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Leader-following consensus control of Markov switched multi-AUV recovery system with time-varying delay

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Recovery is the most critical issue of multiple autonomous underwater vehicles (multi-AUV) in an underwater mission execution. It is also an essential step to ensure the successful recovery and reuse of underwater vehicles. In the process of recovery, all states of the multi-AUV system are required to be ultimately consistent eventually [1].

AUVs commonly use underwater acoustic wireless communication. On the one hand, the use of acoustic communication channels brings serious limitations to communication. On the other hand, communication may be delayed or interrupted by the complex and changeable underwater environment. Yang et al. [2] pointed out that multi-AUV system is an interdisciplinary field, and how to find methods to calculate the delay upper bound of the multi-AUV system is one of the urgent problems to be solved in the future. It is also emphasized that although different variables associated with delay upper bound values are given in [3,4], these studies do not explicitly state how the value of the delay upper bound should be calculated.

This study focuses on the leader-following consensus issue of Markov switched discrete multi-AUV recovery system involving time-varying delays. First, we convert the multi-AUV recovery problem into a leader-follower coordination control problem. Then, we introduce a novel discrete Lyapunov-Krasovskii functional (LKF) to establish a sufficient consensus condition for the discrete multi-AUV recovery system over switching network topologies. Thus, a new approach for explicitly calculating an admissible upper bound of the delay for consensus tracking of discrete multi-AUV recovery systems is proposed.

Main results. The nonlinear coupled kinematics equation of the five-degree-of-freedom AUV model can be written as

$$\dot{\eta} = J(\eta)v, \tag{1}$$

$$M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau, \tag{2}$$

where $\eta = [x_e, y_e, z_e, \theta, \psi]^{\mathrm{T}} \in \mathbb{R}^5$, $v = [u_b, v_b, w_b, q, r]^{\mathrm{T}} \in \mathbb{R}^5$. The notations used, if not given in this study, are all listed in Appendixes A–C. According to (1) and (2), the AUV model can be modeled as

$$\begin{bmatrix} \dot{\eta} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} I & \mathbf{0} \\ \mathbf{0} & -M^{-1} \end{bmatrix} \begin{bmatrix} J(\eta)v \\ H(v,\eta)v \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ M^{-1}\chi(\omega) \end{bmatrix} u_\tau, \tag{3}$$

where $H(v,\eta) = C(v) + D(v) + g(\eta)\bar{v}^{\mathrm{T}}$ with $\bar{v}^{\mathrm{T}}v = 1$. Defining $\omega = \mathrm{col}\{\eta, v\}$, the second-order nonlinear model for AUV can be described as

$$\begin{cases} \dot{\omega} = h(\omega) + g(\omega)u_\tau, \\ \rho = f(\omega), \end{cases} \tag{4}$$

where $h(\omega) = [h_i(\omega)]^{\mathrm{T}} = \begin{bmatrix} I & \mathbf{0} \\ \mathbf{0} & -M^{-1} \end{bmatrix} \begin{bmatrix} J(\eta)v \\ H(v,\eta)v \end{bmatrix}$, $i = 1, 2, \ldots, 10$, $g(\omega) = [g_{ij}(\omega)] = \begin{bmatrix} \mathbf{0} \\ M^{-1}\chi(\omega) \end{bmatrix}$, $i, j = 1, 2, \ldots, 10$, $f(\omega) = [f_i(\omega)]^{\mathrm{T}} = \eta$, $i = 1, 2, \ldots, 5$. According to Lemma S1, the two new vectors after the transformation are $x = \mathrm{col}\{f_1(\omega), f_2(\omega), f_3(\omega), f_4(\omega), f_5(\omega)\}$ and $y = \mathrm{col}\{L_h f_1(\omega), L_h f_2(\omega), L_h f_3(\omega), L_h f_4(\omega), L_h f_5(\omega)\}$. The control input u of the linearized AUVs (4) can be expressed as $u = X(\omega) + \Delta(\omega)u_\tau$, where $X(\omega) = \mathrm{col}\{L_h^2 f_1(\omega), L_h^2 f_2(\omega), L_h^2 f_3(\omega), L_h^2 f_4(\omega), L_h^2 f_5(\omega)\}$. Then the second-order linear standard model for AUVs is established: $\begin{cases} \dot{x}_i = y_i, \\ \dot{y}_i = u_i \end{cases}$ for $i = 1, 2, \ldots, n$ and $x_i, y_i, u_i \in \mathbb{R}^5$. Then we use the forward difference approach and define $z_i(k) = \begin{bmatrix} x_i^{\mathrm{T}}(k) & y_i^{\mathrm{T}}(k) \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{10}$, the dynamics of the leader AUV_0 and followers AUV_i can be described as $z_d(k+1) = Az_d(k)$ and $z_i(k+1) = Az_i(k) + Bu_i(k)$.

Considering the limited underwater communication of the multi-AUV system subject to time-varying delay d_k, $d_k \in [h_1, h_2]$, $d_{k+1} \in [h_1, h_2]$ and $0 \leqslant h_1$, the consensus

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algorithm is designed

$$u_i(k) = -K\sum_{j\in N_i} w_{ij}(k)[z_i(k-d_k) - z_j(k-d_k)] \\ - Kd_{i0}[z_i(k-d_k) - z_d(k-d_k)]. \tag{5}$$

The switching topology $\mathcal{G}(r(k))$ of the multi-AUV system is modeled as a common discrete-time Markov chain $\{r(k), k \geqslant 0\}$. The transition probability rate of the switching topologies is $\mathrm{Prob}\{r(k+1) = a \mid r(k) = c\} = \pi_{ac}(k)$, $\forall\, a, c \in S$, where the transition rate $\pi_{ac}(k) \geqslant 0$ describes the switching topologies from a to c, $\sum_{c=1}^{s} \pi_{ac}(k) = 1$, and the communication topology set S is denoted by $S = \{1, 2, \ldots, s\}$. Let $\delta_i(k) = z_i(k) - z_d(k)$. Then we turn the above multi-AUV recovery system into an error analysis system

$$\delta(k+1) = (I_n \otimes A)\delta(k) + (\mathcal{L}^{r(k)} \otimes BK^{r(k)})\delta(k-d_k), \tag{6}$$

where $K^{r(k)}$ is the gain matrix under switching topologies.

Theorem 1. On the basis that Assumption S1 can be satisfied. For given the scalars $h_1 \geqslant 0$ and control gain $K^{r(k)}$, the leader-following consensus of the multi-AUV system (6) under protocol (5) with switching topologies can be achieved for any $d_k \in [h_1, h_2]$, if there exist symmetrical matrices $P_1 > 0$, $P_2 > 0$, $P_3 > 0$, $R_i > 0$, $Q_i > 0$, $U_i > 0$, S_i $(i = 1, 2)$, and any matrices Y_1, Y_2, such that the following linear matrix inequalities (LMIs) hold:

$$\bar{R}_{21} > 0, \quad \bar{R}_{22} > 0, \tag{7}$$

$$\begin{bmatrix} \Xi_1(h_1) + \Xi_2 + \Xi_3(h_1) & \Upsilon_8^{\mathrm{T}} Y_1 \\ * & -\hat{U}_1 \end{bmatrix} < 0, \tag{8}$$

$$\begin{bmatrix} \Xi_1(h_2) + \Xi_2 + \Xi_3(h_2) & \Upsilon_7^{\mathrm{T}} Y_2^{\mathrm{T}} \\ * & -\hat{U}_2 \end{bmatrix} < 0. \tag{9}$$

Remark 1. In $V_1(k)$, the state variable $\delta(k+1)$ is introduced together with $\delta(k)$, since the augmented vector $\xi_1(k)$ contains not only the previous increasing forms, as $\xi_1(k) = \mathrm{col}\{\delta(k), \sum_{i=k-h_1}^{k-1} \delta(i), \sum_{i=k-h_2}^{k-h_1-1} \delta(i)\}$ in [5], but also uses the variable $\delta(k+1-d_{k+1})$. These state variables are always ignored in most studies. Compared with [5], the construction of $V_2(k)$ and $V_3(k)$ seems only slightly modified, but the cross terms $\delta(k+1)$, $\delta(k+2)$, and $\delta(k+1-d_{k+1})$ are allowed to be introduced, such that the additional forward time information of the system can be utilized and a larger upper bound on the delay can be obtained.

Remark 2. It should be noted that d_k and d_{k+1} may not reach bounds h_1 or h_2 at the same time, since the delay forward difference $d_{k+1} - d_k$ is a hidden variation, which is assumed to be unknown. In the previous studies [5], the criteria are all dependent on d_k or dependent on (d_k, d_{k+1}). However, the d_{k+1}-dependent condition has not been received attention. Because the d_{k+1}-dependent condition may contain the d_k-dependent condition, such that our techniques can extend the solution range of LMIs by exploiting the d_{k+1}-dependent information.

Simulation experiment. The initial parameter values of position and attitude for the mothership are $x_d = [30, 0, 0, -\frac{\pi}{12}, \frac{\pi}{6}]$ and the initial parameter values of velocity for the mothership are $y_d = [1.86, 0, 0, 0, 0]$. The initial values of x_e are haphazardly distributed in the $[0, 30]$, y_e are haphazardly distributed in the $[0, 30]$, z_e are haphazardly distributed in the $[-20, 0]$. Pitch angles θ are in the interval $[-\frac{\pi}{12}, \frac{\pi}{12}]$, and yaw angles ψ are in the interval $[0, \frac{\pi}{6}]$. The original velocities u_b, v_b, w_b, q, r of follower AUVs are zero. The recovery path of AUVs is a helical curve

$$\begin{cases} x_e(k) = 30\cos(0.02\pi k), \\ y_e(k) = 30\sin(0.02\pi k), \\ z_e(k) = -0.03k. \end{cases}$$

Under fixed topology, we set the sampling period $T = 0.1$ s and the controller gain matrix is given as $K = -[0.015 \quad 0.355]$. According to Theorem 1, the upper bound of the time delay calculated under $h_1 = 0.1$ is $h_2 = 0.7379$. Compared with the upper bound of the time delay 0.3 in Theorem 2 of [4], our method provides a larger upper bound value for the time delay.

Under switching topologies, the controller gain K is determined as follows: $K = -[0.024 \quad 0.870]$. According to Theorem 1, the upper bound of the time delay calculated under $h_1 = 0$ is $h_2 = 0.7820$. Then, to prove the validity of our method, we give the 3-D trajectories of the multi-AUV system (6) with $h_2 = 0.7820$ in Figure 1. From Figure 1, we can see that under the time delay and Markov switching topologies, the position states and velocity states between the AUV_i and the mothership AUV_0 still tend to be consistent. Therefore, the follower AUVs can follow the mothership AUV_0 to converge to the desired recovery path.

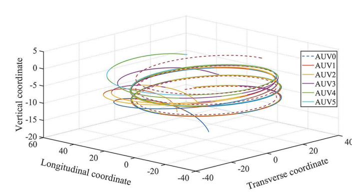

Figure 1 (Color online) Recovery trajectories of multiple AUVs.

Conclusion. The leader-following consensus condition for a discrete multi-AUV system subject to time-varying delay has been examined. Considering the complexity of marine environments, Markov chain switching topology has been used to describe information communication. By introducing a discrete LKF and employing the integral inequality methods, a novel theoretical method for calculating the delay upper bound of a discrete multi-AUV recovery system has been provided.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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