

# Photon-wave traveling model for analyzing spectral characteristics of light sources without resonant cavities

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In general, light sources can be classified into two categories: one with resonant cavities, such as Fabry-Perot (FP) cavity lasers, plasmon lasers, and optical frequency combs [1]; and the other with no resonant cavity, such as light-emitting diodes (LEDs), superluminescent light emitting diodes (SLEDs), and random lasers. SLEDs, also called superluminescent diodes (SLDs), have been an indispensable light source since they were demonstrated by Kurbatov in 1971 [2]. SLEDs exhibit a high degree of spatial coherence and a low degree of temporal coherence. As such, SLEDs are the ideal light sources for optical gyroscopes, optical coherence tomography (OCT), wavelength-division-multiplexing (WDM) system testing, optical channel monitoring, and visible light communication [3, 4].

The free spectral range (FSR), the spectral linewidth (SLW) and the output spectrum are crucial characteristics of light sources. For light sources with resonant cavities, the spectral characteristics can be analyzed using classical resonant cavity theory (see Supporting information). However, for light sources without cavities, the FSR and the SLW cannot be resolved by current theoretical frameworks. Therefore, it is necessary to establish a new model for analyzing the spectral characteristics of light sources without cavities.

In this study, we present a novel theoretical model: the photon-wave traveling (PWT) model. Based on the transit time of photons in the active region of a light source, Parseval's relation, and Rayleigh's criterion, the model provides a theoretical method for analyzing the spectral characteristics of optical sources without resonant cavities. As an illustrative example, we solve the SLW, the FSR, and the output spectrum of an SLED. Our model lays the groundwork for future studies of the spectra of the light sources without resonant cavities.

**Results and discussion.** Unlike conventional semiconductor lasers, SLEDs have a single-pass gain because their facet feedback coefficients are as low as  $10^{-6}$  to avoid the appearance of FP resonant cavity modes [5]. This is obtained through the joint action of tilted waveguides and

anti-reflection coated (ARC) facets in SLEDs. The measured spectrum of an SLED is presented in Figure 1(a).

We assume that the length of the active region in an SLED is  $\mathcal{L}$ ; the intensity-reflection coefficient  $R$  tends to zero at both facets of the active region, and the total number of uniformly distributed photons in the active region is  $N(t)$ . The time of the photons crossing the active region in the traveling-wave state is defined as the transit time  $\tau = \mathcal{L}/(c/n)$ , where  $c$  is the speed of light in vacuum, and  $n$  is the refractive index of the active region. We name this model the PWT model (see Supporting information).

During the time interval  $dt$ , the number of photons escaping from the output facet of the active region is  $dN(t) = [-N(t)/\tau]dt$ . The general solution of the differential equation is  $N(t) = N_0 e^{-t/\tau}$ , where  $N_0$  is the initial number of photons in the active region. The output power of a light source is proportional to the number of photons in its active region; thus, the electric field of the output light can be written as  $E(t) = E_0 e^{-t/(2\tau)} \cdot e^{j\omega_m t}$ ,  $0 \leq t \leq \tau$ , where  $E_0$  is the initial value of the electric field amplitude,  $\omega_m$  is the angular frequency of spontaneous emission photons, and  $m$  is the mode number. The frequency spectrum density of the electric field, within the active region, can be obtained from the Fourier transform

$$\begin{aligned} \tilde{E}(\omega) &= \int_{-\infty}^{+\infty} E(t) \cdot e^{-j\omega t} dt \\ &= \frac{E_0}{\frac{1}{2\tau} + j(\omega - \omega_m)} \left\{ 1 - e^{-[\frac{1}{2} + j(\omega - \omega_m)\tau]} \right\}. \end{aligned} \quad (1)$$

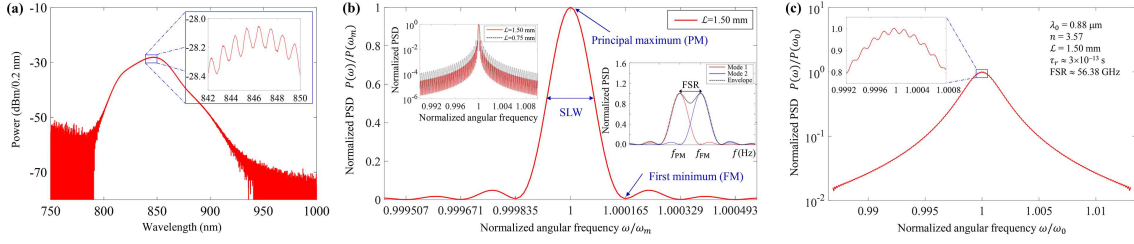
According to Parseval's relation, the energy is determined by

$$\text{Energy} = \int_{-\infty}^{+\infty} |E(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |\tilde{E}(\omega)|^2 d\omega. \quad (2)$$

Within the transit time  $\tau$ , the output power of an SLED is

$$\text{Power} = \frac{1}{2\pi\tau} \int_{-\infty}^{+\infty} |\tilde{E}(\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{+\infty} P(\omega) d\omega. \quad (3)$$

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**Figure 1** (Color online) (a) Spectrum of an SLED (EXALOS) measured by an optical spectrum analyzer (YOKOGAWA AQ6370C). The inset shows the ripple of the SLED. (b) Theoretical spectral lines of PWT modes. Parameters:  $L = 1.50$  mm,  $\lambda_0 = 0.88$   $\mu\text{m}$  and  $n = 3.57$ . Left inset: log-scale normalized PSD for  $L = 1.50$  and  $0.75$  mm, respectively. Right inset: two PWT modes are just resolved. (c) Theoretical output spectrum of an SLED.

Hence the power spectrum density (PSD)  $P(\omega)$  can be expressed as

$$P(\omega) = \frac{1}{\tau} \tilde{E}(\omega) \cdot \tilde{E}^*(\omega) \\ = 4\tau E_0^2 \frac{1 - \frac{2}{\sqrt{e}} \cos[\omega_m \tau (\frac{\omega}{\omega_m} - 1)] + e^{-1}}{1 + 4(\omega_m \tau)^2 (\frac{\omega}{\omega_m} - 1)^2}, \quad (4)$$

where  $\omega_m \tau = 2\pi nL/\lambda_m \approx 2\pi nL/\lambda_0$ , and  $\lambda_0$  is the central wavelength of the light source in vacuum.

Applying (4), the spectral lines of the PWT modes can be plotted, as shown in Figure 1(b). The PSD of the PWT mode is symmetric about  $\omega = \omega_m$ , and drops rapidly as the angular frequency deviates from  $\omega_m$ . The spectrum has a principal maximum at  $\omega = \omega_m$  and an infinite number of extrema. According to (4), the SLW of the PWT mode is given by

$$\text{SLW(Hz)} = \Delta f_{-3 \text{ dB}} \approx 2.80443 \times \left( \frac{c}{\pi n L} \right). \quad (5)$$

The SLW of an SLED is inversely proportional to its active region length  $L$ , as shown in (5) and the left inset of Figure 1(b). Due to the absence of a resonant cavity in an SLED, its quality factor cannot be determined by classical resonant cavity theory. Based on the relationship between the  $-3$  dB linewidth of the PSD and the equivalent on-load quality factor  $Q_e$ , we have

$$Q_e = \frac{f_0}{\Delta f_{-3 \text{ dB}}} = \frac{c/\lambda_0}{\Delta f_{-3 \text{ dB}}} \approx \frac{1}{2.80443} \left( \frac{\pi n L}{\lambda_0} \right). \quad (6)$$

According to (5) and (6), if the parameters of an SLED are set as  $\lambda_0 = 0.88$   $\mu\text{m}$ ,  $n = 3.57$  and  $L = 1.50$  or  $0.75$  mm, the corresponding SLW is approximately 50 or 100 GHz, and  $Q_e$  is approximately 6817 or 3408. This indicates that  $Q_e$  of an SLED is directly proportional to both the length and the refractive index of its active region.

The PWT modes of SLEDs are temporally incoherent. According to Rayleigh's criterion, the adjacent PWT modes are just resolved when the principal maximum of one mode falls on the first minimum of the adjacent mode, as shown in the right inset of Figure 1(b). That is to say the frequency interval of adjacent PWT modes, namely FSR, should be equal to the absolute value of the frequency difference between the principal maximum and the first minimum.

$$\text{FSR(Hz)} = |f_{\text{PM}} - f_{\text{FM}}| \approx 3.1618 \times \left( \frac{c}{\pi n L} \right). \quad (7)$$

The active region materials of SLEDs are usually III-V semiconductor materials, and the gain curve  $g(\omega)$  can be approximated as a Lorentz profile

$$g(\omega) = \frac{g_0}{1 + (\omega_0 \tau_r)^2 (\frac{\omega}{\omega_0} - 1)^2}, \quad (8)$$

where  $g_0 = g(\omega_0)$  is the peak value of the gain curve,  $\omega_0$  is the central angular frequency of the SLED, and  $\tau_r$  is the relaxation time of the charge carriers. Applying (4), (7) and (8), the normalized output spectrum of an SLED is plotted in Figure 1(c), which is consistent with the measured spectrum. The inset shows the ripple of the SLED. The smaller the spectral ripple, the higher the SLED quality.

**Conclusion.** A novel theoretical model, the PWT model, is proposed for analyzing the spectral characteristics of light sources with no cavity. Based on the transit time of photons within the active region, Parseval's relation and Rayleigh's criterion, the PWT model provides a theoretical method for analyzing the spectral parameters of the optical sources without resonant cavities. With the model, the SLW, the FSR, the  $Q_e$ , and the output spectrum of an SLED are determined. Our theoretical model is consistent with experimental results. Furthermore, this model can also be used to analyze the spectral characteristics of other optical sources without resonant cavities, opening up a new avenue for future studies of the spectra of light sources.

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**Supporting information** Appendixes A–C. The supporting information is available online at [info.scichina.com](http://info.scichina.com) and [link.springer.com](http://link.springer.com). The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

## References

- Hu Y, Yu M, Buscaino B, et al. High-efficiency and broadband on-chip electro-optic frequency comb generators. *Nat Photon*, 2022, 16: 679–685
- Feng N, Lu M, Sun S, et al. Light-emitting device based on amplified spontaneous emission. *Laser Photon Rev*, 2023, 17: 2200908
- Zhang Z Y, Hogg R A, Lv X Q, et al. Self-assembled quantum-dot superluminescent light-emitting diodes. *Adv Opt Photon*, 2010, 2: 201–228
- Malinverni M, Castiglia A, Rossetti M, et al. InAlN cladding implementation in green superluminescent diodes and lasers. *Appl Phys Lett*, 2023, 122: 201104
- Zhuravleva O V, Kurnosov V D, Kurnosov K V, et al. Study of the spectral and power characteristics of superluminescent diodes. *Quantum Electron*, 2004, 34: 15–19