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Experimental quantum randomness certification via Einstein-Podolsky-Rosen steering in prepare-and-measure scenario

Xiaoting HUANG, Siyao HUANG, Zehong CHANG*, Yunlong WANG*, Yu XIANG*, Hong GAO, Fuli LI & Pei ZHANG*

Ministry of Education Key Laboratory for Nonequilibrium Synthesis and Modulation of Condensed Matter, Shaanxi Province Key Laboratory of Quantum Information and Quantum Optoelectronic Devices, School of Physics, Xi'an Jiaotong University, Xi'an 710049, China

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Abstract Quantum randomness, characterized by inherent unpredictability, is a critical resource for secure communications and statistical applications. While quantum entanglement-based randomness offers enhanced privacy through monogamy properties of entanglement, existing research mainly focuses on entanglement distribution (ED) scenarios, where maintaining entanglement faces serious challenges, especially in high-dimensional systems. The prepare-and-measure (PM) scenarios can solve this problem by directly characterizing the conditional probabilities. Einstein-Podolsky-Rosen (EPR) steering is a special type of entanglement and can certify randomness in a one-sided device-independent manner. Here, we propose a protocol to certify quantum randomness through EPR steering in the PM scenario, and experimentally realize it in high-dimensional systems. Our approach constructs a two-setting steering inequality and demonstrates that its violation certifies randomness generation. We find that with the increase of steering dimensions, this method enables higher noise tolerance. Experimental validation achieved a certified randomness of 1.2804 ± 0.0044 bits under genuine 4-dimensional steering, surpassing the one-bit threshold by 63.7 standard deviations. This result highlights the protocol's robustness against noise and scalability in practical settings. Compared to ED-based schemes, our PM scenario simplifies experimental implementation while maintaining high security, making it particularly suitable for resource-efficient applications in cryptography and secure communication.

Keywords quantum randomness, Einstein-Podolsky-Rosen steering, prepare-and-measure scenario, one-sided device-independent, randomness certification

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1 Introduction

Randomness is an important resource and has many applications [1, 2] in areas such as secure communications, statistical analysis, and Monte Carlo simulation. The widely used random numbers generated by classical algorithms are pseudo-random numbers with questionable security [3]. Quantum randomness comes from the intrinsic properties of quantum systems and has perfect unpredictability, thus it has attracted extensive research interest. The generation can be roughly classified into two ways according to the source of quantum randomness. One way is generated by the measurement of random variables with the intrinsic randomness of the quantum system, e.g., the radioactive decay time of radioactive substances [4, 5], the indivisible spatial path state of photons [6, 7], the photon detection time [8, 9], the laser phase noise [10, 11], and the Raman scattering noise [12]. The other is generated from quantum entanglement-based, e.g., randomness from Bell nonlocality [13], randomness from quantum entanglement [14], and randomness from Einstein-Podolsky-Rosen (EPR) steering [15].

Randomness certified via quantum entanglement-based is private because entanglement has monogamy [16, 17], and as a result, it has attracted broad research interest. Woodhead et al. [18] studied randomness in tripartite Mermin-Bell inequality tests; Wooltorton et al. [19] studied the maximum randomness

* Corresponding author (email: czh1694381715@163.com, yunlong.wang@mail.xjtu.edu.cn, xiangy.phy@gmail.com, zhangpei@mail.ustc.edu.cn)

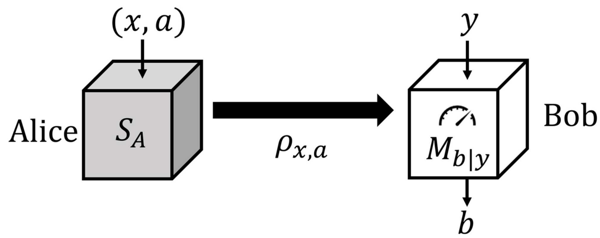

Figure 1 EPR steering in the PM scenario: Alice's device is regarded as a black box, and Bob's device is assumed to be trusted. Alice's source (S_A) can emit quantum states $\rho_{x,a}$ depending on the input $(x,a) \in \{(1,0),\ldots,(1,d-1),(2,0),\ldots,(2,d-1)\}$. Bob's measurement device performs one of the two measurements depending on a choice of input $y \in \{0,1\}$ and outputs $b \in \{0,\ldots,d-1\}$, corresponding to the measurement operator $M_{b|y}$.

certified when violating the Clauser-Horne-Shimony-Holt inequality; Sarkar et al. [20] certified quantum randomness via a self-testing scheme in a quantum steering scenario. EPR steering [21–23] represents a class of quantum correlations between entanglement [24,25] and Bell nonlocality [26–28]. It describes the effect where measurements performed by one party can remotely adjust the state of the other party [29,30]. In this scenario, entanglement can be verified without trust in the steering party's devices. Consequently, steering can be used to certify randomness in a one-sided device-independent (1SDI) manner. Passaro et al. [31] proposed a randomness certification scheme based on quantum steering. However, previous experiments on randomness certification via EPR steering have concentrated on entanglement distribution (ED) scenarios [32, 33]. Preparing and distributing high-dimensional (HD) entangled states with high fidelity is challenging [34–37]. Unlike in ED scenarios, where randomness certification requires the preparation and distribution of high-dimensional entangled states [38–40], the prepare-and-measure (PM) scenario enables randomness certification without relying on shared entanglement resources, significantly reducing experimental complexity.

The prepare-and-measure scenario [41–43], which is completely characterized by conditional probabilities and is experimental-friendly to implement than the ED scenario, has drawn wide research interest. Poderini et al. [44] demonstrated that transferring quantum or classical information of finite dimension between two parties in a PM scenario can generate non-classical properties and give a non-classical criterion. This was subsequently verified experimentally by Luo et al. [45]. Tavakoli [46] explored the quantization of the trade-off between the degree of mistrust of the two parties and the nonclassical properties in a PM scenario. We are interested in how quantum randomness can be certified by showing the existence of EPR steering in the PM scenario.

In this paper, we study randomness certification through violating EPR steering inequalities in the PM scenario. Firstly, we construct a two-setting steering inequality in the PM scenario and show that its violation can result in randomness. This approach can tolerate noise within quantum states and applies to arbitrary-dimensional systems. Secondly, we experimentally prepared a high-dimensional state encoded with orbital angular momentum (OAM) and polarization to enhance robustness against environmental noise. We demonstrate that for certifying the same amount of randomness with higher-dimensional steering, a lower state fidelity is required, that is, a higher noise intensity can be tolerated. Finally, we experimentally certified 1.2803 ± 0.0044 bits of randomness under the proof of genuine 4-dimensional (4D) steering, exceeding one-bit of randomness by 63.7 standard deviations.

2 Prepare-and-measure steering and randomness certification

Consider two-setting EPR steering in the PM scenario, as shown in Figure 1. Alice has a source that can emit different quantum states $\rho_{x,a}$, which depend on the input (x,a). Alice randomly chooses (x,a) (where $x \in \{0,1\}$ is not necessarily equal probability, and $a \in \{0,\ldots,d-1\}$ is equal probability), whereby the corresponding quantum state systematic $\rho_{x,a}$ is sent to Bob. Bob then chooses the measurement setup according to $y \in \{0,1\}$ and outputs $b \in \{0,\ldots,d-1\}$, corresponding to the measurement operator $M_{b|y}$.

The steering inequality in PM scenario takes the form of

$$S \equiv \sum_{x=y=1}^{2} \sum_{a=b=0}^{d-1} \mathrm{Tr}\left(M_{b|y}\rho_{x,a}\right) \leqslant S_{\mathrm{LHS}}. \tag{1}$$

Mutually unbiased bases (MUBs), characterized by their maximal incompatibility, are selected as measurement bases to derive the steering function bounds, where $S_{\mathrm{LHS}} = 1 + 1/\sqrt{d}$ denotes the local hidden state (LHS) bound of the steering function, $S_Q = 2$ represents the quantum bound of the steering function. Once the outcomes violate the steering inequality, the system's randomness can be quantified in the following way [31]. Suppose an eavesdropper (Eve) exists, holding a measurement device E, then the probability that Eve guesses Alice's choice of a is

$$\begin{gathered} P_{\text{guess}}\left(x^*\right) = \max_{\rho_{x,a}^e, p(e), M_{a|x}} \sum_e p(e) p\left(a = e \mid x^*, e\right), \\ p(e) \geqslant 0, \ \sum_e p(e) = 1, \end{gathered} \tag{2}$$

where Eve assigns $\rho_{x,a}^e$ on device E with probability $p(e)$. Given Alice's choice of $x = x^*$, Eve will guess that $a = e$, and the probability that Eve's guess is correct in the distribution of the $\rho_{x,a}^e$ state is

$$p\left(a = e \mid x^*, e\right) = \operatorname{tr}\left(\rho_{x=x^*, a=e}^e\right). \tag{3}$$

P_{guess} can reach its maximum only if Eve's strategy agrees with the observations. This implies the satisfaction of

$$\operatorname{tr} \sum_{a,x} M_{a|x} \sum_e p(e) \operatorname{tr}\left(\rho_{x,a}^e\right) = S_{\text{obs}}, \tag{4}$$

where S_{obs} is the experimentally observed value of the steering function. Provided that P_{guess} is below 1, Eve cannot accurately guess Alice's outcome, and hence there is some randomness. Quantifying the randomness by min-entropy is

$$H_{\min}\left(x^*\right) = -\log P_{\text{guess}}\left(x^*\right). \tag{5}$$

To study the effect of noise in the quantum state preparation process in the task of randomness certification, we consider the case of quantum mixed states and assume that the state-system synthesis that Alice prepares for Bob is an isotropic state:

$$\rho_{x,a}^{\text{iso}} = \mu \rho_{x,a} + (1-\mu) \frac{\mathbb{1}}{d}, \tag{6}$$

while $\mu \in [0,1]$ represent the proportion of quantum state $\rho_{x,a}$. The steering function's upper bound for isotropic states S_{iso} is derived using MUBs as measurement bases.

$$S_{\text{iso}} = S_Q\left(\mu + \frac{1-\mu}{d}\right) = 2\left(\mu + \frac{1-\mu}{d}\right). \tag{7}$$

Eve's guess probability can be computed through Semidefinite programming [15]

$$\begin{aligned} P_{\text{guess}}\left(x^*\right) = \max_{\left\{\rho_{x,a}^e\right\}} \quad & \operatorname{tr} \sum_e \rho_{x=x^*, a=e}^e \\ \text{s.t.} \quad & \operatorname{tr} \sum_{a,x} M_{a|x} \sum_e p(e) \operatorname{tr}\left(\rho_{x,a}^e\right) = S_{\text{obs}}, \\ & \sum_a \rho_{x,a}^e = \sum_a \rho_{x^*,a}^e, \quad \forall e, x, \\ & \operatorname{tr} \sum_{a,e} \rho_{x,a}^e = 1, \quad \rho_{x,a}^e \geqslant 0, \quad \forall a, e, x. \end{aligned} \tag{8}$$

The first constraint makes the average of Eve's prepared quantum states agree with the observed steering function. The second constraint is the no-signaling principle. The third constraint ensures that the $\rho_{x,a}^e$ is semi-positively normalized. Accordingly, when $P_{\text{guess}} < 1$ ($H_{\min} > 0$), randomness in the noise-containing system can be verified.

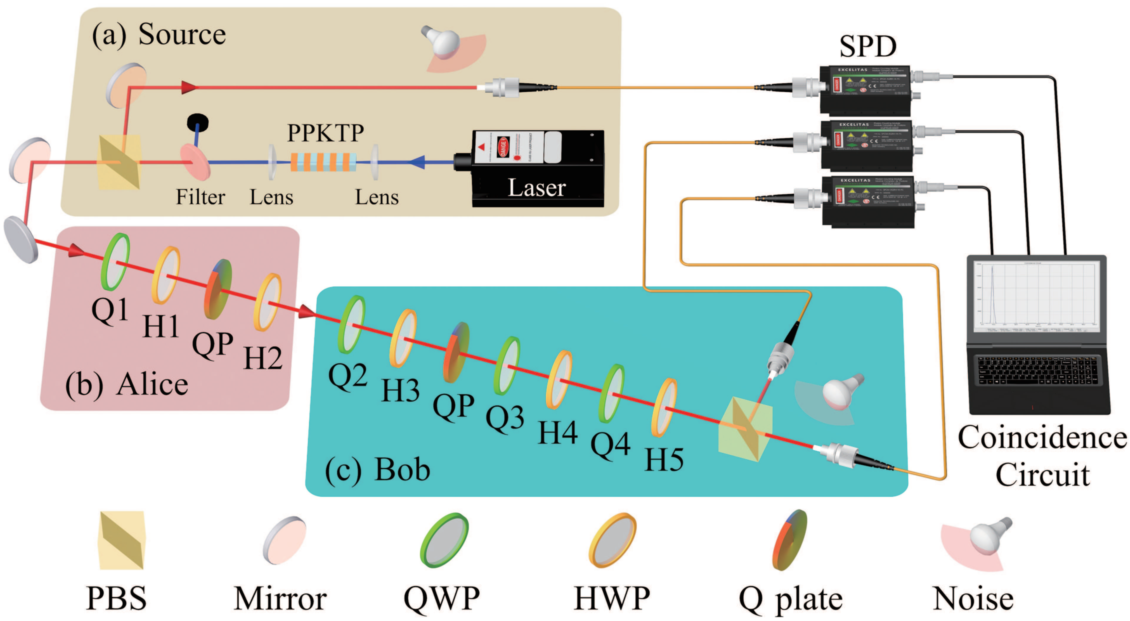

Figure 2 (Color online) Experimental setup. (a) Single photon source; (b) Alice prepares quantum states $\rho_{x,a}$; (c) Bob's measurement settings $M_{b|y}$. PPKTP: periodically poled potassium titanyl phosphate; PBS: polarizing beam splitter; HWP: half-wave plate; QWP: quart-wave plate; QP: Q-plate; SPD: single photon detector; noise: variable-intensity LED.

3 Experimental setup and results

The experimental setup is shown in Figure 2, which is divided into three parts: the single-photon source, the preparation of Alice's states, and Bob's measurement.

In the laser source part, a laser beam with a wavelength of 405 nm is coupled to a 5-mm-long nonlinear periodically poled potassium titanyl phosphate (PPKTP) crystal. Through a type-II phase-matched spontaneous parametric down-conversion process, this crystal generates a pair of correlated photon pairs with orthogonal polarization at a wavelength of 810 nm. A filter is used to remove the pump. Vertically polarized photons are idle photons used to predict single photons, while horizontally polarized photons are used as signal photons to complete the experiment. According to the choice of (x, a), Alice rotates the waveplate in Figure 2(b) to generate the corresponding quantum state $\rho_{x,a}$ sent to Bob. Correspondingly, based on the choice of y, Bob rotates the waveplate in Figure 2(c) to measure the quantum state. To complete the experiment for isotropic states, we place two variable-intensity LEDs in front of the two SPDs. This allows us to regulate the noise fraction $\mathrm{NF} = 1 - \mu$ without affecting $\rho_{x,a}$. Quantum states are encoded using the polarization and OAM of the photon, where $|H\rangle$ stands for horizontal polarization and $|V\rangle$ for vertical polarization; $|\pm 2\rangle$ represents the OAM of each photon along the optical axis. In the 4D quantum state space, define the computational base:

$$\begin{aligned} &|1\rangle = |H, +2\rangle,\ |2\rangle = |H, -2\rangle, \\ &|3\rangle = |V, +2\rangle,\ \ |4\rangle = |V, -2\rangle. \end{aligned} \tag{9}$$

In the experiment, the quantum state sent by Alice's device is $\rho^{\mathrm{iso}}_{x,a} = \mu\rho_{x,a} + (1-\mu)\frac{\mathbb{1}}{d}$. where $\rho_{x,a} = |\phi_{x,a}\rangle\langle\phi_{x,a}|$, $|\phi_{x,a}\rangle$ is shown in Table 1, where $\{|\phi_{1,a}\rangle\}$ and $\{|\phi_{2,a}\rangle\}$ denote two sets of MUBs.

When Bob chooses y, his measurement is set to $M_{b|y} = |\phi_{y,b}\rangle\langle\phi_{y,b}|$, where $\{|\phi_{1,b}\rangle\}$ and $\{|\phi_{2,b}\rangle\}$ denote two sets of MUBs. Table 1 gives the angle of the waveplate setup corresponding to the preparation of the quantum state $\rho_{x,a}$ and the measurement of $M_{b|y}$ in the experiment. The experimental observation

Table 1 During the preparation and measurement of the experimental states, the angle of each waveplate (see Figure 2) was set. The 0° orientation of the waveplates was aligned with the horizontal axis of the laboratory. The incident state of the photons was horizontally polarized, with an orbital angular momentum (OAM) of 0.

State	Alice's preparation			Bob's measurement					
	Q1	H1	H2	Q2	H3	Q3	H4	Q4	H5
$\lvert\phi_{1,0}\rangle = \frac{1}{2}(\lvert 1\rangle + \lvert 2\rangle + i\lvert 3\rangle - i\lvert 4\rangle)$	0	0	–	–	–	–	0	0	0
$\lvert\phi_{1,1}\rangle = \frac{1}{2}(\lvert 1\rangle - \lvert 2\rangle + i\lvert 3\rangle + i\lvert 4\rangle)$	0	+45	–	–	–	–	0	0	+45
$\lvert\phi_{1,2}\rangle = \frac{1}{2}(\lvert 1\rangle + \lvert 2\rangle - i\lvert 3\rangle + i\lvert 4\rangle)$	0	0	+45	–	0	–	0	0	0
$\lvert\phi_{1,3}\rangle = \frac{1}{2}(\lvert 1\rangle - \lvert 2\rangle - i\lvert 3\rangle - i\lvert 4\rangle)$	0	+45	+45	–	0	–	0	0	+45
$\lvert\phi_{2,0}\rangle = \frac{1}{2}(\lvert 1\rangle + i\lvert 2\rangle + \lvert 3\rangle - i\lvert 4\rangle)$	0	+22.5	–	+45	–	−45	+45	+45	+22.5
$\lvert\phi_{2,1}\rangle = \frac{1}{2}(\lvert 1\rangle + i\lvert 2\rangle - \lvert 3\rangle + i\lvert 4\rangle)$	0	−22.5	–	+45	–	−45	+45	+45	−22.5
$\lvert\phi_{2,2}\rangle = \frac{1}{2}(\lvert 1\rangle - i\lvert 2\rangle + \lvert 3\rangle + i\lvert 4\rangle)$	0	+22.5	+45	+45	0	−45	+45	+45	+22.5
$\lvert\phi_{2,3}\rangle = \frac{1}{2}(\lvert 1\rangle - i\lvert 2\rangle - \lvert 3\rangle - i\lvert 4\rangle)$	0	−22.5	+45	+45	0	−45	+45	+45	−22.5

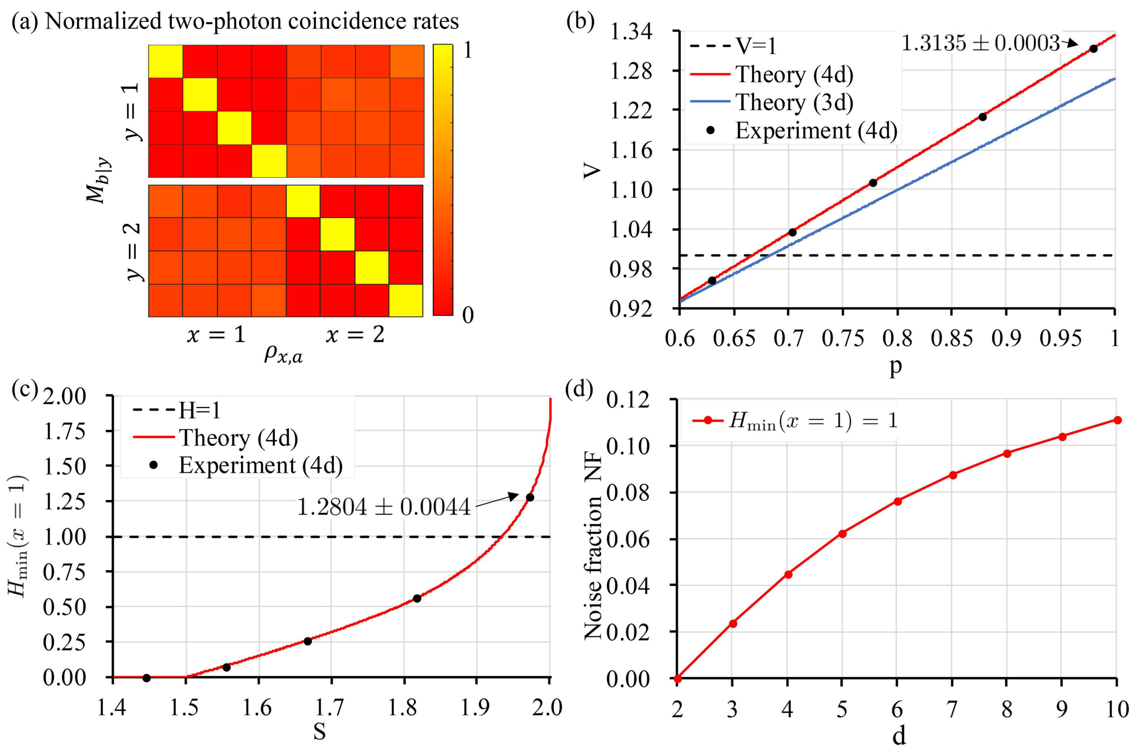

Figure 3 (Color online) (a) Normalized two-photon coincidence rates in 4D, when Alice's source emits the quantum state $\rho_{x,a}(x \in \{1,2\}; a \in \{0,1,2,3\})$ and Bob performs the measurement $M_{b|y}(y \in \{1,2\}; b \in \{0,1,2,3\})$; (b) the relationship between the degree of steering violation $V = S_{\rm iso}/S_{\rm LHS}$ and the p of the isotropic state; (c) the certification of randomness in different isotropic states; (d) the maximum noise fraction $\mathrm{NF} = 1-\mu$ that can be tolerated when certifying randomness with $H_{\min}(x=1) = 1$ bit in different steering dimensions.

of the steering function is

$$\begin{aligned} S_{\rm obs} &\equiv \sum_{x=y=1}^{2} \sum_{a=b=0}^{d-1} \mathrm{Tr}\left(M_{b|y}\rho_{x,a}^{\rm iso}\right) \\ &= \sum_{x=y=1}^{2} \sum_{a=b=0}^{d-1} P(b \mid x,a,y) \\ &= \sum_{x=y=1}^{2} \sum_{a=0}^{d-1} \frac{N_{a|x,a,y}}{\sum_{b=0}^{d-1} N_{b|x,a,y}}, \end{aligned} \tag{10}$$

where $P(b \mid x,a,y)$ represents the conditional probability of obtaining b when Alice's preparation of the $\rho_{x,a}^{\rm iso}$ and and Bob chooses the measurement $M_{b|y}$, while $N_{b|x,a,y}$ denotes the coincidence counting.

Figure 3 shows the experimental results. Since the standard deviation is on the order of 10^{-3}, the error bars cannot be shown. Figure 3(a) shows the normalized two-photon coincidence rates in 4D, when Alice's source emits the quantum state $\rho_{x,a}$ ($x \in \{1,2\}; a \in \{0,1,2,3\}$) and Bob's measurement is $M_{b|y}(y \in \{1,2\}; b \in \{0,1,2,3\})$. Figure 3(b) shows the relationship between the degree of steering violation $V = S_{\rm iso}/S_{\rm LHS}$ and the μ of the isotropic state. The steering inequality is violated when $V > 1$. The red line represents the theoretical prediction V in 4D quantum systems. There are four black experimental points where the steering inequality is violated, thus proving the existence of quantum steering. Additionally, we plot the steering inequality violation for 3D systems. It can be seen that the violations of our experiments are beyond the 3D case for a given μ. Especially, when $\mu = 98.02\%$, $V = 1.3135 \pm 0.0003 > V(d=3)_{\max} = 1.2679$. This shows the existence of genuine 4D steering.

Figure 3(c) shows the randomness certification in different isotropic states. The five black dots denote the experimental observations in the 4D case. Randomness is only certified when the steering is shown, i.e., when $S > S_{\rm LHS} = 1.5$. The maximum steering function in the experiment is 1.9702 ± 0.0004, which is close to the quantum upper bound of 2. At this point, the minimum entropy of randomness is 1.2804 ± 0.0044 bits, which exceeds the one-bit limit of quantum bits by 63.7 standard deviations. In Figure 3(d), we calculate the fraction of the maximum noise fraction, $\mathrm{NF} = 1-\mu$, that can be tolerated when certifying randomness with $H_{\min}(x=1) = 1$ bit in different steering dimensions. In addition to the white noise depicted in the figure, we further analyzed the robustness of our scheme against amplitude-damping and phase-damping noise[1)]. The results show that the noise tolerance for certifying randomness increases as the steering dimension increases. We can efficiently certify randomness using HD EPR steering.

4 Discussion and conclusion

In this work, we have considered HD EPR steering in a PM scenario and calculated the quantum randomness certification. The experiment is completed by using the photon's polarization and OAM degrees of freedom. In our experiments, we observe true 4D EPR steering with a steering violation degree of $V = 1.3135 \pm 0.0003$, which corresponds to the certification of randomness with a minimum entropy of 1.2804 ± 0.0044 bits, exceeding the one-bit bound by 63.7 standard deviations of quantum bits. By adding appropriate noise to the quantum state, we also demonstrate that HD EPR steering offers better noise tolerance than lower-dimensional steering; that is, to certify the same amount of randomness, HD steering can tolerate more noise compared to lower-dimensional steering. Our scheme applies to systems of arbitrary dimensions and can be further demonstrated in higher-dimensional scenarios.

HD quantum steering is an important resource for quantum computing and quantum information due to its noise robustness and system asymmetry, and the PM scenario of quantum steering has the potential to be generalized to other application scenarios, mainly including 1SDI quantum key distribution, quantum secret sharing, and quantum teleportation. In addition, our proposed scheme enables quantum cryptography applications in fiber optics [47], free space [48], and underwater environments [49], and contributes to the construction of hybrid quantum networks [50, 51]. Exploring HD randomness authentication in PM scenarios is conducive to the development of 1SDI quantum random number applications that are compatible with existing quantum networks.

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1) For amplitude damping noise, our scheme exhibits a robustness threshold of 0.425 at $d = 4$, while for phase damping noise, the threshold reaches 0.333 under the same dimension ($d = 4$).

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