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Event-triggered tracking control for a class of nonlinear systems based on high-order fully actuated system theory

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High-order fully actuated (HOFA) system theory has recently received increasing attention since Duan [1] introduced the concept of the HOFA system and the advantages of controller designing. It was pointed out that the HOFA method is a description form of a dynamic system and a control-oriented model. Different from the traditional nonlinear analysis methods, the HOFA system approach supports to construct a straightforward controller and provides freedom in the designing, in which the characteristic of full actuation allows counteract known items. It should be noted that the trajectory tracking problem of the uncertain nonlinear system based on the HOFA method has not been solved, we will consider it in this work.

As an effective signal transmission way, the event-triggered mechanism only works when the triggering condition is satisfied. As the neural networks, radial basis function neural networks (RBFNNs) can approximate any continuous function and accelerate the learning speed of neural networks, especially when dealing with complex nonlinear features. These advantages motivate us to use event-triggered mechanism and RBFNNs.

In this study, an event-triggered neural network adaptive control strategy is proposed for the tracking issue of a class of nonlinear systems. The main contributions are simplified as follows: (1) the tracking control problem for high-order nonlinear systems is first studied by using HOFA theory; (2) the adaptive control algorithm proposed is directly designed, in which RBFNNs are utilized to estimate the unknown nonlinear function of the system; (3) to reduce the consumption of energy and the continuous update of the controller, an event-triggered control scheme is used.

Notations. R stands for the set of real numbers; $\mathbb{C}^1$ represents a function with a first-order continuous partial derivative; $\lambda_{\max}(P)$ means the maximum eigenvalues of P; $\|\cdot\|$ denotes Euclidean norm; $\mathrm{tr}(A)$ denotes the trace of A. $A^{0\sim n-1} = [A_0, A_1, \ldots, A_{n-1}]$, $B = [O, O, \ldots, O, I]^{\mathrm{T}}$ and the following matrices are used in this study:

$$e^{(0\sim n-1)}=\begin{bmatrix} e \\ \dot{e} \\ \vdots \\ e^{(n-1)} \end{bmatrix}, \Lambda(k^{(0\sim n-1)})=\begin{bmatrix} O & I & \cdots & O \\ & & \ddots & \vdots \\ & & & I \\ -k_0 & -k_1 & \cdots & -k_{n-1} \end{bmatrix},$$

$$\Phi(A^{0\sim n-1}) = \begin{bmatrix} O & I & \cdots & O \\ & & \ddots & \vdots \\ & & & I \\ -A_0 & -A_1 & \cdots & -A_{n-1} \end{bmatrix}.$$

Nonlinear system. Consider the following strict-feedback nonlinear system:

$$\begin{cases} \dot{\boldsymbol{x}}(t)=\begin{bmatrix} x_2(t) \\ x_3(t) \\ \vdots \\ f(\boldsymbol{x}(t))+\Delta f(\boldsymbol{x}(t))+H^{\mathrm{T}}(\boldsymbol{x}(t))\theta(t)+L(\boldsymbol{x}(t))u(t) \end{bmatrix}, \\ y(t)=x_1(t), \end{cases} \tag{1}$$

where $\boldsymbol{x}(t) = [x_1(t); x_1(t); \ldots; x_n(t)] \in \mathbb{R}^{nr}$ denotes the state vector with $x_i(t) \in \mathbb{R}^r$ $(i = 1, 2, \ldots, n)$; $f(\boldsymbol{x}(t)) \in \mathbb{R}^r$ is the known $\mathbb{C}^1$ function, $\Delta f(\boldsymbol{x}(t)) \in \mathbb{R}^r$ is an unknown continuous function, $H(\boldsymbol{x}(t)) \in \mathbb{R}^{m\times r}$ is a known $\mathbb{C}^1$ matrix function, $\theta(t) \in \mathbb{R}^m$ is an unknown time-varying parameter, $L(\boldsymbol{x}(t)) \in \mathbb{R}^{r\times r}$ is a known $\mathbb{C}^1$ control matrix function, $u(t) \in \mathbb{R}^r$ is the controller to be designed and $y(t) \in \mathbb{R}^r$ is the output of the system.

Assumption 1. Suppose that $\det(L(\boldsymbol{x}(t))) \neq 0$ for all $\boldsymbol{x}(t) \in \mathbb{R}^{nr}$ and $t \geqslant 0$.

Assumption 2. Suppose that $\|\theta(t) - \theta_0(t)\| \leqslant \delta_0$, $\|\dot{\theta}(t) - \dot{\theta}_0(t)\| \leqslant \delta_1$ for all $t \geqslant 0$, where $\theta_0(t)$ is a pre-estimation of $\theta(t)$, δ_0 and δ_1 are positive constants.

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Define $\tilde{\theta}(t) = \theta(t) - \hat{\theta}(t)$, $\tilde{\theta}_{r0} = \theta(t) - \theta_0(t)$, $\tilde{\theta}_{e0} = \hat{\theta}(t) - \theta_0(t)$. Based on Assumption 2, the relations hold that $\tilde{\theta}_{r0} - \tilde{\theta}_{e0} - \tilde{\theta}(t) = 0$, $\tilde{\theta}_{r0}\tilde{\theta}(t) \leqslant \frac{1}{2}(\delta_0^2 + \|\theta(t)\|^2)$ and $\dot{\tilde{\theta}}_{r0}\tilde{\theta}(t) \leqslant \frac{1}{2}(\delta_1^2 + \|\theta(t)\|^2)$.

Remark 1. Assumption 1 is the usual condition for HOFA theory analysis, it ensures the system is controllable. Assumption 2 is a common hypothesis when handling the unknown function. It indicates that the differences between the unknown function with its derivative and their initial estimations are bounded, which reserves the nonlinear of the system.

Define $x_1(t) = z(t)$, we have $x_i(t) = z^{(i-1)}(t), i = 2, 3, \ldots, n-1$. Replace the last equation of system (1) with each component, the strict-feedback nonlinear system (1) can be converted into the following HOFA system:

$$\begin{cases} z^{(n)}(t) = f(z^{(0\sim n-1)}(t)) + \Delta f(z^{(0\sim n-1)}(t)) \\ \qquad + H^{\mathrm{T}}(z^{(0\sim n-1)}(t))\theta(t) + L(z^{(0\sim n-1)}(t))u(t), \\ y(t) = z(t). \end{cases} \tag{2}$$

Control objective. The objective of this work is to design the adaptive controller for system (1) such that the output $y(t)$ can track the target tracking trajectory $y_d(t) \in \mathbb{R}^r$, which is a smooth function vector.

Tracking error is defined as $e(t) = y_d(t) - x_1(t) = y_d(t) - z(t)$. Naturally, the following expression can be obtained: $e^{(i)}(t) = y_d^{(i)}(t) - \dot{x}_i(t) = y_d^{(i)}(t) - z^{(i)}(t), i = 1, 2, 3, \ldots, n$.

Remark 2. Because the HOFA system approach uses only the full-actuation structure, the approach shows advantages compared with traditional approaches, such as lower computation, simple control structure, efficient control capability and high design flexibility. Based on this, it motivates us to do this work.

RBFNNs. For a continuous function $f(X) : \mathbb{R}^n \to R$, there must exist neural networks leading to $f(X) = W^{*\mathrm{T}}S(X) + \varepsilon^*(X)$, where $X = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$ is an input vector, $W^{*\mathrm{T}} \in \mathbb{R}^{r\times l}$ is the weight matrix; $S(X) = [S_1(X), S_2(X), \ldots, S_l(X)]^{\mathrm{T}} \in \mathbb{R}^{l\times 1}$ is a basis function; $\varepsilon^*(X)$ is the approximation error which satisfies $\|\varepsilon^*(X)\| \leqslant \bar{\varepsilon}$ with $\bar{\varepsilon} > 0$ is a bound unknown parameter; $S_i(X) = \exp\left(\frac{-\|X-c_i\|^2}{v_i^2}\right)(i = 1, 2, \ldots, l)$ is used as Gaussian function, where c_i and v_i are the center vector and the width of the Gaussian function.

Event-triggered mechanism. In this work, we introduce the following event-triggered mechanism:

$$\begin{cases} u(t) = \omega(t_k), \quad \forall t \in [t_k, t_{k+1}), \\ t_{k+1} = \inf\{t \in R \mid \|\xi(t)\| \geqslant \|m\|\}, \end{cases} \tag{3}$$

where $\xi(t) = \omega(t) - u(t)$ is the measurement error matrix, $u(t)$ is the controller of the system, $\omega(t)$ is the adaptive controller designed in (4), m is a constant threshold matrix. Based on the triggered strategy, the controller $u(t)$ holds as $\omega(t_k)$ when $t \in [t_k, t_{k+1})$. Once $\|\xi(t)\| \geqslant \|m\|$, the event-triggered strategy is powered-up and the controller is updated. From event-triggered mechanism (3), it is easily to be found that there exists a parameter $\lambda(t)$ satisfying $\lambda(t_k) = 0$, $\lambda(t_{k+1}) = \pm 1$ and $|\lambda(t)| \leqslant 1$ for all $t \in [t_k, t_{k+1})$, resulting in the relationship between $\omega(t)$ and $u(t)$ as $\omega(t) = u(t) + \lambda(t)m$, such that $u(t) = \omega(t) - \lambda(t)m$.

Adaptive controller. An adaptive control law is designed by using the fully actual theory as follows:

$$\begin{cases} \omega(t) = L^{-1}(z^{(0\sim n-1)}(t))[-f(z^{(0\sim n-1)}(t)) + K^{\mathrm{T}}e^{(0\sim n-1)} \\ \qquad + v_1(t) + v_2(t) + y_d^{(n)}(t)] + v_3(t), \\ v_1(t) = -H^{\mathrm{T}}(z^{(0\sim n-1)}(t))\hat{\theta}(t), \\ \dot{\hat{\theta}}(t) = -((e^{(0\sim n-1)})^{\mathrm{T}}PBH^{\mathrm{T}}(z^{(0\sim n-1)}(t)))^{\mathrm{T}} \\ \qquad - (\mu+1)\tilde{\theta}_{e0}(t) + \dot{\theta}_0(t), \\ v_2(t) = -\hat{W}^{\mathrm{T}}(t)S(z^{(0\sim n-1)}(t)), \\ \dot{\hat{W}}(t) = -S(z^{(0\sim n-1)})(e^{(0\sim n-1)})^{\mathrm{T}}PB - \sigma_1\hat{W}(t), \\ v_3(t) = \bar{m}\tanh\dfrac{L^{\mathrm{T}}(z^{(0\sim n-1)}(t))B^{\mathrm{T}}Pe^{(0\sim n-1)}\bar{m}}{\epsilon}, \end{cases} \tag{4}$$

where $K = [k_0, k_1, \ldots, k_n]^{\mathrm{T}} \in \mathbb{R}^{nr\times r}$ is the designed controller gain matrix; σ_1 and ϵ are positive constants; $\hat{W}(t)$ is the estimation of W^*, $\tilde{W}(t) = W^* - \hat{W}(t)$ is the estimation error.

By substituting (4) into the system (2), the general HOFA system can be transformed into the following HOFA error system:

$$\begin{aligned} e^{(n)}(t) = &-Ke^{(0\sim n-1)}(t) - H^{\mathrm{T}}(z^{(0\sim n-1)}(t))\tilde{\theta}(t) \\ &- [\Delta f(z^{(0\sim n-1)}(t)) - \hat{W}^{\mathrm{T}}(t)S(z^{(0\sim n-1)}(t))] \\ &+ \lambda(t)L(z^{(0\sim n-1)}(t))m - L(z^{(0\sim n-1)}(t))v_3(t), \end{aligned} \tag{5}$$

which can be further converted into the following form:

$$\begin{aligned} e^{(1\sim n)}(t) = &\Lambda(k^{(0\sim n-1)})e^{(0\sim n-1)}(t) \\ &- B\{H^{\mathrm{T}}(z^{(0\sim n-1)}(t))\tilde{\theta}(t) \\ &+ [\Delta f(z^{(0\sim n-1)}(t)) - \hat{W}^{\mathrm{T}}(t)S(z^{(0\sim n-1)}(t))] \\ &- \lambda(t)L(z^{(0\sim n-1)}(t))m + L(z^{(0\sim n-1)}(t))v_3(t)\}. \end{aligned} \tag{6}$$

Theorem 1. For HOFA system (1) satisfying Assumptions 1 and 2 with positive parameters ϵ, μ, σ_1, σ_2, controller gain matrix K, positive definite matrix P, $\mathrm{Re}\lambda_i(\Lambda(k^{(0\sim n-1)})) < -\frac{\mu}{2}$ and $\mu - \frac{1}{\sigma_2\lambda_{\max}(P)} > 0$, then the adaptive event-triggered control laws (4) and (6) for system (1) guarantee that the tracking error is uniformly ultimate bounded.

The proof of Theorem 1 can be found in Appendix A.

Conclusion. This study has proposed a novel event-triggered adaptive neural networks control algorithm using the HOFA approach for the tracking problem of nonlinear systems. Based on the property of direct control structure, the nonlinear systems will be further studied based on the HOFA approach. The simulations are performed as shown in Appendix B, demonstrating the effectiveness of the proposed method.

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Supporting information Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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