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# Eigenvalue-based distributed target detection in compound-Gaussian clutter

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Target detection in clutter emerges as a particularly pertinent issue for a radar system [1, 2]. When a radar system employs pulsed Doppler processing to identify potential targets amidst such clutter, the target components often undergo fluctuations, which subsequently allows them to be embedded within the covariance matrix of the data under examination. In a recent study [3], the authors have devised innovative detectors for targets within compound-Gaussian clutter. These detectors leverage the eigenvalues of the covariance matrix, demonstrating superior detection performance compared to existing methods.

However, the detectors presented in [3] exhibit two notable limitations. First, they are created without the capability to suppress clutter, making them vulnerable to intense clutter. Second, these detectors are tailored for point targets, whereas modern radar systems, with wide bandwidth, often encounter targets spanning multiple range bins, necessitating a different approach. To address these issues, we introduce eigenvalue-based detectors specifically designed for detecting distributed targets amidst compound-Gaussian clutter. These detectors incorporate a clutter suppression mechanism, enabling them to enhance detection performance. The effectiveness of our proposed detectors has been validated using both simulated and real-world data.

*Problem formulation.* For a distributed target, if present, occupying $K$ range cells, the test data gathered by a radar system over a coherent processing interval (CPI) can be represented by $N \times 1$ column vectors $\boldsymbol{z}_l, l = 1, \ldots, K$, with $N$ being the number of pulses in a CPI. $\boldsymbol{z}_l$ typically contains clutter $\boldsymbol{c}_l$ and noise $\boldsymbol{n}_l$. The covariance matrix of the clutter and noise is unknown. Estimation of this covariance matrix necessitates the use of training data, which are commonly acquired adjacent to the test data. We assume that the secondary data $\boldsymbol{z}_l$s, $l = K+1, \ldots, K+L$, with $L$ being the number of secondary data, share the same covariance matrix structure with the test data. Under hypothesis $\mathrm{H}_0$, all the data only contain noise and clutter. Conversely, under hypothesis $\mathrm{H}_1$, the test data also encompass signal components. Hence, the detection problem is formulated as

$$\begin{cases} \mathrm{H}_0 : \boldsymbol{z}_l = \boldsymbol{c}_l + \boldsymbol{n}_l, l = 1, \ldots, K+L, \\ \mathrm{H}_1 : \begin{cases} \boldsymbol{z}_l = \boldsymbol{s}_l + \boldsymbol{c}_l + \boldsymbol{n}_l, l = 1, \ldots, K, \\ \boldsymbol{z}_l = \boldsymbol{c}_l + \boldsymbol{n}_l, l = K+1, \ldots, K+L, \end{cases} \end{cases} \tag{1}$$

where the signal has the form $\boldsymbol{s}_l = \beta_l \boldsymbol{p}$, $l = 1, \ldots, K$, $\beta_l$ is the unknown target amplitude, $\boldsymbol{p} = [1, \mathrm{e}^{-\mathrm{j}2\pi f_d}, \ldots, \mathrm{e}^{-\mathrm{j}2\pi f_d(N-1)}]^{\mathrm{T}}$ denotes the signal steering vector, and $f_d$ is the normalized target Doppler frequency. The clutter $\boldsymbol{c}_l$ is modeled as a spherically invariant random vector (SIRV), described as $\boldsymbol{c}_l = \sqrt{\tau_l}\boldsymbol{\eta}_l, l = 1, \ldots, K+L$, where $\boldsymbol{c}_l$ is expressed as the product of the square root of the slowly varying texture $\tau_l$ and the quickly varying speckle $\boldsymbol{\eta}_l$. Here, $\tau_l$ is a nonnegative real random variable, representing the local power of the clutter in the $l$th range cell. The texture $\tau_l$ is considered to be unknown and deterministic since the statistics of the texture are difficult to obtain in practice. The speckle $\boldsymbol{\eta}_l$ is characterized as an independent, zero-mean, complex circular Gaussian random vector with an $N \times N$ covariance matrix $\boldsymbol{R}$.

*Detector design.* To design effective detectors, we need the sample covariance matrix (SCM), given as $\hat{\boldsymbol{R}} = \frac{1}{L}\sum_{l=K+1}^{K+L} \boldsymbol{z}_l \boldsymbol{z}_l^{\mathrm{H}}$. Then we whiten the data as $\tilde{\boldsymbol{z}}_l = \hat{\boldsymbol{R}}^{-\frac{1}{2}} \boldsymbol{z}_l$, resulting in the whitened test data ($l = 1, \ldots, K$) and whitened training data ($l = K+1, \ldots, K+L$). The data whitening process has the function of clutter suppression and hence enhances detection performance.

Note that if we whiten the data with the actual covariance matrix $\boldsymbol{R}$, resulting in the whitened data $\overline{\boldsymbol{z}}_l = \boldsymbol{R}^{-\frac{1}{2}} \boldsymbol{z}_l$,

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then the covariance matrix of $\overline{\boldsymbol{z}}_l$ can be written as

$$\boldsymbol{R}_{\overline{\boldsymbol{z}}_l} = \mathrm{E}\left[\overline{\boldsymbol{z}}_l \overline{\boldsymbol{z}}_l^{\mathrm{H}}\right] = \begin{bmatrix} r_{0,l} & \cdots & r_{N-1,r}^* \\ \vdots & \cdots & \vdots \\ r_{N-1,r} & \cdots & r_{0,r} \end{bmatrix}, \tag{2}$$

where $r_{m,l}$ is the correlation coefficient, expressed as

$$r_{m,l} = \mathrm{E}\left[\bar{z}_{q,l}\bar{z}_{q+m,l}^*\right], 0 \leqslant m \leqslant N-1, 0 \leqslant q \leqslant N-m-1. \tag{3}$$

with $\bar{z}_{m,l}$ being the $m$th element of $\overline{\boldsymbol{z}}_l$. Moreover, the data in the $l$th range cell $\boldsymbol{z}_l$ can be represented by $\boldsymbol{z}_l = \left[z_{0,l}, \ldots, z_{N-1,l}\right]^{\mathrm{T}}$, $l = 1, \ldots, K+L$.

In (2) $\boldsymbol{R}_{\bar{z}_l}$ is unknown. It is assumed that the data received by the radar are wide-sense stationary. It follows that $\boldsymbol{R}_{\bar{z}_l}$ has a Toeplitz Hermitian positive-definite (HPD) structure. Hence, to estimate $\boldsymbol{R}_{\bar{z}_l}$, we use the whitened data in all the range cells. The ergodicity of the stationary Gaussian process permits us to estimate the correlation coefficient $r_{m,l}$ by averaging the whitened data, i.e.,

$$\widehat{r}_{m,l} = \frac{1}{N}\sum_{q=0}^{N-1-m} \widetilde{z}_{q,l}\widetilde{z}_{q+m,l}^*, 0 \leqslant m \leqslant N-1, l = 1, \ldots, K+L, \tag{4}$$

where $\widetilde{z}_{q,l}$ is the $q$th element of $\widetilde{\boldsymbol{z}}_l$. Moreover, the estimated whitened covariance matrix is defined as $\widehat{\boldsymbol{R}}_{\widetilde{\boldsymbol{z}}_l}$.

Eigenvalues play a pivotal role in extracting information about potential targets from the test data. The maximum eigenvalue of the test data, signifies the most significant information pertaining to the potential target [4]. Adopting the maximum, minimum, harmonic mean (HM), arithmetic mean (AM), and geometric mean (GM) of eigenvalues of an HPD matrix, we propose the following eigenvalue-based detector for distributed targets (EDDT):

$$t_{\mathrm{EDDT}} = \frac{\frac{1}{K}\sum_{l=1}^{K} \widetilde{\lambda}_{\max,l}}{\frac{1}{L}\sum_{l=K+1}^{K+L}\left[\frac{1}{N}\sum_{n=1}^{N}\widetilde{\lambda}_{n,l}^{a}\right]^{1/a}}, \tag{5}$$

where $\widetilde{\lambda}_{\max,l} = \max\{\mathrm{eig}(\widehat{\boldsymbol{R}}_{\widetilde{\boldsymbol{z}}_l})\}$, $l = 1, \ldots, K$, $\widetilde{\lambda}_{n,l}$ is the $n$th eigenvalue of $\widehat{\boldsymbol{R}}_{\widetilde{\boldsymbol{z}}_l}$, $l = 1, \ldots, K+L$, $\Lambda$ is the detection threshold, and $a$ is a tunable parameter. We can adjust the power parameter $a$ to obtain different kinds of EDDT. As $a = -1, 1, 0, -\infty, \infty$, the EDDT in (5) can be recast as

$$t_{\mathrm{EDDT}} = \begin{cases} \dfrac{\frac{1}{K}\sum_{l=1}^{K}\tilde{\lambda}_{\max,l}}{\frac{1}{L}\sum_{l=K+1}^{K+L}\left[N/\sum_{n=1}^{N}\left(1/\tilde{\lambda}_{n,l}\right)\right]}, & a = -1, \\ \dfrac{\frac{1}{K}\sum_{l=1}^{K}\tilde{\lambda}_{\max,l}}{\frac{1}{LN}\sum_{l=K+1}^{K+L}\sum_{n=1}^{N}\tilde{\lambda}_{n,l}}, & a = 1, \\ \dfrac{\frac{1}{K}\sum_{l=1}^{K}\tilde{\lambda}_{\max,l}}{\frac{1}{L}\sum_{l=K+1}^{K+L}\left(\prod_{n=1}^{N}\tilde{\lambda}_{n,l}\right)^{1/N}}, & a = 0, \\ \dfrac{\frac{1}{K}\sum_{l=1}^{K}\tilde{\lambda}_{\max,l}}{\frac{1}{L}\sum_{l=K+1}^{K+L}\tilde{\lambda}_{\min,l}}, & a = -\infty, \\ \dfrac{\frac{1}{K}\sum_{l=1}^{K}\tilde{\lambda}_{\max,l}}{\frac{1}{L}\sum_{l=K+1}^{K+L}\tilde{\lambda}_{\max,l}}, & a = \infty, \end{cases} \tag{6}$$

where $\widetilde{\lambda}_{\min,l} = \min\{\mathrm{eig}(\widehat{\boldsymbol{R}}_{\widetilde{\boldsymbol{z}}_l})\}$. For convenience, the proposed EDDT in (5) or (6) with $a = -1, 1, 0, -\infty, \infty$ is referred to as maximum eigenvalue to harmonic mean detector (MHM-D), maximum eigenvalue to arithmetic mean detector (MAM-D), maximum eigenvalue to geometric mean detector (MGM-D), maximum eigenvalue to minimum eigenvalue detector (MME-D), and maximum eigenvalue to maximum eigenvalue detector (MEM-D), respectively.

*Experiments.* We compare the probabilities of detection (PDs) of the proposed detectors with the distributed target version of the MGM detector in [3], referred to as the MGM detector with no whitening (MGM-D-nW). Precisely, the MGM-D-nW can be obtained from the MGM-D in (8) in [3] when the numerator $\lambda_{\max,\mathrm{cut}}$ is replaced by $\frac{1}{K}\sum_{l=1}^{K}\lambda_{\max,\mathrm{cut},l}$, with $\lambda_{\max,\mathrm{cut},l}$ being the maximum eigenvalue of the estimated covariance matrix for the $l$th test data. For the simulation results, the covariance matrix of the speckle component has the structure of $\boldsymbol{R} = \boldsymbol{R}_0 + p_0\boldsymbol{I}_N$, where $\boldsymbol{R}_0$ represents the clutter covariance matrix, $p_0$ represents the thermal noise power, the $(b,n)$th element of $\boldsymbol{R}_0$ is $\boldsymbol{R}_0(b,n) = \sigma_c^2\rho^{|b-n|}$, $b, n = 1, \ldots, N$, and $\rho = 0.9$. We set the probability of a false alarm as $10^{-3}$. The texture component of the compound-Gaussian clutter follows an inverse gamma distribution, with a shape parameter of $\nu = 0.9$ and a scale parameter of $\mu = 1.3$. For the real data, the dataset TFA17_014.03.mat is used [5]. Figure 1 shows that the new detector MHM-D has the highest PD, and when the PD is 0.8, the performance improvement of the MHM-D in terms of output signal-to-clutter ratio (SCR) exceeds 5 dB compared to the MGM-D-nW.

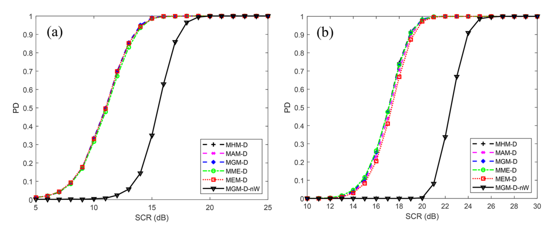


**Figure 1** (Color online) PDs of the detectors under different SCRs. (a) Simulated data; (b) real data.

*Conclusion.* The EDDT first implements a whitening operation to the test data to reduce the impact of the clutter. Then, it performs eigenvalue decomposition to the whitened data to effectively utilize the energy of the potential target embedded in the clutter. Finally, it uses these eigenvalues to form effective detectors. Numerical results showed that the proposed detectors outperform the existing ones.

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**Supporting information** Appendix A. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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