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• **Supplementary File** •

Target defense differential game for autonomous surface vehicles

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Appendix A The proof of Lemmas 1 and 2

Some common analytical steps are outlined before the proofs of Lemmas 1 and 2. First, we address the scenario i)(D-A capturing), considering the cost function $J := -R_A(t_f)$ with the terminal constraint

$$\rho(\boldsymbol{x}, t_f) = R_D(t_f) = 0,$$

where $\boldsymbol{x} := [R_A, R_D]^\top \in G$ is the TDD game state. For conciseness, the time variable t is omitted in the subsequent derivations. Define the Hamiltonian of the TDD game as

$$\begin{aligned} H(\boldsymbol{x}, \phi_D, \phi_A, \phi_T, \boldsymbol{\alpha}) =& \boldsymbol{\alpha}^\top \dot{\boldsymbol{x}} \\ =& \alpha_A(u_T \cos\phi_T - u_A \cos(\theta_A - \phi_A)) - \alpha_D(u_D \cos\phi_D + u_A \cos\phi_A), \end{aligned} \tag{A1}$$

where $\boldsymbol{\alpha} := [\alpha_A, \alpha_D]^\top$ denotes the adjoint vector for the TDD game, and their dynamics are given by

$$\dot{\alpha}_A = -\frac{\partial H}{\partial R_A} = 0, \dot{\alpha}_D = -\frac{\partial H}{\partial R_D} = 0.$$

The terminal adjoint values are obtained from the transversality condition as

$$\alpha^\top(t_f) = \frac{\partial J}{\partial \boldsymbol{x_f}} + \omega \frac{\partial \rho}{\partial \boldsymbol{x_f}} = [-1\ 0] + \omega[0\ 1],$$

which implies

$$\alpha_A(t) = -1, \alpha_D(t) = \omega, \tag{A2}$$

where ω is an additional adjoint variable. It follows from the cost function

$$J := -R_A(t_f). \tag{A3}$$

that the defender-target group (DTG) should strive to minimize the Hamiltonian (A1), whereas the attacker aims to maximize it. Moreover, the control inputs ϕ_A, ϕ_D, ϕ_T in Hamiltonian (A1) are decoupled, which satisfies Isaacs' condition, i.e.,

$$\begin{aligned} & H(\boldsymbol{x}, \phi_D^*, \phi_A^*, \phi_T^*, \boldsymbol{\alpha}) \\ =& \min_{\phi_D, \phi_T} \max_{\phi_A} H(\boldsymbol{x}, \phi_D^*, \phi_A^*, \phi_T^*, \boldsymbol{\alpha}) \\ =& \max_{\phi_A} \min_{\phi_D, \phi_T} H(\boldsymbol{x}, \phi_D^*, \phi_A^*, \phi_T^*, \boldsymbol{\alpha}),\ \forall \boldsymbol{x} \in G. \end{aligned} \tag{A4}$$

Second, we address the scenario ii) (D-T rendezvous) and define the cost functional for the TDD game as

$$\hat{J} := -R_A(t_f). \tag{A5}$$

The terminal constraint for the TDD game are

$$\hat{\rho}(\hat{\boldsymbol{x}}, t_f) = R_T(t_f) = 0,$$

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where $\hat{\boldsymbol{x}} := [R_A, R_T]^\intercal \in G$ is the TDD game state under scenario ii). Define the Hamiltonian of the TDD game as

$$\begin{aligned} H(\hat{\boldsymbol{x}}, \Upsilon_D, \phi_A, \Upsilon_T, \hat{\boldsymbol{\alpha}}) =& \hat{\boldsymbol{\alpha}}^\intercal \dot{\hat{\boldsymbol{x}}} \\ =& -\alpha_A(u_T\cos(\theta_T+\Upsilon_T)+u_A\cos(\theta_A-\phi_A)) - \alpha_T(u_D\cos\Upsilon_D+u_T\cos\Upsilon_T), \end{aligned} \tag{A6}$$

where $\hat{\boldsymbol{\alpha}} := [\alpha_A, \alpha_T]^\intercal$ denotes the adjoint vector for the TDD game, and their dynamics are given by

$$\dot{\alpha}_A = -\frac{\partial H}{\partial R_A} = 0, \dot{\alpha}_T = -\frac{\partial H}{\partial R_T} = 0.$$

The terminal adjoint values are obtained from the transversality condition as

$$\alpha^\intercal(t_f) = \frac{\partial \hat{J}}{\partial \hat{\boldsymbol{x}}_{\boldsymbol{f}}} + \omega\frac{\partial \hat{\rho}}{\partial \hat{\boldsymbol{x}}_{\boldsymbol{f}}} = [-1\ 0] + \hat{\omega}[0\ 1],$$

which implies

$$\alpha_A(t) = -1, \alpha_T(t) = \hat{\omega}, \tag{A7}$$

where $\hat{\omega}$ is an additional adjoint variable. Analogously to (A4), it follows from the cost function (A5) that

$$\begin{aligned} & H(\hat{\boldsymbol{x}}, \Upsilon_D, \phi_A, \Upsilon_T, \hat{\boldsymbol{\alpha}}) \\ =& \min_{\Upsilon_D,\Upsilon_T}\max_{\phi_A} H(\hat{\boldsymbol{x}}, \Upsilon_D, \phi_A, \Upsilon_T, \hat{\boldsymbol{\alpha}}) \\ =& \max_{\phi_A}\min_{\Upsilon_D,\Upsilon_T} H(\hat{\boldsymbol{x}}, \Upsilon_D, \phi_A, \Upsilon_T, \hat{\boldsymbol{\alpha}}),\ \forall\hat{\boldsymbol{x}}. \end{aligned} \tag{A8}$$

Then, a necessary definition is given as

Definition 1. The Defender-failing and winning conditions are defined as

- Defender-failing: $\frac{R_A(t)}{u_A(t)} < \frac{R_D(t)}{u_D(t)}$ and $\frac{R_A(t)}{u_A(t)} < \frac{R_T(t)}{u_D(t)}$.
- Defender-winning: $\frac{R_A(t)}{u_A(t)} \geqslant \frac{R_D(t)}{u_D(t)}$ or $\frac{R_A(t)}{u_A(t)} \geqslant \frac{R_T(t)}{u_D(t)}$.

Remark 1. This paper considers two distinct behavioral modes for the defender: D–A capturing and D–T rendezvous. The success conditions for the defender in these two scenarios are, respectively, $\frac{R_A(t)}{u_A(t)} \geqslant \frac{R_D(t)}{u_D(t)}$ and $\frac{R_A(t)}{u_A(t)} \geqslant \frac{R_T(t)}{u_D(t)}$, both of which can be intuitively interpreted as the defender reaching its destination faster than the attacker. In contrast, the failure condition is unified across both modes and is given by $\frac{R_A(t)}{u_A(t)} < \frac{R_D(t)}{u_D(t)}$ and $\frac{R_A(t)}{u_A(t)} < \frac{R_T(t)}{u_D(t)}$, which implies that the attacker reaches the target faster than the defender, regardless of what kind of strategies adopted.

Now, we are ready to give the proofs of Lemmas 1 and 2. Specifically, Lemmas 1 and 2 formulate the equilibrium state-feedback controller (ESFC) and delineate the winning conditions for the defender in the D–A capturing scenario and the D-T rendezvous scenario.

proof of Lemma 1

Proof. Rewrite the Hamiltonian (A1) as

$$\begin{aligned} & H(\boldsymbol{x}, \phi_D, \phi_A, \phi_T, \boldsymbol{\alpha}) \\ =& -u_T\cos\phi_T - \omega u_D\cos\phi_D + (u_A\cos\theta_A - \omega u_A)\cos\phi_A + u_A\sin\theta_A\sin\phi_A. \end{aligned} \tag{A9}$$

It follows from (A4) that the optimal solutions ϕ_T^*, ϕ_A^* and ϕ_D^* of are the ESFC. Solving (A9), one has

$$\begin{aligned} & \cos\phi_T^* = 1, \cos\phi_D^* = \mathrm{sgn}(\omega), \cos\phi_A^* = \frac{\cos\theta_A - \omega}{\sqrt{(\cos\theta_A-\omega)^2+\sin\theta_A^2}}, \\ & \sin\phi_A^* = \frac{\sin\theta_A}{\sqrt{(\cos\theta_A-\omega)^2+\sin\theta_A^2}} \end{aligned} \tag{A10}$$

and $u_\ell, \ell\in\{D, A, T\}$ satisfy

$$\begin{aligned} & u_T^* = \mathrm{sgn}(\cos\phi_T^*)u_T^{\max}, u_D^* = \mathrm{sgn}(\omega\cos\phi_D)u_D^{\max}, \\ & u_A^* = \mathrm{sgn}((\cos\theta_A-\omega)\cos\phi_A + \sin\theta_A\sin\phi_A)u_A^{\max}. \end{aligned} \tag{A11}$$

It follows from $\cos\phi_D^*$ in (A10) that ω must be positive to drive the defender D chasing the attacker A. The terminal Hamiltonian satisfies

$$H(t_f) = 0.$$

Then, substituting (A10) and (A2), one has

$$H(\boldsymbol{x}, \phi_D^*, \phi_A^*, \phi_T^*, t_f) = -u_T(t_f) - \omega u_D(t_f) + u_A\sqrt{(\cos\theta_A-\omega)^2+\sin^2\theta_A},$$

which yields

$$(u_D^2 - u_A^2)\omega^2 + 2(u_Tu_D + u_A^2\cos\theta_A)\omega + u_T^2 - u_A^2 = 0.$$

It follows from the terminal constraint $R_D(t_f)=0$ that $\theta_A(t_f)=0$, then

$$\omega = \tfrac{-b+\sqrt{b^2-4ac}}{2a} \tag{A12}$$

with

$$a := u_D^2 - u_A^2, b := 2u_T u_D, c := u_T^2 - u_A^2,$$

the condition

$$u_D^2 + u_T^2 > u_A^2, u_D^2 - u_T^2 - u_A^2 > \frac{u_T^2 u_D^2}{u_A^2} \tag{A13}$$

ensuring the existence of the solutions of (A12), and $\omega > 0$.

Furthermore, substituting (A10) and (A12) into

$$\begin{aligned} \dot{R}_D(t) &= -u_D(t)\cos\phi_D(t) - u_A(t)\cos\phi_A(t), \\ \dot{R}_T(t) &= -u_D(t)\cos\Upsilon_D(t) - u_T(t)\cos\Upsilon_T(t), \\ \dot{R}_A(t) &= u_T(t)\cos\phi_T(t) - u_A\cos(\theta_A(t)-\phi_A(t)), \end{aligned} \tag{A14}$$

one has

$$\begin{aligned} \dot{R}_D &= -u_D(t) - u_A(t)\cos\phi_A^*(t), \\ \dot{R}_A &= u_T(t) - u_A(t)\cos(\theta_A(t)-\phi_A^*(t)). \end{aligned} \tag{A15}$$

In other words, for scenario i), the winning and failing conditions for the defender can be expressed as in Lemma 1 according to Definition 1, which completes the proof.

proof of Lemma 2

Proof. Rewrite the Hamiltonian (A6) as

$$\begin{aligned} & H(\hat{\boldsymbol{x}}, \Upsilon_D, \phi_A, \Upsilon_T, \hat{\boldsymbol{\alpha}}) \\ =& (\hat{\omega} u_T + u_T\cos\theta_T)\cos\Upsilon_T - u_T\sin\theta_T\sin\Upsilon_T \\ & -\hat{\omega} u_D\cos\Upsilon_D + u_A\cos\theta_A\cos\phi_A + u_A\sin\theta_A\sin\phi_A. \end{aligned} \tag{A16}$$

It follows from (A4) that the optimal solutions ϕ_T^*, ϕ_A^* and ϕ_D^* of are the ESFC. Solving (A9), one has

$$\begin{aligned} & \sin\Upsilon_T^* = \tfrac{\sin\theta_T}{(\hat{\omega}+\cos\theta_T)^2+\sin^2\theta_T}, \cos\Upsilon_T^* = -\tfrac{\hat{\omega}+\cos\theta_T}{(\hat{\omega}+\cos\theta_T)^2+\sin^2\theta_T}, \\ & \cos\Upsilon_D^* = \mathrm{sgn}(\hat{\omega}), \sin\phi_A^* = \sin\theta_A, \cos\phi_A^* = \cos\theta_A \end{aligned} \tag{A17}$$

and $u_\ell, \ell \in \{D, A, T\}$ should satisfy

$$\begin{aligned} & u_T^* = -\mathrm{sgn}((\hat{\omega}+\cos\theta_T)\cos\Upsilon_T^* - \sin\theta_T\sin\Upsilon_T^*)u_T^{\max}, \\ & u_D^* = \mathrm{sgn}(\hat{\omega}\cos\Upsilon_D^*)u_D^{\max}, u_A^* = u_A^{\max}. \end{aligned} \tag{A18}$$

It follows from (A17) that $\mathrm{sgn}(\hat{\omega}) > 0$ should be satisfied to drive the defender rendezvous with the target.

Then, substituting $\Upsilon_T^* = \arccos\left(-\tfrac{\hat{\omega}+\cos\theta_T}{(\hat{\omega}+\cos\theta_T)^2+\sin^2\theta_T}\right), \phi_A^* = \theta_A, \Upsilon_D^* = 0$, and (A7) into (A16) with $t = t_f$, one has

$$\begin{aligned} & H(\hat{\boldsymbol{x}}, \Upsilon_D^*, \phi_A^*, \Upsilon_T^*, \hat{\boldsymbol{\alpha}}_f) \\ =& (\hat{\omega} u_T + u_T\cos\theta_T)\cos\Upsilon_T^* - u_T\sin\theta_T\sin\Upsilon_T^* - \hat{\omega} u_D + u_A \\ =& u_T\sqrt{(\hat{\omega}+\cos\theta_T)^2+\sin^2\theta_T} - \hat{\omega} u_D + u_A, \end{aligned}$$

which yields

$$\hat{\omega} = \tfrac{-\hat{b}+\sqrt{\hat{b}^2-4\hat{c}}}{2\hat{a}}, \tag{A19}$$

with

$$\hat{a} := u_T^2 - u_D^2, \hat{b} := 2(u_T^2\cos\theta_T + 2u_A u_D), \text{ and } \hat{c} := u_T^2 - u_A^2.$$

It follows from the terminal constraint $R_T(t_f) = 0$ that $\theta_T(t_f) = 0$, which yields

$$\hat{b}^2 - 4\hat{a}\hat{c} = 4(3u_A^2u_D^2 + 4u_Au_Du_T^2 + u_T^2u_D^2 + u_T^2u_A^2) \geqslant 0.$$

Then,

$$\begin{aligned} & \mathrm{sgn}(u_T^2-u_D^2)\sqrt{(u_T^2\cos\theta_T + 2u_Au_D)^2 - (u_T^2-u_D^2)(u_T^2-u_A^2)} \\ >\ & \mathrm{sgn}(u_T^2-u_D^2)u_T^2\cos\theta_T + 2u_Au_D \end{aligned} \tag{A20}$$

immediately leads to $\hat{\omega} > 0$.

Furthermore, substituting (A17) and (A19) into (A14), one has

$$\begin{aligned} \dot{R}_T &= -u_D\cos\Upsilon_D^* - u_T\cos\Upsilon_T^*, \\ \dot{R}_A &= -u_T\cos(\Upsilon_T^*+\theta_T) - u_A. \end{aligned} \tag{A21}$$

In other words, for scenario i), the winning and failing conditions for the defender can be expressed as in Lemma 2 according to Definition 1, which completes the proof.

Appendix B The proof of Theorem 1

Proof. The relationships between the angles are described as

$$\begin{aligned}
&\phi_A = \angle xAD - \psi_A = \arctan\left(\tfrac{y_A-y_D}{x_A-x_D}\right) - \psi_A,\\
&\phi_T = \psi_T - \angle xAT = \psi_T - \arctan\left(\tfrac{y_A-y_T}{x_A-x_T}\right),\\
&\phi_D = \angle ADx - \psi_D = \arctan\left(\tfrac{y_D-y_A}{x_D-x_A}\right) - \psi_D,\\
&\Upsilon_T = \angle xTD - \psi_T = \arctan\left(\tfrac{y_T-y_D}{x_T-x_D}\right) - \psi_T,\\
&\Upsilon_D = \psi_D - (\pi - \angle xTD)\\
&\quad\;\; = \psi_D - \left(\pi - \arctan\left(\tfrac{y_T-y_D}{x_T-x_D}\right)\right).
\end{aligned} \tag{B1}$$

It follows from

$$\begin{aligned}
\dot{x}_\ell &= u_\ell \sin(\psi_\ell),\\
\dot{y}_\ell &= u_\ell \cos(\psi_\ell),\\
\dot{\psi}_\ell &= r_\ell,\\
\dot{r}_\ell &= k_r^\ell r_\ell + r_I^\ell,\\
\dot{u}_\ell &= k_u^\ell u_\ell + u_I^\ell,
\end{aligned} \tag{B2}$$

that

$$\psi_A = \int r_A dt + C \tag{B3}$$

with constant C. Combining with conditions $\psi_A = \angle xAD - \phi_A^*$ in (B1) and ϕ_A^* in Lemmas 1 and 2, one has $C = 0$ and $r_A = \dot{f}_A$, which implies

$$\dot{r}_A = \ddot{f}_A. \tag{B4}$$

Then, substituting (B1) and (B4) into (B2), the lower-level TDD gaming regulator r_I^A for attacker can be obtained as

$$r_I^A = \ddot{f}_A - k_r^A \dot{f}_A,\ r_I^T = \ddot{f}_T - k_r^T \dot{f}_T,\ r_I^D = \ddot{f}_D - k_r^D \dot{f}_D, \tag{B5}$$

where

$$\begin{aligned}
f_A &= \arctan\left(\tfrac{y_A-y_D}{x_A-x_D}\right) - \phi_A^*,\\
f_T &= \begin{cases} \arctan\left(\tfrac{y_A-y_T}{x_A-x_T}\right), & \text{if } \sigma(t)=1,\\ \arctan\left(\tfrac{y_T-y_D}{x_T-x_D}\right) - \Upsilon_T^*, & \text{if } \sigma(t)=2,\end{cases}\\
f_D &= \begin{cases} \arctan\left(\tfrac{y_D-y_A}{x_D-x_A}\right), & \text{if } \sigma(t)=1,\\ \pi - \arctan\left(\tfrac{y_T-y_D}{x_T-x_D}\right), & \text{if } \sigma(t)=2.\end{cases}
\end{aligned}$$

The lower-level TDD game regulators r_I^T, r_I^D, for both the target and the defender, respectively, can be derived using the (B1)–(B4) and are thus omitted here, thereby solving Problem 1.

Remark 2. To focus on theoretical clarity, this work neglects external disturbances in ASV dynamics. Nonetheless, the proposed framework can be extended to handle bounded additive disturbances. Suppose the surge and yaw dynamics are perturbed as:

$$\dot{u}_\ell = k_{u,\ell} u_\ell + u_{\ell I} + d_{u,\ell}(t), \quad \dot{r}_\ell = k_{r,\ell} r_\ell + r_{\ell I} + d_{r,\ell}(t),$$

where $d_{u,\ell}(t), d_{r,\ell}(t)$ are bounded: $|d_{u,\ell}(t)| \leqslant \delta_{u,\ell}, |d_{r,\ell}(t)| \leqslant \delta_{r,\ell}$. The upper-level Hamiltonian can be extended via:

$$H_{\text{robust}} = \min_{u_T, u_D} \max_{u_A} \max_{d\in\mathcal{D}} \alpha^\top f(x,u,d),$$

to preserve Isaacs' condition. For the lower-level, robust tracking (e.g., sliding mode control) compensates external disturbances:

$$r_{\ell I} = -k_r s_\ell - \hat{d}_{r,\ell}(t) + \ddot{\varphi}_\ell^*, \quad s_\ell = \dot{\psi}_\ell - \dot{\varphi}_\ell^*.$$

These modifications do not alter the core structure of the present equilibrium strategy.

Appendix C Numerical experiments

The ASV dynamics are described by [1]

$$\begin{aligned}
\dot{x}_\ell &= w_\ell \sin(\psi_\ell) - v_\ell \cos(\psi_\ell),\\
\dot{y}_\ell &= w_\ell \cos(\psi_\ell) + v_\ell \sin(\psi_\ell),\\
\dot{\psi}_\ell &= r_\ell,\\
\dot{w}_\ell &= k_1^\ell w_\ell + k_2^\ell v_\ell r_\ell + k_3^\ell \tau_\omega^\ell,\\
\dot{v}_\ell &= k_4^\ell v_\ell + k_5^\ell w_\ell r_\ell,\\
\dot{r}_\ell &= k_6^\ell r_\ell + k_7^\ell \tau_r^\ell,
\end{aligned} \tag{C1}$$

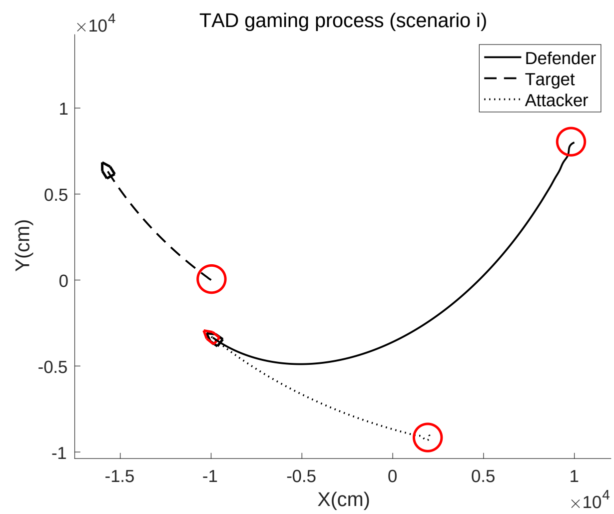

Figure C1 The TDD game process under D-A capturing scenario (scenario i), the red circles denote the initial positions of the three players.

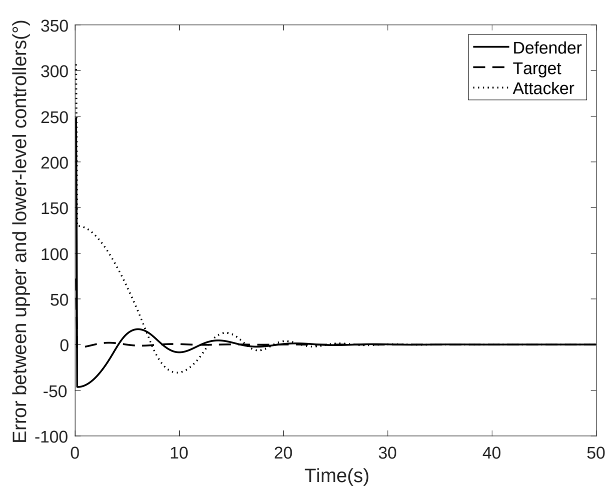

Figure C2 The temporal evolution of the tracking errors $e_\ell = \phi_\ell - \phi_\ell^*$ between the upper-level reference signal and lower-level output.

were identified with the parameters: $k_1^\ell = -1.2235$, $k_2^\ell = 0.95$, $k_3^\ell = 0.00122$, $k_4^\ell = -1.437$, $k_5^\ell = -0.105$, $k_6^\ell = -1.399$, and $k_7^\ell = 0.043$. According to Lemmas 1 and 2, the velocities were picked as $u_D = 150$, $u_A = 80$, and $u_T = 50$, ensuring that the conditions outlined in Eqs. (A13) and (A20) are satisfied.

Appendix C.1 D-A capturing scenario

The initial states of the TDD gaming scenario were given as $[x_D, y_D] = [10000cm, 8000cm]$, $[x_T, y_T] = [-10000cm, 0cm]$, and $[x_A, y_A] = [2000cm, -9000cm]$, resulting in distances of $R_D = 18788cm$, $R_A = 15000cm$, and $R_T = 21541cm$, thus fulfilling the condition specified in

$$\frac{R_D}{u_D(t)+u_A(t)\cos\phi_A^*(t)} \leqslant \frac{R_T}{u_D\cos\Upsilon_D^*+u_T\cos\Upsilon_T^*} \tag{C2}$$

According to Lemma 1, the defender D initially prioritizes capturing the attacker A. During period $[0, t_f]$, the defender-win condition $\frac{h_1(t)}{u_D(t)} - \frac{h_2(t)}{u_A(t)} \leqslant 0$ in Lemma 1 always hold. As shown in the TAD gaming process in Figure C1, D captures A in 30 seconds. As shown in Figure C2, the tracking error $e_\ell = \phi_\ell - \phi_\ell^*$ between the upper and lower levels settles to zero in 30 seconds, which verifies the effectiveness of Theorem 1.

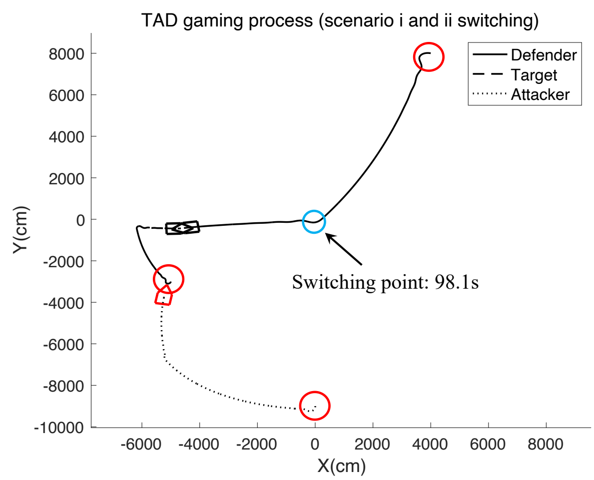

Figure C3 The TDD game process under D-T rendezvous and D-A capturing switching scenario, the red circles denote the initial positions of the three players, and the switching behavior occurs at 98.1s.

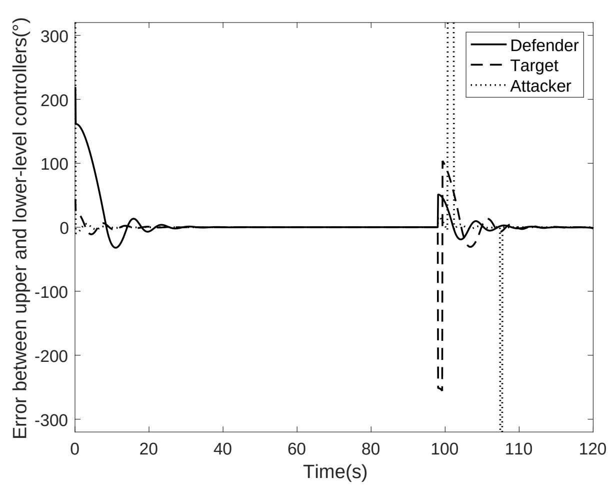

Figure C4 The temporal evolution of the tracking errors $e_\ell = \phi_\ell - \phi_\ell^*$ (scenario i), $e_A = \phi_A - \phi_A^*$, $e_D = \Upsilon_D - \Upsilon_D^*$, $e_T = \Upsilon_T - \Upsilon_T^*$ (scenario ii) between the upper-level reference signal and lower-level output.

Appendix C.2 D-T rendezvous and D-A capturing switch scenario

The initial states of the TDD gaming scenario were given as $[x_D, y_D] = [4000cm, 8000cm]$, $[x_T, y_T] = [-5000cm, -3000cm]$, and $[x_A, y_A] = [0cm, -9000cm]$, resulting in distances of $R_D = 17029cm$, $R_A = 13454cm$, and $R_T = 13601cm$, thus fulfills the condition specified in Eq. (C2) at the initial instant. However, the condition (C2) is violated at 98.1s. Based on Lemma 1, the defender D initially prioritizes capturing the attacker A and rendezvous with the target T after 98.1s, and the TAD gaming process is shown in Figure C3. During period $[0s, 98.1s]$, the defender-win condition $\frac{h_1(t)}{u_D(t)} - \frac{h_2(t)}{u_A(t)} \leqslant 0$ in Lemma 1 holds. During period $[98.1s, t_f]$, the defender-win condition $\frac{\hat{h}_1(t)}{u_D(t)} - \frac{\hat{h}_2(t)}{u_A(t)} \leqslant 0$ in Lemma 2 also holds. As shown in Figure C4, the tracking errors $e_\ell = \phi_\ell - \phi_\ell^*$ (scenario i), $e_A = \phi_A - \phi_A^*$, $e_D = \Upsilon_D - \Upsilon_D^*$, $e_T = \Upsilon_T - \Upsilon_T^*$ (scenario ii) between the upper and lower levels experiences oscillation in 98.1s and then settles to zero in 120 seconds, which verifies the effectiveness of Theorem 1.

Appendix C.3 Comparation

The initial states of the TDD gaming scenario were given as $[x_D, y_D] = [5000cm, 17000cm]$, $[x_T, y_T] = [0cm, 0cm]$, and $[x_A, y_A] = [0cm, -4500cm]$. When the defender uses the proposed switching strategy (between Lemmas 1 and 2), it successfully meets the target before the attacker reaches it (as shown in Figure C6). However, the pure pursuit strategy fails to provide effective defense (as shown in Figure C5), which shows the efficacy of the proposed method.

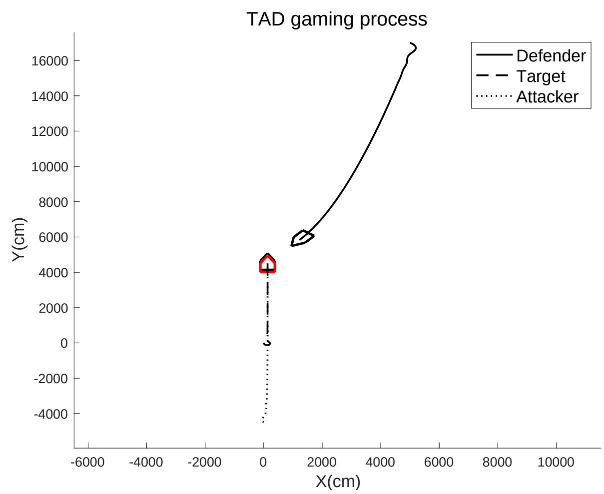

Figure C5 The TDD game process under the pure pursuit method proposed in [2].

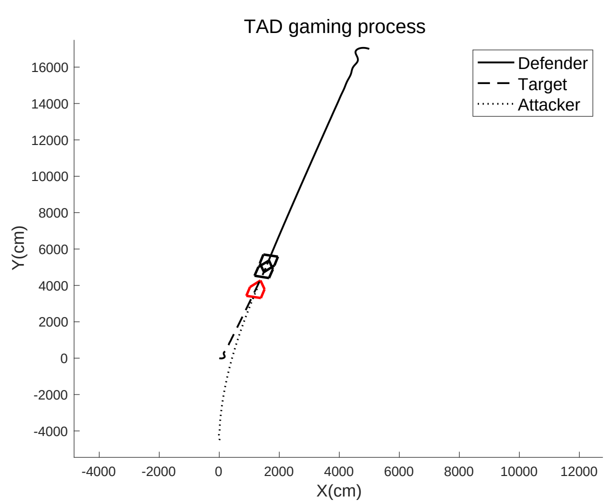

Figure C6 The TDD game process under the method proposed in this letter.

References

1 B. Liu, Z. Chen, H.-T. Zhang, et al. Collective Dynamics and Control for Multiple Unmanned Surface Vessels. IEEE Transactions on Control Systems Technology, 2020, 28(6):2540–2547

2 S. Vishnu, P. Dimitra. Multiplayer Target-Attacker-Defender Differential Game: Pairing Allocations and Control Strategies for Guaranteed Intercept. AIAA Scitech 2019 Forum, 2019.