

Finite-iteration model-free adaptive terminal iterative learning control

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In recent years, terminal iterative learning control (TILC) [1, 2] has attracted more attention for specific control tasks where the only available measurement is the terminal state or terminal output, and the ultimate control objective is also the terminal state or terminal output, instead of the whole trajectory of the system. Since then, TILC has been applied to many applications, such as in station stop control of a train, rapid thermal processing chemical vapor deposition (RTPCVD) system, and the multi-agent systems.

However, the existing results with TILC can achieve tracking error convergence only when the iteration tends to infinity. Nevertheless, in practical applications, more emphasis is placed on maintaining the tracking error within an acceptable range for finite iterations rather than pursuing the theoretical zero error after infinite iterations. Hence, it is very meaningful to carry out the research on finite-iteration learning control. The finite-iteration convergence of linear systems was investigated in exploratory research in [3]. However, its finite convergence can be obtained directly from the existing infinite zero-tracking convergence. Additionally, the algorithm design requires known information about the model of the system.

Based on the above analysis, this study aims to design a novel finite-iteration TILC (FITILC) method for nonlinear discrete-time single input single output (SISO) systems. First, an equivalent linear iterative data model is established for unknown SISO systems. Then, a novel definition of the finite convergence of the iterative domain is given, and an iterative learning control algorithm based on the fractional power of the terminal error is constructed. The main contributions of this study are summarized as follows. (1) This study proposes an iterative learning control algorithm with finite-iteration convergence, where the convergence of tracking errors can be guaranteed within a finite-iteration. In contrast, existing results require an infinite-iteration to ensure convergence. (2) Compared with existing TILCs, the control input update algorithm in this study uses the fractional power of terminal tracking error. Besides, the frac-

tional power update has faster convergence through simulation results. (3) The finite iteration learning control algorithm designed in this study is conducted under the data-driven framework, where the design and analysis of the controller only require input-output (I/O) data, without the need for any model information. In contrast, the results of [3] require the model information to be known.

Problem formulation. Consider a discrete-time SISO nonlinear system as follows:

$$y_k(t+1) = f(y_k(t), u_k), \quad (1)$$

where k denotes the number of repetitions for the system, $t = 0, 1, \dots, T$ is the sampling time index, u_k is the input of the system, y_k denotes the output of the system, where only $y_k(T)$ can be measured at the end of every iteration and $f(\cdot)$ is an unknown vector valued function.

In each iteration, the relationship between the input and output along the iteration domain can be represented by the following functions:

$$\begin{aligned} y_k(1) &= f(y_k(0), u_k) = w^1(y_k(0), u_k), \\ y_k(2) &= f(w^1(y_k(0), u_k), u_k) = w^2(y_k(0), u_k), \\ &\vdots \\ y_k(T) &= f(w^{T-1}(y_k(0), u_k), u_k) = w^T(y_k(0), u_k), \end{aligned} \quad (2)$$

where $y_k(0)$ is the initial value of system and $w^1(\cdot), \dots, w^T(\cdot)$ are unknown nonlinear functions.

Assumption 1 ([1]). The system (1) is completely controllable.

Assumption 2 ([1]). The initial output value $y_k(0)$ is unchanged at every iteration k , i.e., $\forall k, y_k(0) = y_{k-1}(0)$.

Assumption 3. The nonlinear function $w^T(\cdot)$ satisfies the generalized Lipschitz condition

$$|w^T(y_1, u_1) - w^T(y_2, u_2)| \leq L_y |y_1 - y_2| + L_u |u_1 - u_2|,$$

where L_y and L_u are two positive Lipschitz constants.

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Using the mean value theorem for (2) and combined with Assumption 2, it yields

$$\begin{aligned}\Delta y_k(T) &= y_k(T) - y_{k-1}(T) \\ &= w^T(y_k(0), u_k) - w^T(y_k(0) - \Delta y_k(0), u_k - \Delta u_k) \\ &= w^{T,u}(\chi_k) \Delta u_k = \Phi_k \Delta u_k,\end{aligned}\quad (3)$$

where $\Delta u_k = u_k - u_{k-1}$, $\chi_k \in [u_k, u_{k-1}]$, and $\Phi_k = w^{T,u}(\chi_k)$.

Assumption 4. $w^{T,u}(\chi_k)$ has upper and lower bounds and is strictly nonzero, Φ_k is also bounded and the sign of Φ_k is always consistent with that of Φ_0 . Without loss of generality, we assume $0 < l_1 \leq |\Phi_k| \leq l_2$.

Remark 1. Assumptions 1–3 are common for the design of learning control for nonlinear systems. Note that we only need the existence of the lower and upper bounds in Assumption 4 without requiring their exact values.

Then, Eq. (1) can be rewritten as

$$y_k(T) = y_{k-1}(T) + \Phi_k \Delta u_k. \quad (4)$$

Remark 2. From (4), we find that nonlinear functions $f(\cdot, \cdot)$ are compressed into an iterative-varying parameter Φ_k . Thus, it is very difficult to establish the dynamic equations of Φ_k with mathematical formulas. However, through a large number of simulations, we find that its numerical size is relatively simple and easy to estimate.

Theorem 1 ([4,5]). Consider a dynamic system that can be run repeatedly,

$$h_{k+1}(t) = h_k(t) - ah_k^\sigma(t) - bh_k(t) + O_k(t), \quad (5)$$

where $0 < b < 1$ and $0 < \sigma < 1$ are odd fractions. If $O_k(t)$ satisfies $|O_k(t)| < \lambda$ with λ as a bounded constant, then $h_k(t)$ can converge to a bounded range $|h_k(t)| \leq (1 + \sigma^{\frac{1}{1-\sigma}} - \sigma^{\frac{1}{1-\sigma}}) \cdot \max((\lambda/a)^{\frac{1}{\sigma}}, (\frac{1}{1-b})^{\frac{1}{1-\sigma}})$ in finite-iteration.

Remark 3. Although Theorem 1 is similar to Lemma 3.1 in [5] in terms of mathematical expressions, there is a significant difference in their research focus. Theorem 3 focuses on the change of the system in the iterative domain, which involves recursive updates of the system states and iterative optimization, which is closely related to our study of discrete-time systems and iterative algorithms. In contrast, Lemma 3.1 focuses on the dynamics of the system in the time domain, which studies the behavior of the system state that changes continuously over time. This difference implies that although the two theorems are formally similar, the practical problems and application scenarios they address are completely different.

Objective of control. For the nonlinear system 1 with unknown model information, this study aims to design a FITILC strategy. This strategy can ensure that the terminal output of the system accurately tracks the preset target value within finite-iterations. At the same time, the boundedness of the control input u_k is guaranteed to ensure the stability and reliability of the whole control process.

Controller design. Define the terminal control input index function and iterative estimate algorithm as follows:

$$J(u_k) = |e_{k-1}^\sigma(T) + e_{k-1}(T) - 2\Phi_k \Delta u_k|^2 + \lambda |\Delta u_k|^2, \quad (6)$$

$$J(\hat{\Phi}_k) = |\Delta y_{k-1}(T) - \hat{\Phi}_k \Delta u_{k-1}|^2 + \mu |\hat{\Phi}_k - \hat{\Phi}_{k-1}|^2, \quad (7)$$

where $\lambda > 0$ and $\mu > 0$ are the weighting factors, $e_k(T) = y_d - y_k(T)$ denotes the tracking error, $\hat{\Phi}_k$ is the estimation

of Φ_k at the k th iteration. According to the optimal condition of $\partial J(u_k)/\partial u_k = 0$ and $\partial J(\hat{\Phi}_k)/\partial \hat{\Phi}_k = 0$, then we get the FITILC for system (1) as follows:

$$\hat{\Phi}_k = \hat{\Phi}_{k-1} + \frac{\eta(\Delta y_{k-1}(T) - \hat{\Phi}_{k-1} \Delta u_{k-1}) \Delta u_{k-1}}{\mu + |\Delta u_{k-1}|^2}, \quad (8)$$

$$\hat{\Phi}_k = \hat{\Phi}_0, \quad \text{if } |\hat{\Phi}_k| < \varepsilon \quad \text{or} \quad \text{sign}(\hat{\Phi}_k) \neq \text{sign}(\hat{\Phi}_0),$$

$$u_k = u_{k-1} + \frac{2\rho \hat{\Phi}_k}{\lambda + 4|\hat{\Phi}_k|^2} (e_{k-1}^\sigma(T) + e_{k-1}(T)), \quad (9)$$

where $0 < \eta < 2$, $\mu > 0$. ε is a very small positive integer, usually $\varepsilon = 10^{-4}$ or $\varepsilon = 10^{-5}$.

Theorem 2. For the discrete-time SISO nonlinear system (1) satisfies Assumptions 1–4, and the system uses the proposed FITILC scheme (8) and (9) can guarantee the following.

- (i) The estimation of $\hat{\Phi}_k$ is bounded for every iterations k .
- (ii) The terminal error $e_k(T)$ converges to a bounded region within finite-iteration.

The proofs of Theorems 1 and 2 are included in Appendixes A and B, respectively. The proposed results are illustrated by a numerical simulation, shown in Appendix C.

Conclusion. This study addresses the problem of fast terminal output tracking control in nonlinear systems. Based on the existing TILC schemes, this study ingeniously integrates finite-iteration control strategies with terminal iterative learning control to propose an innovative FITILC scheme. The controller design and analysis rely entirely on measured I/O data of the system. No additional model information is required. A rigorous mathematical analysis ensures that the effectiveness of the proposed method can be theoretically guaranteed. Finally, the efficacy and superiority of the designed TFILC method are validated through a simulation study. Future research work will mainly focus on extending the devised FITILC approach to multi-input multi-output nonlinear systems as well as multi-agent systems.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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