SCIENCE CHINA
Information Sciences

• LETTER •

November 2025, Vol. 68, Iss. 11, 219301:1–219301:2
https://doi.org/10.1007/s11432-024-4556-9

Enhanced matrix information geometry detection for weak targets in heterogeneous clutter environment

Zheng YANG[1], Yongqiang CHENG[1*], Hao WU[1], Yang YANG[1], Yuliang QIN[2], Hongqiang WANG[1] & Xiang LI[1]

[1] *College of Electronic Science and Technology, National University of Defense Technology, Changsha 410073, China*
[2] *Department of Advanced Remote Sensing Technology, Tiansun Laboratory, Changsha 410028, China*

Received 25 June 2024/Revised 10 November 2024/Accepted 13 August 2025/Published online 25 September 2025

Citation Yang Z, Cheng Y Q, Wu H, et al. Enhanced matrix information geometry detection for weak targets in heterogeneous clutter environment. Sci China Inf Sci, 2025, 68(11): 219301, https://doi.org/10.1007/s11432-024-4556-9

In many radar detection scenes, the clutter environment is usually heterogeneous, which exhibits statistical variations in properties in the spatiotemporal domains. Detecting weak targets embedded in such heterogeneous clutter environments has received considerable attention. In the last decades, target detection issues have been widely studied in many important works. Recently, a rapidly advancing target detection technique based on the theory of matrix information geometry (MIG) has been garnering attention for its advantages in dealing with scenarios involving short pulses and complicated clutter [1, 2]. This technique does not require any a priori knowledge of clutter; instead, it takes into account the intrinsic Riemannian geometry of the resulting manifold. From a geometric viewpoint, the target detection problem is transformed into a discriminative problem between the target and clutter on the manifold [3]. Specifically, the covariance matrices constructed by the received data represent points on a Hermitian positive definite (HPD) manifold. Then, the detection decision is made based on geometric distances that measure the separability between the cell under test (CUT) and the clutter centroid of reference cells. Nevertheless, in actual environments, the clutter is arbitrarily distributed, and the signal-to-clutter ratio (SCR) is relatively low. Hence, the characteristics of a target on the manifold are nearly similar to those of the clutter, resulting in poor separability. To address the challenges in actual environments, we aimed to fully exploit the actual discriminative information of the manifold and develop an enhancement scheme of data separability from a geometric viewpoint to achieve improved detection performance that meets the actual requirements. Unlike the earlier work performed in a locally flat Euclidean space by using the log-Euclidean metric [4], we examine the non-Euclidean manifold structure by using affine invariant geometric distances and exploit the real geometric structure to formulate the discriminative transformation to enhance the detection performance [5]. The heterogeneity of the clutter distribution is also considered. Specifically, a weighted centroid is developed by employing a shift procedure on the HPD manifold, and the corresponding anisotropy is analyzed. Then, we devise a manifold transformation scheme that maps the HPD manifold into a lower-dimensional and more discriminative one by optimizing the separability between the CUT and the weighted centroid. In this study, the discriminative transformation is formulated as a joint optimization problem on a Stiefel manifold. We devise a two-step optimization method based on the Riemannian gradient descent algorithm to solve the optimization problem. Thus, a manifold transformation-based MIG (MT-MIG) detector is developed. Finally, numerical experiments are performed based on simulated data and real radar data to validate the superior performance of the proposed method.

Preliminaries of MIG detection. In a pulse Doppler (PD) radar system, a Toeplitz covariance matrix can be constructed from the received pulse data $\boldsymbol{z} = (z_1, \ldots, z_n)^{\mathrm{T}}$ under the framework of MIG detection [3], namely,

$$\boldsymbol{R} = \mathbb{E}\left[\boldsymbol{z}\boldsymbol{z}^{\mathrm{H}}\right] = \begin{bmatrix} r_0 & \bar{r}_1 & \cdots & \bar{r}_{n-1} \\ r_1 & r_0 & \cdots & \bar{r}_{n-2} \\ \vdots & \ddots & \ddots & \vdots \\ r_{n-1} & r_{n-2} & \cdots & r_0 \end{bmatrix}, \tag{1}$$

where $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ denote the transpose and conjugate transpose, respectively. Thus, the covariance matrices of the CUT and reference cells are given as $\boldsymbol{R}_D$ and $\{\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_K\}$, respectively, where K denotes the number of reference cells. Hence, the set of such HPD matrices forms an HPD manifold, which is defined as follows:

$$\mathcal{M}^+(n) = \left\{\boldsymbol{R} | \boldsymbol{R} = \boldsymbol{R}^{\mathrm{H}}, \boldsymbol{R} \succ 0, \boldsymbol{R} \in \mathbb{C}^{n\times n}\right\}, \tag{2}$$

where $\mathbb{C}^{n\times n}$ denotes the set of $n \times n$-dimensional complex matrices. The detection decision of the MIG detector is

$$\mathcal{D}^2(\boldsymbol{R}_D, \bar{\boldsymbol{R}}_{\mathcal{G}}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma, \tag{3}$$

* Corresponding author (email: nudtyqcheng@gmail.com)

where $\mathcal{H}_0$ and $\mathcal{H}_1$ denote target presence and absence, respectively, $\boldsymbol{R}_D$ is the HPD matrix of the CUT, and $\bar{\boldsymbol{R}}_{\mathcal{G}}$ is the geometric centroid of the HPD matrices in the reference cells. $\mathcal{D}^2(\cdot,\cdot)$ denotes the geometric distance, and γ is the detection threshold. More details are given in Appendix A.

Proposed enhanced MIG detector. To improve the separability of manifold space, we perform a discriminative analysis of the HPD manifold. In this analysis, the following aspects were considered. (1) Weighted centroid. Clutter is arbitrarily distributed on the manifold in the actual environment, and a weighted centroid is defined to estimate the clutter centroid equipped with different weights. (2) Discriminative manifold transformation. To enhance the separability between the target and clutter, a discriminative manifold transformation scheme that maps the original manifold into a more discriminative one is devised. See Appendix B.

Firstly, for the arbitrarily distributed reference cells of clutter $\{\boldsymbol{R}_1,\ldots,\boldsymbol{R}_K\}$ on the manifold, the weighted centroid $\bar{\boldsymbol{R}}_{\mathcal{W}} = \mathcal{W}(\boldsymbol{R}_1,\ldots,\boldsymbol{R}_K)$ is defined as

$$\mathcal{W}(\boldsymbol{R}_1,\ldots,\boldsymbol{R}_K) = \arg\max_{\tilde{\boldsymbol{R}}\in\mathcal{M}^+(n)} \sum_{\kappa=1}^{K} k\left(\frac{\mathcal{D}^2\left(\tilde{\boldsymbol{R}},\boldsymbol{R}_\kappa\right)}{h^2}\right), \tag{4}$$

where $k(\cdot)$ is the weight function, and h denotes the scaling parameter. The weight function decreases monotonically and satisfies $0 < k(x) \leqslant 1$ for $x \geqslant 0$. The specific solution of the weighted centroid in (4) is given in Appendix C.

The separability of the manifold in a heterogeneous environment with a low SCR is poor. Therefore, for targets located close to the clutter, this study defines a manifold to manifold transformation to seek a more discriminative space. Specifically, for a high-dimensional space $\mathcal{M}^+(n)$, the manifold transformation is given by

$$\mathcal{F}_{\boldsymbol{W}}:\mathcal{M}^+(n)\to\mathcal{M}^+(m),\mathcal{F}_{\boldsymbol{W}}(\boldsymbol{R}) = \boldsymbol{W}^{\mathrm{H}}\boldsymbol{R}\boldsymbol{W}, \tag{5}$$

where $\boldsymbol{R}$ denotes an arbitrary point on $\mathcal{M}^+(n)$, $\boldsymbol{W}\in\mathbb{C}^{n\times m}$ $(m \leqslant n)$ is an orthogonal matrix on a Stiefel manifold $\mathrm{St}(n,m)$ [6], and m denotes the dimension of the manifold after transformation. More details are given in Appendix D.

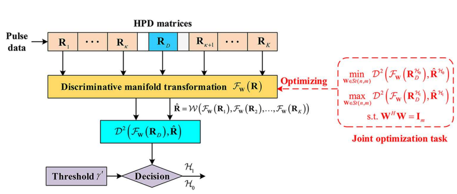

Figure 1 (Color online) Flowchart of the proposed MT-MIG detector.

Then, by utilizing the discriminative manifold $\mathcal{M}^+(m)$, we can obtain the transformed HPD matrix of the CUT $\mathcal{F}_{\boldsymbol{W}}(\boldsymbol{R}_D)$ and the corresponding weighted centroid $\hat{\boldsymbol{R}} = \bar{\boldsymbol{R}}_{\mathcal{W}}$. Thus, an enhanced MIG detector based on manifold transformation (i.e., MT-MIG) is derived, as shown in Figure 1. The detection decision is expressed as follows:

$$\mathcal{D}^2\left(\mathcal{F}_{\boldsymbol{W}}(\boldsymbol{R}_D),\hat{\boldsymbol{R}}\right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma', \tag{6}$$

where $\hat{\boldsymbol{R}} = \mathcal{W}(\mathcal{F}_{\boldsymbol{W}}(\boldsymbol{R}_1),\mathcal{F}_{\boldsymbol{W}}(\boldsymbol{R}_2),\ldots,\mathcal{F}_{\boldsymbol{W}}(\boldsymbol{R}_K))$ is the weighted centroid, γ' is the corresponding threshold. Herein, the computational complexity grows cubically with respect to the pulse length because of the inverse matrix operator. As shown in Figure 1, the vital issue is to formulate a joint optimization task and optimize the transformation matrix.

Suppose two sets data are available under $\mathcal{H}_0$ and $\mathcal{H}_1$ hypotheses: $\mathcal{M}^+_{\mathcal{H}_{\mathbb{I}}}(n) = \left\{\boldsymbol{R}_D^{\mathcal{H}_{\mathbb{I}}},\boldsymbol{R}_1^{\mathcal{H}_{\mathbb{I}}},\boldsymbol{R}_2^{\mathcal{H}_{\mathbb{I}}},\ldots,\boldsymbol{R}_K^{\mathcal{H}_{\mathbb{I}}}\right\}$, $\mathbb{I} = 0,1$. Here, $\mathcal{M}^+_{\mathcal{H}_0}(n)$ indicates that only clutter exists and $\mathcal{M}^+_{\mathcal{H}_1}(n)$ indicates $\mathcal{H}_0$ data plus a target in the CUT. Specifically, we attempt to seek the transformation matrix by optimizing the geometric distance between the CUT and the weighted centroid in the presence and absence of the target. This situation can be formulated as the following joint optimization problems:

$$\min_{\boldsymbol{W}\in\mathrm{St}(n,m)} \mathcal{D}^2\left(\mathcal{F}_{\boldsymbol{W}}\left(\boldsymbol{R}_D^{\mathcal{H}_0}\right),\hat{\boldsymbol{R}}^{\mathcal{H}_0}\right), \tag{7}$$

$$\max_{\boldsymbol{W}\in\mathrm{St}(n,m)} \mathcal{D}^2\left(\mathcal{F}_{\boldsymbol{W}}\left(\boldsymbol{R}_D^{\mathcal{H}_1}\right),\hat{\boldsymbol{R}}^{\mathcal{H}_1}\right), \tag{8}$$

where $\hat{\boldsymbol{R}}^{\mathcal{H}_{\mathbb{I}}}$ denotes the weighted centroid of the reference cells $\left\{\mathcal{F}_{\boldsymbol{W}}\left(\boldsymbol{R}_\kappa^{\mathcal{H}_{\mathbb{I}}}\right)\right\}_{\kappa=1}^{K}$. By combining the above optimizations, we can derive the following optimization problem:

$$\min_{\boldsymbol{W}\in\mathrm{St}(n,m)} \mathcal{D}^2\left(\mathcal{F}_{\boldsymbol{W}}\left(\boldsymbol{R}_D^{\mathcal{H}_0}\right),\hat{\boldsymbol{R}}^{\mathcal{H}_0}\right) - \lambda\mathcal{D}^2\left(\mathcal{F}_{\boldsymbol{W}}\left(\boldsymbol{R}_D^{\mathcal{H}_1}\right),\hat{\boldsymbol{R}}^{\mathcal{H}_1}\right), \tag{9}$$

where λ is a trade-off between parameters. We propose a two-step optimization method to solve (9) and obtain the MT-MIG detector (see Appendix E).

Performance assessment. Appendix F gives the numerical experiments based on simulated data and real radar data to validate the performance of the proposed method.

Conclusion. We proposed an enhanced MIG detector based on manifold transformation to address the challenge of detecting weak targets in a heterogeneous clutter environment. Specifically, a weighted centroid was utilized to estimate the clutter centroid of the manifold. Then, we devised a discriminative manifold transformation and formulated a joint optimization problem. We solved this problem by designing a two-step optimization method to obtain the MT-MIG detector. Through numerical experiments based on simulated data and real radar data, we validated the performance of the proposed method. Future research will focus on studying the constant false alarm rate properties and designing the detector for an extended target.

Acknowledgements This work was supported by Distinguished Youth Science Foundation of Hunan Province (Grant No. 2022JJ10063) and National Natural Science Foundation of China (Grant No. 61921001).

Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Arnaudon M, Barbaresco F, Yang L. Riemannian medians and means with applications to radar signal processing. IEEE J Sel Top Signal Process, 2013, 7: 595–604
2 Hua X, Ono Y, Peng L, et al. Target detection within nonhomogeneous clutter via total Bregman divergence-based matrix information geometry detectors. IEEE Trans Signal Process, 2021, 69: 4326–4340
3 Cheng Y, Hua X, Wang H, et al. The geometry of signal detection with applications to radar signal processing. Entropy, 2016, 18: 381
4 Yang Z, Cheng Y, Wu H, et al. Adaptive log-euclidean metric on HPD manifold for target detection in dynamically changing clutter environments. IEEE Trans Aerosp Electron Syst, 2024, 60: 9261–9274
5 Pennec X. Intrinsic statistics on Riemannian manifolds: basic tools for geometric measurements. J Math Imag Vis, 2006, 25: 127–154
6 Udriste C. Convex Functions and Optimization Methods on Riemannian Manifolds. Dordrecht: Springer, 2013