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Time-delay effects on the dynamical behavior of switched nonlinear time-delay systems

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The switched system is a standard hybrid system comprising a finite number of subsystems, a switching signal and their interactions [1]. Over the past few decades, interest in switched systems has grown significantly owing to two primary reasons. First, many practical systems operate with multiple systems or modes, such as energy systems, multi-unmanned vehicle systems, and robotics [2]. Second, industrial processes often have complex characteristics involving multiple control objectives and presenting switching behaviors [3]. Switched system theories and approaches address these complexities, overcoming the limitations of traditional control methods and enabling better overall system performance. However, despite extensive research on switched systems, the stability of switched systems was not fully studied and remained an open issue.

Given the inherent nonlinearities in many modern industrial systems, research into switched nonlinear systems has garnered growing attention, yielding many intriguing results [4]. However, considering the complexity of nonlinear systems, a comprehensive exploration of their dynamics remains incomplete and challenging.

The time-delay phenomenon is inevitable in signal transmission and digital sampling [5]. Meanwhile, time delays can degrade system performance or even cause instability. For instance, the presence of a time delay can cause a stable delay-free system to be unstable. However, it is challenging and difficult to apply an unstable system to practical applications, and stability analysis of a practical system is significant. Therefore, addressing time delays is essential for switched nonlinear time-delay systems (SNTDSs). Meanwhile, the effects of large and small delays on SNTDSs have not been thoroughly investigated, thereby motivating the subsequent research.

This study focuses on these challenges, analyzing how large and small delays impact the control of SNTDSs. The main technical contributions can be outlined as follows. (1) Utilizing the mode-dependent average dwell time (MDADT) technique, multiple Lyapunov functions, and the delay integral inequality, we establish stability criteria for SNTDSs. These results reveal that increasing the time delay can stabilize originally unstable SNTDSs. (2) We investigate how different time delays influence the exponential convergence rate of SNTDSs. Interestingly, within a certain range, an increase in delay enhances the convergence rates of SNTDSs. We extend our results to hybrid rate functions that contain both stabilizing and destabilizing switches. (3) Our findings enable the coefficients used for upper-bound estimation of multiple Lyapunov functions to be mode-dependent, irrespective of their sign. This demonstrates that the Lyapunov-Krasovskii technique conditions presented in this study are more general and less restrictive compared to existing studies. Finally, a numerical example is provided to showcase the practical application of the theoretical findings.

Notation. $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^+ = \mathbb{R}/(-\infty, 0)$, define $\mathbb{N}$ as nonnegative integer numbers, $\mathbb{N}^+ = \mathbb{N}/\{0\}$. $|\cdot|$ denotes the Euclidean norm. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. C represents the set of continuous functions. Let $(\cdot^-)$ be the left limit operator. The function $x_t \in C([-\tau, +\infty), \mathbb{R}^n)$ with $\tau > 0$ is defined as follows: $x_t(\theta) = x(t+\theta)$, $\theta \in [-\tau, 0]$. $\|\phi\| = \sup\{|\phi(\theta)| : -\tau \leqslant \theta \leqslant 0\}$.

Problem formulation. Consider the following SNTDS:

$$\begin{cases} \dot{x}(t) = f_{\sigma(t)}(t, x(t), x(t-\tau)) \\ \qquad\quad + g_{\sigma(t)}(t, x(t), x(t-\tau)) u_{\sigma(t)}(t, x(t)), t \neq t_k, \\ x_{t_0}(\theta) = \phi(\theta), \theta \in [-\tau, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $\dot{x}(t)$ denotes the right-hand derivative of $x(t)$, $u_{\sigma(t)}(t, x(t)) \in \mathbb{R}^q$, $q \in \mathbb{N}^+$ is the system input. The switching signal $\sigma(t) : [t_0, +\infty) \to \mathbb{S} = \{1, 2, \ldots, m\}$

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is right-piecewise continuous at each switching time, i.e., $\sigma(t_k) = \sigma(t_k^+)$. The positive constant m represents the number of the subsystems, and $\{t_k\}$ indicates the switching sequence. For any $i \in \mathbb{S}$, $f_i(t, x_t)$ and $g_i(t, x_t)$ are nonlinear functions and satisfy the locally Lipschitz condition.

Next, we introduce the concepts of globally uniform exponential stability and the MDADT switching signal.

Definition 1 ([1]). The SNTDSs are called globally uniformly exponentially stable (GUES), if there exist $M, \vartheta \in \mathbb{R}^+$ such that, for any $\phi \in C([-\tau, 0], \mathbb{R}^n)$, $\|x(t, t_0, \phi)\| \leqslant M\mathrm{e}^{-\vartheta(t-t_0)}\|\phi\|$, $\forall t \geqslant t_0$.

Definition 2 ([4]). For any $t_2 \geqslant t_1 \geqslant t_0$, and $\sigma(t)$, $N_{\sigma j}(t_2, t_1)$, $T_j(t_2, t_1)$ denote the number of switches and the total runtime of the jth subsystem over $(t_1, t_2]$, respectively. There are constants N_{0j} and τ_{aj}, $j \in S$ satisfying

$$N_{\sigma j}(t_2, t_1) \leqslant N_{0j} + \frac{T_j(t_2, t_1)}{\tau_{aj}}, \tag{2}$$

where τ_{aj} are called the MDADT.

Subsequently, we present the following crucial assumption. For simplicity, we define the Lyapunov functions $V_{\sigma(t)}(t, x(t)) = H_{\sigma(t)}(t)$ in this study.

Assumption 1. There exist functions $H_{\sigma(t)}(t)$, constants $\alpha_{\sigma(t)} \in \mathbb{R}$, $\beta_{\sigma(t)} \in \mathbb{R}^+$, $\gamma_{\sigma(t)} \in \mathbb{R}^+$, and $a_{\sigma(t)}, b_{\sigma(t)} \in \mathbb{R}^+$, $\eta \in \mathbb{R}^+$ such that

$$a_{\sigma(t)}\|x(t)\|^\eta \leqslant H_{\sigma(t)}(t) \leqslant b_{\sigma(t)}\|x(t)\|^\eta, \tag{3}$$

$$D^+H_{\sigma(t)}(t) \leqslant \alpha_{\sigma(t)}H_{\sigma(t)}(t) + \beta_{\sigma(t)}H_{\sigma(t-\tau)}(t-\tau), t \neq t_k, \tag{4}$$

and

$$H_{\sigma(t)}(t) \leqslant \gamma_{\sigma(t)}H_{\sigma(t^-)}(t^-),\ t = t_k. \tag{5}$$

In this study, we will analyze the effects of two time-delay cases on the stability of SNTDSs. Specifically, two time-delay cases imply that time delay is small ($\tau \leqslant t_{k+1} - t_k$, for any $t \in [t_k, t_{k+1})$), and large ($\tau > t_{k+1} - t_k$ is bounded, for $t \in [t_k, t_{k+1})$).

Lemma 1. Assume Assumption 1 holds with rate coefficient $\alpha_i \in \mathbb{R}$, $\beta_i \in \mathbb{R}^+$, and $\gamma_i \in (0, 1)$, $i \in S$. Consider a piecewise function

$$F(t) = \begin{cases} H_{i_k}(t)\exp[-\varrho(t - t_k)], t \in [t_k, t_{k+1}), k \in \mathbb{N}, \\ H_{i_0}(t), t_0 - \tau \leqslant t \leqslant t_0, \end{cases} \tag{6}$$

where $\alpha = \sup_{i\in S}\{\alpha_i\}$, $\beta = \sup_{i\in S}\{\beta_i\}$, $\gamma = \sup_{i\in S}\{\gamma_i\}$, and $\varrho > \alpha + \beta$ such that $\alpha + \frac{\beta}{\gamma}\mathrm{e}^{-\varrho\tau} - \varrho < 0$, then $D^+F(q) < 0$ holds for any instant $q \geqslant t_0$, which satisfies either case (i) or case (ii) given as follows:

(i) if $q \in [t_0, t_1)$, then

$$F(s) \leqslant F(q),\ t_0 - \tau \leqslant s \leqslant q; \tag{7}$$

(ii) for any $q \in [t_i, t_{i+1})$, $i \in \mathbb{N}^+$, then

$$F(s) \leqslant F(q),\ t_i \leqslant s \leqslant q, \tag{8}$$

and

$$F(s)\mathrm{e}^{\varrho(t_i - t_{i+1})} \leqslant \frac{F(q)}{\gamma},\ t_{i-1} \leqslant s \leqslant t_i. \tag{9}$$

Proof. Please see Appendix A.

Lemma 2. Assuming assumption 1, if rate coefficients $\alpha_i \in \mathbb{R}$, $\beta_i \in \mathbb{R}^+$, $i \in S$ satisfy $\alpha_i + \beta_i > 0$, and $\gamma_i \in (0, 1)$, denoting $\varrho^* = \max\{\alpha_i + \beta_i, \varrho_0\}$, where ϱ_0 is the solution of the following equation:

$$\alpha + \frac{\beta}{\gamma}\mathrm{e}^{-s\tau} - s = 0. \tag{10}$$

Then we have the following implications:

$$H_{i_k}(t) \leqslant \varpi_k \mathrm{e}^{\varrho^*(t-t_0)},\ t \in [t_k, t_{k+1}), \tag{11}$$

where $\varpi_k = \gamma^k \tilde{H}_{i_0}(t_0)$, $k \in \mathbb{N}$, and $\tilde{H}_{i_0}(t_0) = \sup_{t_0 - h \leqslant s \leqslant t_0} H_{i_0}(s)$.

Proof. Please see Appendix B.

First, we research the potential effects of small delays on the stability of SNTDSs. Then, we analyze how increasing the time delay contributes to their stabilizing behavior.

Theorem 1. Considering that Assumption 1 holds with rate coefficients $\alpha_i \in \mathbb{R}$, $\beta_i \in \mathbb{R}^+$, and $\gamma_i \in (0, 1)$ such that $\alpha_i + \beta_i > 0$, then the SNTDS (1) is GUES if the MDADT satisfies

$$\varrho^* \sum_{i=1}^{m} \tau_{ai} + \ln\gamma < 0, \tag{12}$$

where ϱ^* is defined in Lemma 2.

Proof. Please see Appendix C.

Remark 1. It is obvious that the exponential convergence rate $\xi = -(\frac{\ln\gamma}{\sum_{i=1}^{m}\tau_{ai}} + \varrho^*)/\eta$ is monotonously decreasing with respect to the delay τ, i.e., the system time delay affects the system exponential convergence rate. Furthermore, the larger the system time delay is, the faster the system exponential convergence rate.

Next, we address the stability problem of SNTDSs under large delays, where the dwell time is shorter than the delay, and present stability criteria for these systems.

Theorem 2. Suppose that Assumption 1 holds with rate coefficients $\alpha_i \in \mathbb{R}$, $\beta_i \in \mathbb{R}^+$, and $\gamma_i \in (0, 1)$. Then the SNTDS (1) is GUES if the following condition is satisfied

$$\alpha + \prod_{i=1}^{m}\gamma_i^{N_{0i}}\beta + \sum_{i=1}^{m}\frac{\ln\gamma_i}{\tau_{ai}} < 0. \tag{13}$$

Proof. Please see Appendix D.

Simulation example. The simulation examples are included in Appendix E.

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Supporting information Appendixes A–E. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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