepsilon}}(\rho_{1,x} + \|x^*\|^2) + \rho_{\bar{y}} + \rho_{\bar{z}}$, and the definition of $\bar{\varepsilon}$ is shown in Appendix B.

Next, the boundedness of $\bar{x}_1(t)$ and $\xi(t)$ is analyzed. We will prove that there exists $\omega_2^* \geqslant \omega_1^*$ such that

$$(\bar{x}_1(t), \xi(t)) \in \Omega = \left\{(\bar{x}_1, \xi) \in R^{N(n+2)} \middle| V \leqslant \rho_{v_k},\ \sqrt{V_1} \leqslant \sqrt{\bar{c}_{12}}\rho_{2,\xi}\right\} \tag{C21}$$

is satisfied for $\omega_o \geqslant \omega_2^*$ and $t \geqslant t_u$, where $\rho_{2,\xi} = \max\{\frac{\bar{\eta}_{1,\xi}^*}{\sqrt{\bar{c}_{11}}}, \frac{\bar{\eta}_{2,\xi}^*}{\sqrt{\bar{c}_{11}}}\}$, and ρ_{v_k} can be found in Appendix B. By reductio, the proof of (C21) is composed of the following two steps.

(S1) Assuming there exists $t^* \geqslant t_u$, such that $V(t^*) \leqslant \rho_{v_k}$, $\sqrt{V_1(t^*)} = \sqrt{\bar{c}_{12}}\rho_{2,\xi}$ and $\frac{d(\sqrt{V_1(t^*)})}{dt} > 0$. Then $\|\xi(t^*)\| \leqslant \frac{\sqrt{\bar{c}_{12}}}{\sqrt{\bar{c}_{11}}}\rho_{2,\xi}$, $\|\vartheta(t^*)\|^2 \leqslant \frac{\rho_{v_k}}{\lambda_{\min}}$. Since $\|\bar{x}_1(t^*)\|^2 + \sum_{l=2}^{n}\frac{1}{\varepsilon^{l-1}}\|\bar{x}_l(t^*)\|^2 \leqslant \frac{\rho_{v_k}}{\lambda_{\min}}$, there are $\|\bar{x}(t^*)\|^2 \leqslant \frac{\bar{\varepsilon}\rho_{v_k}}{\lambda_{\min}}$ and $\|x(t^*)\|^2 \leqslant \frac{\bar{\varepsilon}\rho_{v_k}}{\lambda_{\min}} + \|x^*\|^2$.

From Lemma A2, we have $\|\bar{\mathbf{P}}_1\|\|\dot{g}\| \leqslant \tilde{\Phi}\left(\sqrt{\frac{\tilde{\varepsilon}\rho_{v_k}}{\lambda_{\min}} + \|x^*\|^2}\right)$. Then the derivative of $\sqrt{V_1(t^*)}$ is written as

$$\frac{d}{dt}\sqrt{V_1(t^*)} \leqslant \frac{-\omega_o}{2\sqrt{\bar{c}_{12}}}\rho_{2,\xi} + \frac{\tilde{\Phi}\left(\sqrt{\frac{\tilde{\varepsilon}\rho_{v_k}}{\lambda_{\min}} + \|x^*\|^2}\right)}{\sqrt{\bar{c}_{11}}}. \tag{C22}$$

According to (C22), there exists $\omega_2^* \geqslant \omega_1^*$ such that $\frac{d}{dt}\sqrt{V_1(t^*)} \leqslant 0, \omega_o \geqslant \omega_2^*$. Hence, the assumption for proof by contradiction is invalid.

(S2) Assuming there exists $t^* \geqslant t_u$ such that $V(t^*) = \rho_{v_k}$, $\sqrt{V_1(t^*)} \leqslant \sqrt{\bar{c}_{12}}\rho_{2,\xi}$, and $\frac{\mathrm{d}}{\mathrm{d}t}V(t^*) > 0$. The derivative of $V(t^*)$ is $\frac{d}{dt}V(t^*) \leqslant -\sigma\frac{\rho_{v_k}}{\lambda_{\max}} + \pi_0\frac{\bar{c}_{12}}{\bar{c}_{11}}\rho_{2,\xi}^2$. According to the definition of ρ_{v_k}, we have $\frac{d}{dt}V(t^*) < 0$. The assumption $\frac{d}{dt}V(t^*) > 0$ does not hold.

Due to (S1) and (S2), there exists $\omega_2^* \geqslant \omega_1^*$ such that $(\bar{x}_1(t), \xi(t)) \in \Omega$ for $t \geqslant t_u$ and $\omega_o \geqslant \omega_2^*$. Based on (C21), the following inequalities hold for any $\omega_o \geqslant \omega_2^*$.

$$\begin{cases} \sup\limits_{t\geqslant t_u} \|\xi(t)\| \leqslant \frac{\sqrt{\bar{c}_{12}}}{\sqrt{\bar{c}_{11}}}\rho_{2,\xi} \triangleq \rho_{3,\xi}, \\ \sup\limits_{t\geqslant t_u} \|\vartheta(t)\| \leqslant \frac{\sqrt{V(t)}}{\lambda_{\min}} \leqslant \frac{\sqrt{\rho_{v_k}}}{\sqrt{\lambda_{\min}}}. \end{cases} \tag{C23}$$

Remark 2. In order to prove that the tracking error and estimation error are bounded for $t \in [t_u, +\infty)$, the procedures (S1) and (S2) are presented. Since the derivatives of Lyapunov functions are non-positive, the uniform boundedness of estimation error and tracking error is proved by reductio ad absurdum.

Step 3: Analyzing the convergence of the system.

By the analyses in Step 1 and Step 2, $\bar{x}_1(t)$ and $\xi(t)$ are proved to be bounded. In this step, a detailed analysis of the boundaries of $\bar{x}_1(t)$ and $\xi(t)$ will be delved into.

Based on Lemma A2 and (C21), the derivative of $\sqrt{V_1(t)}$ is shown as follows.

$$\frac{d}{dt}\sqrt{V_1(t)} \leqslant \frac{-\omega_o}{2\bar{c}_{12}}\sqrt{V_1(t)} + \frac{\tilde{\Phi}\left(\sqrt{\frac{\tilde{\varepsilon}\rho_{v_k}}{\lambda_{\min}} + \|x^*\|^2}\right)}{\sqrt{\bar{c}_{11}}}. \tag{C24}$$

According to Bellman-Grownwall inequality, the inequality (C24) indicates the following inequality.

$$\sqrt{V_1(t)} \leqslant e^{\frac{-\omega_o}{2\bar{c}_{12}}(t-t_u)}\sqrt{\bar{c}_{12}}\rho_{3,\xi} + \frac{2\bar{c}_{12}}{\omega_o}\frac{\tilde{\Phi}\left(\sqrt{\frac{\tilde{\varepsilon}\rho_{v_k}}{\lambda_{\min}} + \|x^*\|^2}\right)}{\sqrt{\bar{c}_{11}}}. \tag{C25}$$

Then, the bound of ξ is obtained as follows.

$$\|\xi(t)\| \leqslant \gamma_1(e^{-\gamma_2\omega_o(t-t_u)} + \frac{1}{\omega_o}), \tag{C26}$$

where the definition of γ_1 and γ_2 can be found in Appendix B.

Based on (C26), the inequality of the derivative of $V(t)$ (C20) implies that

$$V(t) \leqslant e^{\frac{-\sigma}{\lambda_{\max}}(t-t_u)}V(t_u) + \int_{t_u}^{t} e^{\frac{-\sigma}{\lambda_{\max}}(t-\tau)}\pi_0\|\xi(\tau)\|^2\mathrm{d}\tau. \tag{C27}$$

According to (C23) and (C26), the bound for the estimation error satisfies that $\|\xi(t)\|^2 \leqslant \frac{\bar{c}_{12}}{\bar{c}_{11}}\rho_{2,\xi}^2$ for $t_u \leqslant t < t_u + \frac{ln\omega_o}{\gamma_2\omega_o}$ and $\|\xi(t)\| \leqslant \frac{2\gamma_1}{\omega_o}$ for $t \geqslant t_u + \frac{ln\omega_o}{\gamma_2\omega_o}$. Then, by the inequality scaling method in [26], the following bound for the integral term in inequality (C27) can be derived.

$$\int_{t_u}^{t} e^{\frac{-\sigma}{\lambda_{\max}}(t-\tau)}\pi_0\|\xi(\tau)\|^2\mathrm{d}\tau \leqslant \frac{\lambda_{\max}\pi_0\bar{c}_{12}\rho_{2,\xi}^2}{\gamma_2\sigma\bar{c}_{11}}\frac{ln\omega_o}{\omega_o} + \frac{4\pi_0\lambda_{\max}(\gamma_1)^2}{\sigma}\frac{1}{\omega_o^2}. \tag{C28}$$

Finally, combing (C27) and (C28) with $V(t){=}\vartheta^T\mathbf{\Phi}\vartheta$, the following results are obtained.

$$\|\bar{x}_1(t)\| \leqslant \sqrt{e^{\frac{-\sigma}{\lambda_{\max}}(t-t_u)}\frac{\lambda_{\max}}{\lambda_{\min}}\rho_{1,v} + \frac{\tilde{\sigma}}{\lambda_{\min}}\max(\frac{1}{\omega_o^2}, \frac{ln\omega_o}{\omega_o})}, \tag{C29}$$

$$\|\xi(t)\| \leqslant \gamma_1(e^{-\gamma_2\omega_o(t-t_u)} + \frac{1}{\omega_o}). \tag{C30}$$

This completes the proof of Theorem 1.

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