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Special Topic: Mean-Field Game and Control of Large Population Systems: From Theory to Practice

Target defense differential game for autonomous surface vehicles

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Recent advances in autonomous surface vehicles (ASVs) and maritime communication technologies have significantly enhanced ASV performance in dynamic marine environments [1, 2]. These capabilities make ASVs valuable in maritime security missions like interception, escort, and area denial, where interactions are often modeled as target-defense-differential (TDD) games involving a target, an attacker, and a defender [3, 4]. Most existing TDD studies assume linear individual dynamics and restrict the defender's role to attacker interception, limiting model realism. In practice, the defender usually frequently chooses between intercepting the attacker or rendezvousing with the target. This strategic flexibility is indispensable, as optimal actions depend on the players' positions and velocities. To address this issue, we propose a dual-mode defender model with two strategies: (i) attacker capturing and (ii) rendezvous with the target. This manner enriches game dynamics and switching equilibria. We analyze the outcomes under both strategies and thereby provide conditions for determining favorable defender decisions. Additionally, since real ASVs exhibit coupled nonlinear dynamics, we describe all three roles nonlinearly and introduce a model transformation approach that enables Hamiltonian-based analysis. The contributions of this study are two-fold: (i) establishing a dual-strategy defender model for TDD games, and (ii) proposing a nonlinear ASV transformation enabling tractable equilibrium computation.

Consider the kinematics of the defender, the target, and the attacker ASVs as in [5]

$$\begin{aligned}\dot{x}_\ell &= w_\ell \sin(\psi_\ell) - v_\ell \cos(\psi_\ell),\\ \dot{y}_\ell &= w_\ell \cos(\psi_\ell) + v_\ell \sin(\psi_\ell),\\ \dot{\psi}_\ell &= r_\ell,\ \dot{w}_\ell = k_1^\ell w_\ell + k_2^\ell v_\ell r_\ell + k_3^\ell \tau_\omega^\ell,\\ \dot{v}_\ell &= k_4^\ell v_\ell + k_5^\ell w_\ell r_\ell,\ \dot{r}_\ell = k_6^\ell r_\ell + k_7^\ell \tau_r^\ell,\end{aligned} \tag{1}$$

where the subscript $\ell \in \{D, A, T\}$ is the label of the defender, attacker and target, respectively; $[x_\ell, y_\ell]^{\rm T}$ and ψ are the positions and the moving direction of the ASV, respectively; w, v, r denote the surge, sway, and yaw velocities of the ASV in a body-fixed reference frame, respectively; $\boldsymbol{\tau} = [\tau_\omega, \tau_r]^{\rm T}$ the control input, τ_u the actuator power, τ_r the actuator jetting nozzle angle, and w, v, r the nominal forward, sway and angular velocities, respectively. Moreover, $k_1^\ell = -\frac{d_1^\ell}{m_1^\ell}$, $k_2^\ell = \frac{m_2^\ell}{m_1^\ell}$, $k_3^\ell = \frac{1}{m_1^\ell}$, $k_4^\ell = -\frac{d_2^\ell}{m_2^\ell}$, $k_5^\ell = -\frac{m_1^\ell}{m_2^\ell}$, $k_6^\ell = -\frac{d_3^\ell}{m_3^\ell}$, $k_7^\ell = \frac{1}{m_3^\ell}$, where $m_1^\ell, m_2^\ell, m_3^\ell$ are ASV inertia including added mass effects; and $d_1^\ell, d_2^\ell, d_3^\ell$ the hydrodynamic damping coefficients in the surge, sway, and yaw, respectively.

Letting the actual speed $u_\ell := \sqrt{\omega_\ell^2 + v_\ell^2}$, one has $\dot{u}_\ell = \frac{\omega_\ell \dot{\omega}_\ell + v_\ell \dot{v}_\ell}{u_\ell}$. Letting $\tau_r^\ell = \frac{1}{k_7^\ell}(-k_6^\ell r_\ell + k_r^\ell r_\ell + r_I^\ell)$,

$$\begin{aligned}\tau_\omega^\ell = \frac{1}{k_3^\ell}\Big(& -k_1^\ell \omega_\ell - k_2^\ell v_\ell r_\ell + \frac{1}{\omega_\ell}(-k_4^\ell v_\ell^2 - k_5^\ell v_\ell \omega_\ell r_\ell \\ & + k_u^\ell u_\ell^2 + u_I^\ell u_\ell)\Big),\end{aligned} \tag{2}$$

where r_I^ℓ and u_I^ℓ are the control inputs with $k_r^\ell < 0$, $k_\nu^\ell < 0$. Then, Eq. (1) can be rewritten as

$$\begin{aligned}\dot{x}_\ell &= u_\ell \sin(\psi_\ell),\ \dot{y}_\ell = u_\ell \cos(\psi_\ell),\\ \dot{\psi}_\ell &= r_\ell,\ \dot{r}_\ell = k_r^\ell r_\ell + r_I^\ell,\ \dot{u}_\ell = k_u^\ell u_\ell + u_I^\ell.\end{aligned} \tag{3}$$

In the TDD game, the variables R_D, R_T and R_A represent the distance $\|DA\|$, $\|DT\|$ and $\|AT\|$, respectively; $\theta_A = \angle TAD$, $\theta_T = \angle DTA$ with $\theta_A \in [0, \pi], \theta_D \in [0, \pi]$. In this letter, R_D, R_T and R_A are used to determine the result of the TDD game. The TDD gaming state space is denoted by $G = \{R_A > 0, R_D > 0, R_T > 0\}$, with initial conditions $R_{D0} > 0, R_{T0} > 0, R_{A0} > 0$. Specifically, the winning and failing conditions for the defender are defined as follows. By virtue of (3), one has

$$\begin{aligned}&\dot{R}_D(t) = -u_D(t)\cos\phi_D(t) - u_A(t)\cos\phi_A(t),\\ &\dot{R}_T(t) = -u_D(t)\cos\Upsilon_D(t) - u_T(t)\cos\Upsilon_T(t),\\ &\dot{R}_A(t) = u_T(t)\cos\phi_T(t) - u_A\cos(\theta_A(t) - \phi_A(t))\end{aligned} \tag{4}$$

with $\lambda = \angle XAT$, $\phi_A = \theta_A + \lambda - \psi_A$, $\phi_D = \pi - \lambda - \theta_A + \psi_D$, $\phi_T = \psi_T - \lambda$, $\psi_A = \arctan\left(\frac{\dot{y}_A}{\dot{x}_A}\right)$, $\psi_D = \arctan\left(\frac{\dot{y}_D}{\dot{x}_D}\right)$,

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$\psi_T = \arctan\left(\frac{\dot{y}_T}{\dot{x}_T}\right)$, $\Upsilon_D = \psi_D - (\pi - \angle xTD)$, $\Upsilon_T = \angle xTD - \psi_T$.

Define the cost function for the TDD game as

$$J := -R_A(t_f). \tag{5}$$

The terminal time t_f is defined as the moment when one of the following three conditions is first satisfied: $R_A(t) = 0$, $R_D(t) = 0$, or $R_T(t) = 0$. The defender and the target aim to minimize (5), whereas the attacker aims to maximize it. Now, we are ready to give the main problem addressed by this study.

Problem 1 (TDD game problem)**.** Considering the TDD game governed by (4), calculate the equilibrium state-feedback controllers (ESFC) $r_I^\ell = f_r(x_\ell, y_\ell, t)$, $u_I^\ell = f_u(x_\ell, y_\ell, t)$, $\ell \in \{D, A, T\}$ according to the cost function (5). Then, reveal the conditions to determine the winner of the TDD game and determine the defender's destination (T or A).

To solve Problem 1, the following Lemmas and Theorem are proposed, the proofs of Lemmas 1, 2 and Theorem 1 are given in Appendixes A and B.

Lemma 1. Considering the TDD gaming governed by (4), under the D-A capturing scenario, and the condition $u_D^2 + u_T^2 > u_A^2, u_D^2 - u_T^2 - u_A^2 > \frac{u_T^2 u_D^2}{u_A^2}$ and the ESFC

$$\phi_D^* = 0,\ \phi_T^* = 0,$$
$$\phi_A^* = \arccos\left(\frac{\cos\theta_A - \omega}{\sqrt{(\cos\theta_A - \omega)^2 + \sin\theta_A^2}}\right), \tag{6}$$

and

$$u_T^* = \mathrm{sgn}(\cos\phi_T^*)u_T^{\max},\ u_D^* = \mathrm{sgn}(\omega\cos\phi_D)u_D^{\max},$$
$$u_A^* = \mathrm{sgn}((\cos\theta_A - \omega)\cos\phi_A + \sin\theta_A\sin\phi_A)u_A^{\max}, \tag{7}$$

the conditions to determine the defender-winning D_W or failing D_L can be given as $D_L = \{\boldsymbol{x} \mid \frac{h_1(t)}{u_D(t)} - \frac{h_2(t)}{u_A(t)} > 0\}$, $D_W = \{\boldsymbol{x} \mid \frac{h_1(t)}{u_D(t)} - \frac{h_2(t)}{u_A(t)} \leqslant 0\}$ with distances $h_1(t) := R_D(0) + \int_0^t \dot{R}_D(t)\mathrm{d}t, h_2(t) := R_A(0) + \int_0^t \dot{R}_A(t)\mathrm{d}t$.

Lemma 2. Considering the TDD gaming governed by (4), under D-T rendezvous scenario and the condition $\mathrm{sgn}(u_T^2 - u_D^2)\eta > \mathrm{sgn}(u_T^2 - u_D^2)u_T^2\cos\theta_T + 2u_Au_D$ with $\eta = \sqrt{(u_T^2\cos\theta_T + 2u_Au_D)^2 - (u_T^2 - u_D^2)(u_T^2 - u_A^2)}$, and ESFC

$$\Upsilon_T^* = \arccos\left(-\frac{\hat{\omega} + \cos\theta_T}{(\hat{\omega} + \cos\theta_T)^2 + \sin^2\theta_T}\right), \phi_A^* = \theta_A, \Upsilon_D^* = 0, \tag{8}$$

and

$$u_T^* = -\mathrm{sgn}((\hat{\omega} + \cos\theta_T)\cos\Upsilon_T^* - \sin\theta_T\sin\Upsilon_T^*)u_T^{\max},$$
$$u_D^* = \mathrm{sgn}(\hat{\omega}\cos\Upsilon_D^*)u_D^{\max}, u_A^* = u_A^{\max}, \tag{9}$$

the conditions to determine the defender-winning D_W or failing D_L can be given as $D_L = \{\boldsymbol{x} \mid \frac{\hat{h}_1(t)}{u_D(t)} - \frac{\hat{h}_2(t)}{u_A(t)} > 0\}$, $D_W = \{\boldsymbol{x} \mid \frac{\hat{h}_1(t)}{u_D(t)} - \frac{\hat{h}_2(t)}{u_A(t)} \leqslant 0\}$ with $\hat{h}_1(t) := R_T(0) + \int_0^t \dot{R}_T(t)\mathrm{d}t, \hat{h}_2(t) := R_A(0) + \int_0^t \dot{R}_A(t)\mathrm{d}t$.

Lemma 3. Considering the TDD gaming governed by (4), if

$$\frac{R_D}{u_D(t) + u_A(t)\cos\phi_A^*(t)} \leqslant \frac{R_T}{u_D\cos\Upsilon_D^* + u_T\cos\Upsilon_T^*} \tag{10}$$

holds, the defender D will capture the attacker A; otherwise, it proceeds rendezvous with the target T.

Theorem 1. Considering the TDD gaming governed by (4), the upper-level signals $\psi_T^*, \psi_A^*, \psi_D^*$ of target, attacker and defender governed by (3) can be governed to the optimal equilibrium state-feedback controllers (6) and (8) under the lower-level TDD gaming regulators $r_I^A = \ddot{f}_A - k_r^A\dot{f}_A$, $r_I^T = \ddot{f}_T - k_r^T\dot{f}_T$, $r_I^D = \ddot{f}_D - k_r^D\dot{f}_D$, where $f_A = \arctan\left(\frac{y_A - y_D}{x_A - x_D}\right) - \phi_A^*$,

$$f_T = \begin{cases} \arctan\left(\dfrac{y_A - y_T}{x_A - x_T}\right), & \text{if } \sigma(t) = 1,\\ \arctan\left(\dfrac{y_T - y_D}{x_T - x_D}\right) - \Upsilon_T^*, & \text{if } \sigma(t) = 2, \end{cases}$$

$$f_D = \begin{cases} \arctan\left(\dfrac{y_D - y_A}{x_D - x_A}\right), & \text{if } \sigma(t) = 1,\\ \pi - \arctan\left(\dfrac{y_T - y_D}{x_T - x_D}\right), & \text{if } \sigma(t) = 2. \end{cases}$$

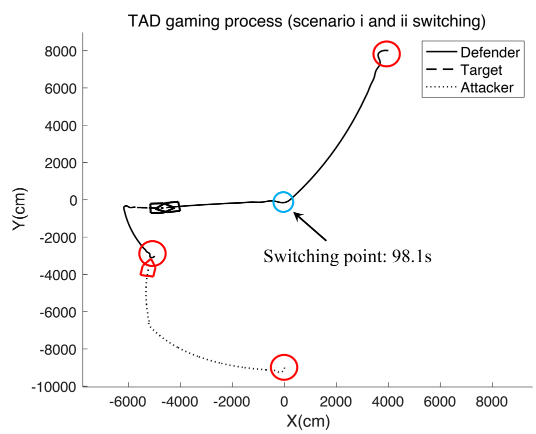

Figure 1 (Color online) The TDD game process under D-T rendezvous and D-A capturing switching scenario, where the red circles denote the initial positions of the three players.

Numerical experiments. The TDD gaming process of D-A capturing and D-A rendezvous switch scenario is given in Figure 1, the switch behavior occurs at 98.1 s. More details can be found in Appendix C.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Cao H, Hu B B, Mo X, et al. The immense impact of reverse edges on large hierarchical networks. Engineering, 2024, 36: 240–249
2 Zhai C, Zhang H T, Xiao G. Cooperative Coverage Control of Multi-Agent Systems and its Applications. Berlin: Springer, 2021
3 Liang L, Deng F, Peng Z, et al. A differential game for cooperative target defense. Automatica, 2019, 102: 58–71
4 Moll A V, Meir P, Daigo S, et al. Circular target defense differential games. IEEE Trans Autom Control, 2023, 68: 4065–4078
5 Liu B, Chen Z, Zhang H T, et al. Collective dynamics and control for multiple unmanned surface vessels. IEEE Trans Contr Syst Technol, 2020, 28: 2540–2547