SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

December 2025, Vol. 68, Iss. 12, 222301:1–222301:22
https://doi.org/10.1007/s11432-024-4278-0

Joint power allocation and beamforming for active IRS-aided secure directional modulation network

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Received 13 August 2024/Revised 18 November 2024/Accepted 17 January 2025/Published online 22 September 2025

Abstract To boost the secrecy rate (SR) of the conventional directional modulation (DM) network and overcome the "double fading" effect of the cascade channels of the passive intelligent reflecting surface (IRS), a novel active IRS-aided DM system with a power adjustment strategy between transmitter and active IRS is proposed in this study. Then, the SR is maximized by jointly optimizing the power allocation (PA) factors, transmit beamforming, receive beamforming, and reflect beamforming at the active IRS, subject to the IRS power constraint. To solve the formulated non-convex optimization problem, a high-performance scheme of maximizing SR based on successive convex approximation (SCA) and Schur complement, called Max-SR-SS, is proposed, where the derivative operation is employed to optimize the PA factors, the generalized Rayleigh-Ritz theorem is adopted to derive the receive beamforming, and the SCA strategy is utilized to design the transmit beamforming and IRS phase shift matrix. To reduce the high complexity, a low-complexity scheme of maximizing SR based on equal amplitude reflecting (EAR) and majorization-minimization (MM), called Max-SR-EM, is developed, where the EAR and MM methods are adopted to derive the amplitude and phase of the IRS phase shift matrix, respectively. In particular, when the receivers are equipped with single antenna, a scheme of maximizing SR based on alternating optimization, called Max-SR-AO, is proposed, where the PA factors, transmit beamforming, and reflect beamforming are derived by the fractional programming and SCA algorithms. The simulation results show that, with the same power constraint, the SR gains achieved by the proposed schemes outperform those of the fixed PA and passive IRS schemes.

Keywords directional modulation, secrecy rate, active intelligent reflecting surface, power allocation, beamforming

Citation Dong R E, Shu F, Li Y Z, et al. Joint power allocation and beamforming for active IRS-aided secure directional modulation network. Sci China Inf Sci, 2025, 68(12): 222301, https://doi.org/10.1007/s11432-024-4278-0

1 Introduction

The broadcast nature of wireless communication makes confidential messages vulnerable to eavesdropping by illegal users, leading to the security issue of confidential message leakage. Directional modulation (DM), as an advanced and promising physical layer security technology, has attracted the research interest of many researchers [1–5]. DM provides security via directives and is suitable for line-of-sight (LoS) channels, such as millimeter waves, unmanned aerial vehicles, intelligent transportation, maritime communication, and satellite communication [6]. The main ideas of DM are as follows. In the LoS channels, DM transmits confidential messages to legitimate users along the desired direction via beamforming and interferes with illegal user eavesdropping by sending artificial noise (AN) in the undesired direction, hence enhancing the secure performance of the system [7]. Thus far, the research on DM technology is mainly focused on the radio frequency frontend (RF) and baseband.

To enhance the secrecy rate (SR) of the DM network with an eavesdropper, in [8], in accordance with the convex optimization method, a sparse array of DM was synthesized, and the proposed approach achieved better flexibility in terms of control security performance and power efficiency. A DM network with

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hybrid active and passive eavesdroppers was considered in [9], and a scheme, which used the frequency division array with assisted AN technique at the transmitter to achieve secure transmission with angle-range dependence, was proposed. In contrast to the aforementioned single legitimate user networks, in [10], the authors investigated a multi-legitimate user DM network and designed a security-enhancing symbol-level precoding vector, which outperformed the benchmark method in terms of both the power efficiency and security enhancement. Multi-beam DM networks were investigated in [11, 12]. Moreover, a generalized synthesis method and an AN-aided zero-forcing synthesis method were proposed in [11, 12] to enhance the system performance, respectively. However, the aforementioned works mainly focused on the scenario where the legitimate user and the eavesdropper have different directions. To ensure secure transmission of the system when the eavesdropper was in the same direction as the legitimate user, secure precise wireless transmission DM systems were investigated in [13, 14], which sent confidential messages to a specific direction and distance to ensure secure wireless transmission.

With the development of wireless communication, the demand for networks increases dramatically [15, 16]. The use of a large number of active devices will lead to serious energy consumption problems; fortunately, the emergence of an intelligent reflecting surface (IRS) provides a novel paradigm to overcome this problem. IRS is a planar array of a large number of electromagnetic elements, each of which is capable of independently adjusting the amplitude and phase of the incident signal [17–20]. Because of this capability, the signal strength at the receiver can be significantly enhanced by properly tuning the reflected signal. Recently, various wireless communication scenarios assisted by IRS have been extensively investigated, including the multicell communications [15], unmanned aerial vehicle communications [21], simultaneous wireless information and power transfer (SWIPT) network [22], non-orthogonal multiple access network [23], and wireless-powered communication network [24].

Given the advantages of the IRS in wireless communication, in recent years, the passive IRS-aided DM network has also been investigated. With the help of the IRS, the DM can overcome the limitation of being able to transmit only one confidential bit stream and significantly enhance the SR performance. In [25], an IRS-aided DM system was considered, and two confidential bit streams were transmitted from Alice to Bob at the same time. Based on the system model proposed in [25, 26], to enhance the SR performance, two low-complexity algorithms were proposed to jointly design the transmit and reflect beamforming vectors of the IRS-aided DM network. An IRS-aided DM network equipped with single antenna for both legitimate user and eavesdropper was investigated in [27], and the SR closed-form expression was derived. Moreover, in [28], the authors proposed two beamforming algorithms to enhance the SR in the DM network aided by IRS and achieved approximately 30% SR gains over no-IRS and random phase shift IRS schemes. The aforementioned studies showed that the passive IRS can boost the SR performance of the conventional DM network.

However, the "double fading" effect that accompanies passive IRS is inevitable, which is caused by the fact that the signal reflected through the IRS needs to pass through the transmitter-to-IRS and IRS-to-receiver cascade links [29–31]. To overcome this physical limitation, an emerging IRS structure called active IRS has been proposed. In contrast to the passive IRS, which can only adjust the phase of the incident signal, the active IRS integrates active reflection-type amplifiers that can simultaneously tune the amplitude and phase of incident signals. Hence the "double fading" effect of the cascade link can be effectively attenuated, enabling better performance than the passive IRS [29]. Notably, although the active IRS can both amplify and reflect incident signals, it is fundamentally different from the full-duplex amplify-and-forward relay. The active IRS does not require RF chains, has no signal processing capability, and has lower hardware cost [32]. Moreover, the relay takes two time slots to accomplish the transmission of one signal, whereas the active IRS only requires one time slot [29].

Similar to the passive IRS, in recent years, many researchers have investigated various wireless communication scenarios with the help of the active IRS [33–37]. For example, to maximize the rate of the IRS-aided downlink/uplink communication system, the placement of the active IRS was investigated in [38], which revealed that the system rate was optimal when the active IRS was placed close to the receiver. An active IRS-aided single input multiple output network was considered in [39], and an alternating optimization (AO) approach was proposed to obtain the IRS reflection coefficient matrix and received beamforming, which achieved better performance compared with the passive IRS-aided network with the same power budget. An active IRS-aided SWIPT network was proposed in [40], an alternating iteration method was employed to maximize the weighted sum rate, and the high-performance gain was achieved. In a single input single output system, in [41], the authors provided a theoretical and numerical comparison of the performance between passive and active IRSs with the same power budget, which showed that the active IRS performed better than the passive IRS when the power budget is not very low and the number of IRS elements is not very large. The aforementioned studies presented the benefits of the active IRS for wireless network performance gains.

Motivated by the aforementioned discussions, to further enhance the SR performance of the passive IRS-aided DM system, an active IRS-aided DM network with an eavesdropper is investigated in this paper. Given that the confidential message beamforming and AN powers of the base station (BS) and active IRS power are subject to the total power constraint of the system, to investigate the impact of the power allocation (PA) among them and beamforming optimization on the system performance, we focus on maximizing the SR by jointly optimizing the PA factors, transmit beamforming, receive beamforming, and reflect beamforming at the active IRS. The main contributions of this paper are summarized as follows.

(1) To enhance the SR performance of the conventional DM system, a novel DM network with the introduction of the active IRS is proposed in this paper. In particular, a PA strategy is proposed to adjust the power fraction between BS and active IRS to further harvest the rate performance gain achieved by active IRS, which does not exist in a passive IRS-aided network. Then, an active IRS-aided DM system with PA is presented. Finally, we formulate the SR maximization problem by jointly optimizing the PA factors, transmit beamforming, receive beamforming, and IRS phase shift matrix for the active IRS-aided secure DM system in the presence of an eavesdropper, subject to the IRS power constraint. By optimizing the PA between BS and IRS as well as beamforming, the SR of the system is significantly boosted.

(2) To address the formulated non-convex maximum SR optimization problem in which the five variables are coupled with each other, a high-performance AO scheme, called maximizing SR based on successive convex approximation (SCA) and Schur complement (Max-SR-SS), is proposed. In this scheme, the derivative operation is employed to calculate the optimal PA factor of the confidential message and the PA factor of power allocated to the BS. The corresponding transmit and receive beamforming are derived by the SCA method and generalized Rayleigh-Ritz theorem, respectively, and the IRS phase shift matrix is calculated by the SCA and Schur complement methods. Moreover, a low-complexity scheme, called maximizing SR based on equal amplitude reflecting (EAR) and majorization-minimization (MM) (Max-SR-EM), is proposed to address the formulated problem, where the EAR and MM strategies are adopted to obtain the amplitude and phase of the IRS phase shift matrix, respectively.

(3) In particular, when the receivers are equipped with single antenna, the optimization problem can be simplified without receive beamforming. To solve the problem, a scheme of maximizing SR based on alternating optimization (Max-SR-AO) is proposed, where the PA factors, transmit beamforming, and IRS phase shift matrix are designed by the fractional programming (FP) and SCA algorithms. The simulation results show that, with the same total power, the SRs harvested by the three proposed schemes are higher than those of the benchmark schemes. In addition, when the number of phase shift elements tends to be large, the gap in terms of the SR between Max-SR-SS and Max-SR-EM schemes is trivial.

The remainder of this paper is organized as follows. Section 2 describes the system model of the active IRS-aided DM network and formulates the maximum SR problem. Section 3 introduces the proposed Max-SR-SS and Max-SR-EM schemes. Section 4 presents the proposed Max-SR-AO scheme. Sections 5 and 6 provide the numerical simulation results and conclusions, respectively.

Notations. In this work, the scalars, vectors, and matrices are marked in lowercase, boldface lowercase, and uppercase letters, respectively. The symbols $(\cdot)^{\mathrm{T}}$, $(\cdot)^{*}$, $(\cdot)^{\mathrm{H}}$, $\partial(\cdot)$, $\mathrm{Tr}(\cdot)$, $\mathrm{rank}(\cdot)$, $(\cdot)^{\dagger}$, $\lambda_{\max}(\cdot)$, $\Re\{\cdot\}$, $\mathrm{diag}\{\cdot\}$, and $\mathrm{blkdiag}\{\cdot\}$ refer to the transpose, conjugate, conjugate transpose, partial derivative, trace, rank, pseudo-inverse, maximum eigenvalue, real part, diagonal, and block diagonal matrix operations, respectively. The sign $|\cdot|$ stands for the absolute value of the scalar or the determinant of the matrix. The notations $\boldsymbol{I}_Q$ and $\mathbb{C}^{P\times Q}$ mean the identity matrix of $Q\times Q$ and complex-valued matrix space of $P\times Q$, respectively. The symbol $\mathcal{CN}(\mathbf{0},\boldsymbol{I})$ denotes the circularly symmetric complex Gaussian random vector with mean $\mathbf{0}$ and covariance $\boldsymbol{I}$.

2 System model

As illustrated in Figure 1, we investigate an active IRS-assisted secure DM network, where the BS (Alice) sends a confidential message to the legitimate user (Bob) with the assistance of active IRS, while sending AN to the eavesdropper (Eve) to reduce the risk of confidential information being intercepted by Eve. There are N, N_{b}, and N_{e} antennas at Alice, Bob, and Eve, respectively. There are M reflection elements

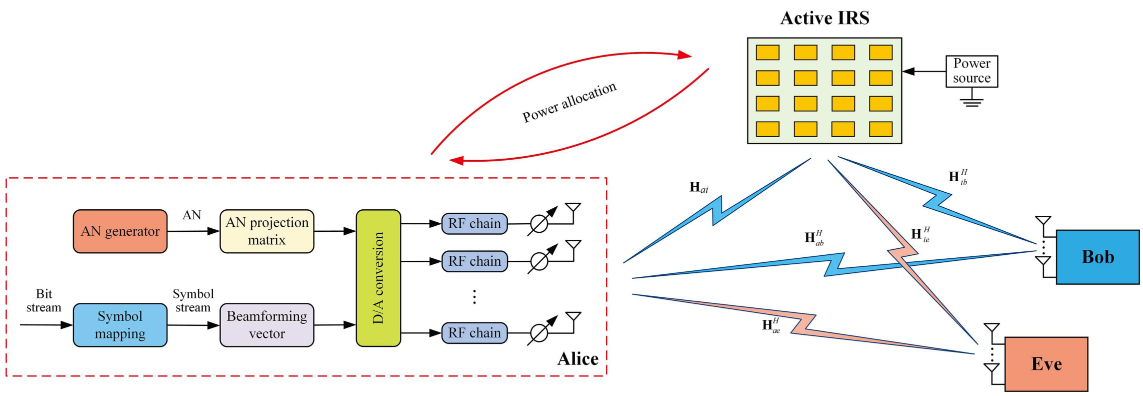

Figure 1 (Color online) System diagram of the active IRS-assisted DM network.

on the active IRS with tunable amplitude and phase. In this paper, it is assumed that the active IRS reflects signal only once and there exists the line-of-sight channels. Moreover, all channel state information is assumed to be available owing to the channel estimation[1)].

The transmit signal at Alice is expressed as

$$\boldsymbol{s} = \sqrt{\beta l P}\boldsymbol{v}x + \sqrt{(1-\beta)lP}\boldsymbol{T}_{\mathrm{AN}}\boldsymbol{z}, \tag{1}$$

where P stands for the total power; $\beta \in (0,1]$ and $(1-\beta)$ refer to the PA parameters of the confidential message and AN; $l \in (0,1)$ means the PA factor of the total power allocated to the BS; $\boldsymbol{v} \in \mathbb{C}^{N\times 1}$ and x refer to the beamforming vector and confidential message intent to Bob, and they satisfy $\boldsymbol{v}^{\mathrm{H}}\boldsymbol{v} = 1$ and $\mathbb{E}[|x|^2] = 1$, respectively; $\boldsymbol{T}_{\mathrm{AN}} \in \mathbb{C}^{N\times N}$ and $\boldsymbol{z} \in \mathbb{C}^{N\times 1}$ represent the projection matrix and vector of AN, and they meet $\mathrm{Tr}(\boldsymbol{T}_{\mathrm{AN}}\boldsymbol{T}_{\mathrm{AN}}^{\mathrm{H}}) = 1$ and $\boldsymbol{z} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_N)$, respectively.

Given the existence of path loss, the received signal at Bob is formulated as

$$\begin{aligned} y_{\mathrm{b}} &= \boldsymbol{u}_{\mathrm{b}}^{\mathrm{H}}(\sqrt{g_{\mathrm{ab}}}\boldsymbol{H}_{\mathrm{ab}}^{\mathrm{H}} + \sqrt{g_{\mathrm{aib}}}\boldsymbol{H}_{\mathrm{ib}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{H}_{\mathrm{ai}})\boldsymbol{s} + \sqrt{g_{\mathrm{ib}}}\boldsymbol{u}_{\mathrm{b}}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{ib}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{n}_r + n_{\mathrm{b}} \\ &= \sqrt{\beta l P}\boldsymbol{u}_{\mathrm{b}}^{\mathrm{H}}(\sqrt{g_{\mathrm{ab}}}\boldsymbol{H}_{\mathrm{ab}}^{\mathrm{H}} + \sqrt{g_{\mathrm{aib}}}\boldsymbol{H}_{\mathrm{ib}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{H}_{\mathrm{ai}})\boldsymbol{v}x + \sqrt{(1-\beta)lP}\boldsymbol{u}_{\mathrm{b}}^{\mathrm{H}}(\sqrt{g_{\mathrm{ab}}}\boldsymbol{H}_{\mathrm{ab}}^{\mathrm{H}} \\ &\quad + \sqrt{g_{\mathrm{aib}}}\boldsymbol{H}_{\mathrm{ib}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{H}_{\mathrm{ai}})\boldsymbol{T}_{\mathrm{AN}}\boldsymbol{z} + \sqrt{g_{\mathrm{ib}}}\boldsymbol{u}_{\mathrm{b}}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{ib}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{n}_r + n_{\mathrm{b}}, \end{aligned} \tag{2}$$

where $\boldsymbol{u}_{\mathrm{b}} \in \mathbb{C}^{N_{\mathrm{b}}\times 1}$ refers to the receive beamforming. g_{ab} and g_{ib} stand for the path loss parameters of Alice-to-Bob and IRS-to-Bob channels, respectively. $g_{\mathrm{aib}} = g_{\mathrm{ai}}g_{\mathrm{ib}}$ means the equivalent path loss parameter of Alice-to-IRS and IRS-to-Bob channels. $\boldsymbol{\Psi} = \mathrm{diag}\{\psi_1, \ldots, \psi_m, \ldots, \psi_M\} \in \mathbb{C}^{M\times M}$ and $\boldsymbol{\psi} = [\psi_1, \ldots, \psi_m, \ldots, \psi_M]^{\mathrm{T}} \in \mathbb{C}^{M\times 1}$ refer to the reflection coefficient matrix and vector of the active IRS, $\psi_m = \alpha_m \mathrm{e}^{\mathrm{j}\phi_m}$. α_m and ϕ_m are the amplitude and phase of the m-th reflecting element, respectively. $\boldsymbol{n}_r \sim \mathcal{CN}(\boldsymbol{0}, \sigma_r^2\boldsymbol{I}_M)$ and $n_{\mathrm{b}} \sim \mathcal{CN}(0, \sigma_{\mathrm{b}}^2)$ mean the complex additive white Gaussian noise (AWGN) at IRS and Bob, respectively. $\boldsymbol{H}_{\mathrm{ab}}^{\mathrm{H}} = \boldsymbol{h}_{\mathrm{ba}}\boldsymbol{h}_{\mathrm{ab}}^{\mathrm{H}} \in \mathbb{C}^{N_{\mathrm{b}}\times N}$, $\boldsymbol{H}_{\mathrm{ib}}^{\mathrm{H}} = \boldsymbol{h}_{\mathrm{bi}}\boldsymbol{h}_{\mathrm{ib}}^{\mathrm{H}} \in \mathbb{C}^{N_{\mathrm{b}}\times M}$, and $\boldsymbol{H}_{\mathrm{ai}} = \boldsymbol{h}_{\mathrm{ia}}\boldsymbol{h}_{\mathrm{ai}}^{\mathrm{H}} \in \mathbb{C}^{M\times N}$ denote the Alice-to-Bob, IRS-to-Bob, and Alice-to-IRS channels, respectively. It is assumed that $\boldsymbol{h}_{tr} = \boldsymbol{h}(\theta_{tr})$ for simplicity, and the normalized steering vector is $\boldsymbol{h}(\theta) \triangleq \frac{1}{\sqrt{N}}[\mathrm{e}^{\mathrm{j}2\pi\Phi_\theta(1)}, \ldots, \mathrm{e}^{\mathrm{j}2\pi\Phi_\theta(n)}, \ldots, \mathrm{e}^{\mathrm{j}2\pi\Phi_\theta(N)}]^{\mathrm{T}}$, where $\Phi_\theta(n) = -(n - \frac{N+1}{2})\frac{d\cos\theta}{\lambda}, n = 1, 2, \ldots, N$, θ represents the direction angle of the signal departure or arrival, n stands for the antenna index, d indicates the distance between adjacent transmitting antennas, and λ refers to the wavelength.

Similarly, the received signal at Eve is cast as

$$\begin{aligned} y_{\mathrm{e}} &= \boldsymbol{u}_{\mathrm{e}}^{\mathrm{H}}(\sqrt{g_{\mathrm{ae}}}\boldsymbol{H}_{\mathrm{ae}}^{\mathrm{H}} + \sqrt{g_{\mathrm{aie}}}\boldsymbol{H}_{\mathrm{ie}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{H}_{\mathrm{ai}})\boldsymbol{s} + \sqrt{g_{\mathrm{ie}}}\boldsymbol{u}_{\mathrm{e}}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{ie}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{n}_r + n_{\mathrm{e}} \\ &= \sqrt{\beta l P}\boldsymbol{u}_{\mathrm{e}}^{\mathrm{H}}(\sqrt{g_{\mathrm{ae}}}\boldsymbol{H}_{\mathrm{ae}}^{\mathrm{H}} + \sqrt{g_{\mathrm{aie}}}\boldsymbol{H}_{\mathrm{ie}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{H}_{\mathrm{ai}})\boldsymbol{v}x + \sqrt{(1-\beta)lP}\boldsymbol{u}_{\mathrm{e}}^{\mathrm{H}}(\sqrt{g_{\mathrm{ae}}}\boldsymbol{H}_{\mathrm{ae}}^{\mathrm{H}} \\ &\quad + \sqrt{g_{\mathrm{aie}}}\boldsymbol{H}_{\mathrm{ie}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{H}_{\mathrm{ai}})\boldsymbol{T}_{\mathrm{AN}}\boldsymbol{z} + \sqrt{g_{\mathrm{ie}}}\boldsymbol{u}_{\mathrm{e}}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{ie}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{n}_r + n_{\mathrm{e}}, \end{aligned} \tag{3}$$

where $\boldsymbol{u}_{\mathrm{e}} \in \mathbb{C}^{N_{\mathrm{e}}\times 1}$ denotes the receive beamforming. g_{ae} and g_{ie} stand for the path loss parameters of Alice-to-Eve and IRS-to-Eve channels, respectively. $g_{\mathrm{aie}} = g_{\mathrm{ai}}g_{\mathrm{ie}}$ means the equivalent path loss parameter of Alice-to-IRS and IRS-to-Eve channels. n_{e} represents the AWGN at Eve that satisfies the distribution

1) The results in this paper can serve as a performance benchmark in the presence of imperfect channel state information.

$n_{\rm e} \sim \mathcal{CN}(0, \sigma_{\rm e}^2)$. $\boldsymbol{H}_{\rm ae}^{\rm H} = \boldsymbol{h}_{ea}\boldsymbol{h}_{\rm ae}^{\rm H} \in \mathbb{C}^{N_{\rm e}\times N}$ and $\boldsymbol{H}_{\rm ie}^{\rm H} = \boldsymbol{h}_{\rm ei}\boldsymbol{h}_{\rm ie}^{\rm H} \in \mathbb{C}^{1\times M}$ refer to the Alice-to-Eve and IRS-to-Eve channels, respectively.

It is assumed that AN is transmitted to Eve for jamming eavesdropping only and does not impact Bob, based on the criterion of null-space projection, and $\boldsymbol{T}_{\rm AN}$ should meet

$$\boldsymbol{H}_{\rm ai}\boldsymbol{T}_{\rm AN} = \boldsymbol{0}_{M\times N},\ \boldsymbol{H}_{\rm ab}^{\rm H}\boldsymbol{T}_{\rm AN} = \boldsymbol{0}_{N_{\rm b}\times N}. \tag{4}$$

Let $\boldsymbol{H}_{\rm CM} = [\boldsymbol{H}_{\rm ai}; \boldsymbol{H}_{\rm ab}^{\rm H}]_{(M+N_{\rm b})\times N}$ denote an equivalent virtual channel matrix of the confidential message. Then, $\boldsymbol{T}_{\rm AN}$ can be designed as

$$\boldsymbol{T}_{\rm AN} = \boldsymbol{I}_N - \boldsymbol{H}_{\rm CM}^{\rm H}[\boldsymbol{H}_{\rm CM}\boldsymbol{H}_{\rm CM}^{\rm H}]^{\dagger}\boldsymbol{H}_{\rm CM}. \tag{5}$$

At this point, Eqs. (2) and (3) can be rewritten as

$$y_{\rm b} = \sqrt{\beta l P}\boldsymbol{u}_{\rm b}^{\rm H}\left(\sqrt{g_{\rm ab}}\boldsymbol{H}_{\rm ab}^{\rm H} + \sqrt{g_{\rm aib}}\boldsymbol{H}_{\rm ib}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai}\right)\boldsymbol{v}x + \sqrt{g_{\rm ib}}\boldsymbol{u}_{\rm b}^{\rm H}\boldsymbol{H}_{\rm ib}^{\rm H}\boldsymbol{\Psi}\boldsymbol{n}_r + n_{\rm b} \tag{6}$$

and

$$\begin{aligned} y_{\rm e} =& \sqrt{\beta l P}\boldsymbol{u}_{\rm e}^{\rm H}(\sqrt{g_{\rm ae}}\boldsymbol{H}_{\rm ae}^{\rm H} + \sqrt{g_{\rm aie}}\boldsymbol{H}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai})\boldsymbol{v}x \\ &+ \sqrt{(1-\beta)lPg_{\rm ae}}\boldsymbol{u}_{\rm e}^{\rm H}\boldsymbol{H}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\boldsymbol{z} + \sqrt{g_{\rm ie}}\boldsymbol{u}_{\rm e}^{\rm H}\boldsymbol{H}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\boldsymbol{n}_r + n_{\rm e}, \end{aligned} \tag{7}$$

respectively.

Based on (6) and (7), the achievable rates at Bob and Eve are given by

$$R_{\rm b} = \log_2\left(1 + \frac{\beta l P|\boldsymbol{u}_{\rm b}^{\rm H}\left(\sqrt{g_{\rm ab}}\boldsymbol{H}_{\rm ab}^{\rm H} + \sqrt{g_{\rm aib}}\boldsymbol{H}_{\rm ib}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai}\right)\boldsymbol{v}|^2}{\sigma_r^2\|\sqrt{g_{\rm ib}}\boldsymbol{u}_{\rm b}^{\rm H}\boldsymbol{H}_{\rm ib}^{\rm H}\boldsymbol{\Psi}\|^2 + \sigma_{\rm b}^2}\right) \tag{8}$$

and

$$R_{\rm e} = \log_2(1+\gamma), \tag{9}$$

respectively, where

$$\gamma = \frac{\beta l P|\boldsymbol{u}_{\rm e}^{\rm H}\left(\sqrt{g_{\rm ae}}\boldsymbol{H}_{\rm ae}^{\rm H} + \sqrt{g_{\rm aie}}\boldsymbol{H}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai}\right)\boldsymbol{v}|^2}{(1-\beta)lPg_{\rm ae}\|\boldsymbol{u}_{\rm e}^{\rm H}\boldsymbol{H}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\|^2 + \sigma_r^2\|\sqrt{g_{\rm ie}}\boldsymbol{u}_{\rm e}^{\rm H}\boldsymbol{H}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\|^2 + \sigma_{\rm e}^2}.$$

Due to the fact that Alice and Bob cannot capture Eve's received beamforming $\boldsymbol{u}_{\rm e}$ in general, an upper bound of (9) can be obtained by

$$\begin{aligned} R_{\rm e} \leqslant& \log_2(1 + {\rm Tr}((1-\beta)lPg_{\rm ae}\boldsymbol{H}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\boldsymbol{T}_{\rm AN}^{\rm H}\boldsymbol{H}_{\rm ae} \\ &+ \sigma_r^2 g_{\rm ie}\boldsymbol{H}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\boldsymbol{\Psi}^{\rm H}\boldsymbol{H}_{\rm ie} + \sigma_{\rm e}^2\boldsymbol{I}_{N_{\rm e}})^{-1}(\beta l P(\sqrt{g_{\rm ae}}\boldsymbol{H}_{\rm ae}^{\rm H} \\ &+ \sqrt{g_{\rm aie}}\boldsymbol{H}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai})\boldsymbol{v}\boldsymbol{v}^{\rm H}(\sqrt{g_{\rm ae}}\boldsymbol{H}_{\rm ae}^{\rm H} + \sqrt{g_{\rm aie}}\boldsymbol{H}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai})^{\rm H})) \\ \triangleq& \widetilde{R}_{\rm e}. \end{aligned} \tag{10}$$

The detailed derivation is available in Appendix A.

At this point, the lower bound of SR for the system is expressed as

$$R_s = \max\{0, R_{\rm b} - \widetilde{R}_{\rm e}\}. \tag{11}$$

Moreover, the transmitted power at active IRS can be formulated as follows:

$$P_r = {\rm Tr}\left(\boldsymbol{\Psi}(g_{\rm ai}\beta l P\boldsymbol{H}_{\rm ai}\boldsymbol{v}\boldsymbol{v}^{\rm H}\boldsymbol{H}_{\rm ai}^{\rm H} + \sigma_r^2\boldsymbol{I}_M)\boldsymbol{\Psi}^{\rm H}\right). \tag{12}$$

In this paper, we maximize the SR by jointly deriving the PA factors β and l, transmit beamforming $\boldsymbol{v}$, receive beamforming $\boldsymbol{u}_{\rm b}$, and active IRS phase shift matrix $\boldsymbol{\Psi}$. The overall optimization problem is formulated as follows:

$$\max_{\beta,l,\boldsymbol{v},\boldsymbol{u}_{\rm b},\boldsymbol{\Psi}} \quad R_s \tag{13a}$$

$$\text{s.t.} \quad \boldsymbol{v}^{\rm H}\boldsymbol{v} = 1,\ \boldsymbol{u}_{\rm b}^{\rm H}\boldsymbol{u}_{\rm b} = 1, \tag{13b}$$

$$0 < \beta \leqslant 1,\ 0 < l < 1, \tag{13c}$$

$$|\boldsymbol{\Psi}(m,m)| \leqslant \psi^{\max}, m = 1,\ldots,M, \tag{13d}$$

$$P_r \leqslant (1-l)P, \tag{13e}$$

where $\psi^{\max}$ means the amplification gain threshold of the active IRS elements, and $(1-l)P$ refers to the maximum transmit power of the active IRS. It is obvious that this optimization problem has a non-convex objective function and constraints, and the optimization variables are highly coupled with each other, which makes it a challenge to address it directly in general. Hence, the alternating iteration strategy is taken into account for solving this optimization problem in what follows.

3 Proposed Max-SR-SS and Max-SR-EM schemes

In this section, to streamline the solution of the problem, we aim at maximizing SR and decompose the problem (13) into five subproblems. In what follows, the parameters β, l, $\boldsymbol{v}$, $\boldsymbol{u}_{\rm b}$, and $\boldsymbol{\Psi}$ are sequentially optimized by fixing the other variables.

3.1 Optimization of the PA factor β

In this subsection, the transmit beamforming $\boldsymbol{v}$, receive beamforming $\boldsymbol{u}_{\rm b}$, and IRS phase shift matrix $\boldsymbol{\Psi}$ are given for the sake of simplicity. We re-arrange the IRS power constraint (13e) as

$$\beta l {\rm Tr}\left(\boldsymbol{\Psi}(g_{\rm ai}P\boldsymbol{H}_{\rm ai}\boldsymbol{v}\boldsymbol{v}^{\rm H}\boldsymbol{H}_{\rm ai}^{\rm H})\boldsymbol{\Psi}^{\rm H}\right) + {\rm Tr}(\sigma_r^2\boldsymbol{\Psi}\boldsymbol{\Psi}^{\rm H}) \leqslant (1-l)P. \tag{14}$$

For the sake of simplicity, let $A_{\rm b} = P|\boldsymbol{u}_{\rm b}^{\rm H}(\sqrt{g_{\rm ab}}\boldsymbol{H}_{\rm ab}^{\rm H} + \sqrt{g_{\rm aib}}\boldsymbol{H}_{\rm ib}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai})\boldsymbol{v}|^2$, $B_{\rm b} = \sigma_r^2\|\sqrt{g_{\rm ib}}\boldsymbol{u}_{\rm b}^{\rm H}\boldsymbol{H}_{\rm ib}^{\rm H}\boldsymbol{\Psi}\|^2 + \sigma_{\rm b}^2$. Then, Eq. (8) can be degenerated to

$$R_{\rm b} = \log_2\left(\frac{\beta l A_{\rm b} + B_{\rm b}}{B_{\rm b}}\right). \tag{15}$$

Let us define

$$\boldsymbol{A} = P(\sqrt{g_{\rm ae}}\boldsymbol{H}_{\rm ae}^{\rm H} + \sqrt{g_{\rm aie}}\boldsymbol{H}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai})\boldsymbol{v}\boldsymbol{v}^{\rm H}(\sqrt{g_{\rm ae}}\boldsymbol{H}_{\rm ae}^{\rm H} + \sqrt{g_{\rm aie}}\boldsymbol{H}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai})^{\rm H}, \tag{16}$$

$$\boldsymbol{B} = \sigma_r^2 g_{\rm ie}\boldsymbol{H}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\boldsymbol{\Psi}^{\rm H}\boldsymbol{H}_{\rm ie} + \sigma_{\rm e}^2\boldsymbol{I}_{N_{\rm e}}, \tag{17}$$

and based on

$$(1-\beta)lPg_{\rm ae}\boldsymbol{H}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\boldsymbol{T}_{\rm AN}^{\rm H}\boldsymbol{H}_{\rm ae} = (1-\beta)l\,\underbrace{Pg_{\rm ae}\boldsymbol{H}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\boldsymbol{T}_{\rm AN}^{\rm H}\boldsymbol{h}_{ea}}_{\boldsymbol{q}}\,\boldsymbol{h}_{\rm ae}^{\rm H}, \tag{18}$$

Eq. (10) can be reformulated as

$$\widetilde{R}_{\rm e} = \log_2(1 + {\rm Tr}[(\boldsymbol{B} + (1-\beta)l\boldsymbol{q}\boldsymbol{h}_{\rm ae}^{\rm H})^{-1}(\beta l\boldsymbol{A})]). \tag{19}$$

Due to the presence of inverse operation, the Sherman-Morrison theorem is taken into account for the simplification, i.e.,

$$(\boldsymbol{Z} + \boldsymbol{x}\boldsymbol{y}^{\rm T})^{-1} = \boldsymbol{Z}^{-1} - \frac{\boldsymbol{Z}^{-1}\boldsymbol{x}\boldsymbol{y}^{\rm T}\boldsymbol{Z}^{-1}}{\boldsymbol{y}^{\rm T}\boldsymbol{Z}^{-1}\boldsymbol{x} + 1}. \tag{20}$$

Then, we have

$$(\boldsymbol{B} + (1-\beta)l\boldsymbol{q}\boldsymbol{h}_{\rm ae}^{\rm H})^{-1} = \boldsymbol{B}^{-1} - \frac{(1-\beta)l\boldsymbol{B}^{-1}\boldsymbol{q}\boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{B}^{-1}}{(1-\beta)l\boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{B}^{-1}\boldsymbol{q} + 1}, \tag{21}$$

and Eq. (19) becomes

$$\widetilde{R}_{\rm e} = \log_2\left(1 + \beta l{\rm Tr}[\boldsymbol{B}^{-1}\boldsymbol{A}] - \frac{\beta(1-\beta)l^2{\rm Tr}[\boldsymbol{B}^{-1}\boldsymbol{q}\boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{B}^{-1}\boldsymbol{A}]}{(1-\beta)l\boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{B}^{-1}\boldsymbol{q} + 1}\right). \tag{22}$$

Let $A_{\rm e} = {\rm Tr}[\boldsymbol{B}^{-1}\boldsymbol{A}]$, $B_{\rm e} = {\rm Tr}[\boldsymbol{B}^{-1}\boldsymbol{q}\boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{B}^{-1}\boldsymbol{A}]$, and $C_{\rm e} = \boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{B}^{-1}\boldsymbol{q}$. Then, Eq. (10) can be reformulated as

$$\widetilde{R}_{\rm e} = \log_2\left(\frac{(1-\beta)lC_{\rm e} + 1 + \beta(1-\beta)l^2(A_{\rm e}C_{\rm e} - B_{\rm e}) + \beta lA_{\rm e}}{(1-\beta)lC_{\rm e}+1}\right). \tag{23}$$

In what follows, we handle the optimization of the PA parameters β and l successively.

Define $E_1 = l^2(A_{\rm e}C_{\rm e} - B_{\rm e})$, $E_2 = l^2(A_{\rm e}C_{\rm e} - B_{\rm e}) - lC_{\rm e} + lA_{\rm e}$, $E_3 = lC_{\rm e} + 1$, and $E_4 = lC_{\rm e}$. Given l, in accordance with (13), (15), and (23), the optimization problem with respect to β can be simplified as follows:

$$\max_{\beta} \quad f_1(\beta) = \frac{\beta^2 A_1 - \beta B_1 - C_1}{\beta^2 D_1 - \beta F_1 - C_1} \tag{24a}$$

$$\text{s.t.} \quad \beta K_1 \leqslant L_1, 0 < \beta \leqslant 1, \tag{24b}$$

where $A_1 = lA_{\rm b}E_4$, $B_1 = lA_{\rm b}E_3 - B_{\rm b}E_4$, $C_1 = B_{\rm b}E_3$, $D_1 = E_1B_{\rm b}$, $K_1 = l{\rm Tr}(\boldsymbol{\Psi}(g_{\rm ai}P\boldsymbol{H}_{\rm ai}\boldsymbol{v}\boldsymbol{v}^{\rm H}\boldsymbol{H}_{\rm ai}^{\rm H})\boldsymbol{\Psi}^{\rm H})$, $F_1 = E_2B_{\rm b}$, and $L_1 = (1-l)P - {\rm Tr}(\sigma_r^2\boldsymbol{\Psi}\boldsymbol{\Psi}^{\rm H})$.

Then, Eq. (24) can be recast as

$$\max_{\beta} \quad f_1(\beta) = \frac{\beta^2 A_1 - \beta B_1 - C_1}{\beta^2 D_1 - \beta F_1 - C_1} \tag{25a}$$

$$\text{s.t.} \quad 0 < \beta \leqslant \beta^{\max}, \tag{25b}$$

where $\beta^{\max} \triangleq \min\{\frac{L_1}{K_1}, 1\}$. Given that the denominator $\beta^2D_1 - \beta F_1 - C_1 \neq 0$, we can obtain that the objective function of problem (25) is continuous and differentiable in the interval $(0, \beta^{\max}]$. Then, taking its partial derivative and making it equal to 0 yield

$$\frac{\partial f_1(\beta)}{\partial \beta} = \frac{1}{(\beta^2D_1 - \beta F_1 - C_1)^2}\left[\beta^2(B_1D_1 - A_1F_1) + 2\beta(C_1D_1 - A_1C_1) + (B_1C_1 - C_1F_1)\right] = 0, \tag{26}$$

which can be simplified as

$$\beta^2(B_1D_1 - A_1F_1) + 2\beta(C_1D_1 - A_1C_1) + (B_1C_1 - C_1F_1) = 0. \tag{27}$$

3.1.1 *When* $B_1D_1 - A_1F_1 \neq 0$

Eq. (27) is a quadratic. Let

$$\Delta_\beta = 4(C_1D_1 - A_1C_1)^2 - 4(B_1D_1 - A_1F_1)(B_1C_1 - C_1F_1). \tag{28}$$

If $\Delta_\beta \geqslant 0$, based on the formula for the roots of a quadratic function, we can get its roots as

$$\beta_1 = \frac{-2(C_1D_1 - A_1C_1) + \sqrt{\Delta_\beta}}{2(B_1D_1 - A_1F_1)}, \tag{29}$$

$$\beta_2 = \frac{-2(C_1D_1 - A_1C_1) - \sqrt{\Delta_\beta}}{2(B_1D_1 - A_1F_1)}. \tag{30}$$

3.1.2 *When* $B_1D_1 - A_1F_1 = 0$

Eq. (27) can be degraded to

$$2\beta(C_1D_1 - A_1C_1) + (B_1C_1 - C_1F_1) = 0, \tag{31}$$

which yields

$$\beta_3 = -\frac{B_1 - F_1}{2(D_1 - A_1)}. \tag{32}$$

Next, we judge whether these candidate solutions of β are in the interval $(0, \beta^{\max}]$. Finally, the optimal value of β can be obtained by comparing the values of $f_1(\beta)$ at endpoints and candidate solutions. The detailed procedure for deriving the PA factor β is shown in Algorithm 1.

Algorithm 1 Algorithm for optimizing β.

1: If $B_1D_1 - A_1F_1 \neq 0$ and $\Delta_\beta \geqslant 0$, the four different scenarios are considered as follows:

(1) If $\beta_1, \beta_2 \in (0, \beta^{\max}]$, then compare the values of $f_1(0)$, $f_1(\beta_1)$, $f_1(\beta_2)$, and $f_1(\beta^{\max})$;

(2) If $\beta_1 \in (0, \beta^{\max}]$ and $\beta_2 \notin (0, \beta^{\max}]$, then compare the values of $f_1(0)$, $f_1(\beta_1)$, and $f_1(\beta^{\max})$;

(3) If $\beta_1 \notin (0, \beta^{\max}]$ and $\beta_2 \in (0, \beta^{\max}]$, then compare the values of $f_1(0)$, $f_1(\beta_2)$, and $f_1(\beta^{\max})$;

(4) If $\beta_1, \beta_2 \notin (0, \beta^{\max}]$, then compare the values of $f_1(0)$ and $f_1(\beta^{\max})$;

2: If $B_1D_1 - A_1F_1 \neq 0$ and $\Delta_\beta < 0$, the optimal PA parameter has been shown in aforementioned (4);

3: If $B_1D_1 - A_1F_1 = 0$, the two different scenarios are taken into account as follows:

(1) If $\beta_3 \in (0, \beta^{\max}]$, then compare the values of $f_1(0)$, $f_1(\beta_3)$, and $f_1(\beta^{\max})$;

(2) If $\beta_3 \notin (0, \beta^{\max}]$, then compare the values of $f_1(0)$ and $f_1(\beta^{\max})$;

4: Output the optimal PA factor β^{opt}.

3.2 Optimization of the PA factor l

Fixed $\boldsymbol{v}$, $\boldsymbol{u}_{\text{b}}$, and $\boldsymbol{\Psi}$, given that the optimal β has been found in Subsection 3.1, we shift our focus to solving l. Let $E_5 = \beta(1-\beta)(A_{\text{e}}C_{\text{e}} - B_{\text{e}})$, $E_6 = (1-\beta)C_{\text{e}} + \beta A_{\text{e}}$, and $E_7 = (1-\beta)C_{\text{e}}$. In accordance with (15) and (23), by neglecting the constant terms, the optimization problem with respect to l can be simplified as follows:

$$\max_{l} \quad f_2(l) = \frac{l^2A_2 + lB_2 + C_2}{l^2D_2 + lF_2 + C_2} \tag{33a}$$

$$\text{s.t.} \quad lK_2 \leqslant L_2, 0 < l < 1, \tag{33b}$$

where $A_2 = \beta A_{\text{b}}E_7$, $B_2 = \beta A_{\text{b}} + E_7B_{\text{b}}$, $C_2 = B_{\text{b}}$, $D_2 = E_5B_{\text{b}}$, $K_2 = \beta\text{Tr}(\boldsymbol{\Psi}(g_{\text{ai}}P\boldsymbol{H}_{\text{ai}}\boldsymbol{v}\boldsymbol{v}^{\text{H}}\boldsymbol{H}_{\text{ai}}^{\text{H}})\boldsymbol{\Psi}^{\text{H}}) + P$, $F_2 = E_6B_{\text{b}}$, and $L_2 = P - \text{Tr}(\sigma_r^2\boldsymbol{\Psi}\boldsymbol{\Psi}^{\text{H}})$. Further simplification yields

$$\max_{l} \quad f_2(l) = \frac{l^2A_2 + lB_2 + C_2}{l^2D_2 + lF_2 + C_2} \tag{34a}$$

$$\text{s.t.} \quad 0 < l \leqslant l^{\max}, \tag{34b}$$

where $l^{\max} \triangleq \min\left\{\frac{L_2}{K_2}, 1\right\}$. Because the denominator $l^2D_2 + lF_2 + C_2 \neq 0$, we can obtain that the objective function of problem (34) is continuous and differentiable in the interval $(0, l^{\max}]$. Then, taking its partial derivative and making it equal to 0 yield

$$\frac{\partial f_2(l)}{\partial l} = \frac{1}{(l^2D_2 + lF_2 + C_2)^2}\left[l^2(A_2F_2 - B_2D_2) + 2l(A_2C_2 - C_2D_2) + (B_2C_2 - C_2F_2)\right] = 0, \tag{35}$$

which yields

$$l^2(A_2F_2 - B_2D_2) + 2l(A_2C_2 - C_2D_2) + (B_2C_2 - C_2F_2) = 0. \tag{36}$$

3.2.1 *When $A_2F_2 - B_2D_2 \neq 0$*

Eq. (36) is quadratic. Let

$$\Delta_l = 4(A_2C_2 - C_2D_2)^2 - 4(A_2F_2 - B_2D_2)(B_2C_2 - C_2F_2). \tag{37}$$

If $\Delta_l \geqslant 0$, based on the formula for the roots of a quadratic function, we can get its roots as

$$l_1 = \frac{-2(A_2C_2 - C_2D_2) + \sqrt{\Delta_l}}{2(A_2C_2 - C_2D_2)}, \tag{38}$$

$$l_2 = \frac{-2(A_2C_2 - C_2D_2) - \sqrt{\Delta_l}}{2(A_2C_2 - C_2D_2)}. \tag{39}$$

3.2.2 *When* $A_2F_2 - B_2D_2 = 0$

Eq. (36) can be recast as

$$2l(A_2C_2 - C_2D_2) + (B_2C_2 - C_2F_2) = 0. \tag{40}$$

We have

$$l_3 = -\frac{B_2 - F_2}{2(A_2 - D_2)}. \tag{41}$$

Next, an analysis similar to solving for β needs to be performed, and we ignore the procedure for the sake of avoiding repetition.

3.3 Optimization of the transmit beamforming vector $\boldsymbol{v}$

Given β, l, $\boldsymbol{u}_{\mathrm{b}}$, and $\boldsymbol{\Psi}$, we reformulate the IRS power constraint (13e) as follows:

$$\begin{aligned} P_r &= \boldsymbol{v}^{\mathrm{H}}(g_{\mathrm{ai}}\beta l P \boldsymbol{H}_{\mathrm{ai}}^{\mathrm{H}}\boldsymbol{\Psi}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{H}_{\mathrm{ai}})\boldsymbol{v} + \mathrm{Tr}(\sigma_r^2\boldsymbol{\Psi}\boldsymbol{\Psi}^{\mathrm{H}}) \\ &\leqslant (1-l)P. \end{aligned} \tag{42}$$

With ignoring the constant term, Eq. (13) can be re-arranged as the optimization problem with respect to $\boldsymbol{v}$ as follows:

$$\max_{\boldsymbol{v}} \quad \frac{\boldsymbol{v}^{\mathrm{H}}\boldsymbol{C}\boldsymbol{v}}{\boldsymbol{v}^{\mathrm{H}}\boldsymbol{D}\boldsymbol{v}} \tag{43a}$$

$$\text{s.t.} \quad \boldsymbol{v}^{\mathrm{H}}\boldsymbol{v} = 1, (42), \tag{43b}$$

where

$$\begin{aligned} \boldsymbol{C} = \beta l P &\left(\sqrt{g_{\mathrm{ab}}}\boldsymbol{H}_{\mathrm{ab}}^{\mathrm{H}} + \sqrt{g_{\mathrm{aib}}}\boldsymbol{H}_{\mathrm{ib}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{H}_{\mathrm{ai}}\right)^{\mathrm{H}} \boldsymbol{u}_{\mathrm{b}}\boldsymbol{u}_{\mathrm{b}}^{\mathrm{H}}\big(\sqrt{g_{\mathrm{ab}}}\boldsymbol{H}_{\mathrm{ab}}^{\mathrm{H}} \\ &+ \sqrt{g_{\mathrm{aib}}}\boldsymbol{H}_{\mathrm{ib}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{H}_{\mathrm{ai}}\big) / \left(\sigma_r^2\|\sqrt{g_{\mathrm{ib}}}\boldsymbol{u}_{\mathrm{b}}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{ib}}^{\mathrm{H}}\boldsymbol{\Psi}\|^2 + \sigma_{\mathrm{b}}^2\right) + \boldsymbol{I}_N, \end{aligned} \tag{44}$$

$$\begin{aligned} \boldsymbol{D} = \beta l P &\left(\sqrt{g_{\mathrm{ae}}}\boldsymbol{H}_{\mathrm{ae}}^{\mathrm{H}} + \sqrt{g_{\mathrm{aie}}}\boldsymbol{H}_{\mathrm{ie}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{H}_{\mathrm{ai}}\right)^{\mathrm{H}} \big[(1-\beta)lPg_{\mathrm{ae}}\boldsymbol{H}_{\mathrm{ae}}^{\mathrm{H}}\boldsymbol{T}_{\mathrm{AN}}\boldsymbol{T}_{\mathrm{AN}}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{ae}} \\ &+ \sigma_r^2 g_{\mathrm{ie}}\boldsymbol{H}_{\mathrm{ie}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{\Psi}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{ie}} + \sigma_{\mathrm{e}}^2\boldsymbol{I}_{N_{\mathrm{e}}}\big]^{-1}\left(\sqrt{g_{\mathrm{ae}}}\boldsymbol{H}_{\mathrm{ae}}^{\mathrm{H}} + \sqrt{g_{\mathrm{aie}}}\boldsymbol{H}_{\mathrm{ie}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{H}_{\mathrm{ai}}\right) + \boldsymbol{I}_N. \end{aligned} \tag{45}$$

Given that the objective function value in (43a) is insensitive to the scaling of $\boldsymbol{v}$, we relax the equation constraint to $\boldsymbol{v}^{\mathrm{H}}\boldsymbol{v} \leqslant 1$ [25]. Then, in accordance with the first order Taylor approximation, we have

$$\frac{|y|^2}{z} \geqslant -\frac{\bar{y}^*\bar{y}}{\bar{z}^2}z + \frac{2\Re\{\bar{y}^*y\}}{\bar{z}}. \tag{46}$$

Then, the problem (43a) can be recast as

$$\max_{\boldsymbol{v}} \quad -\frac{\bar{\boldsymbol{v}}^{\mathrm{H}}\boldsymbol{C}\bar{\boldsymbol{v}}}{(\bar{\boldsymbol{v}}^{\mathrm{H}}\boldsymbol{D}\bar{\boldsymbol{v}})^2}\boldsymbol{v}^{\mathrm{H}}\boldsymbol{D}\boldsymbol{v} + \frac{2\Re\{\bar{\boldsymbol{v}}^{\mathrm{H}}\boldsymbol{C}\boldsymbol{v}\}}{\bar{\boldsymbol{v}}^{\mathrm{H}}\boldsymbol{D}\bar{\boldsymbol{v}}} \tag{47a}$$

$$\text{s.t.} \quad \boldsymbol{v}^{\mathrm{H}}\boldsymbol{v} \leqslant 1, (42), \tag{47b}$$

where $\bar{\boldsymbol{v}}$ stands for the given vector. This is a convex optimization problem that can be tackled directly with CVX toolbox [42].

3.4 Optimization of the receive beamforming vector $\boldsymbol{u}_{\mathrm{b}}$

Fixed β, l, $\boldsymbol{v}$, and $\boldsymbol{\Psi}$, the optimization problem with respect to $\boldsymbol{u}_{\mathrm{b}}$ can be re-arranged as

$$\max_{\boldsymbol{u}_{\mathrm{b}}} \quad \frac{\boldsymbol{u}_{\mathrm{b}}^{\mathrm{H}}\boldsymbol{A}_1\boldsymbol{u}_{\mathrm{b}}}{\boldsymbol{u}_{\mathrm{b}}^{\mathrm{H}}\boldsymbol{A}_2\boldsymbol{u}_{\mathrm{b}}} \tag{48a}$$

$$\text{s.t.} \quad \boldsymbol{u}_{\mathrm{b}}^{\mathrm{H}}\boldsymbol{u}_{\mathrm{b}} = 1, \tag{48b}$$

where

$$\boldsymbol{A}_1 = \beta l P \left(\sqrt{g_{\mathrm{ab}}}\boldsymbol{H}_{\mathrm{ab}}^{\mathrm{H}} + \sqrt{g_{\mathrm{aib}}}\boldsymbol{H}_{\mathrm{ib}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{H}_{\mathrm{ai}}\right)\boldsymbol{v}\boldsymbol{v}^{\mathrm{H}}\left(\sqrt{g_{\mathrm{ab}}}\boldsymbol{H}_{\mathrm{ab}}^{\mathrm{H}} + \sqrt{g_{\mathrm{aib}}}\boldsymbol{H}_{\mathrm{ib}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{H}_{\mathrm{ai}}\right)^{\mathrm{H}}, \tag{49}$$

$$\boldsymbol{A}_2 = \sigma_r^2 g_{\mathrm{ib}}\boldsymbol{H}_{\mathrm{ib}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{\Psi}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{ib}} + \sigma_{\mathrm{b}}^2\boldsymbol{I}_N. \tag{50}$$

In accordance with the generalized Rayleigh-Ritz theorem, the optimal $\boldsymbol{u}_{\mathrm{b}}$ is given by the eigenvector corresponding to the largest eigenvalue of $\boldsymbol{A}_2^{-1}\boldsymbol{A}_1$.

3.5 Optimization of the IRS phase shift matrix $\boldsymbol{\Psi}$

In Subsections 3.1–3.4, the PA factors β and l, transmit beamforming $\boldsymbol{v}$, and receive beamforming $\boldsymbol{u}_{\rm b}$ have been optimized. In this section, we turn our focus to the optimization of the IRS phase shift matrix $\boldsymbol{\Psi}$. In what follows, two strategies for optimizing $\boldsymbol{\Psi}$ by fixing the variables β, l, $\boldsymbol{v}$, and $\boldsymbol{u}_{\rm b}$ will be proposed.

3.5.1 *Max-SR-SS algorithm*

First, we transform the power constraint (13e) into a constraint on $\boldsymbol{\Psi}$. Based on the fact that $\mathrm{diag}\{\boldsymbol{p}\}\boldsymbol{q} = \mathrm{diag}\{\boldsymbol{q}\}\boldsymbol{p}$ for $\forall \boldsymbol{p}, \boldsymbol{q} \in \mathbb{C}^{M\times 1}$, Eq. (13e) can be re-arranged as follows:

$$\begin{aligned} P_r &= \mathrm{Tr}\left(\boldsymbol{\Psi}(g_{\rm ai}\beta l P \boldsymbol{H}_{\rm ai}\boldsymbol{v}\boldsymbol{v}^{\rm H}\boldsymbol{H}_{\rm ai}^{\rm H} + \sigma_r^2\boldsymbol{I}_M)\boldsymbol{\Psi}^{\rm H}\right) \\ &= \boldsymbol{\psi}^{\rm T}(g_{\rm ai}\beta l P \mathrm{diag}\{\boldsymbol{v}^{\rm H}\boldsymbol{H}_{\rm ai}^{\rm H}\}\mathrm{diag}\{\boldsymbol{H}_{\rm ai}\boldsymbol{v}\} + \sigma_r^2\boldsymbol{I}_M)\boldsymbol{\psi}^* \\ &\leqslant (1-l)P. \end{aligned} \tag{51}$$

Given that the inverse operation in (10), it is difficult to tackle the optimization problem (13) directly. Hence, to transform $\widetilde{R}_{\rm e}$ in (10) into a tractable form, let

$$\boldsymbol{H}_1 = \sigma_r^2 g_{\rm ie}\mathrm{diag}\{\boldsymbol{h}_{\rm ie}\}\mathrm{diag}\{\boldsymbol{h}_{\rm ie}^{\rm H}\}, \tag{52a}$$

$$\boldsymbol{H}_2 = (1-\beta)lPg_{\rm ae}\boldsymbol{H}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\boldsymbol{T}_{\rm AN}^{\rm H}\boldsymbol{H}_{\rm ae} + \sigma_{\rm e}^2\boldsymbol{I}_{N_{\rm e}}, \tag{52b}$$

$$\boldsymbol{H}_3 = \sqrt{\beta l P g_{\rm aie}}\boldsymbol{H}_{\rm ie}^{\rm H}\mathrm{diag}\{\boldsymbol{H}_{\rm ai}\boldsymbol{v}\}, \tag{52c}$$

$$\boldsymbol{e} = \sqrt{\beta l P g_{\rm ae}}\boldsymbol{H}_{\rm ae}^{\rm H}\boldsymbol{v}. \tag{52d}$$

Then, we introduce a slack variable t, which meets

$$t \geqslant (\boldsymbol{H}_3\boldsymbol{\psi} + \boldsymbol{e})^{\rm H}(\boldsymbol{\psi}^{\rm H}\boldsymbol{H}_1\boldsymbol{\psi}\boldsymbol{h}_{\rm ei}\boldsymbol{h}_{\rm ei}^{\rm H} + \boldsymbol{H}_2)^{-1}(\boldsymbol{H}_3\boldsymbol{\psi} + \boldsymbol{e}). \tag{53}$$

In accordance with the nature of Schur complement, we can obtain

$$\boldsymbol{S}(\boldsymbol{\psi}, t) = \begin{bmatrix} \boldsymbol{\psi}^{\rm H}\boldsymbol{H}_1\boldsymbol{\psi}\boldsymbol{h}_{\rm ei}\boldsymbol{h}_{\rm ei}^{\rm H} + \boldsymbol{H}_2 & \boldsymbol{H}_3\boldsymbol{\psi} + \boldsymbol{e} \\ \boldsymbol{\psi}^{\rm H}\boldsymbol{H}_3^{\rm H} + \boldsymbol{e}^{\rm H} & t \end{bmatrix} \succeq \boldsymbol{0}. \tag{54}$$

According to the first-order Taylor approximation of $\boldsymbol{\psi}^{\rm H}\boldsymbol{H}_1\boldsymbol{\psi}$ at feasible point $\bar{\boldsymbol{\psi}}$, we have $\boldsymbol{\psi}^{\rm H}\boldsymbol{H}_1\boldsymbol{\psi} \geqslant 2\Re\{\boldsymbol{\psi}^{\rm H}\boldsymbol{H}_1\bar{\boldsymbol{\psi}}\} - \bar{\boldsymbol{\psi}}^{\rm H}\boldsymbol{H}_1\bar{\boldsymbol{\psi}}$. Then, Eq. (54) can be rewritten as

$$\boldsymbol{S}(\boldsymbol{\psi}, t) \succeq \begin{bmatrix} (2\Re\{\boldsymbol{\psi}^{\rm H}\boldsymbol{H}_1\bar{\boldsymbol{\psi}}\} - \bar{\boldsymbol{\psi}}^{\rm H}\boldsymbol{H}_1\bar{\boldsymbol{\psi}})\boldsymbol{h}_{\rm ei}\boldsymbol{h}_{\rm ei}^{\rm H} + \boldsymbol{H}_2 & \boldsymbol{H}_3\boldsymbol{\psi} + \boldsymbol{e} \\ \boldsymbol{\psi}^{\rm H}\boldsymbol{H}_3^{\rm H} + \boldsymbol{e}^{\rm H} & t \end{bmatrix} \succeq \boldsymbol{0}. \tag{55}$$

At this point, the optimization problem with respect to $\boldsymbol{\Psi}$ can be recast as

$$\max_{\boldsymbol{\psi}, t} \quad R_{\rm b} - \log_2(1+t), \tag{56a}$$

$$\text{s.t.} \quad |\boldsymbol{\psi}(m)| \leqslant \psi^{\max}, \ (51), \ (55). \tag{56b}$$

The objective function of the problem (56) is the difference of two logarithmic functions and is non-convex. To address this problem, let $\boldsymbol{a}^{\rm H} = \sqrt{\beta l P g_{\rm aib}}\boldsymbol{u}_{\rm b}^{\rm H}\boldsymbol{H}_{\rm ib}^{\rm H}\mathrm{diag}\{\boldsymbol{H}_{\rm ai}\boldsymbol{v}\}$, $b_1 = \sqrt{\beta l P g_{\rm ab}}\boldsymbol{u}_{\rm b}^{\rm H}\boldsymbol{H}_{\rm ab}^{\rm H}\boldsymbol{v}$, and $\boldsymbol{C}_1 = \sigma_r\sqrt{g_{\rm ib}}\mathrm{diag}\{\boldsymbol{u}_{\rm b}^{\rm H}\boldsymbol{H}_{\rm ib}^{\rm H}\}$. Then, we have

$$\begin{aligned} R_{\rm b} &= \log_2\left(1 + \frac{|\boldsymbol{a}^{\rm H}\boldsymbol{\psi} + b_1|^2}{\|\boldsymbol{C}_1\boldsymbol{\psi}\|^2 + \sigma_{\rm b}^2}\right) \\ &= \log_2(\boldsymbol{\psi}^{\rm H}(\underbrace{\boldsymbol{a}\boldsymbol{a}^{\rm H} + \boldsymbol{C}_1^{\rm H}\boldsymbol{C}_1}_{\boldsymbol{E}})\boldsymbol{\psi} + 2\Re\{b_1^*\boldsymbol{a}^{\rm H}\boldsymbol{\psi}\} + |b_1|^2 + \sigma_{\rm b}^2) - \log_2(1 + \|\boldsymbol{C}_1\boldsymbol{\psi}\|^2/\sigma_{\rm b}^2) - \log_2(\sigma_{\rm b}^2). \end{aligned} \tag{57}$$

Based on the first-order Taylor approximation of $\boldsymbol{\psi}^{\rm H}\boldsymbol{E}\boldsymbol{\psi}$, i.e., $\boldsymbol{\psi}^{\rm H}\boldsymbol{E}\boldsymbol{\psi} \geqslant 2\Re\{\boldsymbol{\psi}^{\rm H}\boldsymbol{E}\bar{\boldsymbol{\psi}}\} - \bar{\boldsymbol{\psi}}^{\rm H}\boldsymbol{E}\bar{\boldsymbol{\psi}}$ and the result in [43], for a fixed point $\bar{e}_1$,

$$-\ln(1+e_1) \geqslant -\ln(1+\bar{e}_1) - \frac{1+e_1}{1+\bar{e}_1} + 1, \tag{58}$$

after neglecting the constant entries, Eq. (56) can be recast as

$$\max_{\boldsymbol{\psi},t} \ \ln(2\Re\{\boldsymbol{\psi}^{\rm H}\boldsymbol{E}\bar{\boldsymbol{\psi}}\} - \bar{\boldsymbol{\psi}}^{\rm H}\boldsymbol{E}\bar{\boldsymbol{\psi}} + 2\Re\{b_1^*\boldsymbol{a}^{\rm H}\boldsymbol{\psi}\} + |b_1|^2 + \sigma_{\rm b}^2) - \frac{\|\boldsymbol{C}_1\boldsymbol{\psi}\|^2}{\sigma_{\rm b}^2}/\left(1+\|\boldsymbol{C}_1\bar{\boldsymbol{\psi}}\|^2/\sigma_{\rm b}^2\right) - \frac{t}{1+\bar{t}} \tag{59a}$$

$$\text{s.t.} \quad |\boldsymbol{\psi}(m)| \leqslant \psi^{\max},\ (51),\ (55), \tag{59b}$$

where $\bar{t}$ stands for the value obtained at the previous iteration of t. It is noted that the problem (59) is convex, which can be derived directly with the convex optimizing toolbox.

3.5.2 *Max-SR-EM algorithm*

In Subsection 3.5.1, a Max-SR-SS algorithm has been proposed to optimize the IRS phase shift matrix $\boldsymbol{\Psi}$, which has a high computational complexity. To reduce the complexity, a Max-SR-EM algorithm with lower complexity is proposed in this section. Given that $\boldsymbol{\Psi}$ consists of amplitude and phase, we will derive $\boldsymbol{\Psi}$ by solving for them separately in the following.

First, the derivation of the amplitude is taken into account. For the sake of derivation, we assume that $|\boldsymbol{\Psi}(m,m)| \leqslant \psi^{\max}$ in (13) always holds and the amplitude of each IRS phase shift elements is the same, noted as $|\boldsymbol{\Psi}(m,m)| = \alpha_m = \alpha$, and $\boldsymbol{\Theta} = {\rm diag}\{{\rm e}^{{\rm j}\phi_1},\ldots,{\rm e}^{{\rm j}\phi_m},\ldots,{\rm e}^{{\rm j}\phi_M}\} \in \mathbb{C}^{M\times M}$. Then, we have $\boldsymbol{\Psi} = \alpha\boldsymbol{\Theta}$. Based on the IRS power constraint (13e) and the fact that it is optimal when taking the equivalent value, i.e.,

$${\rm Tr}\left(\alpha\boldsymbol{\Theta}(g_{\rm ai}\beta lP\boldsymbol{H}_{\rm ai}\boldsymbol{v}\boldsymbol{v}^{\rm H}\boldsymbol{H}_{\rm ai}^{\rm H} + \sigma_r^2\boldsymbol{I}_M)\alpha\boldsymbol{\Theta}^{\rm H}\right) = (1-l)P, \tag{60}$$

we yield the amplitude:

$$\alpha = \sqrt{\frac{(1-l)P}{{\rm Tr}\left(\boldsymbol{\Theta}(g_{\rm ai}\beta lP\boldsymbol{H}_{\rm ai}\boldsymbol{v}\boldsymbol{v}^{\rm H}\boldsymbol{H}_{\rm ai}^{\rm H} + \sigma_r^2\boldsymbol{I}_M)\boldsymbol{\Theta}^{\rm H}\right)}} = \sqrt{\frac{(1-l)P}{{\rm Tr}\left(g_{\rm ai}\beta lP\boldsymbol{H}_{\rm ai}\boldsymbol{v}\boldsymbol{v}^{\rm H}\boldsymbol{H}_{\rm ai}^{\rm H} + \sigma_r^2\boldsymbol{I}_M\right)}}. \tag{61}$$

In the following, we focus on finding the phase matrix $\boldsymbol{\Theta}$. Let

$$\boldsymbol{\theta} = [{\rm e}^{{\rm j}\phi_1},\ldots,{\rm e}^{{\rm j}\phi_m},\ldots,{\rm e}^{{\rm j}\phi_M}]^{\rm T},\ \boldsymbol{\phi} = [\boldsymbol{\theta};1], \tag{62a}$$

$$\boldsymbol{H}_{\rm e1} = {\rm blkdiag}\big\{\alpha^2\boldsymbol{H}_1, 0\big\},\ \boldsymbol{H}_{\rm e2} = [\alpha\boldsymbol{H}_3\ \ \boldsymbol{e}], \tag{62b}$$

$$\boldsymbol{h}_{\rm b}^{\rm H} = [\alpha\sqrt{\beta lPg_{\rm aib}}\boldsymbol{u}_{\rm b}^{\rm H}\boldsymbol{H}_{\rm ib}^{\rm H}{\rm diag}\{\boldsymbol{H}_{\rm ai}\boldsymbol{v}\}\ \ \sqrt{\beta lPg_{\rm ab}}\boldsymbol{u}_{\rm b}^{\rm H}\boldsymbol{H}_{\rm ab}\boldsymbol{v}], \tag{62c}$$

$$\boldsymbol{H}_{\rm b} = {\rm blkdiag}\big\{\alpha\sigma_r{\rm diag}\{\sqrt{g_{\rm ib}}\boldsymbol{u}_{\rm b}^{\rm H}\boldsymbol{H}_{\rm ib}^{\rm H}\}, 0\big\}. \tag{62d}$$

Then, Eqs. (8) and (10) can be rewritten as

$$R_{\rm b} = \log_2\left(1 + \frac{|\boldsymbol{h}_{\rm b}^{\rm H}\boldsymbol{\phi}|^2}{\|\boldsymbol{H}_{\rm b}\boldsymbol{\phi}\|^2 + \sigma_{\rm b}^2}\right) \tag{63}$$

and

$$\widetilde{R}_{\rm e} = \log_2\left(1 + {\rm Tr}\left[\frac{\boldsymbol{H}_{\rm e2}\boldsymbol{\phi}\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e2}^{\rm H}}{\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e1}\boldsymbol{\phi}\boldsymbol{h}_{\rm ei}\boldsymbol{h}_{\rm ei}^{\rm H} + \boldsymbol{H}_2}\right]\right), \tag{64}$$

respectively.

Next, we perform a transformation of $R_{\rm b}$. By (63) and the fact that for fixed points $\bar{e}_2$ and $\bar{e}_3$,

$$\ln\left(1+\frac{|e_2|^2}{e_3}\right) \geqslant \ln\left(1+\frac{|\bar{e}_2|^2}{\bar{e}_3}\right) - \frac{|\bar{e}_2|^2}{\bar{e}_3} + \frac{2\Re\{\bar{e}_2e_2\}}{\bar{e}_3} - \frac{|\bar{e}_2|^2}{\bar{e}_3(\bar{e}_3+|\bar{e}_2|^2)}\left(e_3+|e_2|^2\right), \tag{65}$$

one obtains

$$R_{\rm b}\cdot\ln 2 = \ln\left(1+\frac{|\boldsymbol{h}_{\rm b}^{\rm H}\boldsymbol{\phi}|^2}{\|\boldsymbol{H}_{\rm b}\boldsymbol{\phi}\|^2+\sigma_{\rm b}^2}\right) = \ln\left(1+\frac{|\boldsymbol{h}_{\rm b}^{\rm H}\bar{\boldsymbol{\phi}}|^2}{\tau}\right) - \frac{|\boldsymbol{h}_{\rm b}^{\rm H}\bar{\boldsymbol{\phi}}|^2}{\tau} + \frac{2\Re\{\boldsymbol{\phi}^{\rm H}\boldsymbol{h}_{\rm b}\boldsymbol{h}_{\rm b}^{\rm H}\bar{\boldsymbol{\phi}}\}}{\tau} + G, \tag{66}$$

where $\tau = \|\boldsymbol{H}_{\rm b}\bar{\boldsymbol{\phi}}\|^2 + \sigma_{\rm b}^2$, and

$$G = -\boldsymbol{\phi}^{\rm H}\left(\underbrace{\frac{|\boldsymbol{h}_{\rm b}^{\rm H}\bar{\boldsymbol{\phi}}|^2}{\tau(\tau+\|\boldsymbol{H}_{\rm b}\bar{\boldsymbol{\phi}}\|^2)}(\boldsymbol{H}_{\rm b}^{\rm H}\boldsymbol{H}_{\rm b}+\boldsymbol{h}_{\rm b}\boldsymbol{h}_{\rm b}^{\rm H})}_{\widetilde{\boldsymbol{M}}}\right)\boldsymbol{\phi}^{\rm H}. \tag{67}$$

With the MM algorithm in [44], i.e.,

$$-\boldsymbol{x}^{\rm H}\boldsymbol{Y}\boldsymbol{x} \geqslant -\boldsymbol{x}^{\rm H}\boldsymbol{Z}\boldsymbol{x} - 2\Re\{\boldsymbol{x}^{\rm H}(\boldsymbol{Y}-\boldsymbol{Z})\overline{\boldsymbol{x}}\} - \overline{\boldsymbol{x}}^{\rm H}(\boldsymbol{Z}-\boldsymbol{Y})\overline{\boldsymbol{x}}, \tag{68}$$

where $\boldsymbol{Z} = \lambda_{\max}(\boldsymbol{Y})\boldsymbol{I}$, Eq. (67) can be recast as

$$-\boldsymbol{\phi}^{\rm H}\widetilde{\boldsymbol{M}}\boldsymbol{\phi} \geqslant -\boldsymbol{\phi}^{\rm H}\lambda_{\max}(\widetilde{\boldsymbol{M}})\boldsymbol{I}_{M+1}\boldsymbol{\phi} - 2\Re\{\boldsymbol{\phi}^{\rm H}(\widetilde{\boldsymbol{M}}-\lambda_{\max}(\widetilde{\boldsymbol{M}})\boldsymbol{I}_{M+1})\bar{\boldsymbol{\phi}}\} - \bar{\boldsymbol{\phi}}^{\rm H}(\lambda_{\max}(\widetilde{\boldsymbol{M}})\boldsymbol{I}_{M+1}-\widetilde{\boldsymbol{M}})\bar{\boldsymbol{\phi}}. \tag{69}$$

Next, we transform $\widetilde{R}_{\rm e}$ in (10) into a form that is tractable to solving. Based on the fact that for $\forall \boldsymbol{X} \in \mathbb{C}^{X\times Y}$ and $\boldsymbol{Y} \in \mathbb{C}^{Y\times X}$, one has

$$|\boldsymbol{I}_X + \boldsymbol{X}\boldsymbol{Y}| = |\boldsymbol{I}_Y + \boldsymbol{Y}\boldsymbol{X}|. \tag{70}$$

Then, we have

$$\begin{aligned}
&\log_2\left(1+{\rm Tr}\left[\frac{\boldsymbol{H}_{\rm e2}\boldsymbol{\phi}\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e2}^{\rm H}}{\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e1}\boldsymbol{\phi}\boldsymbol{h}_{\rm ei}\boldsymbol{h}_{\rm ei}^{\rm H}+\boldsymbol{H}_2}\right]\right)\cdot \ln 2\\
&\quad = \ln|1+\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e2}^{\rm H}(\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e1}\boldsymbol{\phi}\boldsymbol{h}_{\rm ei}\boldsymbol{h}_{\rm ei}^{\rm H}+\boldsymbol{H}_2)^{-1}\boldsymbol{H}_{\rm e2}\boldsymbol{\phi}|\\
&\quad = \ln|\boldsymbol{I}_{N_{\rm e}}+(\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e1}\boldsymbol{\phi}\boldsymbol{h}_{\rm ei}\boldsymbol{h}_{\rm ei}^{\rm H}+\boldsymbol{H}_2)^{-1}\boldsymbol{H}_{\rm e2}\boldsymbol{\phi}\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e2}^{\rm H}|\\
&\quad = \ln|\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e1}\boldsymbol{\phi}\boldsymbol{h}_{\rm ei}\boldsymbol{h}_{\rm ei}^{\rm H}+\boldsymbol{H}_2+\boldsymbol{H}_{\rm e2}\boldsymbol{\phi}\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e2}^{\rm H}| - \ln|\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e1}\boldsymbol{\phi}\boldsymbol{h}_{\rm ei}\boldsymbol{h}_{\rm ei}^{\rm H}+\boldsymbol{H}_2|\\
&\quad = \ln|\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e1}\boldsymbol{\phi}\boldsymbol{h}_{\rm ei}\boldsymbol{h}_{\rm ei}^{\rm H}+\boldsymbol{H}_2+\boldsymbol{H}_{\rm e2}\boldsymbol{\phi}\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e2}^{\rm H}| - \ln(|\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e1}\boldsymbol{\phi}\boldsymbol{h}_{\rm ei}\boldsymbol{h}_{\rm ei}^{\rm H}\boldsymbol{H}_2^{-1}+\boldsymbol{I}_{N_{\rm e}}|\boldsymbol{H}_2|)\\
&\quad = \ln|\underbrace{\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e1}\boldsymbol{\phi}\boldsymbol{h}_{\rm ei}\boldsymbol{h}_{\rm ei}^{\rm H}+\boldsymbol{H}_2+\boldsymbol{H}_{\rm e2}\boldsymbol{\phi}\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e2}^{\rm H}}_{\boldsymbol{J}}| - \ln(1+\underbrace{\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e1}\boldsymbol{\phi}\boldsymbol{h}_{\rm ei}^{\rm H}\boldsymbol{H}_2^{-1}\boldsymbol{h}_{\rm ei}}_{\eta}) - \ln|\boldsymbol{H}_2|.
\end{aligned} \tag{71}$$

To simplify the first term of (71), based on

$$\ln|\boldsymbol{X}| \leqslant \ln|\bar{\boldsymbol{X}}| + {\rm Tr}[\bar{\boldsymbol{X}}^{-1}(\boldsymbol{X}-\bar{\boldsymbol{X}})], \tag{72}$$

we have

$$\begin{aligned}
-\ln|\boldsymbol{J}| &\geqslant -\ln|\bar{\boldsymbol{J}}| - {\rm Tr}[\bar{\boldsymbol{J}}^{-1}(\boldsymbol{J}-\bar{\boldsymbol{J}})]\\
&= -\ln|\bar{\boldsymbol{J}}| + {\rm Tr}[\bar{\boldsymbol{J}}^{-1}\bar{\boldsymbol{J}}] - \boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e1}\boldsymbol{\phi}{\rm Tr}[\bar{\boldsymbol{J}}^{-1}\boldsymbol{h}_{\rm ei}\boldsymbol{h}_{\rm ei}^{\rm H}] - {\rm Tr}[\bar{\boldsymbol{J}}^{-1}\boldsymbol{H}_2] - \boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e2}^{\rm H}\bar{\boldsymbol{J}}^{-1}\boldsymbol{H}_{\rm e2}\boldsymbol{\phi}\\
&= -\ln|\bar{\boldsymbol{J}}| + {\rm Tr}[\bar{\boldsymbol{J}}^{-1}\bar{\boldsymbol{J}}] - {\rm Tr}[\bar{\boldsymbol{J}}^{-1}\boldsymbol{H}_2] - \boldsymbol{\phi}^{\rm H}(\underbrace{\boldsymbol{H}_{\rm e1}{\rm Tr}[\bar{\boldsymbol{J}}^{-1}\boldsymbol{h}_{\rm ei}\boldsymbol{h}_{\rm ei}^{\rm H}]+\boldsymbol{H}_{\rm e2}^{\rm H}\bar{\boldsymbol{J}}^{-1}\boldsymbol{H}_{\rm e2}}_{\boldsymbol{K}})\boldsymbol{\phi},
\end{aligned} \tag{73}$$

where $\bar{\boldsymbol{J}}$ means the solution obtained at the previous iteration of $\boldsymbol{J}$. By utilizing (68), one has

$$-\boldsymbol{\phi}^{\rm H}\boldsymbol{K}\boldsymbol{\phi} \geqslant -\boldsymbol{\phi}^{\rm H}\lambda_{\max}(\boldsymbol{K})\boldsymbol{I}_{M+1}\boldsymbol{\phi} - 2\Re\{\boldsymbol{\phi}^{\rm H}(\boldsymbol{K}-\lambda_{\max}(\boldsymbol{K})\boldsymbol{I}_{M+1})\bar{\boldsymbol{\phi}}\} - \bar{\boldsymbol{\phi}}^{\rm H}(\lambda_{\max}(\boldsymbol{K})\boldsymbol{I}_{M+1}-\boldsymbol{K})\bar{\boldsymbol{\phi}}. \tag{74}$$

To make the second term of (71) tractable, according to (58), we can obtain

$$-\ln(1+\eta) \geqslant -\ln(1+\bar{\eta}) - \frac{1+\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e1}\boldsymbol{\phi}\boldsymbol{h}_{\rm ei}^{\rm H}\boldsymbol{H}_2^{-1}\boldsymbol{h}_{\rm ei}}{1+\bar{\eta}} + 1, \tag{75}$$

where $\bar{\eta}$ is the solution obtained at the previous iteration. Based on the first-order Taylor series expansion, we have

$$\frac{\boldsymbol{\phi}^{\rm H}\boldsymbol{H}_{\rm e1}\boldsymbol{\phi}\boldsymbol{h}_{\rm ei}^{\rm H}\boldsymbol{H}_2^{-1}\boldsymbol{h}_{\rm ei}}{1+\bar{\eta}} \geqslant 2\Re\left\{\boldsymbol{\phi}^{\rm H}\frac{\boldsymbol{H}_{\rm e1}(\boldsymbol{h}_{\rm ei}^{\rm H}\boldsymbol{H}_2^{-1}\boldsymbol{h}_{\rm ei})}{1+\bar{\eta}}\bar{\boldsymbol{\phi}}\right\} - \bar{\boldsymbol{\phi}}^{\rm H}\frac{\boldsymbol{H}_{\rm e1}(\boldsymbol{h}_{\rm ei}^{\rm H}\boldsymbol{H}_2^{-1}\boldsymbol{h}_{\rm ei})}{1+\bar{\eta}}\bar{\boldsymbol{\phi}}. \tag{76}$$

At this point, combined with (66), (69), (74), and (76), after neglecting the constant term, the optimization problem with respect to $\boldsymbol{\phi}$ can be recast as

$$\max_{\boldsymbol{\phi}} \quad 2\Re\{\boldsymbol{\phi}^{\rm H}\boldsymbol{g}\} \tag{77a}$$

$$\text{s.t.} \quad |\boldsymbol{\phi}(m)| = 1, m = 1, \ldots, M, \ \boldsymbol{\phi}(M+1) = 1, \tag{77b}$$

where $\boldsymbol{g} = \left[\frac{\boldsymbol{h}_{\rm b}\boldsymbol{h}_{\rm b}^{\rm H}}{\tau} - (\widetilde{\boldsymbol{M}} - \lambda_{\max}(\widetilde{\boldsymbol{M}})\boldsymbol{I}_{M+1}) - (\boldsymbol{K} - \lambda_{\max}(\boldsymbol{K})\boldsymbol{I}_{M+1}) + \frac{\boldsymbol{H}_{\rm e1}(\boldsymbol{h}_{\rm ei}^{\rm H}\boldsymbol{H}_2^{-1}\boldsymbol{h}_{\rm ei})}{1+\bar{\eta}}\right]\bar{\boldsymbol{\phi}}$. Then, the optimal solution of $\boldsymbol{\theta}$ can be obtain directly by

$$\boldsymbol{\theta}^{\rm opt} = \boldsymbol{\phi}^{\rm opt}(1:M) = {\rm e}^{{\rm j}\arg(\boldsymbol{g}(1:M))}. \tag{78}$$

3.6 Overall scheme and complexity analysis

Up to now, we have completed the derivation of the PA factors β and l, transmit beamforming $\boldsymbol{v}$, receive beamforming $\boldsymbol{u}_{\rm b}$, and IRS phase shift matrix $\boldsymbol{\Psi}$. To make the process clearer, we summarize the entire proposed schemes' ideas below.

The iterative idea of the proposed Max-SR-SS scheme is as follows: (1) the PA factors β and l, transmit beamforming $\boldsymbol{v}$, receive beamforming $\boldsymbol{u}_{\rm b}$, and IRS phase shift matrix $\boldsymbol{\Psi}$ are initialized to feasible solutions; (2) given l, $\boldsymbol{v}$, $\boldsymbol{u}_{\rm b}$, and $\boldsymbol{\Psi}$, based on Algorithm 1, update β; (3) fixed β, $\boldsymbol{v}$, $\boldsymbol{u}_{\rm b}$, and $\boldsymbol{\Psi}$, solve (34) to update l; (4) given β, l, $\boldsymbol{u}_{\rm b}$, and $\boldsymbol{\Psi}$, solve (47) to obtain $\boldsymbol{v}$; (5) fixed β, l, $\boldsymbol{v}$, and $\boldsymbol{\Psi}$, solve (48a) to yield $\boldsymbol{u}_{\rm b}$; (6) given β, l, $\boldsymbol{v}$, and $\boldsymbol{u}_{\rm b}$, solve (59) to yield $\boldsymbol{\psi}$, and $\boldsymbol{\Psi} = {\rm diag}\{\boldsymbol{\psi}\}$. The five variables are updated alternately until the termination condition is realized, i.e., $|R_s^{(k)} - R_s^{(k-1)}| \leqslant \epsilon$, where k and ϵ refer to the iteration number and convergence accuracy, respectively.

The overall procedure of the proposed Max-SR-EM scheme is listed below: (1) the PA factors β and l, transmit beamforming $\boldsymbol{v}$, receive beamforming $\boldsymbol{u}_{\rm b}$, and IRS phase shift matrix $\boldsymbol{\Psi}$ are initialized to feasible solutions; (2) given l, $\boldsymbol{v}$, $\boldsymbol{u}_{\rm b}$, and $\boldsymbol{\Psi}$, β is computed by Algorithm 1; (3) fixed β, $\boldsymbol{v}$, $\boldsymbol{u}_{\rm b}$, and $\boldsymbol{\Psi}$, l is updated by (34); (4) given β, l, $\boldsymbol{u}_{\rm b}$, and $\boldsymbol{\Psi}$, $\boldsymbol{v}$ is updated by (47); (5) fixed β, l, $\boldsymbol{v}$, and $\boldsymbol{\Psi}$, $\boldsymbol{u}_{\rm b}$ is derived via the generalized Rayleigh-Ritz theorem; (6) given β, l, $\boldsymbol{v}$, and $\boldsymbol{u}_{\rm b}$, solve (61) to obtain α, solve (78) to find $\boldsymbol{\theta}$, and $\boldsymbol{\Psi} = \alpha{\rm diag}\{\boldsymbol{\theta}\}$. The iteration is repeated until the termination condition is met.

Because the obtained solutions in Max-SR-SS and Max-SR-EM schemes are sub-optimal, and the objective value sequence $\{R_s(\beta^{(k)}, l^{(k)}, \boldsymbol{v}^{(k)}, \boldsymbol{u}_{\rm b}^{(k)}, \boldsymbol{\Psi}^{(k)})\}$ obtained in each iteration of the alternate optimization method is non-decreasing.

Moreover, $R_s(\beta^{(k)}, l^{(k)}, \boldsymbol{v}^{(k)}, \boldsymbol{u}_{\rm b}^{(k)}, \boldsymbol{\Psi}^{(k)})$ has a finite upper bound since the limited power constraint. Therefore, the convergence of the proposed two schemes can be guaranteed.

Next, we calculate the computational complexity of the two proposed schemes.

(1) For the Max-SR-SS scheme, the overall computational complexity is $C_{\rm SS} = \mathcal{O}\{L_{\rm SS}[(\sqrt{5}(N_{\rm e}M^3 + N_{\rm e}M^2) + N^3M^2 + NM)1/\xi + (N_{\rm b} + N_{\rm e})M^2 + N_{\rm b}^3]\}$ float-point operations (FLOPs), where $L_{\rm SS}$ refers to the maximum number of alternating iterations, and ξ stands for the given accuracy of CVX.

(2) For the Max-SR-EM scheme, the whole computational complexity is $C_{\rm EM} = \mathcal{O}\{L_{\rm EM}[(N^3M^2 + NM)1/\xi + (N_{\rm b}+2N_{\rm e})M^2 + N_{\rm e}M + N_{\rm b}^3]\}$ FLOPs, where $L_{\rm FS}$ represents the maximum number of alternating iterations.

It is not difficult to find that the computational complexity of the two proposed schemes can be listed in decreasing order as $C_{\rm SS} > C_{\rm EM}$.

4 Proposed Max-SR-AO scheme

In this section, we consider a special situation of problem (13), i.e., both Bob and Eve are equipped with single antenna. At this point, the channels $\boldsymbol{H}_{\rm ab}$, $\boldsymbol{H}_{\rm ae}$, $\boldsymbol{H}_{\rm ib}$, and $\boldsymbol{H}_{\rm ie}$ are degenerated to $\boldsymbol{h}_{\rm ab} \in \mathbb{C}^{N\times 1}$, $\boldsymbol{h}_{\rm ae} \in \mathbb{C}^{N\times 1}$, $\boldsymbol{h}_{\rm ib} \in \mathbb{C}^{M\times 1}$, $\boldsymbol{h}_{\rm ie} \in \mathbb{C}^{M\times 1}$, respectively, and the receive beamforming is not done. Then, the receive signal (6) and (7) can be respectively degenerated to

$$y_{\rm b} = \sqrt{\beta l P}\left(\sqrt{g_{\rm ab}}\boldsymbol{h}_{\rm ab}^{\rm H} + \sqrt{g_{\rm aib}}\boldsymbol{h}_{\rm ib}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai}\right)\boldsymbol{v}x + \sqrt{g_{\rm ib}}\boldsymbol{h}_{\rm ib}^{\rm H}\boldsymbol{\Psi}\boldsymbol{n}_r + n_{\rm b}, \tag{79}$$

$$\begin{aligned} y_{\rm e} = {} & \sqrt{\beta l P}\left(\sqrt{g_{\rm ae}}\boldsymbol{h}_{\rm ae}^{\rm H} + \sqrt{g_{\rm aie}}\boldsymbol{h}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai}\right)\boldsymbol{v}x \\ & + \sqrt{(1-\beta)lP}\sqrt{g_{\rm ae}}\boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\boldsymbol{z} + \sqrt{g_{\rm ie}}\boldsymbol{h}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\boldsymbol{n}_r + n_{\rm e}. \end{aligned} \tag{80}$$

Correspondingly, the achievable rates at Bob and Eve are respectively given by

$$R_{\rm b}=\log_2\left(1+\frac{\beta lP\left|\left(\sqrt{g_{\rm ab}}\boldsymbol{h}_{\rm ab}^{\rm H}+\sqrt{g_{\rm aib}}\boldsymbol{h}_{\rm ib}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai}\right)\boldsymbol{v}\right|^2}{\sigma_r^2\|\sqrt{g_{\rm ib}}\boldsymbol{h}_{\rm ib}^{\rm H}\boldsymbol{\Psi}\|^2+\sigma_{\rm b}^2}\right), \tag{81}$$

$$R_{\rm e}=\log_2\left(1+\frac{\beta lP\left|\left(\sqrt{g_{\rm ae}}\boldsymbol{h}_{\rm ae}^{\rm H}+\sqrt{g_{\rm aie}}\boldsymbol{h}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai}\right)\boldsymbol{v}\right|^2}{(1-\beta)lP\|\sqrt{g_{\rm ae}}\boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\|^2+\sigma_r^2\|\sqrt{g_{\rm ie}}\boldsymbol{h}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\|^2+\sigma_{\rm e}^2}\right).$$

In the absence of receive beamforming, the optimization problem (13) can be recast as

$$\max_{\beta,l,\boldsymbol{v},\boldsymbol{\Psi}} \quad R_s=R_{\rm b}-R_{\rm e} \tag{82a}$$

$$\text{s.t.} \quad \boldsymbol{v}^{\rm H}\boldsymbol{v}=1,\ P_r\leqslant(1-l)P, \tag{82b}$$

$$0<\beta\leqslant 1,\ 0<l<1, \tag{82c}$$

$$|\boldsymbol{\Psi}(m,m)|\leqslant\psi^{\max}, m=1,\ldots,M. \tag{82d}$$

In what follows, the alternating iteration strategy is taken into account for solving the variables β, l, $\boldsymbol{v}$, and $\boldsymbol{\Psi}$.

4.1 Optimization of the PA factor β

In this subsection, the beamforming vector $\boldsymbol{v}$ and IRS phase shift matrix $\boldsymbol{\Psi}$ are given for the sake of simplicity. Let $D_{\rm b}=P|(\sqrt{g_{\rm ab}}\boldsymbol{h}_{\rm ab}^{\rm H}+\sqrt{g_{\rm aib}}\boldsymbol{h}_{\rm ib}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai})\boldsymbol{v}|^2$, $D_{\rm e}=P|(\sqrt{g_{\rm ae}}\boldsymbol{h}_{\rm ae}^{\rm H}+\sqrt{g_{\rm aie}}\boldsymbol{h}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai})\boldsymbol{v}|^2$, $E_{\rm b}=\sigma_r^2\|\sqrt{g_{\rm ib}}\boldsymbol{h}_{\rm ib}^{\rm H}\boldsymbol{\Psi}\|^2+\sigma_{\rm b}^2$, $E_{\rm e}=\sigma_r^2\|\sqrt{g_{\rm ie}}\boldsymbol{h}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\|^2+\sigma_{\rm e}^2$, and $F_{\rm e}=P\|\sqrt{g_{\rm ae}}\boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\|^2$. Then, Eqs. (81) and (82a) can be transformed into

$$R_{\rm b}=\log_2\left(\frac{\beta lD_{\rm b}+E_{\rm b}}{E_{\rm b}}\right) \tag{83}$$

and

$$R_{\rm e}=\log_2\left(\frac{\beta lD_{\rm e}+(1-\beta)lF_{\rm e}+E_{\rm e}}{(1-\beta)lF_{\rm e}+E_{\rm e}}\right), \tag{84}$$

respectively. The objective function of the optimization problem (82) can be degenerated as

$$\begin{aligned} R_s&=R_{\rm b}-R_{\rm e}\\ &=\log_2\left(\frac{(\beta lD_{\rm b}+E_{\rm b})[(1-\beta)lF_{\rm e}+E_{\rm e}]}{\beta lD_{\rm e}+(1-\beta)lF_{\rm e}+E_{\rm e}}\right)-\log_2E_{\rm b}\\ &=\log_2\frac{\beta(1-\beta)l^2D_{\rm b}F_{\rm e}+\beta lD_{\rm b}E_{\rm e}+(1-\beta)lE_{\rm b}F_{\rm e}+E_{\rm b}E_{\rm e}}{\beta lD_{\rm e}+(1-\beta)lF_{\rm e}+E_{\rm e}}-\log_2E_{\rm b}. \end{aligned} \tag{85}$$

In what follows, we handle the optimization of the PA parameters β and l successively.

Given l, in accordance with (82) and (85), the optimization problem with respect to β can be simplified as follows:

$$\begin{aligned} \max_{\beta} \quad & \frac{1}{\beta(lD_{\rm e}-lF_{\rm e})+lF_{\rm e}+E_{\rm e}}\Big(-\beta^2l^2D_{\rm b}F_{\rm e}\\ & +\beta\left(l^2D_{\rm b}F_{\rm e}+lD_{\rm b}E_{\rm e}-lE_{\rm b}F_{\rm e}\right)+lE_{\rm b}F_{\rm e}+E_{\rm b}E_{\rm e}\Big) \end{aligned} \tag{86a}$$

$$\text{s.t.} \quad (27), 0<\beta\leqslant 1, \tag{86b}$$

which can be re-arrange as

$$\max_{\beta} \quad \frac{-\beta^2A_3+\beta B_3+C_3}{\beta D_3+K_3} \tag{87a}$$

$$\text{s.t.} \quad \beta F_3\leqslant G_3, 0<\beta\leqslant 1, \tag{87b}$$

where $A_3 = l^2 D_{\rm b} F_{\rm e}$, $B_3 = l^2 D_{\rm b} F_{\rm e} + l D_{\rm b} E_{\rm e} - l E_{\rm b} F_{\rm e}$, $C_3 = l E_{\rm b} F_{\rm e} + E_{\rm b} E_{\rm e}$, $D_3 = l D_{\rm e} - l F_{\rm e}$, $K_3 = l F_{\rm e} + E_{\rm e}$, $F_3 = l {\rm Tr}(\boldsymbol{\Psi}(g_{\rm ai} P \boldsymbol{H}_{\rm ai} \boldsymbol{v}\boldsymbol{v}^{\rm H} \boldsymbol{H}_{\rm ai}^{\rm H})\boldsymbol{\Psi}^{\rm H})$, and $G_3 = (1-l)P - {\rm Tr}(\sigma_r^2 \boldsymbol{\Psi}\boldsymbol{\Psi}^{\rm H})$. It can be found that this problem is non-convex. Notice that this is an FP problem, and the denominator of (87a) is $\beta D_3 + K_3 = \beta l D_{\rm e} + (1-\beta) l F_{\rm e} + E_{\rm e} > 0$. To transform (87) into a convex optimization problem, based on the Dinkelbach's transform in [45], we introduce an auxiliary parameter τ_1 and recast the problem (87) as follows:

$$\max_{\beta,\tau_1} \quad -\beta^2 A_3 + \beta B_3 + C_3 - \tau_1(\beta D_3 + K_3) \tag{88a}$$

$$\text{s.t.} \quad \beta F_3 \leqslant G_3, 0 < \beta \leqslant 1. \tag{88b}$$

The optimal solution can be obtained by taking the root of $-\beta^2 A_3 + \beta B_3 + C_3 - \tau_1(\beta D_3 + K_3) = 0$. At this point, the optimization problem (88) is convex, and we can address it by the CVX toolbox directly.

4.2 Optimization of the PA factor l

Fixed β, $\boldsymbol{v}$, and $\boldsymbol{\Psi}$, we transfer the focus to solving for l. In accordance with (82) and (85), by neglecting the constant terms, the optimization problem with respect to l can be simplified as follows:

$$\max_{l} \quad \frac{l^2\beta(1-\beta)D_{\rm b}F_{\rm e} + l(\beta D_{\rm b}E_{\rm e} + (1-\beta)E_{\rm b}F_{\rm e}) + E_{\rm b}E_{\rm e}}{l(\beta D_{\rm e} + (1-\beta)F_{\rm e}) + E_{\rm e}} \tag{89a}$$

$$\text{s.t.} \quad (27), 0 < l < 1, \tag{89b}$$

which yields

$$\max_{l} \quad \frac{l^2 A_4 + l B_4 + C_4}{l D_4 + K_4} \tag{90a}$$

$$\text{s.t.} \quad l F_4 \leqslant G_4, 0 < l < 1, \tag{90b}$$

where $A_4 = \beta(1-\beta)D_{\rm b}F_{\rm e}$, $B_4 = \beta D_{\rm b} E_{\rm e} + (1-\beta)E_{\rm b}F_{\rm e}$, $C_4 = E_{\rm b}E_{\rm e}$, $D_4 = \beta D_{\rm e} + (1-\beta)F_{\rm e}$, $K_4 = E_{\rm e}$, $F_4 = \beta {\rm Tr}\left(\boldsymbol{\Psi}(g_{\rm ai} P \boldsymbol{H}_{\rm ai}\boldsymbol{v}\boldsymbol{v}^{\rm H}\boldsymbol{H}_{\rm ai}^{\rm H})\boldsymbol{\Psi}^{\rm H}\right) + P$, and $G_4 = P - {\rm Tr}(\sigma_r^2 \boldsymbol{\Psi}\boldsymbol{\Psi}^{\rm H})$. It is noticed that $l D_4 + K_4 > 0$, and this is a non-convex fractional optimization problem. In accordance with the FP method, we introduce an auxiliary parameter τ_2 and recast the problem (90) as

$$\max_{l,\tau_2} \quad l^2 A_4 + l B_4 + C_4 - \tau_2(l D_4 + K_4) \tag{91a}$$

$$\text{s.t.} \quad l F_4 \leqslant G_4, 0 < l < 1. \tag{91b}$$

The optimal solution to this problem is the root of $l^2 A_4 + l B_4 + C_4 - \tau_2(l D_4 + K_4) = 0$. However, the problem (91) is still non-convex and requires further transformation. With the first-order Taylor approximation of $l^2 A_4$ at feasible point $\bar{l}$, i.e., $l^2 A_4 \geqslant 2\bar{l} A_4 l - \bar{l}^2 A_4$, Eq. (91) can be converted to

$$\max_{l,\tau_2} \quad 2\bar{l} A_4 l - \bar{l}^2 A_4 + l B_4 + C_4 - \tau_2(l D_4 + K_4) \tag{92a}$$

$$\text{s.t.} \quad l F_4 \leqslant G_4, 0 < l < 1, \tag{92b}$$

which is a convex optimization problem and can be addressed directly by the convex optimizing toolbox.

4.3 Optimization of the beamforming vector $\boldsymbol{v}$

Given β, l, and $\boldsymbol{\Psi}$ with ignoring the constant term, Eq. (82) can be reformulated as the optimization problem with respect to $\boldsymbol{v}$ as follows:

$$\max_{\boldsymbol{v}} \quad \frac{\boldsymbol{v}^{\rm H}\boldsymbol{F}_1\boldsymbol{v}}{\boldsymbol{v}^{\rm H}\boldsymbol{F}_2\boldsymbol{v}} \tag{93a}$$

$$\text{s.t.} \quad \boldsymbol{v}^{\rm H}\boldsymbol{v} = 1, (42), \tag{93b}$$

where

$$\boldsymbol{F}_1 = \beta l P \left(\sqrt{g_{\rm ab}}\boldsymbol{h}_{\rm ab}^{\rm H} + \sqrt{g_{\rm aib}}\boldsymbol{h}_{\rm ib}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai}\right)^{\rm H}\left(\sqrt{g_{\rm ab}}\boldsymbol{h}_{\rm ab}^{\rm H} + \sqrt{g_{\rm aib}}\boldsymbol{h}_{\rm ib}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai}\right) + \left(\sigma_r^2\|\sqrt{g_{\rm ib}}\boldsymbol{h}_{\rm ib}^{\rm H}\boldsymbol{\Psi}\|^2 + \sigma_{\rm b}^2\right)\boldsymbol{I}_N, \tag{94}$$

$$\boldsymbol{F}_2 = \beta lP\left(\sqrt{g_{\rm ae}}\boldsymbol{h}_{\rm ae}^{\rm H} + \sqrt{g_{\rm aie}}\boldsymbol{h}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai}\right)^{\rm H}\left(\sqrt{g_{\rm ae}}\boldsymbol{h}_{\rm ae}^{\rm H} + \sqrt{g_{\rm aie}}\boldsymbol{h}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\boldsymbol{H}_{\rm ai}\right) + \Big((1-\beta)lP\|\sqrt{g_{\rm ae}}\boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\|^2 + \sigma_r^2\|\sqrt{g_{\rm ie}}\boldsymbol{h}_{\rm ie}^{\rm H}\boldsymbol{\Psi}\|^2 + \sigma_{\rm e}^2\Big)\boldsymbol{I}_N. \tag{95}$$

Based on (46) and relaxed the constraint $\boldsymbol{v}^{\rm H}\boldsymbol{v}=1$ to $\boldsymbol{v}^{\rm H}\boldsymbol{v}\leqslant 1$, the problem (93a) can be recast as

$$\max_{\boldsymbol{v}} \quad -\frac{\bar{\boldsymbol{v}}^{\rm H}\boldsymbol{F}_1\bar{\boldsymbol{v}}}{(\bar{\boldsymbol{v}}^{\rm H}\boldsymbol{F}_2\bar{\boldsymbol{v}})^2}\boldsymbol{v}^{\rm H}\boldsymbol{F}_2\boldsymbol{v} + \frac{2\Re\{\bar{\boldsymbol{v}}^{\rm H}\boldsymbol{F}_1\boldsymbol{v}\}}{\bar{\boldsymbol{v}}^{\rm H}\boldsymbol{F}_2\bar{\boldsymbol{v}}} \tag{96a}$$

$$\text{s.t.} \quad \boldsymbol{v}^{\rm H}\boldsymbol{v}\leqslant 1, (42). \tag{96b}$$

It can be found that this is a convex optimization problem that can be tackled directly with a convex optimizing toolbox.

4.4 Optimization of the IRS phase shift matrix $\boldsymbol{\Psi}$

In this subsection, we turn our target to optimize $\boldsymbol{\Psi}$ with given β, l, and $\boldsymbol{v}$. For the sake of derivation, let $\widetilde{\boldsymbol{\psi}} = [\boldsymbol{\psi};1]^*_{(M+1)\times 1}$, $\boldsymbol{h}_{jj} = [\sqrt{g_{{\rm ai}j}}{\rm diag}\{\boldsymbol{h}_{{\rm i}j}^{\rm H}\}\boldsymbol{H}_{\rm ai}\boldsymbol{v};\sqrt{g_{{\rm a}j}}\boldsymbol{h}_{{\rm a}j}^{\rm H}\boldsymbol{v}]_{(M+1)\times 1}$, $\boldsymbol{H}_{jj} = [\sqrt{g_{{\rm i}j}}{\rm diag}\{\boldsymbol{h}_{{\rm i}j}^{\rm H}\};\boldsymbol{0}^{\rm H}]_{(M+1)\times M}$, $j=$b, e.

Then, the achievable rates (81) and (82a) can be rewritten as

$$R_{\rm b} = \log_2\left(1+\frac{\beta lP|\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{h}_{\rm bb}|^2}{\sigma_r^2\|\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{H}_{\rm bb}\|^2+\sigma_{\rm b}^2}\right) \tag{97}$$

and

$$\begin{aligned} R_{\rm e} &= \log_2\left(1+\frac{\beta lP|\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{h}_{\rm ee}|^2}{\sigma_r^2\|\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{H}_{\rm ee}\|^2+(1-\beta)lP\|\sqrt{g_{\rm ae}}\boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\|^2+\sigma_{\rm e}^2}\right)\\ &= \log_2\left(1+\frac{\beta lP|\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{h}_{\rm ee}|^2+\sigma_r^2\|\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{H}_{\rm ee}\|^2}{(1-\beta)lP\|\sqrt{g_{\rm ae}}\boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\|^2+\sigma_{\rm e}^2}\right) - \log_2\left(1+\frac{\sigma_r^2\|\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{H}_{\rm ee}\|^2}{(1-\beta)lP\|\sqrt{g_{\rm ae}}\boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\|^2+\sigma_{\rm e}^2}\right), \end{aligned} \tag{98}$$

respectively.

Moreover, the power constraint (13e) can be re-arranged as

$$\begin{aligned} P_r &= {\rm Tr}\left(\boldsymbol{\Psi}(g_{\rm ai}\beta lP\boldsymbol{H}_{\rm ai}\boldsymbol{v}\boldsymbol{v}^{\rm H}\boldsymbol{H}_{\rm ai}^{\rm H}+\sigma_r^2\boldsymbol{I}_M)\boldsymbol{\Psi}^{\rm H}\right)\\ &= \widetilde{\boldsymbol{\psi}}^{\rm H}{\rm blkdiag}\big\{g_{\rm ai}\beta lP{\rm diag}\{\boldsymbol{v}^{\rm H}\boldsymbol{H}_{\rm ai}^{\rm H}\}{\rm diag}\{\boldsymbol{H}_{\rm ai}\boldsymbol{v}\}+\sigma_r^2\boldsymbol{I}_M,0\big\}\widetilde{\boldsymbol{\psi}}\\ &\leqslant (1-l)P. \end{aligned} \tag{99}$$

At this point, the optimization problem with respect to $\boldsymbol{\Psi}$ is

$$\begin{aligned} \max_{\widetilde{\boldsymbol{\psi}}} \quad & \log_2\left(1+\frac{\beta lP|\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{h}_{\rm bb}|^2}{\sigma_r^2\|\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{H}_{\rm bb}\|^2+\sigma_{\rm b}^2}\right) + \log_2\left(1+\frac{\sigma_r^2\|\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{H}_{\rm ee}\|^2}{(1-\beta)lP\|\sqrt{g_{\rm ae}}\boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\|^2+\sigma_{\rm e}^2}\right)\\ & - \log_2\left(1+\frac{\beta lP|\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{h}_{\rm ee}|^2+\sigma_r^2\|\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{H}_{\rm ee}\|^2}{(1-\beta)lP\|\sqrt{g_{\rm ae}}\boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\|^2+\sigma_{\rm e}^2}\right) \end{aligned} \tag{100a}$$

$$\text{s.t.} \quad |\widetilde{\boldsymbol{\psi}}(m)|\leqslant \psi^{\max},\ \widetilde{\boldsymbol{\psi}}(m+1)=1,\ (99). \tag{100b}$$

This problem is non-convex and the further transformation is required. According to (65) and (58), by omitting the constant term, the optimization problem (100) can be degenerated to

$$\begin{aligned} \max_{\widetilde{\boldsymbol{\psi}}} \quad & \frac{2\Re\{\bar{a}a^{\rm H}\}}{\bar{b}} - \frac{|\bar{a}|^2(b+|a|^2)}{\bar{b}(\bar{b}+|\bar{a}|^2)} + \frac{2\Re\{\bar{\boldsymbol{c}}^{\rm H}\boldsymbol{c}\}}{\bar{d}} - \frac{|\bar{\boldsymbol{c}}|^2(d+|\boldsymbol{c}|^2)}{\bar{d}(\bar{d}+|\bar{\boldsymbol{c}}|^2)} - \frac{1+e}{1+\bar{e}}\\ \text{s.t.} \quad & (100\text{b}), \end{aligned} \tag{101}$$

where $a=\sqrt{\beta lP}\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{h}_{\rm bb}$, $b=\sigma_r^2\|\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{H}_{\rm bb}\|^2+\sigma_{\rm b}^2$, $\boldsymbol{c}=(\sqrt{\sigma_r^2}\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{H}_{\rm ee})^{\rm H}$, $d=(1-\beta)lP\|\sqrt{g_{\rm ae}}\boldsymbol{h}_{\rm ae}^{\rm H}\boldsymbol{T}_{\rm AN}\|^2+\sigma_{\rm e}^2$, $e=(\beta lP|\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{h}_{\rm ee}|^2+\sigma_r^2\|\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{H}_{\rm ee}\|^2)/d$, $\bar{a}$, $\bar{b}$, $\bar{\boldsymbol{c}}$, $\bar{d}$, and $\bar{e}$ mean the solutions obtained at the previous iteration. Then, the problem (101) degenerates towards the following problem:

$$\min_{\widetilde{\boldsymbol{\psi}}} \quad \widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{W}\widetilde{\boldsymbol{\psi}} - 2\Re\{\widetilde{\boldsymbol{\psi}}^{\rm H}\boldsymbol{u}\}, \tag{102a}$$

$$\text{s.t.}\ \ (100\text{b}), \tag{102b}$$

where

$$\boldsymbol{W} = \frac{|\bar{a}|^2}{\bar{b}(\bar{b}+|\bar{a}|^2)}(\beta l P \boldsymbol{h}_{\rm bb}\boldsymbol{h}_{\rm bb}^{\rm H} + \sigma_r^2 \boldsymbol{H}_{bb}\boldsymbol{H}_{\rm bb}^{\rm H}) + \frac{|\bar{\boldsymbol{c}}|^2}{\bar{d}(\bar{d}+|\bar{\boldsymbol{c}}|^2)} \cdot \sigma_r^2 \boldsymbol{H}_{\rm ee}\boldsymbol{H}_{\rm ee}^{\rm H} + \frac{1}{1+\bar{e}}\frac{\beta l P \boldsymbol{h}_{\rm ee}\boldsymbol{h}_{\rm ee}^{\rm H} + \sigma_r^2 \boldsymbol{H}_{\rm ee}\boldsymbol{H}_{\rm ee}^{\rm H}}{d}, \tag{103}$$

$$\boldsymbol{u} = \beta l P \boldsymbol{h}_{\rm bb}\boldsymbol{h}_{\rm bb}^{\rm H}\widetilde{\boldsymbol{\psi}}_t/\bar{b} + \sigma_r^2 \boldsymbol{H}_{\rm ee}\boldsymbol{H}_{\rm ee}^{\rm H}\widetilde{\boldsymbol{\psi}}_t/\bar{d}, \tag{104}$$

and $\widetilde{\boldsymbol{\psi}}_t$ stands for the solution obtained at the previous iteration. It is noted that the problem (102a) is convex, which can be derived directly with the CVX toolbox.

4.5 Overall scheme and complexity analysis

So far, we have completed the derivation of the PA factors β and l, beamforming vector $\boldsymbol{v}$, and IRS phase shift matrix $\boldsymbol{\Psi}$. To make the procedure of this scheme clearer, we summarize the whole proposed Max-SR-AO algorithm below. (1) Initialize β, l, $\boldsymbol{v}$, and $\boldsymbol{\Psi}^{(0)}$ to feasible solutions; (2) fixed l, $\boldsymbol{v}$, and $\boldsymbol{\Psi}$, solve (88) to update β; (3) given β, $\boldsymbol{v}$, and $\boldsymbol{\Psi}$, solve (92) to update l; (4) fixed β, l, and $\boldsymbol{\Psi}$, optimize (96) to update $\boldsymbol{v}$; (5) given β, l, and $\boldsymbol{v}$, solve (102a) to update $\widetilde{\boldsymbol{\psi}}$, and $\boldsymbol{\Psi} = \text{diag}\{\widetilde{\boldsymbol{\psi}}(1:M)\}^*$. Optimize the four variables alternately until the termination condition is satisfied.

In this scheme, the objective value sequence $\{R_s(\beta^{(k)}, l^{(k)}, \boldsymbol{v}^{(k)}, \boldsymbol{\Psi}^{(k)})\}$ obtained in each iteration of the alternate optimization strategy is non-decreasing, and $R_s(\beta^{(k)}, l^{(k)}, \boldsymbol{v}^{(k)}, \boldsymbol{\Psi}^{(k)})$ has a finite upper bound since the limited power constraint. Therefore, the convergence of the proposed Max-SR-AO scheme can be guaranteed.

The computational complexity of the overall Max-SR-AO algorithm is $C_{\rm AO} = \mathcal{O}\{L_{\rm AO}[M^2 \text{In}(1/\xi) + L_v(N^3+NM^2) + L_\Psi(2\sqrt{2}(M+1)^3 + N(M+1)^2)]\}$ FLOPs, where $L_{\rm AO}$ means the maximum number of alternating iterations, and L_v and L_Ψ mean the iterative numbers of the subproblems (96) and (102a).

5 Simulation results

To verify the performance of the three proposed maximum SR schemes, we perform the simulation comparison in this section. Unless otherwise noted, the parameters of the simulation are listed as follows: $P = 30$ dBm, $N = 8$, $M = 64$, $N_{\rm b} = N_{\rm e} = 4$, $d_{\rm ai} = 110$ m, $d_{\rm ab} = d_{\rm ae} = 120$ m, $\theta_{\rm ai} = 11\pi/36$, $\theta_{\rm ab} = \pi/3$, $\theta_{\rm ae} = 5\pi/12$, and $\sigma_{\rm b}^2 = \sigma_{\rm e}^2 = \sigma_r^2 = -40$ dBm. The path loss model is modeled as $g = \lambda^2/(4\pi d_{tr})^2$, where λ and d_{tr} are the wavelength and reference distance, respectively. For the sake of convenience, we set $(\lambda/(4\pi))^2 = 10^{-2}$. The convergence accuracy is $\epsilon = 10^{-3}$.

To evaluate the performance of the proposed schemes, the semi-definite relaxation (SDR) scheme (no AN) reported in [34], passive IRS scheme (i.e., GAI algorithm) reported in [25], passive IRS scheme reported in [27], passive IRS scheme (i.e., Algorithm 1) reported in [28], and several benchmark schemes are applied for comparison at the same power. These benchmark schemes are listed as follows.

(1) Benchmark scheme I: Setting the PA factor $l = 0.6$, we only optimize the remaining variables alternatively.

(2) Benchmark scheme II: Fixing the PA factor $\beta = 0.5$, we only have to alternately optimize the remaining variables.

(3) Benchmark scheme III: Both the PA factors β and l are fixed at 0.5, i.e., $\beta = l = 0.5$, and only the residual variables need to be optimized alternately.

(4) No-IRS scheme: Set all the active IRS-related channel vectors and matrices to zero vectors and zero matrices; then, we only have to optimize the remaining variables.

5.1 Bob and Eve equipped with multiple antennas

First, we show the convergence of both the proposed AO schemes in Figure 2, where the number $M = 16, 64, 256$ of IRS phase shift elements. Figure 2 shows that the SRs of both proposed schemes increase rapidly with the number of iterations and finally converge to a certain value after a finite number of iterations. The convergence speed of the proposed Max-SR-SS scheme is slightly faster than that of the

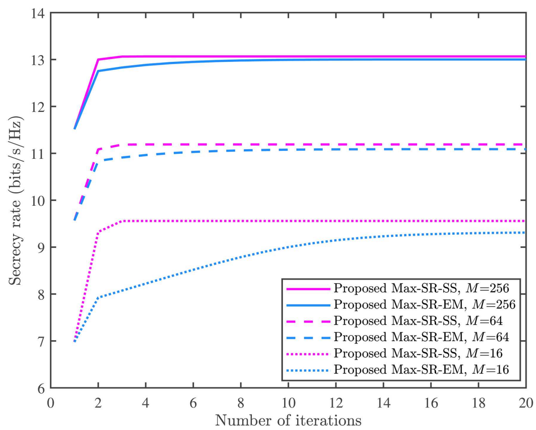

Figure 2 (Color online) Convergence of the proposed schemes.

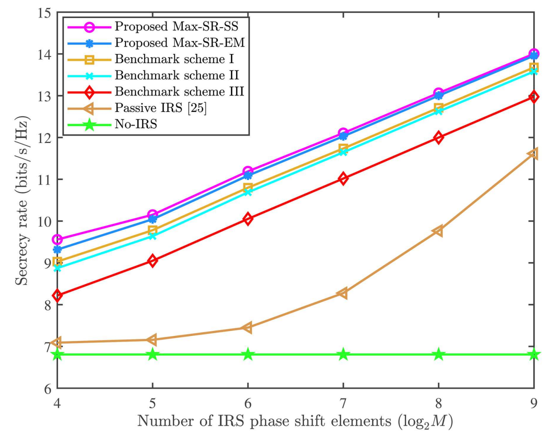

Figure 3 (Color online) SR versus the number M of IRS elements.

proposed Max-SR-EM scheme. In addition, the SRs of both proposed schemes increase with the increase in M, and the SR of the proposed Max-SR-SS scheme is slightly better than that of the proposed Max-SR-EM scheme, regardless of the value of M. Combined with the previous analysis of the computational complexity of both proposed schemes, it can be found that the low-complexity of the Max-SR-EM scheme is achieved at the price of some performance loss. As a result, the proposed Max-SR-EM scheme strikes a good balance between computational complexity and SR performance.

Figure 3 plots the curves of the SR versus the number M of active IRS phase shift elements. Figure 3 shows that the SRs of both the proposed and benchmark schemes gradually increase with the increase in M, and exhibit a decreasing order in terms of SR performance, as follows: proposed Max-SR-SS, proposed Max-SR-EM, benchmark scheme I, benchmark scheme II, benchmark scheme III, passive IRS [25], and no-IRS scheme. The SR difference between the two proposed schemes is trivial with the increase in M, and they make significant SR performance enhancements over the five benchmark schemes at the same total power budget. For example, when $M = 64$, the SR performance enhancements achieved by both the proposed schemes over the benchmark scheme I, benchmark scheme II, benchmark scheme III, passive IRS [25], and no-IRS scheme are above 3%, 4%, 11%, 40%, and 47%, respectively. These further explain the motivation for investigating the active IRS, PA, and beamforming algorithms.

Figure 4 depicts the curves of the SR versus the total power P ranging from 10 to 35 dBm. From Figure 4, we learn that the SRs of the two proposed schemes and five benchmark schemes increase with the increase in P, and the ordering of their achieved SRs is similar to that shown in Figure 3. The difference in SR performance between the proposed Max-SR-SS scheme and benchmark scheme I is slightly less than that between the proposed Max-SR-SS scheme and benchmark scheme II, which means that optimizing the confidential message PA factor β has a more significant performance enhancement for the system compared to optimizing the base station PA factor l in this paper. Compared with the benchmark schemes of no-IRS and passive IRS [25], the SRs achieved by both proposed schemes and the remaining benchmark schemes are remarkable, with the latter being more than one times higher than the former because active IRS elements equipped with power amplifiers enable more SR performance gain. Moreover, the gap between the SRs of the two proposed schemes is trivial when $P \leqslant 20$ dBm.

Figure 5 illustrates the curves of the SR versus the noise ratio η ranging from 1 to 3.5, where $\eta = \sigma_r^2/\sigma_{\rm b}^2$ and $\sigma_{\rm b}^2$ remains constant, i.e., the increase in η is equivalent to that of the noise power at the active IRS. Figure 5 shows that, apart from the no-IRS scheme, the SRs of the two proposed schemes and benchmark schemes I–III decrease gradually with the increase in η. This is because the active IRS helps to transmit the confidential information to Bob and reflects the noise generated at the IRS to him. When η increases, the noise received by Bob also increases, which leads to a decrease in the SR performance for all schemes apart from the no-IRS scheme. Taking the Max-SR-SS scheme as an example, the SR at $\eta = 2$ and $\eta = 3$ are above 8% and 13% lower than those at $\eta = 1$, respectively.

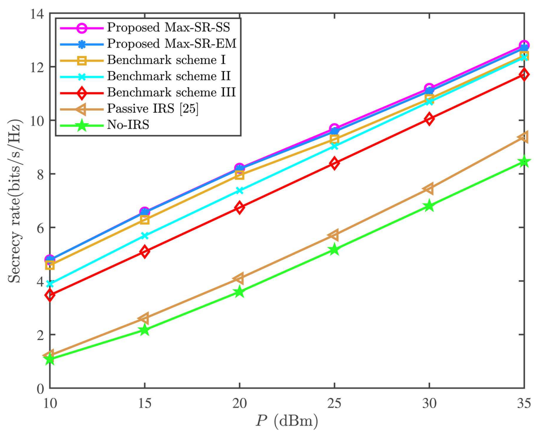

Figure 4 (Color online) SR versus the total power P.

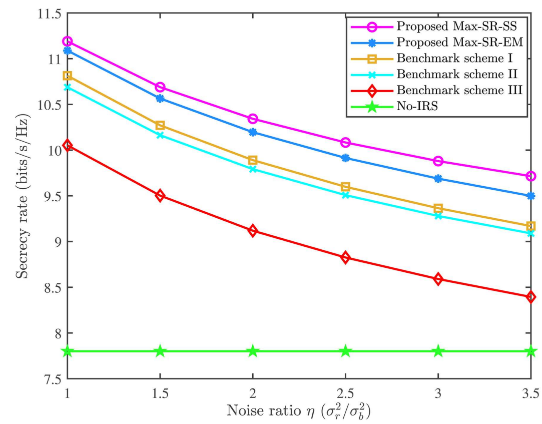

Figure 5 (Color online) SR versus the noise ratio η.

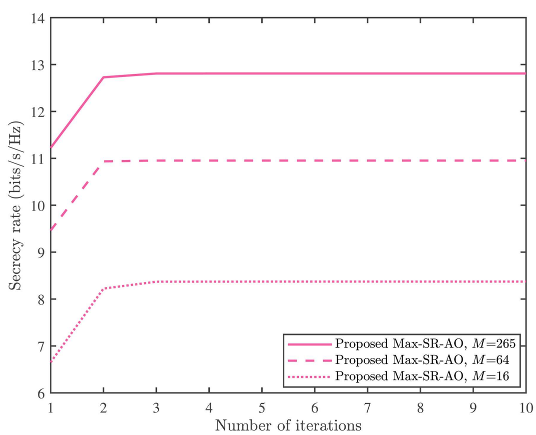

Figure 6 (Color online) Convergence of the proposed Max-SR-AO scheme.

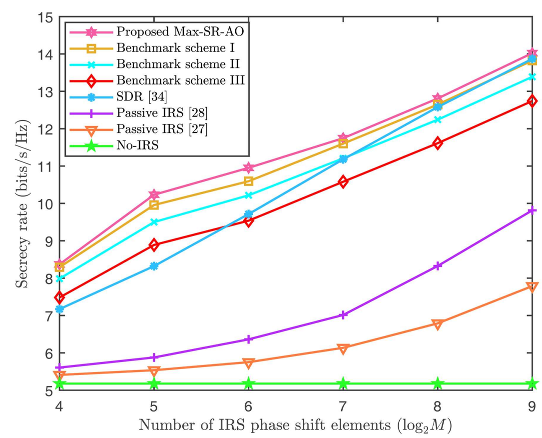

Figure 7 (Color online) SR versus the number M of IRS elements for the proposed scheme and benchmark schemes.

5.2 Bob and Eve equipped with single antenna

Figure 6 shows the SR versus the number of iterations of the proposed Max-SR-AO scheme. Figure 6 shows that, regardless of the value of the number M of IRS phase shift elements, the proposed Max-SR-AO scheme takes approximately four iterations to converge to the SR ceiling. Moreover, as M gradually increases, the SR of the proposed Max-SR-AO scheme increases accordingly.

Figure 7 plots the SR versus the number M of the IRS phase shift elements. Similar to the scenario where both Bob and Eve are equipped with multiple antennas, the SR of the proposed Max-SR-AO scheme is slightly better than those of the fixed PA schemes (i.e., benchmark schemes I–III and SDR [34]) and significantly better than those of the passive IRS [27, 28] schemes because the optimization of the PA factors enables the optimal distribution between power sources. The active IRS with a power amplifier achieves a higher SR performance gain than the passive IRS with only phase tuning. Moreover, the gap between the SRs of the proposed Max-SR-AO and SDR [34] decreases as M increases gradually.

To investigate the impact of Bob's location on SR performance, with fixed positions of Alice, the IRS, and Eve, we assume that Bob moves only along the straight line $L_{\rm ab}$ (i.e., the line connecting Alice and Bob) for simplicity of analysis. At this point, Bob's location only depends on the distance $d_{\rm ab}$ of the Alice-to-Bob link. As $d_{\rm ab}$ increases, Bob first moves closer to the IRS, reaches a peak, and then moves away from it. The detailed diagram of Bob's movement is shown in Figure 8.

Figure 9 presents the curves of the SR versus the distance $d_{\rm ab}$, where $M = 128$. Figure 9 shows that, as Bob moves away from Alice along $L_{\rm ab}$ and closer to the IRS, the SR of the no-IRS scheme gradually decreases with the increase in $d_{\rm ab}$. For the proposed Max-SR-AO scheme, when Bob first moves between Alice and IRS and then away from them, the energy received from Alice decreases, and the SRs gradually

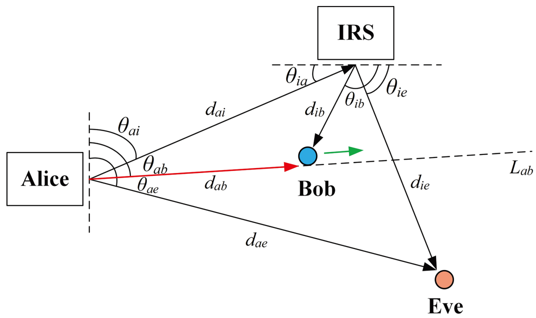

Figure 8 (Color online) Diagram of Bob's movement.

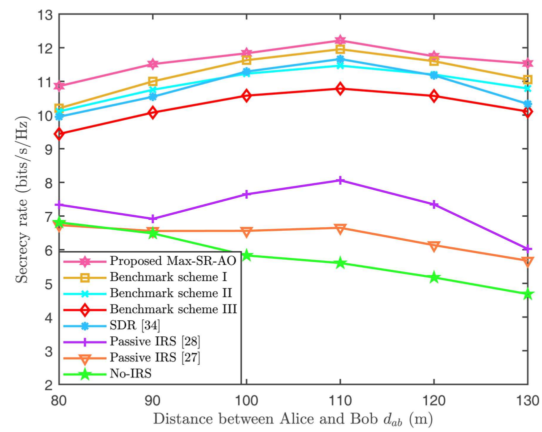

Figure 9 (Color online) SR versus the distance between Alice and Bob $d_{\rm ab}$.

decrease with the increase in $d_{\rm ab}$. Then, as Bob moves away from Alice and closer to the IRS, the energy received from the IRS gradually increases, and the SRs gradually increase and reach a peak when Bob is at the bottom of the IRS. Finally, with Bob moving away from Alice and the IRS, the energy received from Alice and the IRS gradually decreases, and the SRs gradually decrease. Moreover, there are similar SR performance tendencies for the SDR [34] and the passive IRS [27, 28] schemes. After the peak, the gap between the SRs gained by the proposed and passive IRS schemes increases gradually with $d_{\rm ab}$.

6 Conclusion

In this study, we investigated active IRS-aided DM networks and focused on adjusting the PA between IRS and Alice to improve SR performance. To the best of our knowledge, such a PA has not been investigated in the optimization of the PA factors, transmit beamforming, and receive beamforming, and IRS phase shift matrix in the active IRS-aided DM network. To address the formulated problem, two alternating iteration schemes, namely, Max-SR-SS and Max-SR-EM, were proposed. The Max-SR-SS scheme employed the derivative operation, SCA, and generalized Rayleigh-Ritz methods to find the optimal PA factors, transmit beamforming, and receive beamforming, and IRS phase shift matrix. While the Max-SR-EM scheme with a low-complexity derived the closed-form expression of the IRS phase shift matrix based on the criteria of EAR and MM. Moreover, a special case of receivers equipped with single antenna was considered, and the Max-SR-AO scheme was proposed to address the problem. The simulation results showed that the SR of the DM network was dramatically enhanced with the help of the active IRS, and the three proposed schemes have made an obvious SR enhancement over the fixed PA and passive IRS schemes.

Acknowledgements This work was supported in part by National Key Research and Development Program of China (Grant No. 2023YFF0612900), National Natural Science Foundation of China (Grant No. U22A2002), Hainan Province Science and Technology Special Fund (Grant No. ZDYF2024GXJS292), Scientific Research Fund Project of Hainan University (Grant No. KYQD(ZR)-21008), and Collaborative Innovation Center of Information Technology, Hainan University (Grant No. XTCX2022XX-C07).

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Appendix A

Let $\boldsymbol{q}=\sqrt{\beta lP}(\sqrt{g_{\mathrm{ae}}}\boldsymbol{H}_{\mathrm{ae}}^{\mathrm{H}}+\sqrt{g_{\mathrm{aie}}}\boldsymbol{H}_{\mathrm{ie}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{H}_{\mathrm{ai}})\boldsymbol{v}$, $\boldsymbol{Q}_1=\boldsymbol{q}\boldsymbol{q}^{\mathrm{H}}$, $\boldsymbol{Q}_2=(1-\beta)lPg_{\mathrm{ae}}\boldsymbol{H}_{\mathrm{ae}}^{\mathrm{H}}\boldsymbol{T}_{\mathrm{AN}}\boldsymbol{T}_{\mathrm{AN}}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{ae}}+\sigma_r^2g_{\mathrm{ie}}\boldsymbol{H}_{\mathrm{ie}}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{\Psi}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{ie}}+\sigma_{\mathrm{e}}^2\boldsymbol{I}_{N_{\mathrm{e}}}$, $\boldsymbol{Q}_2=\boldsymbol{Q}_2^{1/2}(\boldsymbol{Q}_2^{1/2})^{\mathrm{H}}$, $\boldsymbol{w}=\boldsymbol{Q}_2^{1/2}\boldsymbol{u}_{\mathrm{e}}$, $\boldsymbol{u}_{\mathrm{e}}=\boldsymbol{Q}_2^{-1/2}\boldsymbol{w}$, and

$$\gamma=\frac{\boldsymbol{u}_{\mathrm{e}}^{\mathrm{H}}\boldsymbol{Q}_1\boldsymbol{u}_{\mathrm{e}}}{\boldsymbol{u}_{\mathrm{e}}^{\mathrm{H}}\boldsymbol{Q}_2\boldsymbol{u}_{\mathrm{e}}}=\frac{\boldsymbol{w}^{\mathrm{H}}(\boldsymbol{Q}_2^{-1/2})^{\mathrm{H}}\boldsymbol{Q}_1\boldsymbol{Q}_2^{-1/2}\boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{w}}. \tag{A1}$$

Defining $\widetilde{\boldsymbol{w}}=\frac{\boldsymbol{w}}{\|\boldsymbol{w}\|}$, we have $\boldsymbol{w}=\widetilde{\boldsymbol{w}}\|\boldsymbol{w}\|$. Then, Eq. (A1) can be rewritten as

$$\begin{aligned}\frac{\boldsymbol{w}^{\mathrm{H}}(\boldsymbol{Q}_2^{-1/2})^{\mathrm{H}}\boldsymbol{Q}_1\boldsymbol{Q}_2^{-1/2}\boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{w}}&=\frac{\widetilde{\boldsymbol{w}}^{\mathrm{H}}\|\boldsymbol{w}\|(\boldsymbol{Q}_2^{-1/2})^{\mathrm{H}}\boldsymbol{Q}_1\boldsymbol{Q}_2^{-1/2}\|\boldsymbol{w}\|\widetilde{\boldsymbol{w}}}{\|\boldsymbol{w}\|^2\widetilde{\boldsymbol{w}}^{\mathrm{H}}\widetilde{\boldsymbol{w}}}\\&=\widetilde{\boldsymbol{w}}^{\mathrm{H}}(\boldsymbol{Q}_2^{-1/2})^{\mathrm{H}}\boldsymbol{Q}_1\boldsymbol{Q}_2^{-1/2}\widetilde{\boldsymbol{w}}\\&\leqslant\lambda_{\max}((\boldsymbol{Q}_2^{-1/2})^{\mathrm{H}}\boldsymbol{Q}_1\boldsymbol{Q}_2^{-1/2})\\&\overset{(\varpi)}{=}\mathrm{Tr}(\boldsymbol{Q}_2^{-1}\boldsymbol{Q}_1).\end{aligned} \tag{A2}$$

(ϖ) is due to the fact that $\mathrm{rank}((\boldsymbol{Q}_2^{-1/2})^{\mathrm{H}}\boldsymbol{Q}_1\boldsymbol{Q}_2^{-1/2})=\mathrm{rank}((\boldsymbol{Q}_2^{-1/2})^{\mathrm{H}}\boldsymbol{q}\boldsymbol{q}^{\mathrm{H}}\boldsymbol{Q}_2^{-1/2})=1$.