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Saturation function-based intermittent control on fixed-time output synchronization of multilayered networks

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Recently, output synchronization has been widely applied in communication networks and multiple robotic manipulators [1–3]. Liu et al. [4] introduced an event-triggered aperiodic intermittent control (IC) mechanism to investigate the fixed-time (FT) synchronization of multilayered networks (MLNs). This study focuses on fixed-time output synchronization (FOS) in MLNs by establishing FT stability and developing novel IC schemes. First, a model of MLNs with intra/inter-layer output coupling is proposed to more accurately capture and represent the complexity and precision of MLN structures. Subsequently, a switching-type FT stability theorem is established, with its stability conditions relaxed, and a high-precision estimate of synchronization time is obtained by using the comparison principle. Furthermore, two novel saturation-based IC protocols are designed to mitigate the chattering effects associated with the traditional FT controller that employs a sign function.

Notations. R and R^n respectively denote the set of all real constants and a space formed by all n-dimensional real vectors. $\mathbb{N}$ is defined as a set of natural numbers. $R^{n\times n}$ is the set of all $n\times n$ real matrices, $\hat{n}=\{1,2,\ldots,n\}$, $\bar{P}=\{1,2,\ldots,P\}$ and $\bar{M}=\{1,2,\ldots,M\}$, where n, P and M are positive integers. For any $\xi\in R$, $\mathrm{sign}(\xi)$ is the sign function of ξ, and $\mathrm{sat}(\xi)$ is the saturation function of ξ. For any $\zeta=(\zeta_1,\zeta_2,\ldots,\zeta_n)^{\mathrm{T}}\in R^n$, $\varepsilon=(\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n)^{\mathrm{T}}\in R^n$, $\varepsilon^s=\frac{\varepsilon+\varepsilon^{\mathrm{T}}}{2}$, $[\varepsilon]^\theta=(|\varepsilon_1|^\theta,\ldots,|\varepsilon_n|^\theta)^{\mathrm{T}}$, $\theta>0$, and $\zeta\circ\varepsilon=(\zeta_1\varepsilon_1,\zeta_2\varepsilon_2,\ldots,\zeta_n\varepsilon_n)^{\mathrm{T}}$. $\mathrm{diag}\{\cdot\}$ stands for a diagonal matrix, $\mathbf{0}_n$ is the column vector with all zero elements, $\otimes$ represents the Kronecker product for matrices.

Problem formulation. The r-layer can be visualized by a graph $\mathcal{G}^r=(\nu^r,\wp^r)$, where ν^r and $\wp^r$ denote the set of vertices and undirected edges, respectively. Let $\delta^r=(\delta^r_{pq})\in R^{P\times P}$, where $\delta^r_{pq}=\delta^r_{qp}>0$ if and only if the p-node and the q-node of the r-layer are connected, otherwise, $\delta^r_{pq}=\delta^r_{qp}=0$ $(p\neq q)$ for $p,q\in\bar{P}$. Let $\gamma=(\gamma_{r\alpha})\in R^{M\times M}$ represent the adjacency matrix between layers, in which $\gamma_{r\alpha}=\gamma_{\alpha r}>0$ if and only if an edge exists between the p-node of the r-layer and the p-node of the α-layer, otherwise, $\gamma_{r\alpha}=\gamma_{\alpha r}=0$ $(r\neq\alpha)$ for $r,\alpha\in\bar{M}$. The corresponding Laplacian matrix $\widehat{\delta}^r=(\widehat{\delta}^r_{pq})_{P\times P}$ is given by $\widehat{\delta}^r_{pq}=\widehat{\delta}^r_{qp}=-\delta^r_{pq}<0$ for $p\neq q$, and $\widehat{\delta}^r_{pp}=\sum_{q=1,q\neq p}^{P}\delta^r_{pq}>0$. $\widehat{\gamma}=(\widehat{\gamma}_{r\alpha})_{M\times M}$ is defined as $\widehat{\gamma}_{r\alpha}=\widehat{\gamma}_{\alpha r}=-\gamma_{r\alpha}<0$ for $r\neq\alpha$, and $\widehat{\gamma}_{rr}=\sum_{\alpha=1,\alpha\neq r}^{M}\gamma_{r\alpha}>0$.

Consider a category of MLNs composed of M-layers, which is described as

$$\begin{cases}\dot{x}^r_p(t) &= f^r_p(x^r_p(t))+\sum_{\alpha=1,\alpha\neq r}^{M}f^\alpha_p(x^\alpha_p(t))+u^r_p(t)\\ &\quad +b\sum_{q=1,q\neq p}^{P}\delta^r_{pq}\bar{\mathcal{H}}(y^r_q(t)-y^r_p(t))\\ &\quad +c\sum_{\alpha=1,\alpha\neq r}^{M}\gamma_{r\alpha}\bar{\mathcal{F}}(y^\alpha_p(t)-y^r_p(t)),\\ y^r_p(t) &= Ax^r_p(t),\qquad p\in\bar{P},r\in\bar{M},\end{cases}\tag{1}$$

where $x^r_p(t)=(x^r_{p1}(t),x^r_{p2}(t),\ldots,x^r_{pn}(t))^{\mathrm{T}}\in R^n$ represents the state vector of the p node in the r-layer, $y^r_p(t)\in R^m$ is the output state vector of the p node in the r-layer, $f^r_p(x^r_p)=(f^r_{p1}(x^r_p),\ldots,f^r_{pn}(x^r_p))^{\mathrm{T}}\in R^n$ is a continuous nonlinear vector function that describes the intrinsic dynamic characteristics of node p in the r-layer, and $f^\alpha_p(\cdot):R^n\to R^n$ represents the dynamic behavior of node p as it interacts with the duplicate node at the α-layer. Here, $b>0$ represents the inter-layer coupling intensity, $c>0$ stands for the intra-layer coupling intensity, and $u^r_p(t)$ represents the control input for node p-node at r-layer. Additionally, $A\in R^{m\times n}$ is the output coefficient matrix, $0<\bar{\mathcal{H}}\in R^{n\times m}$ is the inter-layer coupling matrix, and $0<\bar{\mathcal{F}}\in R^{n\times m}$ is the intra-layer coupling matrix.

The dynamic state of the isolated node in the MLN (1) is considered the synchronization target, and it satisfies

$$\begin{cases}\dot{x}_0(t)=f(x_0(t)),\\ y_0(t)=Ax_0(t).\end{cases}\tag{2}$$

Assuming that $x_0(t)$ is bounded, then there exists a time $\mathcal{T}^\dagger(x_0(0))$ for any given initial condition $x_0(0)$, such that $\|x_0(t)\|\leqslant\hat{u},\forall t>\mathcal{T}^\dagger(x_0(0))$, where $\hat{u}>0$. Let $\hbar^r_p(t)=$

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$y_p^r(t) - y_0(t)$, where $p \in \bar{P}$ and $r \in \bar{M}$. The error system can be formulated as

$$\begin{aligned}\dot{\hbar}_p^r(t) = {} & A\big[f_p^r(x_p^r(t)) + \textstyle\sum_{\alpha=1,\alpha\neq r}^{M} f_p^\alpha(x_p^\alpha(t)) - f(x_0(t))\big] \\ & + b\textstyle\sum_{q=1,q\neq p}^{P} \delta_{pq}^r A\bar{\mathcal{H}}(\hbar_q^r(t) - \hbar_p^r(t)) \\ & + c\textstyle\sum_{\alpha=1,\alpha\neq r}^{M} \gamma_{r\alpha} A\bar{\mathcal{F}}(\hbar_p^\alpha(t) - \hbar_p^r(t)) + Au_p^r(t).\end{aligned}$$

Since $x_0(t)$ is bounded, and both f_p^α and f satisfy the Lipschitz condition, it follows that $f_p^\alpha(x_0(t)) - f(x_0(t))$ is bounded above. More precisely, $\sup_{t\to\mathcal{T}^\dagger} M \max_{\alpha} \|f_p^\alpha(x_0(t)) - f(x_0(t))\| = \Theta$ for $\mathcal{T}^\dagger > \mathcal{T}^\dagger(x_0(0))$. The following saturation function is used:

$$\mathrm{sat}(z) = \begin{cases} \mathrm{sign}(z), & |z| \geqslant \Bbbk, \\ g(z), & |z| < \Bbbk, \end{cases} \tag{3}$$

where $\Bbbk > 0$, $g(z)$ is continuous on the interval $(-\Bbbk, \Bbbk)$, $g(0) = 0$, $\lim_{z\to\Bbbk^-} g(z) = 1$ and $\lim_{z\to-\Bbbk^+} g(z) = -1$, and $\mathrm{sign}(z)g(z) \geqslant w|z|^{\beta_1}$ when $z \in (-\Bbbk, \Bbbk)$ for some constants $w > 0$, $0 < \beta_1 < 1$.

The IC strategy in this study is formulated as

$$\begin{cases} u_p^r(t) = -B\hbar_p^r(t) - \beta_2 \mathrm{sat}(\hbar_p^r(t)) \circ [\hbar_p^r(t)]^{\bar{s}_1} \\ \qquad\qquad -\beta_3 \mathrm{sat}(\hbar_p^r(t)) \circ [\hbar_p^r(t)]^{\bar{s}_2}, t \in [t_\iota, z_\iota), \\ u_p^r(t) = 0, t \in [z_\iota, t_{\iota+1}), \end{cases} \tag{4}$$

where $B \in R^{n\times m}$, $\beta_2 > 0$, $\beta_3 > 0$, $0 < \bar{s}_1 < 1$, and $\bar{s}_2 > 1$. The intervals $[t_\iota, z_\iota)$ and $[z_\iota, t_{\iota+1})$ represent the ιth work interval and rest interval, respectively, where $t_0 = 0$ and $t_\iota < z_\iota < t_{\iota+1}$ for $\iota \in \mathbb{N}$.

Definition 1. For any $t_b > t_a \geqslant 0$, there exist $\tau \in (0, 1)$ and $o \geqslant 0$ such that $\mathrm{T}^\dagger(t_b, t_a) \geqslant \tau(t_b - t_a) - o$, where $\mathrm{T}^\dagger(t_b, t_a)$ denotes the total work interval length on $[t_a, t_b)$, τ represents the average control ratio, and o is the elasticity index.

Definition 2. The MLN model given by (1) will achieve FOS provided that there exists a positive constant $\mathcal{T} \in [0, +\infty)$ and a constant $\Psi > 0$ such that

$$\begin{cases} \lim\limits_{t\to\mathcal{T}} \hbar(t) \in \{\hbar(t) | \mathrm{Q}(\hbar(t)) \leqslant \Psi\}, \\ \hbar(t) \in \{\hbar(t) | \mathrm{Q}(\hbar(t)) \leqslant \Psi\},\ \forall t \geqslant \mathcal{T}, \end{cases}$$

and $\{\hbar(t) | \mathrm{Q}(\hbar(t)) \leqslant \Psi\} \triangleq \Xi$ is called a residual set for all $\hbar(t) \in R^{MPm}$, where $\mathcal{T}$ is the synchronization time required to achieve FOS synchronization, and $\hbar(t) = (\hbar^1(t)^{\mathrm{T}}, \ldots, \hbar^M(t)^{\mathrm{T}})^{\mathrm{T}}$.

Assumption 1. For any vectors ρ_1, $\rho_2 \in R^n$, there exists a scalar $\varepsilon_p^r > 0 (p \in \bar{P})$, such that $\|f_p^r(\rho_1) - f_p^r(\rho_2)\| \leqslant \varepsilon_p^r \|\rho_1 - \rho_2\|$.

Lemma 1. Assume that the function $\mathrm{Q}(\hbar(t)) : R^n \to R$ is positive-definite and radially unbounded. If there exist constants $\flat, \backepsilon > 0$, $\kappa \in R$, $1 > \jmath > 0$, and $\imath > 1$, such that when $\hbar(t) \in R^n \backslash \{\mathbf{0}_n\}$, the following differential inequalities hold:

$$\begin{cases} \dot{\mathrm{Q}}(\hbar(t)) \leqslant -\flat \mathrm{Q}^\imath(\hbar(t)) - \backepsilon\, \mathrm{Q}^\jmath(\hbar(t)) + \kappa \mathrm{Q}(\hbar(t)), t \in [t_\iota, z_\iota), \\ \dot{\mathrm{Q}}(\hbar(t)) \leqslant 0, t \in [z_\iota, t_{\iota+1}), \end{cases} \tag{5}$$

and $\kappa < \min\{\flat, \backepsilon\}$. Then the origin is FT stable and the synchronization time is estimated as

$$\begin{cases} T_1^\dagger = \frac{1}{\kappa\tau(\imath-1)} \ln(\frac{\flat}{\flat-\kappa}) + \frac{1}{\kappa\tau(1-\jmath)} \ln(\frac{\backepsilon}{\backepsilon-\kappa}) + \frac{2o}{\tau}, & \kappa \neq 0, \\ T_2^\dagger = \frac{1+\flat(\imath-1)o}{\flat(\imath-1)\tau} + \frac{1+\backepsilon(1-\jmath)o}{\backepsilon(1-\jmath)\tau}, & \kappa = 0. \end{cases}$$

Lemma 2. Given a positive-definite and radially unbounded function $\mathrm{Q}(\hbar(t)) : R^n \to R$, along with constants $\kappa \in R$, $\flat, \backepsilon, \sigma > 0$, $1 > \jmath > 0$, $\imath > 1$, $d > 0$, $1 > \varsigma > 0$, and $\text{ß} > 0$ satisfying

$$\begin{cases} \dot{\mathrm{Q}}(\hbar(t)) \leqslant -\flat \mathrm{Q}^\imath(\hbar(t)) - \backepsilon\, \mathrm{Q}^\jmath(\hbar(t)) + \kappa \mathrm{Q}(\hbar(t)) + \text{ß}, \\ \quad t \in [t_\iota, z_\iota), \\ \dot{\mathrm{Q}}(\hbar(t)) \leqslant \sigma \mathrm{Q}(\hbar(t)) + \text{ß}, t \in [z_\iota, t_{\iota+1}), \end{cases} \tag{6}$$

and $\hat{\kappa} < \min\{\hat{\flat}, \hat{\backepsilon}\}$, where $\hbar(t) \in R^n \backslash \{\mathbf{0}_n\}$. If there exist positive numbers d and τ satisfying $\sigma - d\tau < 0$ and $0 < \tau < 1$, then the systems (1) and (2) are FT stable. Moreover, the residual set can be expressed as $\Xi = \{\lim_{t\to T_r^\dagger} \hbar(t) \mid \mathrm{Q}(\hbar) \leqslant \max\{m_1, m_2, m_3, m_4\}\}$, where $r = 3, 4$, $m_1 = (\text{ß}/(\flat\varsigma))^{1/\imath}$, $m_2 = (\text{ß}/(\backepsilon\,\varsigma))^{1/\jmath}$, $m_3 = \text{ß}/(\kappa\varsigma)$, $m_4 = \text{ß}/(d\tau - \sigma)$. The synchronization time is estimated as

$$\begin{cases} T_3^\dagger = \frac{1}{\hat{\kappa}\tau(\imath-1)} \ln(\frac{\hat{\flat}}{\hat{\flat}-\hat{\kappa}}) + \frac{1}{\hat{\kappa}\tau(1-\jmath)} \ln(\frac{\hat{\backepsilon}}{\hat{\backepsilon}-\hat{\kappa}}) + \frac{2o}{\tau}, & \hat{\kappa} \neq 0, \\ T_4^\dagger = \frac{1+\hat{\flat}(\imath-1)o}{\hat{\flat}(\imath-1)\tau} + \frac{1+\hat{\backepsilon}(1-\jmath)o}{\hat{\backepsilon}(1-\jmath)\tau}, & \hat{\kappa} = 0, \end{cases}$$

where $\hat{\flat} = (1 - \varsigma)\flat \exp\{(1 - \imath)do\}$, $\hat{\backepsilon} = (1 - \varsigma)\ \backepsilon$, and $\hat{\kappa} = (1 + \varsigma)\kappa + d(1 - \tau)$.

Theorem 1. Under Assumption 1 and the control law (4), if the conditions $\bar{\kappa} < \min\{\tilde{\flat}, \bar{\flat}, \bar{\backepsilon}, \tilde{\backepsilon}\}$, $0 < \bar{s}_1 + \beta_1 < 1$, $\wedge_3 - d\tau < 0$ are satisfied where $d > 0$ and $0 < \tau < 1$, then systems (1) and (2) will achieve the FOS, which implies that the synchronization error $\hbar_p^r(t)$ converges to the attraction region Ξ in a fixed-time $T^\dagger$: $\Xi = \{\lim_{t\to T^\dagger} \hbar_p^r(t) \mid \mathrm{Q}(\hbar(t)) \leqslant \max\{m_1, m_2, \hat{m}_1, \hat{m}_2, \hat{m}_3, \hat{m}_4\}\}$, and $T^\dagger$ satisfies the following inequality:

$$T^\dagger \leqslant \begin{cases} \frac{1}{\bar{\kappa}\tau(\bar{\imath}-1)} \ln(\frac{\bar{\flat}}{\bar{\flat}-\bar{\kappa}}) + \frac{1}{\bar{\kappa}\tau(1-\bar{\jmath})} \ln(\frac{\bar{\backepsilon}}{\bar{\backepsilon}-\bar{\kappa}}) \\ + \frac{1}{\bar{\kappa}\tau(\tilde{\imath}-1)} \ln(\frac{\tilde{\flat}}{\tilde{\flat}-\bar{\kappa}}) + \frac{1}{\bar{\kappa}\tau(1-\tilde{\jmath})} \ln(\frac{\tilde{\backepsilon}}{\tilde{\backepsilon}-\bar{\kappa}}) + \frac{4o}{\tau}, \bar{\kappa} \neq 0, \\ \frac{1+\bar{\flat}(\bar{\imath}-1)o}{\bar{\flat}(\bar{\imath}-1)\tau} + \frac{1+\bar{\backepsilon}(1-\bar{\jmath})o}{\bar{\backepsilon}(1-\bar{\jmath})\tau} \frac{1+\tilde{\flat}(\tilde{\imath}-1)o}{\tilde{\flat}(\tilde{\imath}-1)\tau} + \frac{1+\tilde{\backepsilon}(1-\tilde{\jmath})o}{\tilde{\backepsilon}(1-\tilde{\jmath})\tau}, \bar{\kappa} = 0. \end{cases}$$

The MLNs model achieved the best results compared with other approaches under the same conditions in this study. As shown in Appendixes A–E, the Motivation, the proof of Lemmas 1 and 2, the proof and symbols of Theorem 1, and the Simulations are discussed, respectively.

Conclusion. This work investigated the FOS of MLN with intra/inter-layer output coupling based on the IC approach. The analysis of FT synchronization of MLNs against replay attacks using composite antidisturbance control [5] is extremely rare and will be explored in future research.

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Supporting information Appendixes A–E. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Chen Z. Feedforward design for output synchronization of nonlinear heterogeneous systems with output communication. Automatica, 2019, 104: 126–133
2 Huang C, Yan H, Zhang H, et al. Passivity-based output synchronization with switching graphs and transmission delays. IEEE Trans Cybern, 2022, 52: 3370–3379
3 Liu Y, Wang Z, Wang Y. Data-based output synchronization of multi-agent systems with actuator faults. IEEE Trans Neural Netw Learn Syst, 2023, 34: 11013–11020
4 Liu J, Xu Z, Xue L, et al. Practical fixed-time synchronization of multilayer networks via intermittent event-triggered control. IEEE Trans Syst Man Cybern Syst, 2024, 54: 2626–2637
5 Wang J, Wang D, Yan H, et al. Composite antidisturbance control for hidden markov jump systems with multi-sensor against replay attacks. IEEE Trans Automat Contr, 2024, 69: 1760–1766