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Special Topic: Cohesive Clustered Satellites System for 5GA and 6G Networks

Improving connectivity in LEO clustered satellite systems: identify optimal interconnection points

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The satellite communication systems provide seamless wireless access over large areas, supporting remote IoT devices in regions without ground-based stations. Compared to medium Earth orbit (MEO) and geostationary orbit (GEO) satellites, low Earth orbit (LEO) satellites offer lower propagation loss, lower latency, and near-global coverage [1]. The cohesive clustered satellites (CCS) system formed by multiple interconnected cooperative satellites can establish a robust network with high connectivity and great resilience. This feature is particularly advantageous for remote or hard-to-reach areas [2].

However, in certain locally critical areas, satellites may not be fully interconnected, resulting in incomplete communication coverage. To address this, guiding subsequent satellite deployments is essential to enhance CCS connectivity. The main challenge lies in modeling feasible regions and identifying the optimal interconnection point in 3D Earth coordinates. In this study, we propose a general coordinate transformation model to mathematically represent the feasible region. Then, based on the connectivity evaluation method in [3], we aim to find the optimal interconnection point within the region of maximizing satellite connectivity.

Methodology. $\mathcal{L} = \{L_1, L_2, \ldots, L_N\}$ denotes a cluster of N LEO satellites, where communication between any two satellites is possible only if they are within each other's communication range. We first identify pairs of LEO satellites $(L_i, L_j), L_i \neq L_j \in \mathcal{L}$ with overlapping communication ranges but no interconnection. Here, this overlapping region is a feasible region. Next, we perform coordinate transformations for each (L_i, L_j) . Finally, we evaluate the impact of different interconnection points on satellite connectivity through the algebraic connectivity (i.e., the second smallest eigenvalue of the Laplacian matrix) [3].

Communication range. Based on [4], we can calculate the communication range of each LEO satellite in orbit. For each LEO satellite pair (L_i, L_j) with no interconnection, their communication ranges r_i and r_j must satisfy $\max(r_i, r_j) < d < (r_i + r_j)$, where d represents the Euclidean distance between satellites L_i and L_j .

 r_i and r_j can be seen as two spheres. Then for each satellite L_i , we define it using a quadruple $L_i \leftarrow (x_i, y_i, z_i, r_i)$. (x_i, y_i, z_i) represents the 3D coordinate of the sphere center (i.e., the position of satellite L_i), and r_i is the sphere radius.

Coordinate transformation. To facilitate analysis, we need to perform coordinate transformation and set the coordinate origin to L_i . Then the direction from L_i to L_j is defined as the new z-axis. To achieve this, we translate and rotate the coordinates of L_i and L_j . For the translation transformation, the translation matrix is given by

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & -x_i \\ 0 & 1 & 0 & -y_i \\ 0 & 0 & 1 & -z_i \\ 0 & 0 & 0 & 1 \end{bmatrix} . \tag{1}$$

Specifically, we apply matrix \mathbf{T} to translate the coordinates of L_i and L_j , positioning L_i at (0,0,0) and expressing L_j as $\mathbf{v} = (x_j - x_i, y_j - y_i, z_j - z_i)$ in the transformed coordinate system. For the rotation transformation, we apply Rodrigues' rotation formula to align L_j with the z-axis. As a result, the coordinate of L_j after rotation transformation is $\mathbf{v}^{\mathrm{rot}} = (0,0,||\mathbf{v}||)$. This rotation transformation is achieved using the rotation equation $\mathbf{v}^{\mathrm{rot}} = \mathbf{R}_{3\times3} \cdot \mathbf{v}$, where $\mathbf{R}_{3\times3}$ is the rotation matrix. To determine $\mathbf{R}_{3\times3}$, we first calculate the rotation axis \mathbf{k} based on $\mathbf{v}^{\mathrm{rot}} = \mathbf{k}$ and \mathbf{v} , which is given by $\mathbf{k} = (k_x, k_y, k_z)^{\mathrm{T}} = \frac{\mathbf{v}^{\mathrm{rot}} \times \mathbf{v}}{|\mathbf{v}^{\mathrm{rot}} \cdot \mathbf{v}|}$. Next, we calculate the rotation angle θ as $\theta = \arccos(\frac{\mathbf{v}^{\mathrm{rot}} \cdot \mathbf{v}}{|\mathbf{v}^{\mathrm{rot}} \cdot \mathbf{v}|})$.

The skew symmetric matrix of the rotation axis \mathbf{k} is

$$\mathbf{K} = \begin{pmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{pmatrix}.$$
 (2)

As such, the rotation matrix is obtained by $\mathbf{R}_{3\times3} = \mathbf{I}_{3\times3} + \sin(\theta)\mathbf{K} + (1-\cos(\theta))\mathbf{K}^2$, where $\mathbf{I}_{3\times3}$ is an identity matrix.

To obtain the transformation matrix containing translation and rotation operations, we need to extend $\mathbf{R}_{3\times3}$ as

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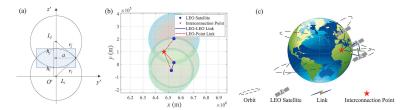


Figure 1 (Color online) (a) Feasible region; (b) connectivity improvement using our identified interconnection point; (c) LEO satellites and optimal interconnection point above the Earth.

 $\mathbf{R} = \begin{pmatrix} \mathbf{R}_{3\times3} & \mathbf{0}_{3\times1} \\ \mathbf{0}_{1\times3} & 1 \end{pmatrix}$. Based on the above, the coordinate transformation matrix \mathbf{M} can be expressed as $\mathbf{M} = \mathbf{R} \cdot \mathbf{T}$. \mathbf{M} is used to map the coordinate of the optimal interconnection point back to the real Earth coordinate. For satellite L_j , its Earth coordinate is $(x_j, y_j, z_j, 1)^T = \mathbf{M}^{-1}(0, 0, ||v||, 1)^T$.

The converted coordinate system is shown in Figure 1(a). D is the region where L_i and L_j intersect (i.e., L_i and L_j have overlapping communication ranges but are not interconnected). D consists of two spherical shells, where the radius a of intersection circle C is given by $a^2 = r_i^2 - h_i^2 =$ $r_j^2 - h_j^2$. h_i and h_j denote the distances from the centers of spheres L_i and L_j to the center of the intersection circle C, respectively. Additionally, h_i and h_j must satisfy $d=h_i+h_j$. Therefore, the radius a is calculated by $a=\sqrt{r_i^2-(\frac{r_i^2-r_j^2+d^2}{2d})^2}$. We can find a cuboid P that tightly encloses region D, with x, y and z satisfying the

constraints of $x, y \in [-a, a]$ and $z \in [d - r_i, r_i]$. The upper and lower bound structure of P is well suited as the tight feasible region for heuristic methods, leading to more stable and efficient solutions.

Identify interconnection point. For the tight feasible region P of (L_i, L_j) , we utilize the differential evolution (DE) algorithm to find the optimal interconnection point in the converted coordinate system. When the optimal point $p = (x_p, y_p, z_p, r_p)$ is determined, we can obtain the real position coordinate (x'_p, y'_p, z'_p) of this point on Earth by performing the inverse coordinate transformation as $(x'_p, y'_p, z'_p, 1)^{\mathrm{T}} = \mathbf{M}^{-1}(x_p, y_p, z_p, 1)^{\mathrm{T}}.$

Consequently, we derive $p' = (x'_p, y'_p, z'_p, r_p)$ and extend \mathcal{L} to $\mathcal{L}' = \mathcal{L} \cup \{p'\}$. For the satellite topology represented by \mathcal{L}' , we model the Laplacian matrix and calculate its second smallest eigenvalue [3], which corresponds to the algebraic connectivity. The difference between the algebraic connectivity corresponding to \mathcal{L} and \mathcal{L}' is the contribution of point p' to the CCS connectivity. When there are multiple feasible regions, we need to utilize DE algorithm to identify candidate optimal interconnection points within each feasible region. Then we rank these candidate points in descending order based on their gain contribution to the second smallest eigenvalue. Depending on the number of interconnection points to be placed (i.e., the number of satellites to be added), we select the corresponding number of optimal interconnection points from the candidates in order.

Simulation and discussion. We select N=3 LEO satellites, denoted as L_1, L_2 and L_3 . According to [4], we set the minimum elevation angle as $\phi_{\min} = 16^{\circ}$. The inclination angles of the LEO satellites are $i_1 = 99.5^{\circ}, i_2 = 45^{\circ}$ and $i_3 = 87^{\circ}$. We adopt the Cartesian coordinate expressions from [4] within an Earth-centered coordinate system to compute satellite positions and communication ranges at a specific time snapshot. Then, we conduct simulation modeling using MATLAB. The satellite coordinates obtained at a specific time snapshot are as follows: L_1 =

(6524604.585, -47691.6206, 284993.887, 178177.5369) m, L_2 = (6544643.566, 204011.7545, 204011.7545, 202308.4246) m, $L_3 = (6544643.566, 15099.7707, 288120.7887, 202308.4246)$ m. We set the communication range of the interconnection point as $r_p = 2 \times 10^5$ m. For the DE algorithm, we employ the most classic DE/best/1/bin strategy, and set the algorithm parameter as CR = 0.7. During each mutation step, Fis randomly selected within the interval (0.5, 1). The maximum number of iterations is set to 2000, and the population size is set as NP = 100.

Figure 1(b) shows the topology of cohesive clustered satellites in the particular snapshot after adding our identified optimal interconnection point. The three blue dots in the topology represent LEO satellites, and the blue solid line represents the bidirectional communication link between the satellites (i.e., LEO-LEO link). We find from Figure 1(b) that one satellite cannot establish communication with the other two. Here, the algebraic connectivity is equal to 0, and the number of communication links is 1. The red pentagram indicates the optimal interconnection point, and the red dash-dotted line indicates the newly established bidirectional communication link. By doing so, the resulting algebraic connectivity is increased to 1, and the number of inter-communication links is increased from 1 to 4. As such, our solution significantly improves the CCS connectivity.

Figure 1(c) shows the positions of satellites and the optimal interconnection point above the Earth in this particular snapshot. We can see that the optimal interconnection point can establish bidirectional communication for the three satellites.

Conclusion. In this study, based on [3], we aimed to enhance the overall connectivity of LEO clustered satellites by identifying the optimal interconnection point. Specifically, we proposed a general coordinate transformation model to mathematically represent the feasible region, and used the DE algorithm to obtain the solution. Experimental results demonstrated that our solution effectively improves the CCS connectivity.

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