SCIENCE CHINA
Information Sciences

• REVIEW •

February 2026, Vol. 69, Iss. 2, 121202:1–121202:17
https://doi.org/10.1007/s11432-025-4647-9

An overview of distributed fixed-time and prescribed-time optimization of multi-agent systems

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Received 25 June 2025/Revised 28 August 2025/Accepted 13 October 2025/Published online 8 January 2026

Abstract Distributed fixed-time and prescribed-time optimization has become a key focus in the study of multi-agent systems (MASs), enabling efficient and scalable solutions to optimization problems with guaranteed convergence within fixed-time and prescribed-time frames, respectively. This survey presents a thorough overview of distributed fixed-time and prescribed-time optimization methodologies, focusing on two key paradigms: time-invariant and time-varying cases. Specifically, the survey begins by exploring fundamental principles of fixed-time optimization of MASs, emphasizing their advantages over asymptotic and finite-time methods, particularly in scenarios with strict convergence-time requirements. Then, recent advances are presented in distributed fixed-time optimization, including second-order MASs, event-triggered control, and application to smart grids. Following that, representative results on distributed prescribed-time optimization are provided, which extend the fixed-time counterpart to a problem with user-defined settling time. Finally, some open challenges in the field, such as handling communication delays, cyber-physical threats, and nonconvexities, are identified that deserve further investigation.

Keywords distributed optimization, multi-agent systems, time-invariant cost functions, time-varying cost functions, fixed-time convergence, prescribed-time convergence

Citation Ning B D, Han Q-L, Luan M, et al. An overview of distributed fixed-time and prescribed-time optimization of multi-agent systems. Sci China Inf Sci, 2026, 69(2): 121202, https://doi.org/10.1007/s11432-025-4647-9

1 Introduction

The optimization of multi-agent systems (MASs) has become a central focus within control theory, particularly with the increasing demand for decentralized coordination in various complex systems, such as robotic swarms, autonomous vehicles, sensor networks, and distributed energy grids [1–12]. These systems are made up of multiple autonomous agents that interact both with each other and with their environment to work toward a common goal or optimize a shared objective function [13]. Traditional optimization techniques, often centralized, suffer from limitations in scalability, robustness, and resilience to failures or communication constraints, making distributed optimization methods more suitable for real-world applications [14, 15].

In distributed optimization, agents aim to collaboratively solve an optimization problem without relying on a centralized protocol [15–18]. Each agent can only access local information and coordinate actions via communication with its neighbors. This decentralized method improves fault tolerance and scalability, offering advantages such as privacy preservation and reduced computational load. For instance, a subgradient approach is established for MASs, where each agent has a local averaging operation to estimate the optimal solution. This estimation is then projected onto set constraints for updating states [19]. Building on this, the optimization problem with global inequality constraints is addressed by solving its associated Lagrangian dual problem [20]. Different from discrete-time algorithms discussed in [19, 20], some recent results have focused on MASs with continuous-time dynamics [21–25]. For instance, a distributed optimization problem under time-varying interconnection topologies

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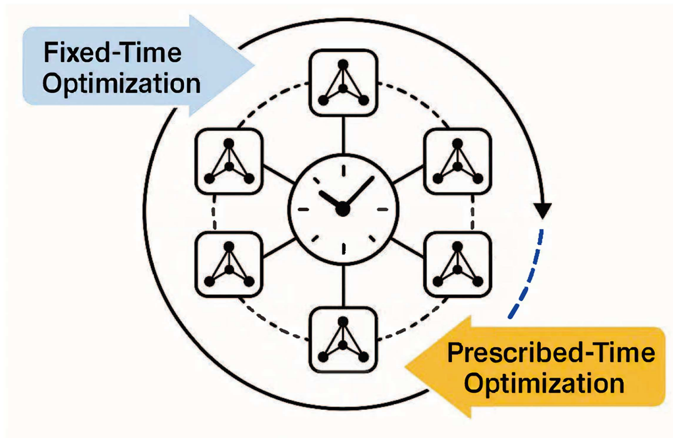

Figure 1 (Color online) Distributed optimization with time requirements.

is solved by transforming the original problem into an intersection computation task [21]. To handle equality, inequality, and bounded constraints, a distributed protocol using the proportional-integral control is designed in [25]. Additionally, the problem with a common set constraint is investigated in [24], where a protocol incorporating local averaging, projection, and subgradient terms is developed.

The distributed optimization problem becomes more challenging when time constraints are imposed, as some applications require solutions within strict deadlines. To address this challenge, one important class of time-constrained optimization methods has emerged: distributed finite-time optimization, where protocols are designed to ensure that agents achieve consensus and optimization in finite-time [26–31]. These methods are essential in applications where rapid convergence is critical, such as real-time control systems and mission-critical operations. For example, a distributed algorithm using an integral sliding-mode approach is designed to address optimization problems in finite-time for MASs with external disturbances [27]. However, in distributed finite-time optimization, the estimated convergence time relies on initial values of MASs, which can be difficult to measure or even unavailable in extreme situations. Thus, a specific convergence time cannot be guaranteed from the finite-time control strategies [26–31]. To tackle this issue, distributed fixed-time optimization, grounded in the fixed-time stability theory [32,33], has gained significant attention in recent years for its ability to ensure guaranteed convergence times, which is crucial for time-sensitive tasks [34–38]. For example, a distributed optimal protocol utilizing the Hessian matrix of local objective functions is proposed to achieve fixed-time optimization under directed communication topologies [34]. The key advantage of these fixed-time methods lies in their capability to eliminate dependence on the initial conditions, ensuring that the agents will reach the solution within a fixed time, regardless of their starting points [39, 40]. Distributed prescribed-time optimization builds on fixed-time optimization by allowing designers to explicitly set the convergence time, which makes it attractive for scenarios where user-defined timing is required [41–44]. A time-varying gain is usually utilized to design the protocols [45–48]. For example, a three-stage distributed algorithm is proposed for a team of unmanned aerial vehicles to achieve the optimal rendezvous formation in a prescribed time [46]. A distributed prescribed-time approach is developed to reach the optimal economic dispatch while satisfying power constraints [48]. Figure 1 provides a conceptual overview of distributed optimization when considering the convergence-time requirement. Each agent cluster represents a distributed network, while the clock indicates the importance of time-constrained convergence. The evolution of distributed optimization including key features is summarized in Figure 2.

This survey focuses on distributed fixed-time and prescribed-time optimization of MASs, which is different from the existing distributed optimization surveys [13, 15, 49–53]. The primary objective of this survey is to present an overview of the methodologies and recent developments in the presence of specific time requirements. More precisely, we offer a comprehensive overview of distributed fixed-time and prescribed-time optimization methodologies, based on two key paradigms: time-invariant and time-varying cases. The time-invariant case is first examined in depth, where fixed-time convergence and prescribed-time convergence are achieved, respectively, ensuring robust and predictable performance. In contrast, the time-varying case is further explored, where dynamic elements are introduced, enabling greater flexibility by incorporating time-dependent parameters to adapt to evolving system demands and environmental changes. After that, we present recent advances in distributed fixed-time and prescribed-time optimization, including event-triggered control, second-order MASs, along their applications to smart grids.

The remaining content is structured as follows. Section 2 gives some preliminaries for distributed optimization

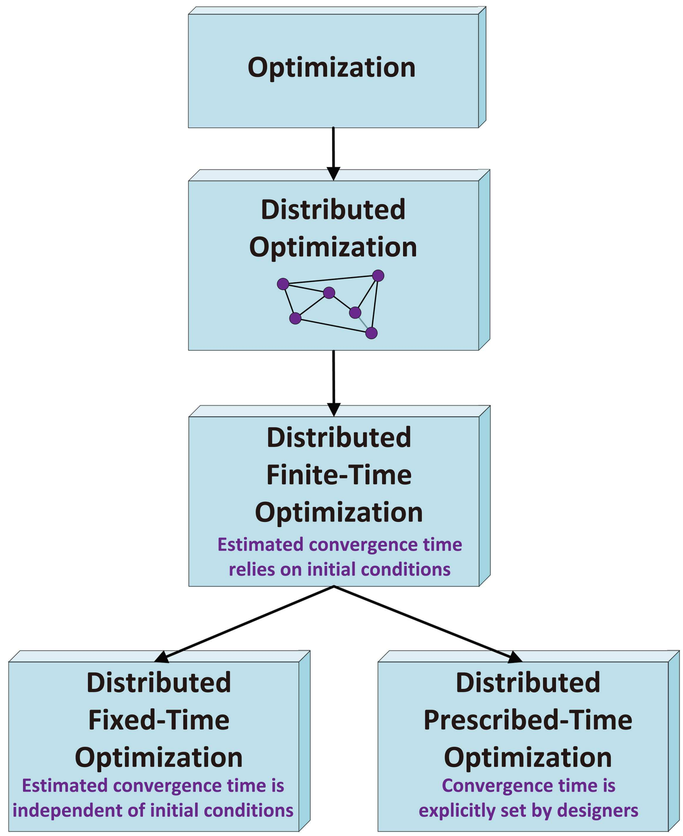

Figure 2 (Color online) The evolution of distributed optimization.

in MASs, including lemmas and their relevance to fixed-time and prescribed-time optimization. Section 3 presents representative results on distributed fixed-time optimization for MASs with time-invariant and time-varying cost functions separately. Section 4 presents some latest trends in distributed fixed-time optimization. Section 5 provides featuring results on distributed prescribed-time optimization. Section 6 discusses the conclusion and outlines several challenging future directions.

2 Preliminaries and lemmas

2.1 Convex functions

Some fundamental concepts of convex analysis are introduced, summarized from [54].

A set $\Omega \subset \mathbb{R}^m$ is convex if for any u, $v \in \Omega$ and $\alpha \in [0,1]$, $\alpha u + (1-\alpha)v \in \Omega$ holds. For a twice differentiable function $f : \mathbb{R}^m \to \mathbb{R}$, symbols $\nabla f(u)$ and $\nabla^2 f(u)$ stand for its gradient and Hessian matrix, respectively. A function $f : \mathbb{R}^m \to \mathbb{R}$ is convex if $f(\alpha u + (1-\alpha)v) \leqslant \alpha f(u) + (1-\alpha) f(v)$ for all u, $v \in \mathbb{R}^m$ and any $\alpha \in (0,1)$. A continuously differentiable function $f : \mathbb{R}^m \to \mathbb{R}$ is l-smooth if $\|\nabla f(u) - \nabla f(v)\| \leqslant l\|u-v\|$ holds for all u, $v \in \mathbb{R}^m$ and some constant $l > 0$, which is equivalent to $\nabla^2 f(u) \preceq l I_m$, where $I_m \in \mathbb{R}^{m\times m}$ denotes the identity matrix. A continuously differentiable function $f : \mathbb{R}^m \to \mathbb{R}$ is strongly convex with convexity parameter $\theta > 0$ if $f(v) \geqslant f(u) + \nabla f(u)^{\mathrm{T}}(v-u) + \frac{\theta}{2}\|v-u\|^2$ holds for all u, $v \in \mathbb{R}^m$, which also implies $(\nabla f(v) - \nabla f(u))^{\mathrm{T}}(v-u) \geqslant \theta\|v-u\|^2$.

2.2 Graph theory

Consider a system of n agents whose interactions are modeled by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Here, $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ corresponds to the set of vertices, each associated with an agent indexed by the set $\Gamma = \{1, 2, \ldots, n\}$. The edge set $\mathcal{E}$ consists of ordered pairs (v_i, v_j), indicating agent j could receive the information of agent i (note that this access is not necessarily reciprocal). As a result, agent i is considered a neighbor of agent j, and let $\mathcal{N}_j = \{v_i \in \mathcal{V} \mid (v_i, v_j) \in \mathcal{E}\}$ denote the set of neighbors of agent j. A graph is deemed undirected if the presence of an edge (v_i, v_j) automatically ensures the existence of (v_j, v_i); if this condition is not met, the graph is considered directed.

The structure of $\mathcal{G}$ can be algebraically captured using two matrices. The first is a weighted adjacency matrix $\mathcal{A} = [a_{ij}]_{n\times n}$ with $a_{ij} > 0$ for $(v_j, v_i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The second is the Laplacian matrix $\mathcal{L} = [l_{ij}]_{n\times n}$ with $l_{ii} = \sum_{j=1}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. It is typically assumed that there are no self-loops. In the case of undirected graphs, the symmetry $a_{ij} = a_{ji}$ holds.

2.3 Fixed-time stability

Consider the following system described by

$$\dot{x}(t) = f(t, x(t)), \quad x(0) = x_0, \tag{1}$$

where $x \in \mathbb{R}^m$ denotes the state, and the function $f : \mathbb{R}^+ \times \mathcal{D} \to \mathbb{R}^m$ represents an upper semi-continuous mapping on an open neighborhood $\mathcal{D}$ of the origin. Additionally, for every $(t, x) \in \mathbb{R}^+ \times \mathcal{D}$, the set $f(t, x)$ is non-empty, and $f(t, 0) = 0$ holds for all $t > 0$.

Lemma 1 ([55]). Given nonnegative numbers $\varpi_1, \varpi_2, \ldots, \varpi_n$, and a real number $\rho > 1$, there holds

$$\sum_{i=1}^{n} \varpi_i^{\rho} \geqslant n^{1-\rho} \left(\sum_{i=1}^{n} \varpi_i\right)^{\rho}.$$

Lemma 2 ([32]). If there exists a continuous, positive definite, and radially unbounded function $V(x) : \mathbb{R}^m \to \mathbb{R}^+ \cup \{0\}$, such that

$$\dot{V}(x(t)) \leqslant -(\tau_1 V^{\nu_1}(x(t)) + \tau_2 V^{\nu_2}(x(t)))^{\psi}, \tag{2}$$

where τ_1, τ_2, ν_1, ν_2, $\psi > 0$ satisfy $\nu_1\psi < 1$, $\nu_2\psi > 1$, the origin of system (1) is fixed-time stable. In particular, the settling time is bounded by

$$T(x_0) \leqslant T_{\max} := \frac{1}{\tau_1^{\psi}(1 - \nu_1\psi)} + \frac{1}{\tau_2^{\psi}(\nu_2\psi - 1)}. \tag{3}$$

Lemma 2 is a Lyapunov-based criterion to guarantee the fixed-time stability, which is often used to derive a fixed settling time bound. Some variations of Lemma 2 are summarized in [56].

2.4 Prescribed-time stability

Lemma 3 ([57]). For a continuously differentiable function $V(x(t), t) : \mathbb{R}^m \times \mathbb{R}_{>0} \to \mathbb{R}_{\geqslant 0}$, if there exists a positive constant $r > 0$ satisfying the following conditions on $[t_0, \infty)$:

$$\begin{cases} V(0, t) = 0 \text{ and } V(x(t), t) > 0, & \text{for } x(t) \in \mathbb{R}^m \setminus \{0\}, \\ \dot{V} = -rV - 2\dfrac{\dot{\kappa}(t)}{\kappa(t)}V, & \text{for } x(t) \in \mathbb{R}^m, \end{cases}$$

with

$$\kappa(t) = \begin{cases} \left(\dfrac{t_{\text{pre}}}{t_{\text{pre}} + t_0 - t}\right)^{h}, & t \in [t_0, t_0 + t_{\text{pre}}), \\ 1, & t \in [t_0 + t_{\text{pre}}, \infty), \end{cases}$$

and

$$\dot{\kappa}(t) = \begin{cases} \dfrac{h}{t_{\text{pre}}}\kappa^{1+\frac{1}{h}}, & t \in [t_0, t_0 + t_{\text{pre}}), \\ 0, & t \in [t_0 + t_{\text{pre}}, \infty), \end{cases}$$

where t_0 represents the initial time, t_{pre} denotes the user-assignable time, and $h > 2$, one can attain

$$\begin{cases} \lim_{t\to(t_0+t_{\text{pre}})^-} V(x(t), t) = 0, \\ V(x(t), t) = 0, \quad \forall t \geqslant t_0 + t_{\text{pre}}. \end{cases}$$

Lemma 3 is often employed to demonstrate the prescribed-time stability. Some variations of Lemma 3 are summarized in [41].

3 Distributed fixed-time optimization of MASs

This section presents the methodologies and significant results in the distributed fixed-time optimization of MASs.

3.1 Distributed fixed-time optimization of MASs with time-invariant cost functions

Consider an MAS with n agents indexed by a set $\Gamma = \{1, 2, \ldots, n\}$. The dynamics of agent i are given by

$$\dot{x}_i(t) = u_i(t),\ i \in \Gamma, \tag{4}$$

where $x_i(t) \in \mathbb{R}^m$ denotes the state of agent i, and $u_i(t) \in \mathbb{R}^m$ represents the control protocol to be designed. In the following, we omit t in $x_i(t)$ and $u_i(t)$ to simplify the presentation.

The goal of distributed optimization in MASs with time-invariant cost functions is to design a control protocol u_i based solely on local information and interaction, ensuring that all agents collaboratively converge to the optimal state $x^* \in \mathbb{R}^m$.

This optimal state solves the following time-invariant optimization problem:

$$\min \ \sum_{i=1}^{n} f_i(x), \tag{5}$$

where $f_i(x) : \mathbb{R}^m \mapsto \mathbb{R}$ is the local cost function of agent i, accessible only to that agent.

This optimization problem (5) corresponds to ensuring consensus among all agents while minimizing the total cost function $\sum_{i=1}^{n} f_i(x_i)$, formulated as

$$\min \ \sum_{i=1}^{n} f_i(x_i) \quad \text{s.t.} \quad x_i = x_j \in \mathbb{R}^m. \tag{6}$$

The objective of distributed fixed-time optimization in MASs is to reach state consensus and/or to address the optimization problem in fixed-time. In what follows, based on different assumptions, typical fixed-time controllers and related theorems are presented.

Assumption 1. Each function $f_i(x)$ is differentiable with respect to x and its gradient satisfies: $\triangledown f_i(x) = \rho x + \varphi_i(x)$ for $\forall x \in \mathbb{R}^m$, where $\rho \geqslant 0$ is a given constant, and the term φ_i satisfies the bounded norm condition $\|\varphi_i\|_2 \leqslant g_0$ with a given constant g_0. Additionally, each $f_i(x)$ has a nonempty optimal solution set, ensuring that a minimizer exists for every agent's cost function.

A distributed protocol for agent i is designed in [58], i.e.,

$$\begin{aligned} u_i = & - k_1 \sum_{j=1}^{n} a_{ij}(x_i - x_j)^{1-\frac{a}{b}} - k_2 \sum_{j=1}^{n} a_{ij}(x_i - x_j)^{1+\frac{a}{b}} \\ & - k_3 \sum_{j=1}^{n} a_{ij}\text{sign}(x_i - x_j) - k_4 \triangledown f_i(x_i) \end{aligned} \tag{7}$$

with positive constants k_1, k_2, k_3, k_4, a positive even integer a and a positive odd integer b such that $a < b$. Let $a_0 = \min\{a_{ij}\colon a_{ij} \neq 0,\ \forall i, j \in \Gamma\}$.

Theorem 1 ([58])**.** Under Assumption 1, for the MAS (4), suppose that the communication topology $\mathcal{G}$ is undirected and connected. If $k_3 \geqslant 2nk_4 g_0/a_0$, by using the protocol (7), state consensus is reached in a fixed time T_1, and the problem (5) is solved as $t \to \infty$.

The protocol (7) is applied in battery energy storage systems (BESSs) to optimize a reference power output to balance the SOC_i for all battery packages as illustrated in Figure 3. Although protocol (7) realizes the fixed-time consensus, the cost function is only minimized when time goes to infinity. To address this concern, distributed fixed-time optimization of MASs is investigated in [59], where the proposed distributed protocol for agent i is

$$u_i = \begin{cases} \left(\triangledown^2 f_i(x_i)\right)^{-1} \left(-\gamma_1 \text{sig}^{\alpha_1}\left(\triangledown f_i(x_i)\right) - \gamma_2 \text{sig}^{\beta_1}\left(\triangledown f_i(x_i)\right)\right), & 0 \leqslant t \leqslant t_1, \\ \left(\triangledown^2 f_i(x_i)\right)^{-1} \left(-k_1 \displaystyle\sum_{j \in \mathcal{N}_i} \text{sig}^{\alpha_2}(x_i - x_j) - k_2 \sum_{j \in \mathcal{N}_i} \text{sig}^{\beta_2}(x_i - x_j)\right), & t > t_1, \end{cases} \tag{8}$$

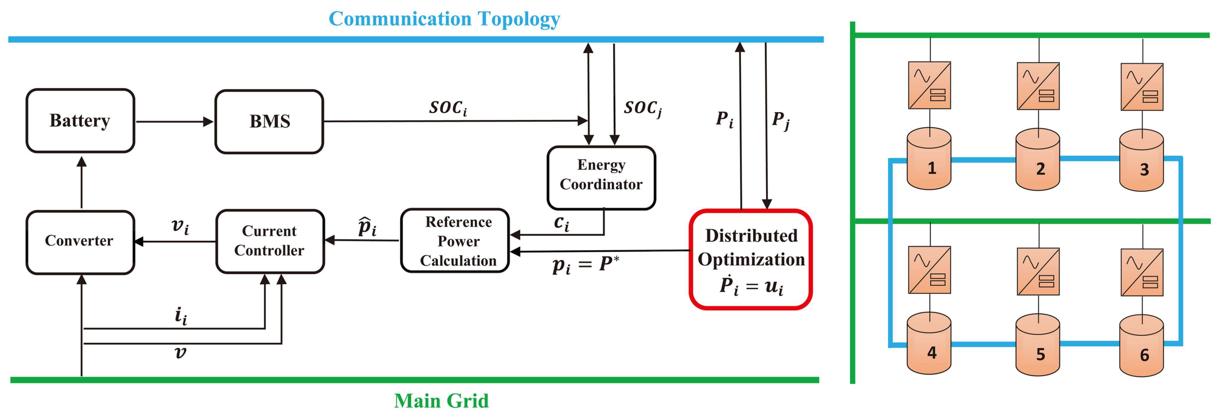

Figure 3 (Color online) Distributed optimization in BESSs.

where $\gamma_1, \gamma_2 > 0$, $0 < \alpha_1 < 1$, $0 < \alpha_2 < 1$, $\beta_1 > 1$, $\beta_2 > 1$, and

$$t_1 = \frac{2^{(1-\alpha_1)/2}}{\gamma_1(1-\alpha_1)} + \frac{m^{(\beta_1-1)/2}}{2^{(\beta_1-1)/2}\gamma_2(\beta_1-1)}.$$

Assumption 2. Each function $f_i(x_i)$, $i \in \Gamma$, is twice continuously differentiable, θ_i-strongly convex and Θ_i-smooth with bounded constants $\theta_i > 0$, $\Theta_i > 0$.

Theorem 2 ([59])**.** Under Assumption 2, for the MAS (4), suppose that the communication topology $\mathcal{G}$ is undirected, connected, and unweighted ($a_{ij} = 1$ if the communication between agents i and j is connected). Then, the distributed fixed-time optimization is achieved for problem (5) by using the distributed protocol (8), i.e., $x_i \to x^*$ in a fixed time T_2, $i \in \Gamma$, where x^* is the unique optimal solution of problem (5).

Note that the protocol (8) consists of two parts. When $0 \leqslant t \leqslant t_1$, the aim is to find the local minimizers of local cost functions in fixed-time. After time t_1, the aim becomes to ensure that all agents reach the minimizer of the global cost function in fixed-time. However, this approach can be restrictive for the requirement of all the local gradients converge to zero in the first stage. In fact, as long as the initial sum gradient of local cost functions is zero in the first stage, the second stage can get started. In other words, the algorithm in [59] is a little conservative. To address this concern, a sliding-mode based approach is developed in [60] with Assumption 3.

Assumption 3. Each function $f_i(x_i)$, $i \in \Gamma$, is twice continuously differentiable, θ_i-strongly convex with $\theta_i > 0$ and has Π_i-Lipschitz gradient, where $\Pi_i > 0$ and $\bar{\Pi} = \max_{i\in\Gamma}\{\Pi_i\}$.

A distributed integral sliding manifold is constructed for agent i in [60]:

$$s_i = \nabla f_i(x_i) + \int_0^t k_1 \sum_{j\in\mathcal{N}_i} a_{ij}\mathrm{sig}^{p_2}(x_i - x_j)\mathrm{d}\tau + \int_0^t k_2 \sum_{j\in\mathcal{N}_i} a_{ij}\mathrm{sig}^{q_2}(x_i - x_j)\mathrm{d}\tau, \tag{9}$$

where $0 < p_2 < 1$, $q_2 > 1$ are tuning parameters. Based on the sliding manifold (9), a distributed protocol is developed as follows [60]:

$$u_i = (\nabla^2 f_i(x_i))^{-1}\left(-k_3\mathrm{sig}^{p_1}(s_i) - k_4\mathrm{sig}^{q_1}(s_i) - k_1\sum_{j\in\mathcal{N}_i} a_{ij}\mathrm{sig}^{p_2}(x_i - x_j) - k_2\sum_{j\in\mathcal{N}_i} a_{ij}\mathrm{sig}^{q_2}(x_i - x_j)\right), \tag{10}$$

where $0 < p_1 < 1$, and $q_1 > 1$.

Theorem 3 ([60])**.** Under Assumption 3, for the MAS (4), suppose that the communication topology $\mathcal{G}$ is undirected and connected. Then, by employing the distributed protocol (10) together with the manifold (9), the distributed fixed-time optimization is realized for problem (5), i.e., $x_i \to x^*$ in a fixed time T_3, $i \in \Gamma$.

3.2 Distributed fixed-time optimization of MASs with time-varying cost functions

Different from the time-invariant optimization problem in (5), a time-varying case is considered for (4) in this section, i.e.,

$$\min \sum_{i=1}^{n} f_i(x, t) \quad \text{s.t.} \quad x \in \mathbb{R}^m, \tag{11}$$

where $f_i(x,t):\mathbb{R}^m\times\mathbb{R}^+\mapsto\mathbb{R}$ denotes the local cost function for agent i at time t.

This section examines the time-varying distributed optimization problem associated with the MAS described by (4). Notably, problem (11) can be reformulated as a consensus-based optimization task, where all the agents reach an agreement while simultaneously minimizing the overall time-dependent cost function:

$$\min \quad \sum_{i=1}^{n} f_i(x_i,t) \quad \text{s.t.} \quad x_i=x_j\in\mathbb{R}^m. \tag{12}$$

Following the protocol (7), a distributed protocol for agent i is developed [58], i.e.,

$$\begin{aligned} u_i=&-k_1\sum_{j=1}^{n}a_{ij}(x_i-x_j)^{1-\frac{a}{b}}-k_2\sum_{j=1}^{n}a_{ij}(x_i-x_j)^{1+\frac{a}{b}}\\ &-k_3\sum_{j=1}^{n}a_{ij}\mathrm{sign}(x_i-x_j)+\eta_i, \end{aligned} \tag{13}$$

where

$$\eta_i=-(\nabla^2 f_i(x_i,t))^{-1}\left(\nabla f_i(x_i,t)+\frac{\partial\nabla f_i(x_i,t)}{\partial t}\right). \tag{14}$$

Assumption 4. Each function $f_i(x_i,t)$ is twice continuously differentiable in $x_i\in\mathbb{R}^m$, and all agents share an identical Hessian matrix: $\nabla^2 f_i(x_i,t)=\nabla^2 f_j(x_j,t)$ for $\forall i,j\in\Gamma$. Additionally, suppose that $\nabla^2 f_i(x_i,t)$ is invertible for all x_i,t. The term η_i can be written as: $\vartheta_i(t)+\zeta x_i$, where $\|\vartheta_i(t)\|_2\leqslant\phi_0$ with ϕ_0 being a positive constant and constant $\zeta>0$.

The difference between protocol (13) and protocol (7) lies in the additional consideration of the Hessian matrix in the time-varying setting. If the cost function $f_i(x_i,t)$ reduces to a time-invariant function $f_i(x_i)$, then the time derivative of the gradient vanishes, i.e., $\frac{\partial\nabla f_i(x_i,t)}{\partial t}=0$ in η_i.

Although Assumption 4, which imposes that all agents have an identical Hessian matrix and the structured form $\eta_i=\zeta x_i+\vartheta_i(t)$, may seem restrictive, it is satisfied by a significant class of cost functions. One notable instance is the objective function commonly used in energy minimization problems [61], such as $f_i(x_i,t)=(g_i(t)+cx_i)^2$, where function $g_i(t)$ is time-varying and $c\in\mathbb{R}$ is a constant.

Theorem 4 ([58])**.** Under Assumption 4, for the MAS (4), suppose that the communication topology $\mathcal{G}$ is undirected and connected. Then the protocol (13) with $k_3\geqslant 2n\phi_0/a_0$ guarantees that the states reach consensus within a fixed time T_4, and solves the problem (11) as $t\to\infty$.

Although the protocol (13) achieves the fixed-time consensus, the cost function is only minimized when time goes to infinity. To handle this concern, distributed fixed-time optimization of MASs is further investigated in [62], where a distributed integral sliding manifold for agent i is described as

$$s_i=\nabla f_i(x_i,t)+\int_0^t\left(k_1\sum_{j\in\mathcal{N}_i}\mathrm{sig}^{\alpha_2}(x_i-x_j)+k_2\sum_{j\in\mathcal{N}_i}\mathrm{sig}^{\beta_2}(x_i-x_j),+\gamma\sum_{j\in\mathcal{N}_i}\mathrm{sgn}(x_i-x_j)\right)\mathrm{d}\tau, \tag{15}$$

where $0<\alpha_2<1$, $\beta_2>1$, and γ is a parameter to be designed. Based on the sliding manifold (15), a distributed protocol for agent i is established as [62]

$$\begin{aligned} u_i=\left(\nabla^2 f_i(x_i,t)\right)^{-1}\Bigg(&-k_3\mathrm{sig}^{\alpha_1}(s_i)-k_4\mathrm{sig}^{\beta_1}(s_i)-k_1\sum_{j\in\mathcal{N}_i}\mathrm{sig}^{\alpha_2}(x_i-x_j)-k_2\sum_{j\in\mathcal{N}_i}\mathrm{sig}^{\beta_2}(x_i-x_j)\\ &-\gamma\sum_{j\in\mathcal{N}_i}\mathrm{sgn}(x_i-x_j)-\frac{\partial}{\partial t}\nabla f_i(x_i,t)\Bigg), \end{aligned} \tag{16}$$

where $0<\alpha_1<1$, $\beta_1>1$.

Assumption 5. Each function $f_i(x_i,t)$, $i\in\Gamma$, satisfies the following properties. It is twice continuously differentiable; it has an ω-Lipschitz continuous gradient with respect to x_i for $\forall t\geqslant 0$ with $\omega>0$; it is θ-strongly convex with $\theta>0$; the Hessian matrices of all local cost functions are the same for any state, i.e., $\nabla^2 f_i(x_i,t)=\nabla^2 f_j(x_j,t)$, $\forall i,j\in\Gamma$; the time partial derivatives of their gradients are uniformly bounded, i.e., $\|\frac{\partial}{\partial t}\nabla f_i(x_i,t)\|_\infty\leqslant\sigma$, for $t\geqslant 0$.

Table 1 Representative distributed fixed-time optimization results.

Ref.	Objective function	Feature	Topology	Constraint	Disturbances
[34]	Time-invariant	Sliding-mode control	Directed & strongly connected	–	✓
[58]	Time-invariant	Edge-based consensus	Undirected & connected	–	×
[59]	Time-invariant	Two-stage control	Undirected & connected	–	×
[60]	Time-invariant	Sliding-mode control	Undirected & connected	–	×
[63]	Time-invariant	Two-hop communication	Undirected & connected	Equality	×
[64]	Time-invariant	Event-triggered control	Undirected & connected	–	×
[65]	Time-invariant	Economic dispatch	Directed & strongly connected	Equality	×
[66]	Time-invariant	Resource management	Undirected & connected	(In)equality	×
[67]	Time-invariant	Resource management	Undirected & connected	(In)equality	×
[58]	Time-varying	Edge-based consensus	Undirected & connected	–	×
[62]	Time-varying	Sliding-mode control	Undirected & connected	–	×
[68]	Time-varying	Second-order MASs	Directed & strongly connected	–	✓
[69]	Time-varying	Average tracking	Undirected & connected	–	×

Theorem 5 ([62])**.** For the MAS (4), the undirected communication topology $\mathcal{G}$ is connected and unweighted ($a_{ij} = 1$ if the communication between agents i and j is connected). Then, if $\gamma > (n-1)\omega\sigma/\theta$, the distributed fixed-time optimization is realized with the distributed protocol (16) for problem (11) under Assumption 5, i.e., $\lim_{t\to T}\|x_i(t) - x^*(t)\|_2 = 0$ and $\|x_i(t) - x^*(t)\|_2 = 0, \forall t \geqslant T$ with T upper bounded by a fixed time T_5, $i \in \Gamma$, where $x^*(t)$ is the unique solution to problem (11).

It is worth noting that this survey focuses on employing fixed-time consensus methods to address the distributed optimization problem. This differs from the results in [19–25], which primarily deal with more general optimization problems with or without constraints. Rather than allowing $x \in \mathbb{R}^m$ in problems (5) and (11), an interesting avenue for future research could involve imposing specific constraints on x, restricting it to a range. More discussions about the constrained optimization are provided in Section 6.

In Table 1, we summarize some representative distributed fixed-time optimization results [34, 58–60, 62–69]. Generally speaking, the assumptions made for time-varying cost functions are more restrictive compared to those for time-invariant ones. For example, Assumption 5 requires the identical Hessian matrices for all local cost functions. By contrast, there are no identical Hessian requirements in Assumptions 1–3. These kinds of restrictions do make sense as time-varying functions are more complex than the time-invariant counterparts. How to relax the assumptions for time-varying cost functions is an interesting future direction.

4 Recent trends in distributed fixed-time optimization of MASs

This section explores recent trends in the fixed-time optimization of MASs, focusing on second-order systems, high-order systems, event-triggered control, and the handling of constraints. Each of these aspects represents a critical area of research aimed at enhancing the practical applicability of fixed-time optimization methods. The integration of these strategies into real-world systems promises to substantially improve the performance and efficiency of multi-agent coordination in various applications.

4.1 Distributed fixed-time optimization for second-order MASs

Second-order MASs, characterized by agents described using both position and velocity states, are prominent in real-world applications including autonomous vehicles and unmanned surface vessels (USVs) as illustrated in Figure 4. These systems offer more complexity than first-order MASs due to the possible coupling between position and velocity, necessitating advanced control strategies to ensure stability and convergence within a fixed time.

Recent work on fixed-time optimization of second-order systems has focused on designing protocols that can drive both the velocity and position states of agents to their desired values in a fixed time, while solving an optimization problem [68, 70, 71]. For instance, the dynamics of a second-order nonlinear MAS are described as [68]

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = u_i + h_i(x_i, v_i, t) + d_i, \\ y_i = x_i, \end{cases} \tag{17}$$

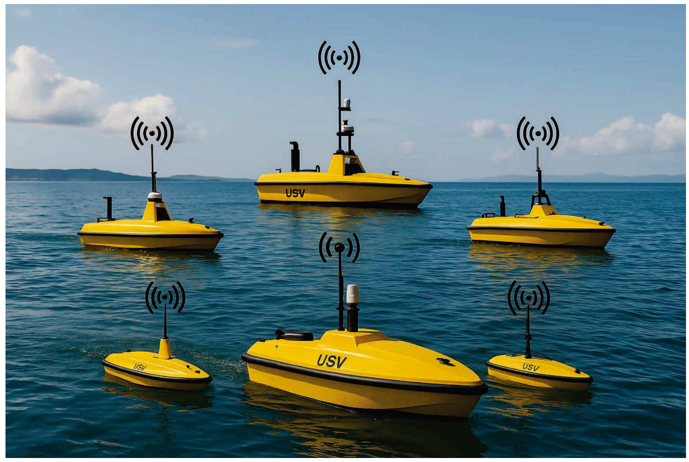

Figure 4 (Color online) Distributed optimization in USVs.

where for agent i, $i \in \Gamma$, $v_i \in \mathbb{R}^m$ is the velocity, $x_i \in \mathbb{R}^m$ is the position, $u_i \in \mathbb{R}^m$ and $y_i \in \mathbb{R}^m$ denote the control input and output, respectively. Additionally, the function $h_i(x_i, v_i, t) \in \mathbb{R}^m$ is nonlinear, and $d_i \in \mathbb{R}^m$ accounts for external disturbance. The local cost function $f_i(y_i, t) \in \mathbb{R}$ of agent i is time-varying and accessible only to itself. Consider the following optimization problem:

$$\min \sum_{i=1}^{n} f_i(y, t) \quad \text{s.t.} \quad y \in \mathbb{R}^m, \tag{18}$$

which corresponds to the problem of achieving output consensus among all agents while minimizing the global cost function $\sum_{i=1}^{n} f_i(y_i, t)$, i.e.,

$$\min \sum_{i=1}^{n} f_i(y_i, t) \quad \text{s.t.} \quad y_i = y_j \in \mathbb{R}^m. \tag{19}$$

The distributed fixed-time optimization objective for the nonlinear MAS in (17) is to construct a suitable control input u_i, such that $\lim_{t \to T} \|y_i(t) - y^*(t)\|_2 = 0$ and $\|y_i(t) - y^*(t)\|_2 = 0$ for $\forall t \geqslant T$, $i \in \Gamma$, where $y^*(t)$ is the unique solution of problem (18), and T is upper bounded by a fixed time.

Let $\mathbb{F} = \{\frac{m}{2(m+n)-1} : m, n \in \mathbb{N}^+\}$. Suppose γ_1, γ_2, and $\gamma_3 \in \mathbb{F}$. First, an optimization term of agent i for $i \in \Gamma$ is designed as

$$\begin{aligned}\phi_i = {} & \left(\left[\sum_{j \in \mathcal{N}_i} \varpi_{j,1}^i, \sum_{j \in \mathcal{N}_i} \varpi_{j,2}^i, \ldots, \sum_{j \in \mathcal{N}_i} \varpi_{j,n}^i \right] \right)^{-1} \\ & \times \left[\alpha_3 \mathrm{sign} \left(\sum_{j \in \mathcal{N}_i} \varpi_{j,n+1}^i \right) + \beta_3 \left(\sum_{j \in \mathcal{N}_i} \varpi_{j,n+1}^i \right)^{2\gamma_3+1} + \sum_{j \in \mathcal{N}_i} \varpi_{j,n+2}^i \right],\end{aligned}$$

where α_3, $\beta_3 > 0$, and $\varpi_j^i = [(\varpi_{j,1}^i)^{\mathrm{T}}, \ldots, (\varpi_{j,n+2}^i)^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n(n+2)}$ is the estimation of ω_j for agent i. Note that $\omega_j = [([\nabla^2 f_j]_1)^{\mathrm{T}}, \ldots, ([\nabla^2 f_j]_n)^{\mathrm{T}}, (\nabla f_j)^{\mathrm{T}}, (\frac{\partial}{\partial t} \nabla f_j)^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n(n+2)}$. Let $\bar{\mathcal{A}}_j = \mathrm{diag}(\bar{a}_{j1}, \ldots, \bar{a}_{ji}, \ldots, \bar{a}_{jN}) \in \mathbb{R}^{N \times N}$, where $\bar{a}_{ji} = 0$ for $\forall i \neq j \in \Gamma$ and $\bar{a}_{jj} > 0$. Let $\mathcal{H}_j = \mathcal{L} + \bar{\mathcal{A}}_j$.

The dynamics of ϖ_j^i evolves according to

$$\dot{\varpi}_j^i = -b_j \mathrm{sign}(\varepsilon_j^i) - c_j (\varepsilon_j^i)^{2\theta_j+1}, \tag{20}$$

where $\theta_j \in \mathbb{F}$, $b_j \geqslant \sup_t \|\dot{\omega}_j\|_2 + \delta_j$ with $\delta_j > 0$, $c_j > 0$, and $\varepsilon_j^i = \sum_{k \in \mathcal{N}_i} a_{ik}(\varpi_j^i - \varpi_j^k) + \bar{a}_{ji}(\varpi_j^i - \omega_j)$. Suppose that $\dot{\omega}_j$ is bounded.

Now, it is ready to present the distributed optimization protocol:

$$u_i = -\chi_i \mathrm{sign}(v_i - v_i^*) - \beta_1 (v_i - v_i^*)^{2\gamma_1+1} - \beta_2 (2\gamma_2 + 1)(e_{y_i})^{2\gamma_2} e_{v_i} - \dot{\phi}_i, \tag{21}$$

where $\chi_i=\alpha_1+\xi_i(1+\|\varphi_i\|_2)$ with $\xi_i\geqslant\max\{\sigma_i+\varsigma_i,\varrho_i\}$, $v_i^*=-\alpha_2\mathrm{sign}(\sum_{j\in\mathcal{N}_i}a_{ij}(y_i-y_j))-\beta_2(\sum_{j\in\mathcal{N}_i}a_{ij}(y_i-y_j))^{2\gamma_2+1}-\phi_i$, and

$$\begin{aligned}\dot{\phi}_i=&\left(\left[\sum_{j\in\mathcal{N}_i}\varpi_{j,1}^i,\sum_{j\in\mathcal{N}_i}\varpi_{j,2}^i,\ldots,\sum_{j\in\mathcal{N}_i}\varpi_{j,n}^i\right]\right)^{-1}\Bigg[\beta_3(2\gamma_3+1)\\&\times\left(\sum_{j\in\mathcal{N}_i}\varpi_{j,n+1}^i\right)^{2\gamma_3}\sum_{j\in\mathcal{N}_i}\dot{\varpi}_{j,n+1}^i+\sum_{j\in\mathcal{N}_i}\dot{\varpi}_{j,n+2}^i\\&-\phi_i\left[\sum_{j\in\mathcal{N}_i}\dot{\varpi}_{j,1}^i,\sum_{j\in\mathcal{N}_i}\dot{\varpi}_{j,2}^i,\ldots,\sum_{j\in\mathcal{N}_i}\dot{\varpi}_{j,n}^i\right]\Bigg].\end{aligned}\tag{22}$$

Assumption 6. There exists a known function $\varphi_i(x_i,v_i,t)\in\mathbb{R}^n$, along with nonnegative constants σ_i, ϱ_i, ς_i, satisfying $\|d_i\|_2\leqslant\varsigma_i$ and $\|h_i\|_2\leqslant\sigma_i+\varrho_i\|\varphi_i\|_2$ for $\forall i\in\Gamma$. Additionally, the global cost function $\sum_{i=1}^n f_i(y,t)$ is convex and twice continuously differentiable with respect to y, and the Hessian matrix $\sum_{i=1}^n\nabla^2 f_i(y,t)$ is invertible for all y,t.

Theorem 6 ([68])**.** Under Assumption 6, for the MAS (17), suppose that the communication topology $\mathcal{G}$ is directed, strongly connected and detail balanced. Then the distributed optimization problem (18) can be solved in a fixed-time under the distributed protocol (21).

For most of distributed fixed-time optimization results, each local cost function $f_i(x_i)$ or $f_i(x_i,t)$ needs to be (strongly) convex, e.g., in Assumptions 1–5. In contrast, Assumption 6 requires that the global cost function $\sum_{i=1}^n f_i(y,t)$ is convex, which is a more relaxed condition.

4.2 Distributed fixed-time optimization using an event-triggered strategy

Event-triggered control is a recent trend that has gained attention in the area of distributed control for MASs. Unlike traditional time-based control methods, where agents continuously exchange information at fixed intervals, event-triggered control allows agents to communicate or update their control inputs only when certain conditions or events are triggered. This approach significantly reduces communication frequency, making it well-suited for systems with bandwidth or energy limitations, for instance, wireless sensor networks or mobile robots [72–78].

In recent years, event-triggered control strategies have been successfully integrated into fixed-time optimization frameworks for MASs [64, 79, 80]. These strategies ensure that agents only communicate when necessary, while still guaranteeing the optimization problem is handled within a fixed time. To achieve this, event-triggered conditions are carefully designed to ensure that the agents maintain their convergence towards the optimal solution even in the absence of continuous communication. For instance, a two-stage protocol is designed in [64], where the first stage (fixed-time local optimization) is similar to that in [59]. In the second stage, the distributed protocol

$$u_i(t)=-\gamma_1\big(\nabla^2 f_i(x_i(t))\big)^{-1}\sum_{j\in\mathcal{N}_i}\mathrm{sig}^{\alpha}\big(x_i(t_k^i)-x_j(t_k^i)\big)-\gamma_2\big(\nabla^2 f_i(x_i(t))\big)^{-1}\sum_{j\in\mathcal{N}_i}\mathrm{sig}^{\beta}\big(x_i(t_k^i)-x_j(t_k^i)\big).\tag{23}$$

Theorem 7 ([64])**.** For the MAS (4), the communication topology $\mathcal{G}$ is undirected and connected. Suppose that for agent $i\in\Gamma$, its local cost function $f_i(x_i(t))$ is m_i-strongly convex, has a smoothness measure M_i, and quadratically continuously differentiable. Moreover, there is two constants m, M satisfying $m_i\geqslant m$, $M_i\leqslant M$ for all $i\in\Gamma$. Then, by employing the distributed protocol (23), the distributed fixed-time optimization is achieved for problem (18).

4.3 Distributed fixed-time optimization with application to smart grids

Smart grids represent the next generation of power systems, integrating advanced information and communication technologies to enhance the reliability, efficiency, and sustainability of electricity distribution. The inherent complexity of smart grids, arising from their decentralized and dynamic nature, necessitates the development of efficient distributed optimization techniques to manage energy resources, ensure grid stability, and meet demand in real time. By ensuring convergence to the optimal solution in a fixed time frame, regardless of initial conditions, fixed-time optimization enhances the responsiveness and predictability of smart grid operations.

Economic dispatch as illustrated in Figure 5 involves minimizing the cost of power generation while meeting the electricity demand across the grid. In distributed smart grids, distributed energy resources (DERs) must coordinate their power outputs to achieve global cost minimization while respecting operational constraints such as generation

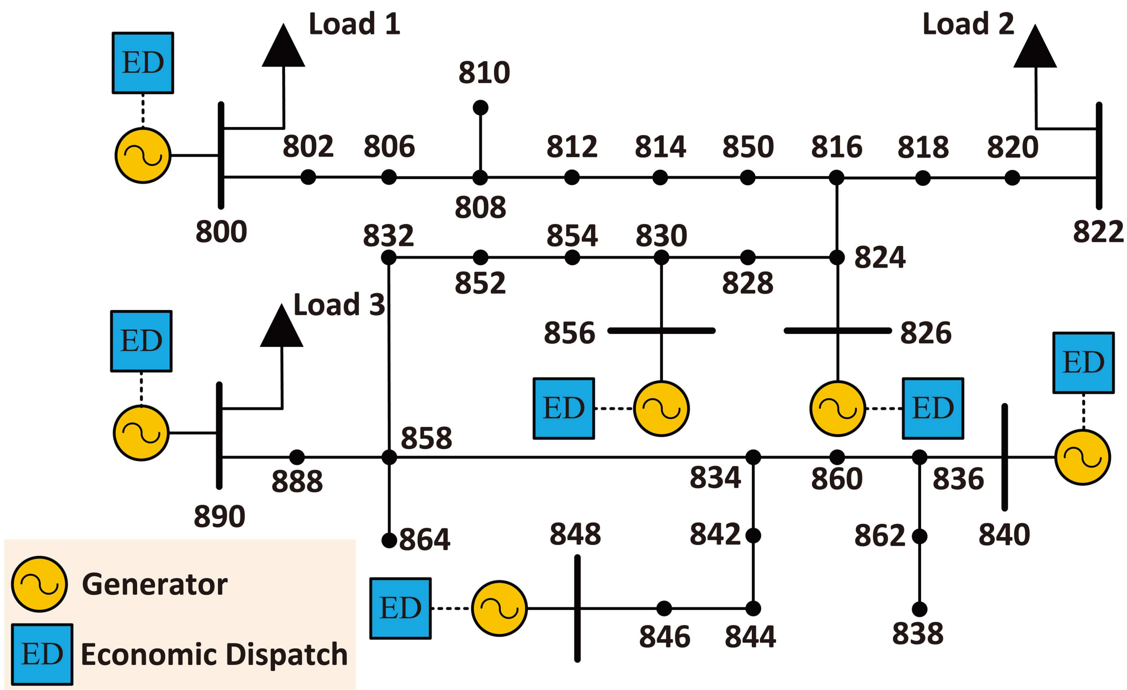

Figure 5 (Color online) Distributed optimization in economic dispatch.

limits and ramp rates. Distributed fixed-time optimization enables DERs to deal with economic dispatch problems efficiently by leveraging peer-to-peer communication networks. Algorithms based on fixed-time consensus ensure that all agents achieve the global optimal dispatch strategy in a fixed time, regardless of their initial power levels. This rapid convergence is crucial for handling the variability and intermittency of renewable energy sources like wind power and solar.

Specifically, the economic dispatch problem is presented as

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} C_i(P_i(t)) = \sum_{i=1}^{n} (\tilde{\alpha}_i P_i^2(t) + \tilde{\beta}_i P_i(t) + \tilde{\gamma}_i) \\ \text{s.t.} \quad & \sum_{i=1}^{n} P_i(t) = \sum_{i=1}^{n} P_i(0) = D, \end{aligned} \tag{24}$$

where $P_i(t) \in \mathbb{R}$ is the active power of generator i at time t, $P_i(0)$ is the initial setting, $C_i(P_i(t)) : \mathbb{R} \to \mathbb{R}$ is the local objective function, D is the total demand, and $\tilde{\alpha}_i$, $\tilde{\beta}_i$, $\tilde{\gamma}_i$ are the constant coefficients.

A distributed fixed-time algorithm is designed as [65]

$$\begin{cases} P_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij}(\zeta_j(t) - \zeta_i(t)) + P_i(0), \\ \dot{\zeta}_i = \kappa \Phi_i(t) + r\mathrm{sig}(\Phi_i(t))^{\omega} + s\mathrm{sig}(\Phi_i(t))^{\xi}, \\ \Phi_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij}(\varphi_j(t) - \varphi_i(t)), \end{cases} \tag{25}$$

where $\varphi_i(t) = \frac{\partial C_i(P_i(t))}{\partial P_i(t)}$, κ, r, $s > 0$, $0 < \omega < 1$, $\xi > 1$, and $\zeta_i(t)$, $\Phi_i(t)$, $\varphi_i(t)$ serve as auxiliary variables.

Theorem 8 ([65])**.** If the undirected communication topology $\mathcal{G}$ is connected, the optimization problem in (24) is solved in a fixed-time with the distributed protocol (25).

Another key aspect for smart grids is demand response management, which encourages consumers to adjust their energy consumption behaviors in reaction to grid signals including price changes or peak demand periods [66,81,82]. Distributed fixed-time optimization facilitates real-time coordination among consumers and grid operators, enabling the efficient scheduling of loads to minimize energy costs and prevent grid overload. For instance, by applying fixed-time optimization techniques, smart appliances and electric vehicles can adjust their charging or operating schedules autonomously while ensuring global grid objectives are met. These methods provide predictable response times, making them ideal for time-sensitive demand response scenarios [66].

5 Distributed prescribed-time optimization of MASs

This section presents representative methodologies and results in distributed prescribed-time optimization of MASs.

5.1 Distributed prescribed-time optimization of MASs with time-invariant cost functions

The objective of distributed prescribed-time optimization of MASs with time-invariant cost functions is to accomplish state consensus and to address the optimization problem (6) within prescribed-time. For example, a distributed protocol for agent i is designed in [83], i.e.,

$$u_i = \begin{cases} -\left(\hat{k}_1 + \hat{c}_1 \frac{\dot{\mu}_1(t)}{\mu_1(t)}\right) \left(\nabla^2 f_i(x_i)\right)^{-1} \times \nabla f_i(x_i), & 0 \leqslant t < t_{\text{pre1}}, \\ -\left(\hat{k}_2 + \hat{c}_2 \frac{\dot{\mu}_2(t)}{\mu_2(t)}\right) \left(\nabla^2 f_i(x_i)\right)^{-1} \times \sum_{j=1}^{n} a_{ij}(x_i - x_j), & t \geqslant t_{\text{pre1}}, \end{cases} \tag{26}$$

where $\hat{k}_1$, $\hat{k}_2$, $\hat{c}_1$, $\hat{c}_2$ are positive constants, and

$$\begin{cases} \mu_1 = \frac{T_{\text{pre1}}^{h_1}}{(T_{\text{pre1}} + t_0 - t)^{h_1}}, & t_0 \leqslant t < t_0 + T_{\text{pre1}}, \\ \mu_2 = \begin{cases} \frac{T_{\text{pre2}}^{h_2}}{(T_{\text{pre2}} + t_{\text{pre1}} - t)^{h_2}}, & t_{\text{pre1}} \leqslant t < t_{\text{pre1}} + T_{\text{pre2}}, \\ 1, & t \geqslant t_{\text{pre1}} + T_{\text{pre2}}, \end{cases} \end{cases} \tag{27}$$

with $h_1, h_2 > 0$, $t_{\text{pre1}} = t_0 + T_{\text{pre1}}$, T_{pre1} and T_{pre2} being the prescribed time instants defined in $\mu_1(t)$ and $\mu_2(t)$, respectively.

Theorem 9 ([83]). Under Assumption 2, for the MAS (4), assume that the communication topology $\mathcal{G}$ is undirected and connected. If $h_1, h_2 > 1$, $\hat{c}_1 \geqslant 1$, and $\hat{c}_2 \geqslant \frac{2\Theta_{\max}}{\lambda_2(\mathcal{L})}$ with $\Theta_{\max} = \max\{\Theta_1, \Theta_2, \ldots, \Theta_n\}$, then by using the distributed protocol (26), the distributed optimization problem is solved within a prescribed time $t_{\text{pre1}} + T_{\text{pre2}}$, i.e., $x_i \to x^*$ as $t \to t_{\text{pre1}} + T_{\text{pre2}}$ for all $i \in \Gamma$, where x^* is the unique solution of problem (5).

Similar to the distributed fixed-time optimization protocol (8), the prescribed-time counterpart involves staged operations. The difference of these two protocols is the terms of $\text{sig}(\cdot)$ in protocol (8) have been replaced by the time-varying terms of μ_1 and μ_2 in protocol (26), which guarantee the prescribed-time convergence. Another three-stage distributed prescribed-time protocol is proposed in [46], where the estimation of average group variables, consensus among networked agents, as well as convergence to the global optimal solution are all achieved in prescribed-time.

5.2 Distributed prescribed-time optimization of MASs with time-varying cost functions

The objective of distributed prescribed-time optimization of MASs with cost functions being time-varying is to attain state consensus and to deal with the optimization problem (12) in prescribed-time. For example, a distributed protocol for agent i is developed as follows [84]:

$$\begin{cases} u_i = \bar{u}_i + \hat{u}_i, \\ \bar{u}_i = -\sum_{j=1}^{n} \left(c_1 + \beta_{ij} \frac{\dot{\rho}(t; T_{\text{pre4}})}{\rho(t; T_{\text{pre4}})}\right) a_{ij}(x_i - x_j), \\ \hat{u}_i = -\theta_{i2}^{-1} \left[\left(c_2 + \frac{\dot{\rho}(t; T_{\text{pre5}})}{\rho(t; T_{\text{pre5}})}\right) \theta_{i1} + \theta_{i3}\right], \\ \theta_i = \Pi_i + h_i, \\ \dot{\varkappa}_i = -\sum_{j=1}^{n} \left(c_3 + \alpha_{ij} \frac{\dot{\rho}(t; T_{\text{pre3}})}{\rho(t; T_{\text{pre3}})}\right) a_{ij}(\theta_i - \theta_j) - c_4 \sum_{j=1}^{n} a_{ij} \text{sign}(\theta_i - \theta_j), \end{cases} \tag{28}$$

where $\varkappa_i$ is an auxiliary variable, $h_i = [\nabla f_i, \nabla^2 f_i, \frac{\partial}{\partial t}\nabla f_i]^{\mathrm{T}}$ denotes the gradient information of the local cost function $f_i(x_i, t)$, $\theta_i = [\theta_1, \theta_2, \theta_3]^{\mathrm{T}}$ is the estimate of h_i, c_1, c_2, c_3, c_4 are some constants, $T_{\mathrm{pre3}}, T_{\mathrm{pre4}}, T_{\mathrm{pre5}}$ are prescribed time instants defined in a type of time-varying functions ρ [84], and α_{ij}, β_{ij} are adaptive parameters with

$$\begin{cases} \alpha_{ij} = \dfrac{1}{2}\rho(t; T_{\mathrm{pre3}})\dot{\rho}(t; T_{\mathrm{pre3}})e^{\gamma t}a_{ij}(\theta_i - \theta_j)^{\mathrm{T}}(\theta_i - \theta_j), \\ \beta_{ij} = \dfrac{1}{2}\rho(t; T_{\mathrm{pre4}})\dot{\rho}(t; T_{\mathrm{pre4}})e^{st}a_{ij}(x_i - x_j)^{\mathrm{T}}(x_i - x_j), \end{cases}$$

where γ and s are some positive constants.

Assumption 7. The optimal trajectory of state $x^*(t)$ exists and is continuous. Moreover, the state $x_i^*(t) = x_j^*(t)$ of this trajectory can minimize the total cost function $\sum_{i=1}^n f_i(x^*, t)$.

Assumption 8. For each local cost function $f_i(x_i, t)$ and the total cost function $f_0 = \sum_{i=1}^n f_i(x_i, t)$, the two continuous functions should be twice differentiable for x and piecewise differentiable for t. For any x and t, the Hessian matrix of f_0 that can be expressed as $\sum_{i=1}^n \nabla^2 f_i(x_i, t)$ is a nonsingular matrix. Moreover, f_0 is m_f-strongly convex with constant $m_f > 0$, i.e., $\sum_{i=1}^n \nabla^2 f_i(x_i, t) \geqslant m_f I_n$.

Theorem 10 ([84]). Under Assumptions 7 and 8, for the MAS (4), assume that the communication topology $\mathcal{G}$ is undirected and connected. Then, if each agent's local cost function satisfies $\|\dot{h}_i - \dot{h}_j\| < \epsilon$ with $\epsilon > 0$, $i, j \in \{1, 2, \ldots, n\}$, $c_1, c_2, c_3 > 0$, and $c_4 \geqslant \epsilon(n-1)/2$, by using the distributed protocol (28), the optimization problem (6) is solved in a prescribed-time of $T = T_{\mathrm{pre3}} + T_{\mathrm{pre4}} + T_{\mathrm{pre5}}$.

As discussed in Subsection 3.2, the assumptions made for time-varying cost functions such as Assumption 8 are usually more restrictive compared to those for time-invariant ones. These stronger conditions are introduced to enable rigorous convergence and stability analysis. A notable characteristic of certain prescribed-time optimization algorithms is that the time-varying control gains may approach infinity as the preassigned convergence deadline approaches. This behavior can introduce undesirable singularities and numerical instability. Consequently, it is critical to ensure that the time-varying gains are designed in a manner that guarantees boundedness over the entire time horizon [83]. To address this issue, recent studies have proposed bounded gain frameworks specifically constructed to circumvent singularity-related complications [85–89]. For example, one approach employs a prescribed-time controller equipped with uniformly bounded gains, thereby improving robustness against model uncertainties and non-vanishing disturbances [86]. More recently, emerging directions in distributed prescribed-time optimization include the consideration of agents with general linear dynamics [90], as well as communication-efficient strategies such as event-triggered coordination protocols [91], reducing communication load.

It is worth mentioning that a prescribed-time T_{pre} can be theoretically set as any positive value. In practice, however, the choice of T_{pre} is restricted by factors such as physical constraints. For example, selecting an overly small T_{pre} can lead to impractically high control gains, steep gradients, or even numerical instability, which may violate actuator constraints or communication bandwidth limits in real-world systems. Conversely, choosing a large T_{pre} may undermine the benefits of fast convergence and system responsiveness. Moreover, the performance and feasibility of the designed protocol are highly sensitive to the chosen T_{pre}, yet there is currently no general framework to optimize T_{pre} automatically while guaranteeing both convergence and constraint satisfaction. This inherent trade-off introduces a challenge in applying prescribed-time methods to dynamic, uncertain, or resource-limited multi-agent systems.

6 Conclusion and challenging future directions

While distributed fixed-time and prescribed-time optimization has shown significant promise in enhancing the efficiency of the coordination of MASs, several challenges need to be addressed to unlock its full potential. Below is a detailed discussion of these challenges and future research directions.

(i) Handling nonconvexities. Most of the existing distributed fixed-time and prescribed-time optimization problems with convex cost functions are solved. However, practical optimization problems often involve nonconvex objectives, such as minimizing generation costs or optimizing traffic flow [92–95]. The fixed-time and prescribed-time optimization methods are traditionally more suited to convex problems, making their application to nonconvex scenarios challenging. Hence, how to develop distributed fixed-time or prescribed-time optimization algorithms that leverage gradient tracking, convex relaxation, or consensus over nonconvex domains can expand their applicability. One possible approach is to utilize adaptive gain mechanisms to handle the local curvature of nonconvex cost functions, which can enhance convergence accuracy without sacrificing fixed-time or prescribed-time guarantees [92].

(ii) Constrained optimization. Some optimization problems can have constrained feasibility domain [96–99]. For example, smart grids often need to minimize costs while maximizing renewable energy usage, under strict operational constraints like voltage limits and power balance. Generally, constraints can be categorized into two main forms: equality constraints and inequality ones. In [97], a distributed finite-time primal-dual method is designed for optimization problems with both inequality and equality constraints. In [98], a distributed approach utilizing column and row stochastic weight matrices, structured according to the underlying graph topology, is designed for optimization problems with general constraints, including nonidentical set constraints, multiple inequality, and equality constraints. Note that these results are achieved without considering the fixed-time or prescribed-time convergence. Hence, it would be an interesting future direction for distributed fixed-time or prescribed-time optimization with general constraints.

(iii) Communication constraints. Distributed fixed-time and prescribed-time optimization heavily relies on communication among agents for exchanging information, such as local states, control inputs, and gradient estimates. However, communication networks in practice may experience delays, packet losses, and bandwidth limitations, especially during high-load scenarios [100]. One possible approach is to design delay-tolerant algorithms to ensure reliable performance under real-world communication constraints [101, 102]. Another strategy is to employ event-triggered mechanisms where agents communicate only when necessary can reduce the communication burden while maintaining convergence guarantees [73, 74, 103, 104], with preliminary results being reported in [64, 79, 80].

(iv) Robustness to cyber-physical threats. Networked systems are increasingly vulnerable to cyberattacks (e.g., false data injection, denial of service) and physical disruptions (e.g., extreme weather events) [105]. These threats can compromise the integrity of the optimization process by disrupting communication links or altering system parameters [106–108]. Hence, how to develop secure fixed-time or prescribed-time optimization algorithms that can detect and mitigate the impact of cyberattacks is essential. For instance, we can incorporate anomaly detection mechanisms can prevent malicious data from influencing the optimization process, or design optimization strategies that account for possible node or link failures, thus improving the networked agent's ability to recover from disruptions [109].

(v) Beyond cooperation: integrating competitive and cooperative optimization. Recent developments highlight the importance of addressing both cooperative and competitive behaviors in multi-agent systems [44]. Unlike conventional distributed optimization, which assumes collective alignment, real-world applications often involve agents with divergent objectives. This gives rise to scenarios better captured by game-theoretic models [49]. Cooperative games aim to minimize global costs, whereas non-cooperative formulations focus on individual rationality and equilibrium strategies. The interplay between these paradigms introduces new algorithmic challenges, particularly in dynamic settings with partial information. Integrating distributed fixed-time and prescribed-time optimization with game-theoretic principles offers a promising direction for studying strategic interactions, with potential applications in energy management, federated learning, and multi-agent autonomy under heterogeneous objectives.

(vi) Experimentation and real-world deployment. Most results presented in this paper lack testing in real-world environments. While theoretical research has provided valuable insights into distributed fixed-time and prescribed-time optimization, experimental validation remains a necessary but challenging area. Factors such as hardware limitations, environmental uncertainties, and human-in-the-loop interactions can significantly affect performance. In the existing literature, some preliminary experiment of quadrotors has been conducted for formation control and target tracking in the lab level [110]. Future research should prioritize the design and execution of robust experimental tests to validate distributed control strategies under real-world operating conditions, thus bridging the gap between theory and practice of distributed fixed-time and prescribed-time optimization.

Acknowledgements This work was supported in part by International Leader Fellowship from Royal Society of New Zealand (Grant No. ILF-AUT2501) and National Natural Science Foundation of China (Grant Nos. 62325304, U22B2046).

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