

# Event-based nonsingular fixed-time containment control for nonlinear multiagent systems with dynamic uncertainties

Yuanbo SU<sup>1</sup>, Qihe SHAN<sup>1\*</sup>, Tieshan LI<sup>2</sup> & C. L. Philip CHEN<sup>3</sup>

<sup>1</sup>Navigation College, Dalian Maritime University, Dalian 116026, China

<sup>2</sup>School of Automation Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China

<sup>3</sup>School of Computer Science and Engineering, South China University of Technology, Guangzhou 510641, China

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Owing to the extensive applications in many areas such as networked systems, formation flying of unmanned air vehicles, and coordinated manipulation of multiple robots, the distributed containment control for nonlinear multiagent systems (MASs) has received considerable attention, for example [1,2]. Although the valued studies in [1,2] investigate containment control problems for MASs subject to nonlinearities, the proposed distributed nonlinear protocols only achieve the asymptotic stability. As a crucial performance indicator for distributed containment control of MASs, the fast convergence is conducive to achieving better control accuracy [3]. The work in [4] first addresses the backstepping-based adaptive fuzzy fixed-time containment tracking problem for nonlinear high-order MASs with unknown external disturbances. However, the designed fixed-time control protocol [4] cannot escape the singularity problem in the backstepping-based adaptive control scheme. As is well known, the singularity problem has become an inherent problem in the adaptive fixed-time control design, which may cause the unbounded control inputs and even the instability of controlled systems. Therefore, how to solve the nonsingular fixed-time containment control problem for nonlinear MASs is still open and awaits breakthrough to the best of our knowledge.

Since the energy and communication resources for network agents in some real scenes are limited, how to save resources is very considerable for enhancing the practicability of MASs. Thus, the event-triggered control has been employed to reduce the resource consumption, which has a more flexible sampling mechanism than the traditional control design based on the periodic sampling rule. However, in practical control mechanisms, the actuators of agents may undergo abrupt faults individually or simultaneously during operation. These faults can disrupt the transmission of sampled control signals, potentially leading to system instability.

Unmodeled dynamics usually occurs in physical systems

due to modeling errors and measurement noise, which may lead to system instability. The classical technique of dynamic signal function proposed in [5] cannot directly address the adaptive fixed-time control problem for nonlinear MASs with unmodeled dynamics since the introduced dynamic signal function is only exponentially input-to-state stable. Therefore, how to design an event-based distributed controller to guarantee the fixed-time stability of nonlinear MASs with unmodeled dynamics and actuator faults while avoiding the singularity problem, is a challenging question to be discussed in this study.

**Control design.** Motivated by the aforementioned observations, an event-triggered nonsingular fixed-time fuzzy control method is proposed for nonlinear networked MASs with unmodeled dynamics and actuator faults. Consider the following class nonlinear MASs consisting of  $N$  followers. The dynamic equation of the  $i$ th follower ( $i = 1, 2, \dots, N$ ) can be described as

$$\begin{cases} \dot{q}_i = p_i(q_i, \mathbf{x}_i), \\ \dot{x}_{il} = x_{i(l+1)} + f_{il}(\bar{\mathbf{x}}_{il}) + \Delta_{il}(q_i, \mathbf{x}_i, t), \\ \dot{x}_{in} = u_i + f_{in}(\mathbf{x}_i) + \Delta_{in}(q_i, \mathbf{x}_i, t), \\ y_i = x_{i1}, \end{cases} \quad (1)$$

where  $1 \leq l \leq n-1$ ,  $\bar{\mathbf{x}}_{il} = [x_{i1}, \dots, x_{il}]^T \in \mathbb{R}^l$ .  $\mathbf{x}_i = [x_{i1}, \dots, x_{in}]^T \in \mathbb{R}^n$  represents the state vector of the  $i$ th follower.  $f_{il}(\cdot)$  ( $l = 1, 2, \dots, n$ ) are unknown smooth nonlinear functions.  $u_i$  and  $y_i$  denote the outputs of actuator and system, respectively.  $q_i$ -dynamics is the unmodeled dynamics.  $p_i(\cdot)$  and  $\Delta_{il}(\cdot)$  are assumed as uncertain smooth functions, which satisfy the local Lipschitz condition.

In this study, the model of actuator faults is described as

$$u_i(t) = \lambda_i \bar{w}_i(t), \quad t \geq t^f, \quad (2)$$

where  $\bar{w}_i(t)$  represents the event-triggered control input.  $0 < \lambda_i \inf \leq \lambda_i \leq 1$  is the unknown control effectiveness rate when the actuator fault occurs.  $t^f$  denotes the beginning

\* Corresponding author (email: shanqihe@dlmu.edu.cn)

time of the actuator fault. Then, we define the unknown parameters as  $l_i = \lambda_{i \inf}$  and  $\mu_i = \frac{1}{\bar{\mu}_i}$  where  $\bar{\mu}_i = \mu_i - \hat{\mu}_i$ .

A graph  $\mathcal{G} = (\mathcal{L}, \Xi, \Lambda)$  is introduced to describe the communication network among  $N + M$  agents with the node set  $\mathcal{L} = \{1, 2, \dots, N, \dots, N + M\}$ , where  $\Xi = \{(j, i) \in \mathcal{L} \times \mathcal{L}\}$  denotes a set of edges. The adjacency matrix is  $\mathbf{A} = [a_{ij}^*] \in \mathbb{R}^{(N+M) \times (N+M)}$ , and  $(j, i) \in \mathcal{L}$  represents that  $j$  and  $i$  can exchange information with each other.  $a_{ij}^*$  is defined as

$$a_{ij}^* = \begin{cases} 0, & \text{if } (j, i) \notin \Xi, \\ 1, & \text{if } (j, i) \in \Xi, \end{cases} \quad (3)$$

where the neighbor set of node  $i$  is defined as  $F_i = \{j \in \mathcal{L} : (j, i) \in \Xi\}$ . The Laplacian matrix  $\mathbf{L} = [l_{ij}]_{(N+M) \times (N+M)} = \mathbf{D} - \mathbf{A} \in \mathbb{R}^{(N+M) \times (N+M)}$  is defined as

$$l_{ij} = \begin{cases} -a_{ij}^*, & \text{if } i \neq j, \\ \sum_{j \in F_i} a_{ij}^*, & \text{if } i = j, \end{cases} \quad (4)$$

where  $\mathbf{D} = \text{diag}\{d_1, \dots, d_N\}$  is the degree matrix with  $d_i = \sum_{j \in F_i} a_{ij}^*$ .

In our work,  $N + M$  agents consisting of  $N$  followers and  $M$  leaders are considered under a directed communication topology. Supposing that the followers have one neighbor at least, we have

$$\mathbf{L} = \begin{bmatrix} \mathbf{L}_1 & \mathbf{L}_2 \\ \mathbf{0}_{M \times N} & \mathbf{0}_{M \times M} \end{bmatrix}, \quad (5)$$

where  $\mathbf{L}_1 \in \mathbb{R}^{N \times N}$ ,  $\mathbf{L}_2 \in \mathbb{R}^{N \times M}$ .

Based on the above preliminaries, a distributed adaptive fuzzy fixed-time controller can be designed (see Appendix A.1).

Then, the result of stability analysis can be concluded in the following Theorem.

**Theorem 1.** Suppose that the directed communication topology is fixed. Considering the controlled nonlinear MASs with unmodeled dynamics and actuator faults (1), under the intermediate control signal, the actual control input, the event-triggered control signal, and the adaptive laws, all the signals of closed-loop systems are bounded, and the outputs of all followers converge to the dynamic convex hull formed by the leaders. Meanwhile, the local neighborhood containment errors are guaranteed to converge to a small neighborhood of zero with a fixed-time convergence rate.

*Proof.* The proof of Theorem 1 is listed in Appendix A.2.

*Simulations.* To demonstrate the effectiveness of the developed control scheme, we consider a group of autonomous underwater vehicles (AUVs) (see Appendix B). The network communication topology is shown in Figure B1(a), where each edge weight is set as 1. The containment control performance in 3D space is plotted in Figure B1(b). It is observed that four AUVs can converge to the convex hull formed by the given leaders' signals that are not affected by actuator faults owing to the design of fault-tolerant controllers. Figure B1(c) depicts the curves of transformed errors  $\xi_{ilx}$  ( $i = 1, 2, 3, 4, l = 1, 2$ ), which shows the transformed

errors can be constrained to an interval greater than zero to avoid the singularity problem caused by  $\xi_{ilx}^{2v_1-1}$  in designed virtual controllers  $\alpha_{ilx}$ . The comparisons of error convergence between the adaptive fixed-time containment control scheme of this study and the traditional adaptive containment scheme are described in Figure B1(d). It is obvious that the convergence times of containment synchronization errors under the traditional containment control are slower than the proposed fixed-time control method in this study. Figure B1(e) depicts the curves of outputs of the actuators  $u_{ix}$  and event-triggered inputs  $\bar{\varpi}_{ix}$  in surge. It can be observed that the actuator confronts partial loss of effectiveness starting at  $t = 35$  s. Correspondingly, the fault compensation mechanism is activated at  $t = 35$  s to realize the fault-tolerant control based on the compensation parameters shown in Figure B1(f).

**Conclusion and future work.** In this study, we have investigated the event-based nonsingular adaptive fuzzy fixed-time control problem for networked MASs with unmodeled dynamics and actuator faults. By combining the modified error transformation mechanism with a symmetrical barrier Lyapunov function, the converted errors can be confined to an interval greater than zero, which is a constructive method to avoid the singularity problem in the backstepping design. Using the fixed-time stability theory, we can show that both the containment control performance and the stability of the closed-loop system can be ensured within a fixed time. In the future, we will possibly concentrate on extending this control method to nonlinear multiagent systems with communication delays.

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**Supporting information** Appendixes A, B, and Figure B1. The supporting information is available online at [info.scichina.com](http://info.scichina.com) and [link.springer.com](http://link.springer.com). The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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